Syntax and Semantics:
Exercise Session 6

Exercise 1.
Give context-free grammars that generate the following languages with the alphabet $\Sigma = \{0, 1\}$.

$L_1 = \{w \in \Sigma^* \mid w \text{ contains at least three occurrences of } 1\}$
$L_2 = \{w \in \Sigma^* \mid w \text{ starts and ends with the same symbol}\}$
$L_3 = \{w \in \Sigma^* \mid \text{the length of } w \text{ is odd and its middle symbol is } 0\}$
$L_4 = \emptyset$
$L_5 = \{\varepsilon\}$

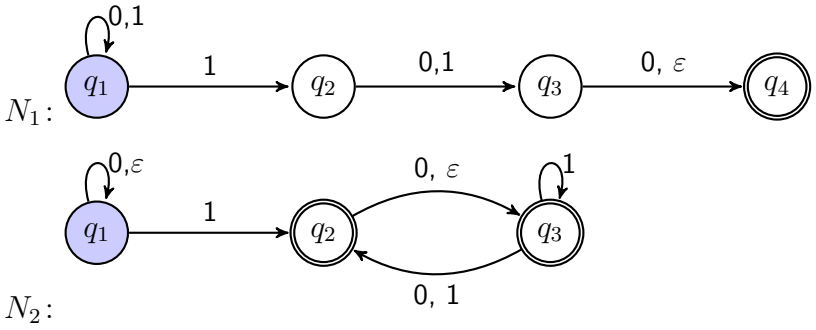
Exercise 2.
Consider the following CFG G with start variable A.

$$A \to X\,A\,X \mid S \quad S \to a\,T\,b \mid b\,T\,a \quad T \to X\,T\,X \mid X \mid \varepsilon \quad X \to a \mid b$$

1. Describe G formally by giving all its components.
2. Give five strings in $\mathcal{L}(G)$.
3. Give five strings not in $\mathcal{L}(G)$.
4. Which of the following derivations is allowed in G?

$T \Rightarrow aba$	$T \Rightarrow^* aba$	$XXX \Rightarrow^* aba$
$T \Rightarrow^* XX$	$T \Rightarrow^* XXX$	$S \Rightarrow^* \varepsilon$

Exercise 3.
Provide a CFG equivalent to the following NFAs



Exercise 4.
Provide CFGs equivalent to each of the following regular expressions:

$$0^*10^* \qquad 1 \cup 0^*\emptyset^* \qquad (01^+)^+$$