Syntax and Semantics
Exercise Session 10

Exercise 1.
Is there a big-step transition for each of the following cases? If so, prove it.

(i) $\langle \texttt{x} := 2;\ (\texttt{skip};\ \texttt{y} := 3),\ [x \mapsto 3, y \mapsto 5] \rangle \to ?$

(ii) $\langle \texttt{if x} < \texttt{y then z} := 5 \texttt{ else S},\ [x \mapsto 2, y \mapsto 3, z \mapsto 5] \rangle \to ?$
where $\texttt{S} = (\texttt{if x} + 1 < \texttt{y then z} := 2 \texttt{ else z} := 3)$

(iii) $\langle \texttt{skip};\ \texttt{x} := 3;\ \texttt{while x} \leq 5 \texttt{ do } (\texttt{x} := \texttt{x} + 1;\ \texttt{y} := 2),\ \sigma \rangle \to ?$ where $\sigma = [x \mapsto 2, y \mapsto 0]$.

Exercise 2.
Let $\texttt{S} = \texttt{while } \neg(2 < (1 + 1)) \texttt{ do } (\texttt{if x} < \texttt{x then x} := 2 \texttt{ else skip})$.

(i) Prove that S loops forever in the small-step semantics.

(ii) Prove that there exists $k \geq 0$ s.t. in the SS-semantics for any state $s \in \mathbb{S}tates$, $\langle \texttt{S},\ s \rangle \Rightarrow^k \langle \texttt{S},\ s \rangle$.

Exercise 3.
Find all the transitions (if there are any) in the transition sequence starting from $\langle \texttt{S},\ s \rangle$ in the small-step semantics, for each of the following cases:

(i) $\texttt{S} = \texttt{if } (\neg(\texttt{x} > 3) \vee \texttt{y} > 2) \texttt{ then } (\texttt{x} := \texttt{x} + 3;\ \texttt{y} := 2) \texttt{ else skip}$ and $s = [x \mapsto 0, y \mapsto 0]$.

(ii) $\texttt{S} = \texttt{while } \neg(\texttt{x} < \texttt{y}) \texttt{ do } (\texttt{x} := \texttt{x} - 1;\ \texttt{y} := \texttt{y} + 1)$ and $s = [x \mapsto 3, y \mapsto 0]$.

Exercise 4.
Prove that regular expressions are closed under intersection.