Exercise 1

i)

a) $(a^* \cup b^+) \cup b^* = a^* \cup b^*$, hence

$\mathcal{L}(a^* \cup b^*) \supseteq \{\varepsilon, a, aa, b, bb\}$

$\emptyset = \mathcal{L}(a^* \cup b^*) \cap \{ab, ba, abba, baba, aaaaab\}$

b) $(\varepsilon \cup a)b^* \cup b^+b^* = ab^* \cup b^*$, so

$\mathcal{L}(ab^* \cup b^*) \supseteq \{a, ab, abb, \varepsilon, b\}$

$\emptyset = \mathcal{L}(ab^* \cup b^*) \cap \{ba, baa, bab, baaa, abba\}$

c) $\underbrace{(a \cup ba \cup bb)(a \cup b)}^{*} = \{a, b\}^* \setminus \{\varepsilon, b\}$

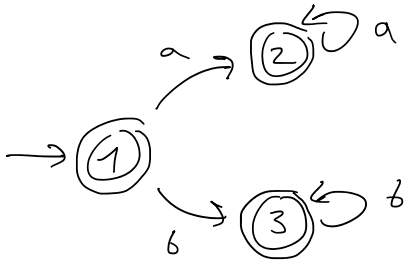
$\mathcal{L}(\ldots) \supseteq \{a, ba, bb, abba, bba\}$

$\emptyset = \mathcal{L}(\ldots) \cap \{\varepsilon, b\}$

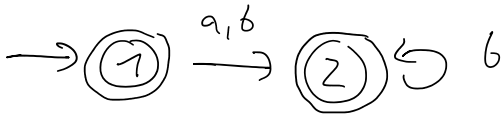
d) $(a^* \cup b^*)^+ \cup (b^+a^*)^* \cup (a^*b^*)^+ = (a \cup b)^* = \{a, b\}^*$

ii) We use the simplified regular expressions from above to construct the NFAs.

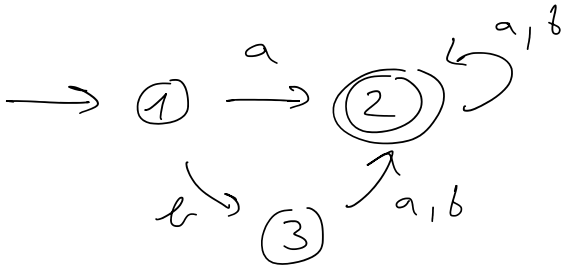
a)



b)



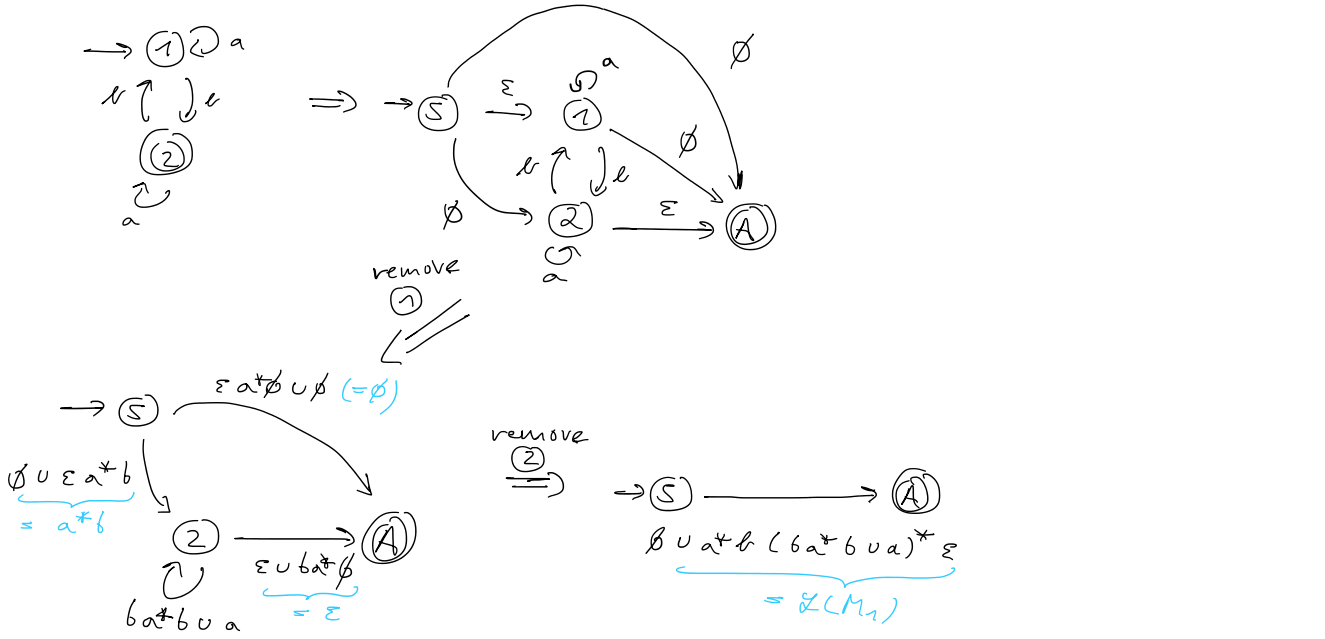
c)



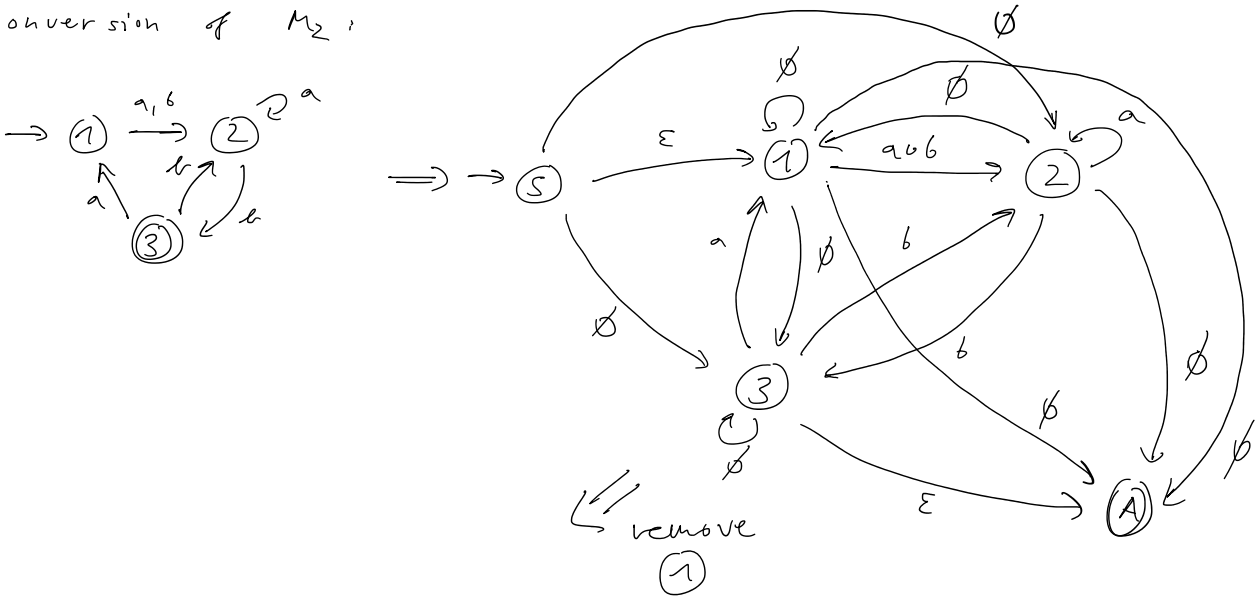
d)

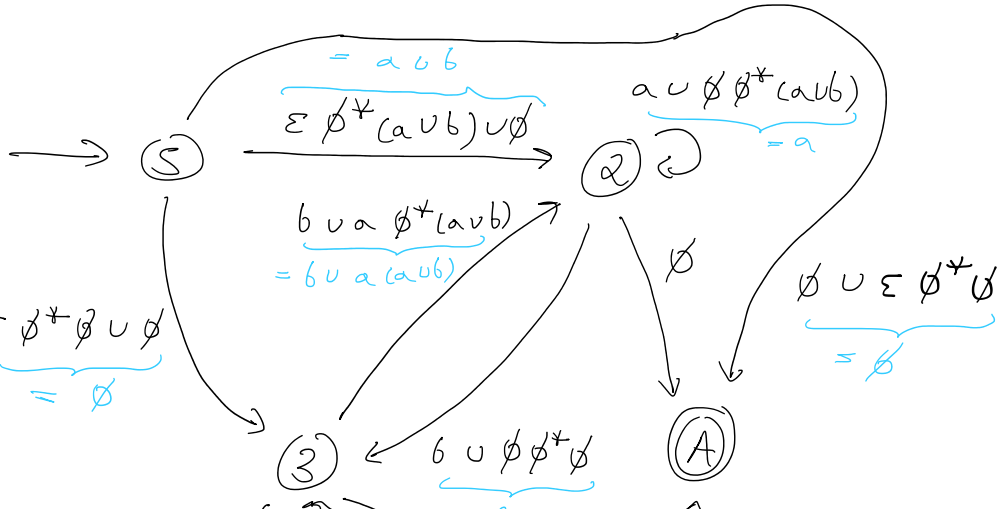
Exercise 2

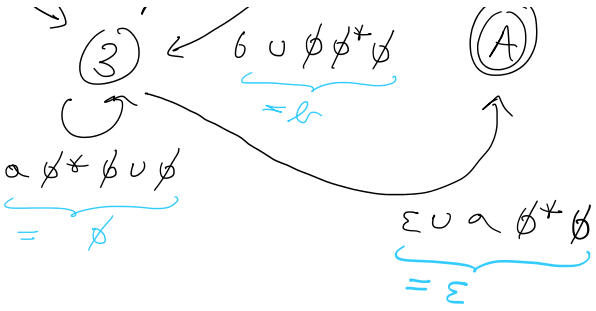
Conversion of M_1:



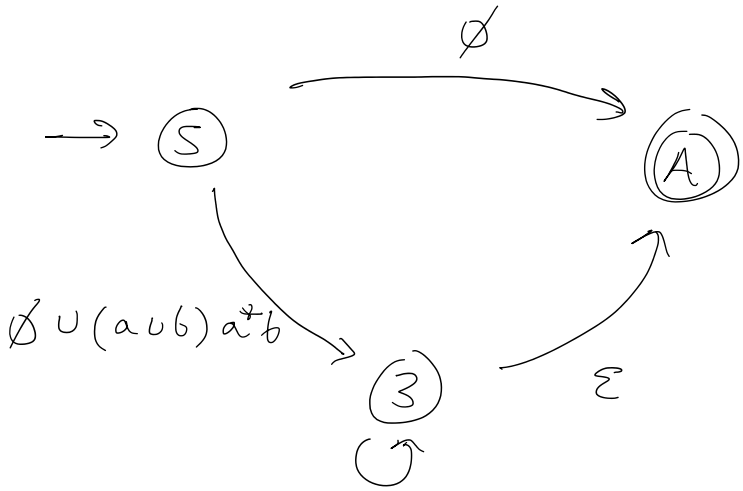
Conversion of M_2:





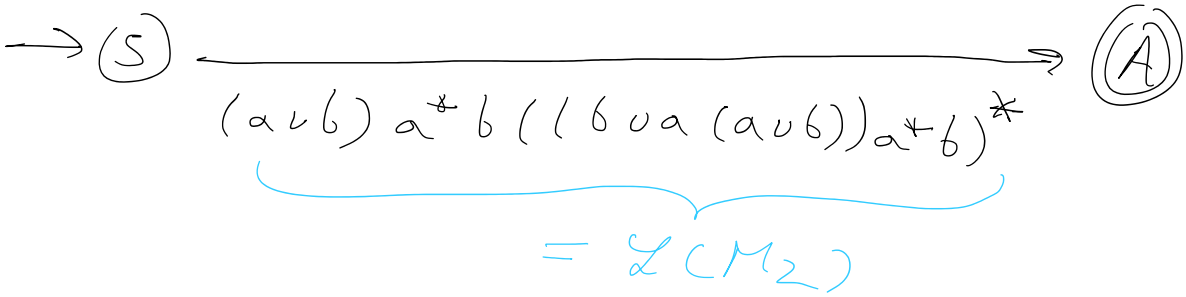


remove (2)

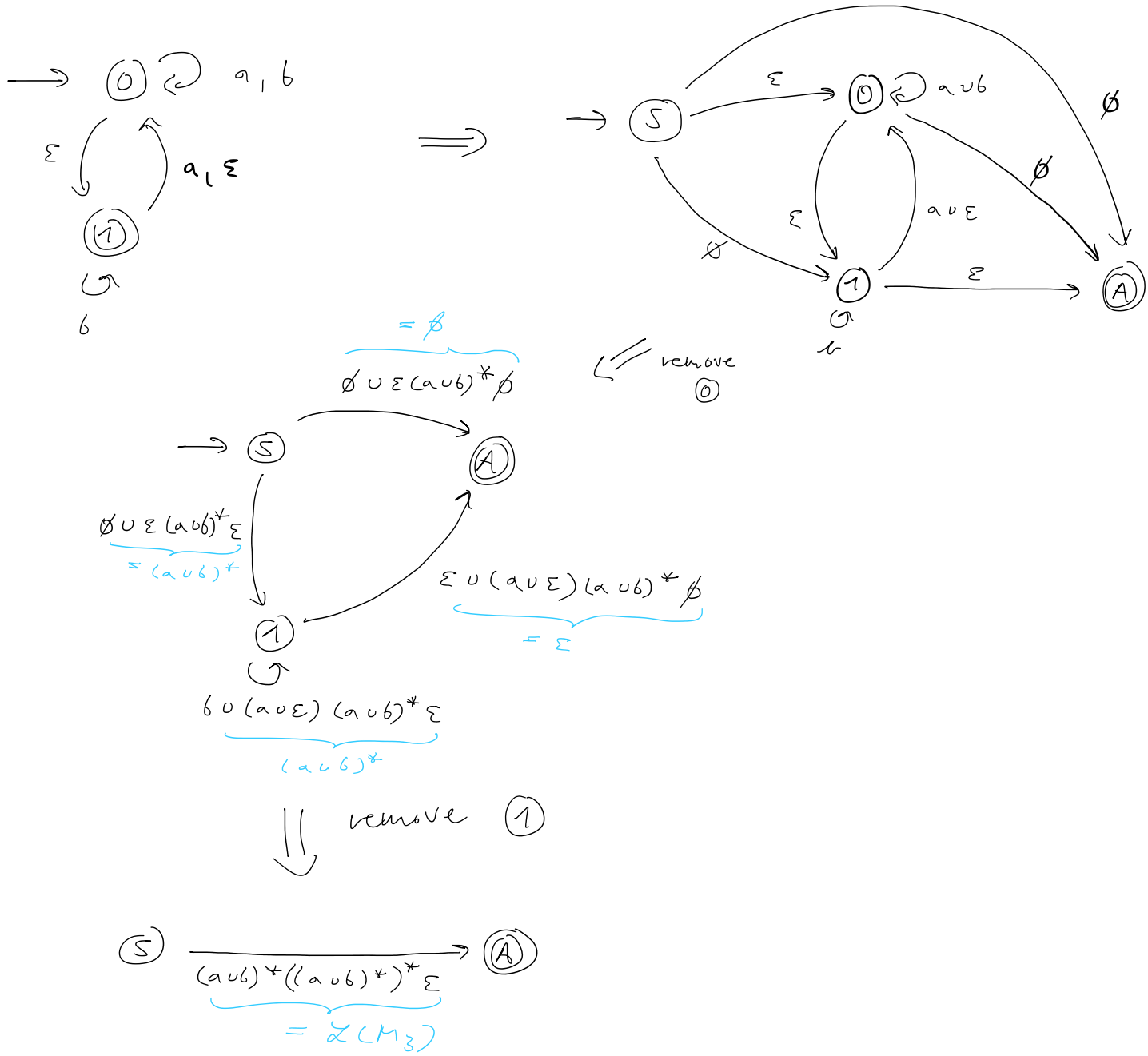


$(b \cup a(a \cup b))a^*b$

remove (3)

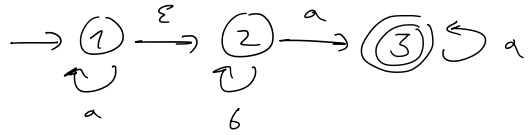


Conversion of M_3:

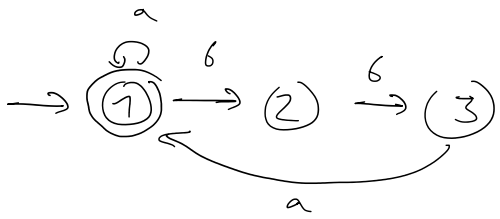

a, b
ε
a, ε
b
S
0
1
A
ε
a ∪ b
∅
a ∪ ε
remove
0
∅ ∪ ε (a ∪ b)* ∅
= ∅
∅ ∪ ε (a ∪ b)* ε
= (a ∪ b)*
ε ∪ (a ∪ ε) (a ∪ b)* ∅
= ε
b ∪ (a ∪ ε) (a ∪ b)* ε
(a ∪ b)*
remove 1
(a ∪ b)* ((a ∪ b)*)* ε
= L(M3)

Exercise 3

a) $a^+b^*a^+$ with 3 states:



b) $a^*(bba^+)^*$ with 3 states:



c) $L_1 = \{ w \mid w \text{ contains } abab \}$ with 5 states:

d) $L_2 = \{ w \mid w$ has even number of a or exactly two b $\}$ with 6 states: