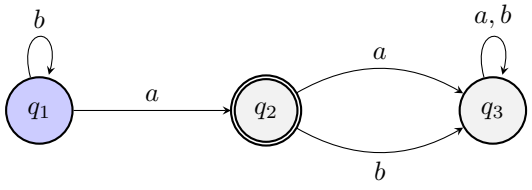
Syntax and Semantics:
Exercise Session 2

Exercise 1.
Let $L_1 = \{aa, bb, bbb\}$, $L_2 = \{abba, aab, bb\}$
Specify the following languages:

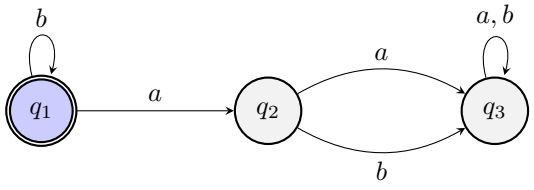
1. $L_1 \circ L_2$
2. $L_1 \cup L_2$
3. $L_1 \cap L_2$
4. $L_1 \setminus L_2$
5. Provide a few strings of L_2^*

Exercise 2.
Describe the following automata.

M_1



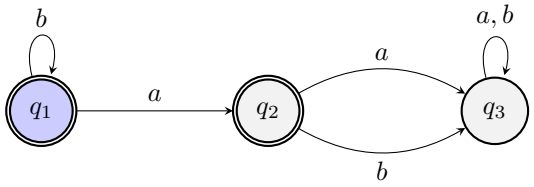
M_2



(a) Describe the sequence of states of M_1 for the following inputs:

(1) abbbab

M_3



(2) abababb

(3) aaaaa

(4) ε

(b) Which of the previous sequences are final in M_1, M_2 and M_3?

(c) Describe the languages accepted by each of the three machines.

Exercise 3.
Give the state diagram for the following automaton and describe its language. $M_4 = (Q, \Sigma, \delta, q_o, F)$ where:

$Q = \{s, q_1, q_2, r_1, r_2\}$

$\Sigma = \{a, b\}$

$q_0 = s$

$F = \{q_1, r_1\}$

δ	a	b
s	q_1	r_1
q_1	q_1	q_2
q_2	q_1	q_2
r_1	r_2	r_1
r_2	r_2	r_1

Exercise 4.
Give the state diagram for an automaton that recognizes the following language:

(i) $L_1 = \{w \in \{1, 22\}^* \mid 11 \text{ is a prefix of } w\}$

(ii) $L_2 = \emptyset \subseteq \{0, 1, 2\}^*$

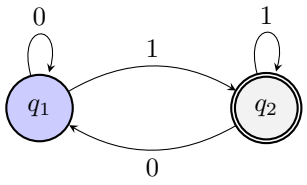
(iii) $L_3 = \{\varepsilon\} \subseteq \{0, 1, 2\}^*$

(iv) $L_4 = \{w \in \{\text{go}, \text{stop}\}^* \mid w = \varepsilon \text{ or ends with stop}\}$

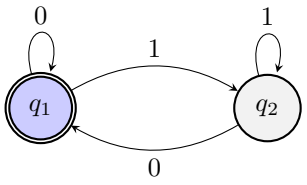
(v) $L_5 = \{w \in \{0, 1\}^* \mid w \text{ has 001 as a prefix or 11 as a suffix}\}$

Exercise 5.
Consider the automata M_1 and M_2 drawn below.

1. Construct an automaton that recognizes the language $L(M_1) \cap L(M_2)$
2. Prove that the regular languages are closed under intersection.



M_1



M_2

3. Construct an automaton for each of the following languages:

   (i) $\{0,1\}^* \setminus L(M_1)$

   (ii) $\{0,1\}^* \setminus L(M_2)$

   (iii) $\{0,1\}^* \setminus (L(M_1) \cap L(M_2))$

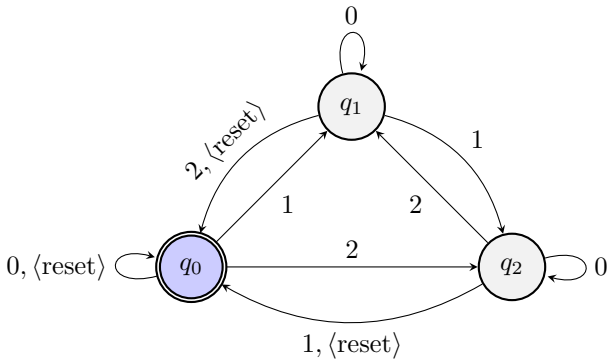
4. Prove that the set of regular languages is closed under complement.

Hint:

2. Similar construction with the one for union, only that $F = F_1 \times F_2$
4. Change the final states in not-final and reverse

Exercise 6.

Describe the following automaton.



(i) Give examples of accepted and nonaccepted words (at least five for each).

(ii) Prove that the language $L(M)$ can be characterized as follows: Suppose that M keeps a running count of the sum of the numerical input symbols it reads and it reset the count to 0 every time it reads ⟨reset⟩. Then, $L(M) = \{w \mid count(w) = 0 (\text{mod } 3)\}$.

(iii) Generalize the automaton such that $L(M) = \{w \mid count(w) = 0 (\text{mod } 4)\}$