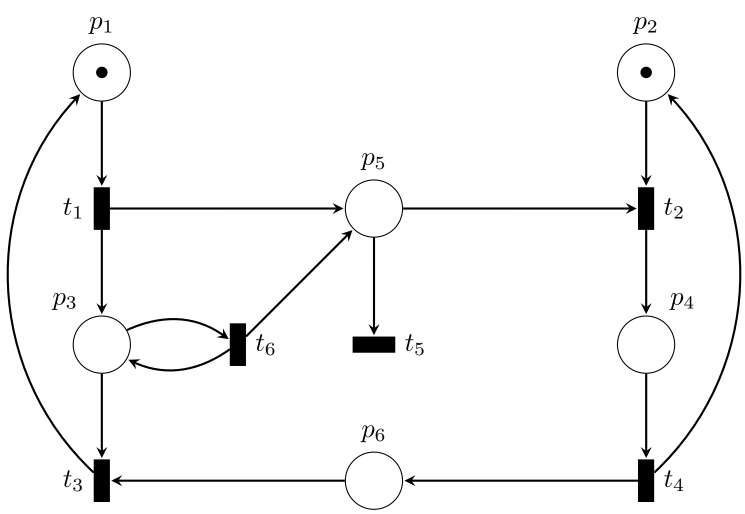
Exercises — Petri Nets: Modelling, Properties and Analysis

Jiri Srba, Aalborg University

Advanced Topics in Semantics and Verification

1 Exercise

Consider the following Petri net from the slides.



- Modify the net so that the place p_5 has a bounded capacity of at most three tokens and the place p_6 of at most two tokens.
- Modify the net so that the total number of tokens in both of the places p_5 and p_6 is guaranteed to be at most five.

2 Exercise

Draw a marked Petri net and define a set of its final markings such that the net accepts the language $\{(ab)^n c^n \mid n \geq 1\}$.

3 Exercise

a) Draw a marked Petri net that is not reversible but it has a home marking.

b) Draw a marked Petri net that has at least two places and two transitions, is reversible and has a deadlock.

c) Draw a marked Petri net that has at least two places and two transitions, is L2-live and has a deadlock.

d) Draw a marked Petri net that has at least two places and two transitions, is L4-live and has a deadlock.

4 Exercise

Improve your traffic light example (modeled in TAPAAL) so that it is possible that both lights are in yellow state at the same time, but never together in the green state. Extra challenge: try to do it in such a way that both traffic lights share only one common (shared) "communication" place, possibly with more tokens.

5 Exercise

We showed that boundedness for Petri nets with inhibitor arcs is undecidable by reduction from boundedness problem for Minsky machine. Prove the undecidability of boundedness of Petri nets with inhibitor arcs by reduction from the halting problem of Minsky machine. (Hint: for a given Minsky machine construct a net that is bounded if and only if the Minsky machine halts.)

6 Exercise

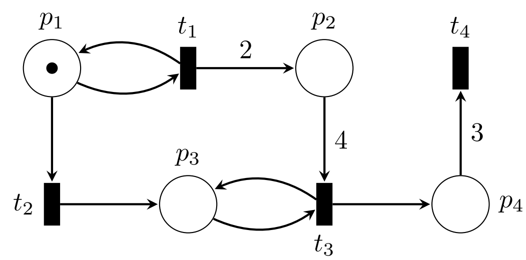
A Petri net with priorities is a classical Petri net $N = (P, T, F, W)$ equipped with a priority function $\pi : T \to \mathbb{N}^0$. Transitions with higher numbers assigned by the function π have a priority over the transitions with lower numbers. A transition can now fire, provided that there is no other enabled transition with a higher priority. Formally, t can fire only if there is no other enabled transition $t' \in T$ such that $\pi(t) < \pi(t')$.

a) Prove that the reachability problem for Petri nets with priorities is undecidable.

b) Prove that the reachability problem for bounded Petri nets with priorities is decidable.

7 Exercise

Construct the coverability tree and coverability graph for the marked Petri net below.



a) Can the coverability tree/graph be used to answer the question whether the place p_4 is unbounded (if yes explain how).

b) Can the coverability tree/graph be used to answer the question whether there is a reachable marking with exactly 117 tokens in the place p_2? Give arguments for your answer.

8 Exercise (slightly challenging)

Show that the coverability tree construction for Petri nets with inhibitor arcs does not work (i.e. that it breaks the soundness or completeness property on some concrete example of a net with inhibitor arcs).

9 Exercise

Draw a Petri net with its initial marking and the corresponding coverability tree such

- the net has a deadlock, but
- there is no node in the coverability graph tagged as deadlock.

If a coverability graph for a given Petri net has a node marked as deadlock, is this a guarantee that the Petri net has a reachable deadlock marking?

10 Exercise

Draw two marked Petri nets that have isomorphic coverability trees but one of them is live and the other one is not live.

11 Exercise

Consider the structural reduction rules a)–f) discussed during the lecture.

a) Do the reduction rules preserve deadlock-freedom?

b) Do the reduction rules preserve reversibility?

If the answer is no at some cases, what rules need to be removed to preserve the properties?

12 Exercise

Construct state equations for the Petri net from Exercise 7 and demonstrate that the marking $(0, 1, 1, 0)$ is not reachable in the net.

13 Exercise

a) Can state equations be used as an argument that a given marking is not coverable from an initial marking?

b) Can state equations be used as an argument that a given transition is dead (L0-live)?

c) Can state equations be used as an argument that a net is L1-live?

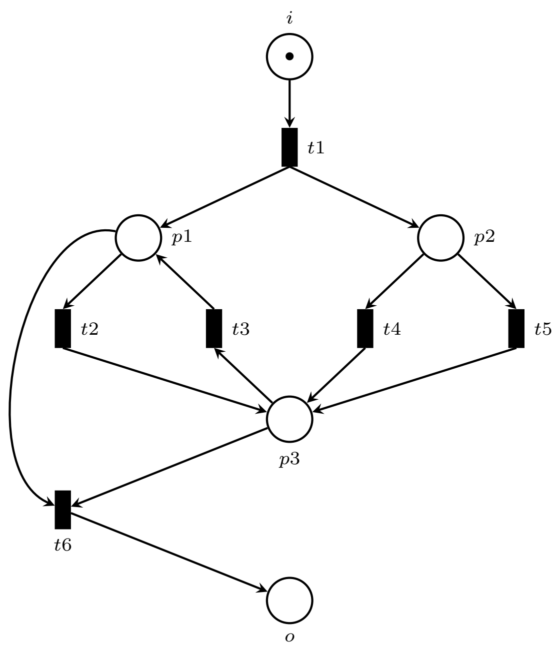
Justify your answers.

14 Exercise

Verify whether the workflow net below is sound by

a) first applying the structural reduction rules as long as possible, and then

b) constructing the coverability graph and arguing that the net satisfies the required soundness conditions.



15 Exercise

a) Is it possible to construct an unbounded workflow net that is sound?

b) Is it possible to construct an unbounded workflow net with inhibitor arcs that is sound?

Give arguments for your answers.