1 Del 1

1.1 Delopgave 1.3

Omskriv problemet (3) til et lineært problem i kanonisk form, og bestem det duale problem.

(3):

$$\begin{aligned} \text{minimize} \quad & \tilde{c} \cdot \tilde{x} \\ \text{subject to} \quad & \tilde{A}\tilde{x} = b, \\ & \tilde{x} \geq 0 \end{aligned} \tag{1}$$

$$\begin{aligned} \text{maximize} \quad & c \cdot x \\ \text{subject to} \quad & Ax \leq b \\ & x \geq 0 \end{aligned} \tag{2}$$

$$\tilde{A}\tilde{x} = b \implies \begin{bmatrix} \tilde{A} \\ -\tilde{A} \end{bmatrix} \tilde{x} \leq \begin{bmatrix} b \\ -b \end{bmatrix} \tag{3}$$

$$\hat{A} = \begin{bmatrix} \tilde{A} \\ -\tilde{A} \end{bmatrix}, \hat{b} = \begin{bmatrix} b \\ -b \end{bmatrix} \tag{4}$$

The primal problem

$$\begin{aligned} \text{maximize} \quad & -\tilde{c} \cdot x \\ \text{subject to} \quad & \hat{A}\tilde{x} \leq \hat{b} \\ & \tilde{x} \geq 0 \end{aligned} \tag{5}$$

The dual problem

$$\begin{aligned} \text{minimize} \quad & \hat{b} \cdot y \\ \text{subject to} \quad & \hat{A}^T y \geq -\tilde{c} \\ & y \geq 0 \end{aligned} \tag{6}$$

1.2 Delopgave 4

Antag at $m = 1$, $n = 5$, $A = [1, 2, 3, 4, 5]$, $b = [10]$. Ved hjælp af en tegning find en løsning til det duale problem I har bestemt i det sidste spørsmål. Brug stærk dualitet til at bestemme en løsning til det primale problem ud fra løsningen til det duale. Løs også det primale problem vha. simplexmetoden, og tjek at I får samme resultat.

$$\hat{b} \cdot y = \begin{bmatrix} 10 \\ -10 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 10y_1 - 10y_2 \tag{7}$$

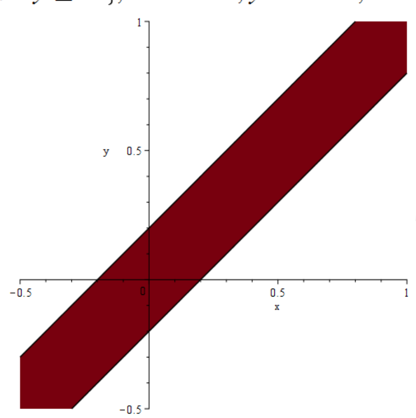
$$\tilde{A} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & -1 & -2 & -3 & -4 & -5 \end{bmatrix} \tag{8}$$

$$\hat{A} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & -1 & -2 & -3 & -4 & -5 \\ -1 & -2 & -3 & -4 & -5 & 1 & 2 & 3 & 4 & 5 \end{bmatrix} \tag{9}$$

$$\hat{A}^T = \begin{bmatrix} 1 & -1 \\ 2 & -2 \\ 3 & -3 \\ 4 & -4 \\ 5 & -5 \\ -1 & 1 \\ -2 & 2 \\ -3 & 3 \\ -4 & 4 \\ -5 & 5 \end{bmatrix} \tag{10}$$

$$\hat{A}^T y = \begin{bmatrix} 1 & -1 \\ 2 & -2 \\ 3 & -3 \\ 4 & -4 \\ 5 & -5 \\ -1 & 1 \\ -2 & 2 \\ -3 & 3 \\ -4 & 4 \\ -5 & 5 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \geq \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = -\tilde{c} \tag{11}$$

inequal({5 *x* − 5 *y* ≥ -1, -5 *x* + 5 *y* ≥ -1}, *x* = -0.5 ..1, *y* = -0.5 ..1, *color* = "Niagara 1")



$$\begin{aligned} (0,0) &\implies 10 \cdot 0 - 10 \cdot 0 = 0 \\ (0,0.2) &\implies 10 \cdot 0 - 10 \cdot 0.2 = -2 \\ (0.2,0) &\implies 10 \cdot 0.2 - 10 \cdot 0 = 2 \end{aligned} \tag{12}$$

$$\begin{aligned} c^T \bar{x} &= b^T \bar{y} \\ -\tilde{c}^T \tilde{x} &= \hat{b}^T y = -2 \end{aligned} \tag{13}$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	s_1	s_2	M	b
s_1	1	2	3	4	5	−1	−2	−3	−4	−5	1	0	0	10
s_2	−1	−2	−3	−4	−5	1	2	3	4	5	0	1	0	−10
M	1	1	1	1	1	1	1	1	1	1	0	0	1	0

(14)

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	s_1	s_2	M	b
s_1	1	2	3	4	5	-1	-2	-3	-4	-5	1	0	0	10
x_5	.2	.4	.6	.8	1	$-.2$	$-.4$	$-.6$	$-.8$	-1	0	$-.2$	0	2
M	1	1	1	1	1	1	1	1	1	1	0	0	1	0

(15)

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	s_1	s_2	M	b
s_1	0	0	0	0	0	0	0	0	0	0	1	1	0	0
x_5	.2	.4	.6	.8	1	$-.2$	$-.4$	$-.6$	$-.8$	-1	0	$-.2$	0	2
M	.8	.6	.4	.2	0	1.2	1.4	1.6	1.8	2	0	.2	1	-2

(16)