1 Exercise 2

We are now using the stochastic matrix, where we will have $t \in \mathbb{N}_0$.

$$P = \begin{bmatrix} 0 & 1/2 & 1/2 & 1/2 & 1/2 \\ 1/4 & 1/8 & 1/8 & 1/8 & 1/8 \\ 1/4 & 1/8 & 1/8 & 1/8 & 1/8 \\ 1/4 & 1/8 & 1/8 & 1/8 & 1/8 \\ 1/4 & 1/8 & 1/8 & 1/8 & 1/8 \end{bmatrix} \tag{1}$$

1.1 Hvad er sandsynligheden for at være i tilstand w_1 til tiden $t = 5$, hvis vi bruger startfordelingen fra (1) og den stokastiske matrix P?

$$w_0 = \begin{bmatrix} 1/2 \\ 1/8 \\ 1/8 \\ 1/8 \\ 1/8 \end{bmatrix} \tag{2}$$

$$x_k = P^k x_0 \text{ for } k = 0, 1 \ldots \tag{3}$$

$$P^5 = \begin{bmatrix} \frac{5}{16} & \frac{11}{32} & \frac{11}{32} & \frac{11}{32} & \frac{11}{32} \\ \frac{11}{64} & \frac{21}{128} & \frac{21}{128} & \frac{21}{128} & \frac{21}{128} \\ \frac{11}{64} & \frac{21}{128} & \frac{21}{128} & \frac{21}{128} & \frac{21}{128} \\ \frac{11}{64} & \frac{21}{128} & \frac{21}{128} & \frac{21}{128} & \frac{21}{128} \\ \frac{11}{64} & \frac{21}{128} & \frac{21}{128} & \frac{21}{128} & \frac{21}{128} \end{bmatrix}$$

$$w_5 = P^5 \cdot w_0 = \begin{bmatrix} 21/64 \\ 43/256 \\ 43/256 \\ 43/256 \\ 43/256 \end{bmatrix} \tag{4}$$

1.2 Har Markov-kæden en stationær fordeling? Hvis ja, bestem sådan en.

$$\begin{gathered} Pq = q \\ Pq - q = 0 \\ (P - I)q = 0 \end{gathered} \tag{5}$$

$$P - I_5 = \begin{bmatrix} -1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & -\frac{7}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{8} & -\frac{7}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & -\frac{7}{8} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & -\frac{7}{8} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$v = x_5 \cdot \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{6}$$

$$x_5(2+1+1+1+1) = 1 \implies x_5 = 1/6 \tag{7}$$

$$q = \begin{bmatrix} 1/3 \\ 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \end{bmatrix} \tag{8}$$

$$P^{25} \cdot w_0 = \begin{bmatrix} \frac{22369621}{67108864} \\ \frac{44739243}{268435456} \\ \frac{44739243}{268435456} \\ \frac{44739243}{268435456} \\ \frac{44739243}{268435456} \end{bmatrix} \xrightarrow{\text{at 5 digits}} \begin{bmatrix} 0.33333 \\ 0.16667 \\ 0.16667 \\ 0.16667 \\ 0.16667 \end{bmatrix}$$