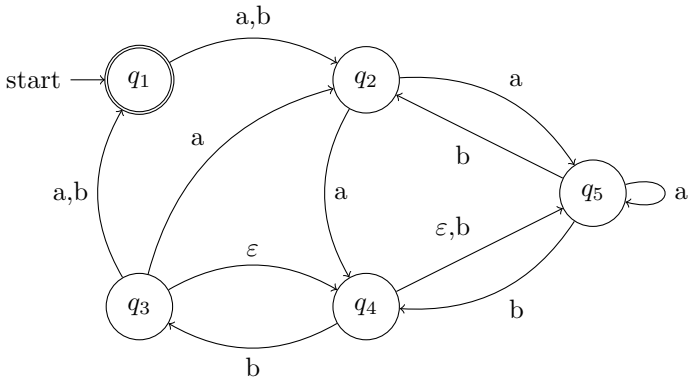
Syntax and Semantics
Exercise Session 3

Exercise 1

Consider the NFA N in the state diagram below.



1. Describe N. What are $Q, \Sigma, \delta, q_0, F$?
   Solution:

   $Q = \{q_1, q_2, q_3, q_4, q_5\}$
   $\Sigma = \{a, b\}$
   $F = \{q_1\}$
   $q_0 = q_1$

δ	a	b	ε
q_1	$\{q_2\}$	$\{q_2\}$	$\emptyset$
q_2	$\{q_4, q_5\}$	$\emptyset$	$\emptyset$
q_3	$\{q_1, q_2\}$	$\{q_1\}$	$\{q_4\}$
q_4	$\emptyset$	$\{q_3, q_5\}$	$\{q_5\}$
q_5	$\{q_5\}$	$\{q_2, q_4\}$	$\emptyset$

2. Find an accepting computation (sequence of states) of N on the input aabba.
   Solution: $q_1 \xrightarrow{a} q_2 \xrightarrow{a} q_4 \xrightarrow{b} q_3 \xrightarrow{\varepsilon} q_4 \xrightarrow{b} q_3 \xrightarrow{a} q_1$

3. Which of the following inputs are accepted?

   (a) aabbbba
   Solution: Accepted

   (b) abbabb
   Solution: Rejected

(c) aaabab
Solution: Rejected

(d) abbbaab
Solution: Rejected

(e) abababa
Solution: Rejected

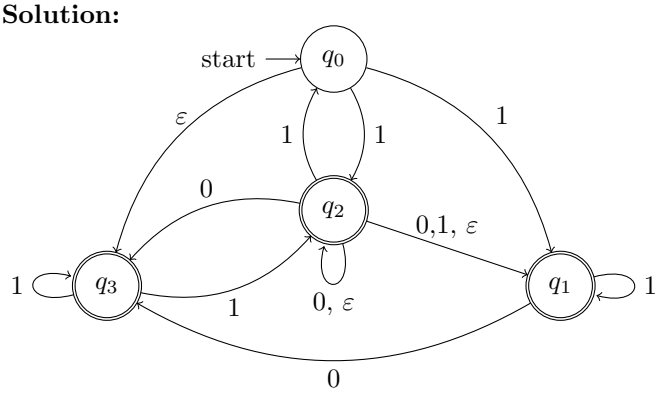
Exercise 2

Construct the state diagram for the NFA $N = (Q, \Sigma, \delta, q_0, F)$ where

$Q = \{q_0, q_1, q_2, q_3\}$
$\Sigma = \{0, 1\}$
$q_0 = q_0$
$F = \{q_1, q_2, q_3\}$

δ	0	1	ε
q_0	$\emptyset$	$\{q_1, q_2\}$	$\{q_3\}$
q_1	$\{q_3\}$	$\{q_1\}$	$\emptyset$
q_2	$\{q_1, q_2, q_3\}$	$\{q_0, q_1\}$	$\{q_1, q_2\}$
q_3	$\emptyset$	$\{q_2, q_3\}$	$\emptyset$

Solution:



Which of the following inputs are accepted by this machine?

(a) 0011010
Solution: Rejected

(b) 0001110
Solution: Rejected

(c) 1010111
Solution: Accepted

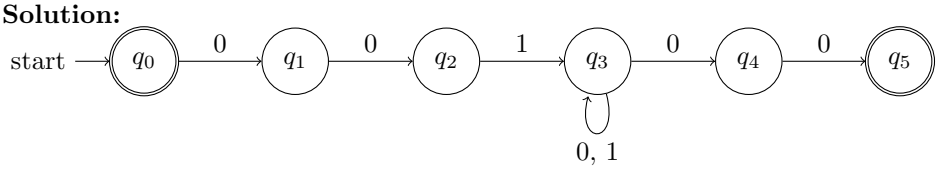
(d) 1101011
Solution: Accepted

Exercise 3

Construct automata that recognize the following languages:

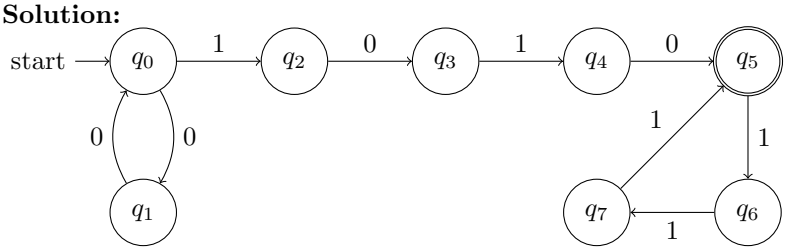
1. $L_1 = \{w \in \{0,1\}^* \mid w \text{ has the prefix } 001 \text{ and the suffix } 00\} \cup \{\varepsilon\}$

   Solution:



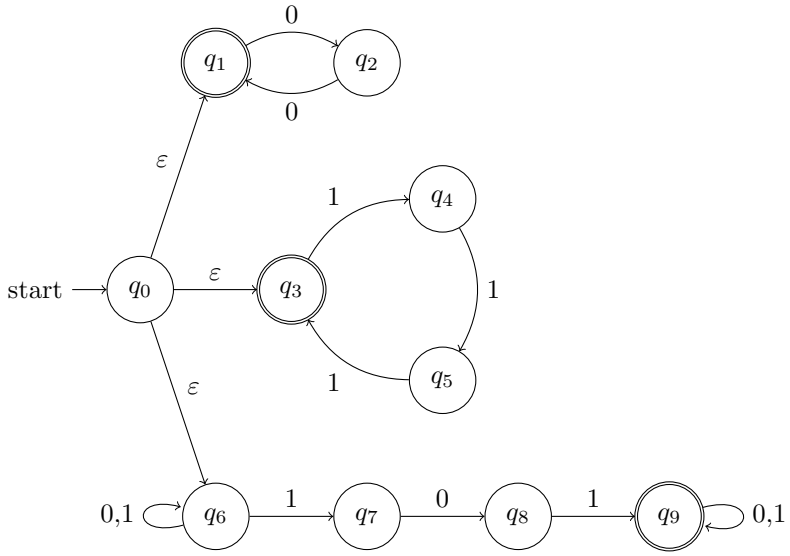
2. $L_2 = \{w \in \{0,1\}^* \mid w = \underbrace{00\ldots0}_{2n}1010\underbrace{11\ldots1}_{3k} \text{ where } n, k \in \mathbb{N}_0\}$

   Solution:



3. $L_3 = \{w \in \{0,1\}^* \mid w = \underbrace{00\ldots0}_{2n} \text{ or } w = \underbrace{11\ldots1}_{3n} \text{ for } n \geq 0 \text{ or } w \text{ contains substring } 101\}$

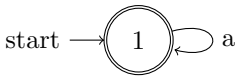
   Solution:



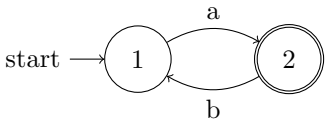
Exercise 4

For each NFA given below, construct an equivalent DFA, where $\Sigma = \{a, b\}$.

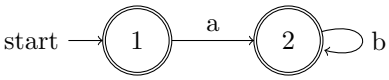
(1)



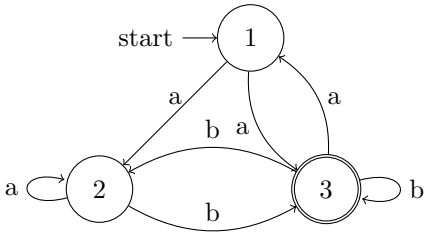
(2)



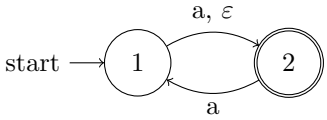
(3)



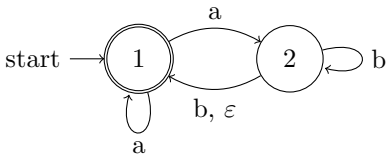
(4)



(5)



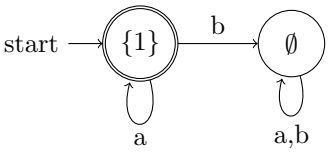
(6)



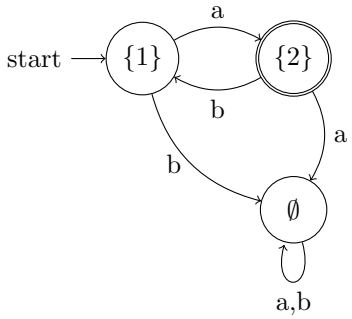
Solution:

Solution:
Only reachable states are shown

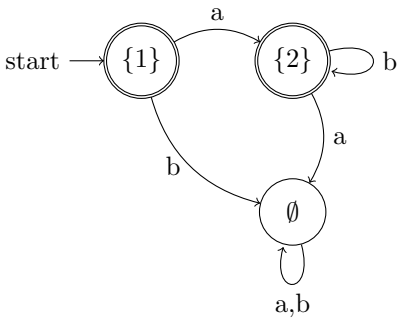
(1)



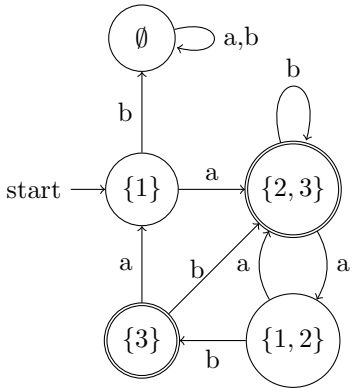
(2)



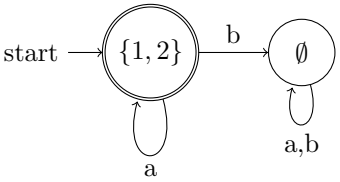
(3)



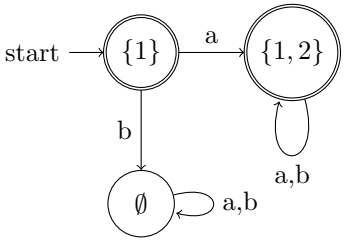
(4)



(5)

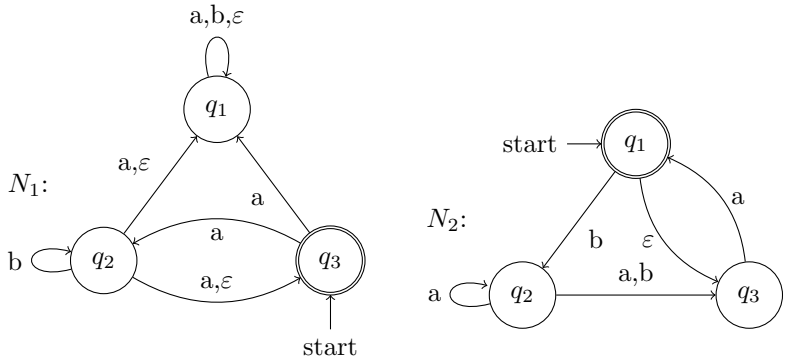


(6)

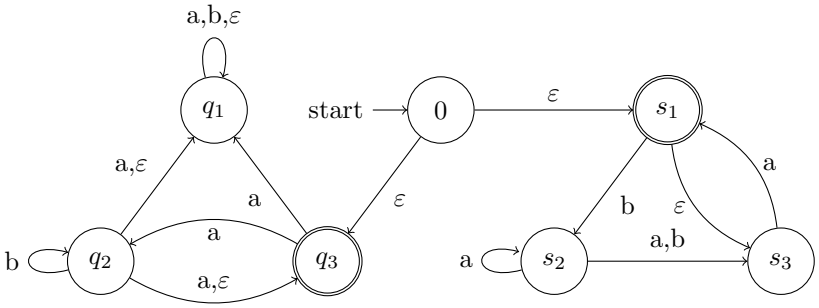


Exercise 5

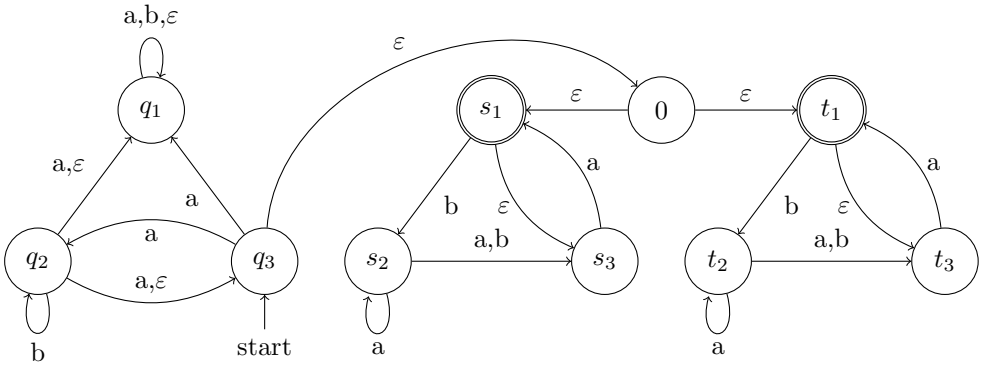
Consider the following automata



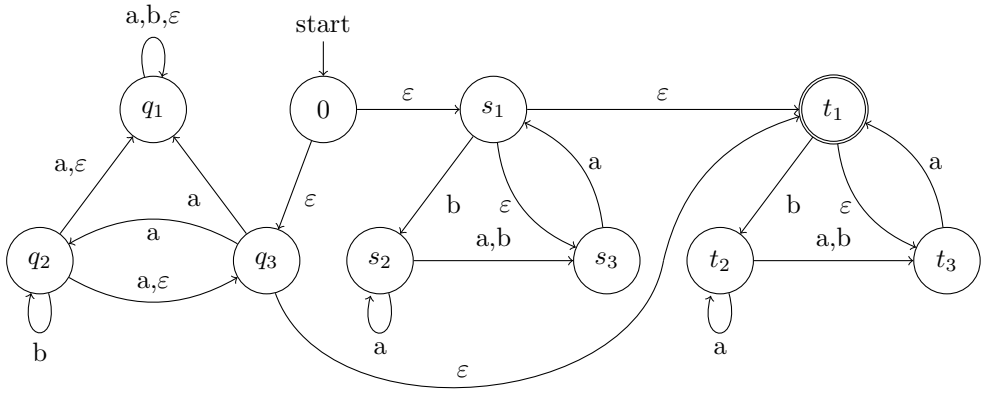
1. Construct an automaton that recognizes $L(N_1) \cup L(N_2)$
   Solution:



2. Construct an automaton that recognizes $L(N_1) \circ (L(N_2) \cup L(N_2))$
   Solution:



3. Construct an automaton that recognizes $(L(N_1) \cup L(N_2)) \circ L(N_2)$
Solution:



4. Construct an automaton that recognizes $(L(N_1) \cup L(N_2)^*) \circ L(N_2)$
Solution: