Syntax and Semantics
Exercise Session 12

Exercise 1.
Given S_1 and S_2 defined as follows

$$\mathtt{x} := 0;\ \mathtt{while}\ (\mathtt{x} \geq 0)\ \mathtt{do}\ \mathtt{x} := \mathtt{x} + 1 \tag{S_1}$$
$$\mathtt{x} := 3;\ \mathtt{y} := 4;\ \mathtt{while}\ (\mathtt{x} \leq \mathtt{y})\ \mathtt{do}\ (\mathtt{x} := 2 * \mathtt{x};\ \mathtt{y} := 3 * \mathtt{y}) \tag{S_2}$$

check whether S_1 and S_2 are semantically equivalent in the big-step semantics, that is $S_1 \sim_{bs} S_2$. Motivate your answer.

Exercise 2.
Given S_3 and S_4 defined as follows

$$\mathtt{y} := \mathtt{n}_1;\ \mathtt{for}\ \mathtt{x} := 1\ \mathtt{to}\ \mathtt{n}_2\ \mathtt{do}\ \mathtt{y} := \mathtt{y} + 1 \tag{S_3}$$
$$\mathtt{y} := \mathtt{n}_1 + \mathtt{n}_2 * (\mathtt{n}_2 + 1)/2;\ \mathtt{x} := \mathtt{n}_2 + 1 \tag{S_4}$$

check whether S_3 and S_4 are semantically equivalent in the big-step semantics, that is $S_3 \sim_{bs} S_4$, for all $\mathtt{n}_1, \mathtt{n}_2 \in \mathbb{N}$. Motivate your answer.

Exercise 3.
Consider the following extension of **Bims** which adds the following formation rule to those of **Stm**, for $m > 0$,

$$S ::= \cdots \mid \mathtt{foreach}\ x\ \mathtt{in}\ [n_1, \ldots, n_m]\ \mathtt{do}\ S\ .$$

Intuitively, the above construct executes the body S m-times, and at each execution of the body S, the value of the variable x is set to v_i, the value of the numeral n_i, for $i = 1 \ldots m$. At the end of the execution of the `foreach` construct, x assumes the value v_m of the numeral n_m. Give both the big-step and the small-step semantics that formalize the above description.

Exercise 4.
Consider the following statements in **Bims**

$$\texttt{y} := \texttt{x} + 4;\ (\texttt{for x} := 1 \texttt{ to } 3 \texttt{ do y} := \texttt{y} * \texttt{x});\ \texttt{y} := \texttt{y} + \texttt{x} \qquad (S_5)$$

$$(\texttt{if x} < 0 \texttt{ then x} := 2 * \texttt{x else x} := 2 + \texttt{x});\ \texttt{x} := \texttt{x} * (-1) \qquad (S_6)$$

$$\texttt{repeat } S_6 \texttt{ until } (\texttt{x} \geq 200) \qquad (S_7)$$

$$\texttt{while } (\texttt{x} < 200) \texttt{ do } S_6 \qquad (S_8)$$

Find all the transitions (if there are any) in the SS-semantics, for each of the following cases:

(i) $\langle S_5, [x \mapsto -2] \rangle \Rightarrow^* ?$

(ii) $\langle S_7, [x \mapsto 100] \rangle \Rightarrow^* ?$

(iii) $\langle S_8, [x \mapsto 100] \rangle \Rightarrow^* ?$

Exercise 5.
Prove or disprove that the following languages are regular or context-free.

$$L_1 = \{ww \mid w \in \{a,b\}^*\}$$
$$L_2 = \{w_1 w_2 \mid w_1, w_2 \in \{a,b\}^*, |w_1| = |w_2|\}$$
$$L_3 = \{a^n w b^n \mid w \in \{a,b\}^*,\ |w| = n\}$$