Syntax and Semantics

Digital Written Exam, June the 14th 2021, 10:00-13:00

Please read the following before solving the exercises.

- This exam contains 5 exercises. Each exercise is compulsory and has an equal contribution to the final grade. The solution has to be composed in English. If you believe that the assignment wording is ambiguous or erroneous, then write down what additional assumption you are using and outline your reasons.
- Each of the five exam exercises has to be solved on separate handwritten A4 pieces of paper, with at most 2 A4 pages per exercise. On each A4 page, state your name, your student number and the question(s) which is (are) solved.
- Solutions have to be uploaded to Digital Exam in form of a single pdf file consisting of at most ten digital pictures of at most ten handwritten A4 pages. Other submission formats will not be considered.
- The submitted digital pictures should be readable, i.e., of sufficient quality (high resolution, enough light, not blurry, etc.).
- Allowed aids are your own notes (made entirely by yourself or as an active participant of a group), lecture slides and exercise sheets, the books of Hans Hüttel and Michael Siepser used during the course. Anything else is rendered illegal, including, in particular, Googling or asking other persons for help.
- In case of emergencies: Students can contact the instructor during the exam by approaching the study secretary, as outlined in the guidelines for online exams. Keep an eye on your student mail for potential announcements during the exam.

Terminology applied in the exam:

- *Provide*: Give something without arguing why it is correct.
- *Prove*: Give a formal proof for the correctness of something.
- *Motivate*: Give an informal argument for the correctness or choice of something.

Last but not least, good luck!

Exercise 1.
Consider the following NFA over alphabet $\Sigma = \{0, 1\}$.

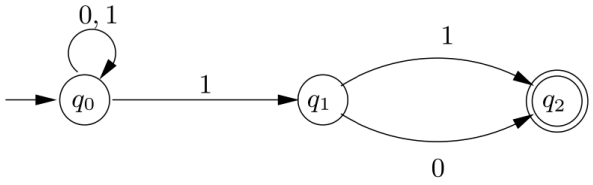


Figure 1: NFA M.

1) Provide two words that are accepted by M and two words that are rejected by M.

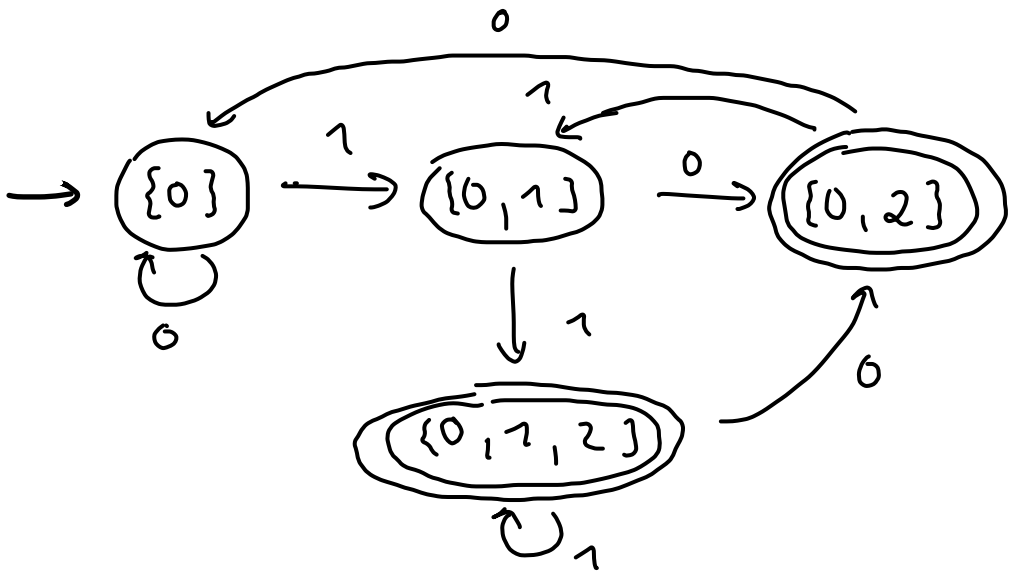
2) Provide a regular expression that describes the same language as M.

3) Convert the NFA M into a DFA that recognizes the same language by using the construction from the course (ad-hoc solutions will be not considered). DFA states that are not reachable from the initial state of the DFA can be omitted.

1) acc.: 10, 11 rej.: 1, 01

2) $\Sigma^* 1 (0 \cup 1)$

3)



Exercise 2.
Consider the language

$$L = \{w \in \{a, b\}^* \mid w = a^n b^m \text{ with } n, m > 0 \text{ and } n \neq m\}.$$

1) Provide a CFG generating L.

2) Provide the CFG derivations of *aabbb* and *aaabb*.

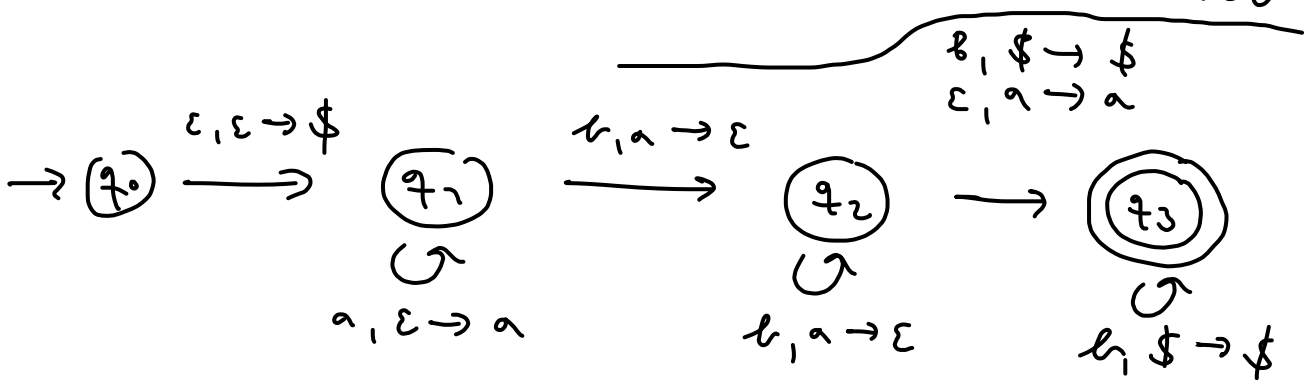
3) Provide a PDA that recognizes L.

1) $V = \underbrace{\{S, A, B\}}_{\text{variables}}$, $\underbrace{\Sigma = \{a, b\}}_{\text{terminals}}$, S is the start var.

$S \to aSb \mid aA \mid Bb$
$A \to aA \mid \varepsilon$
$B \to Bb \mid \varepsilon$

2) $S \to aSb \to aaSbb \to aaBbbb \to aabbb$
$S \to aSb \to aaSbb \to aaaAbb \to aaabb$

3)



[Alternatively, the CFG from 2) can be converted into a PDA using the algorithm from the course]

Exercise 3.
Using the pumping lemma for context-free languages, prove that the following language is not context-free.

$$L' = \{w \in \{a, b, c\}^* \mid w = a^n b^m c^n \text{ with } 0 < n < m\}.$$

Assume that L' is context-free. Then, by the PL, there is a $p \geq 0$ s.t. for $w = a^p b^{p+1} c^p \in L'$ there exist u, v, x, y, z satisfying $w = uvxyz$ and

i) $uv^i xy^i z \in L'$, $\forall i \geq 0$

ii) $|vy| > 0$

iii) $|vxy| \leq p$

Thanks to ii)-iii), vxy has to satisfy one of the following cases:

- vxy has a's but no c's: then uv^0xy^0z has more c's than a's, hence is not in L'.
- vxy has c's but no a's: similar to ↑
- $vxy = b^k$ for some $k \geq 0$. Then, uv^0xy^0z does not have more b's than a's or c's, hence is not in L'.

Exercise 4.
Consider the language $L'' = \{w \in \Sigma^* \mid w = a^n \text{ with } n \geq 1\}$ over the singleton alphabet $\Sigma = \{a\}$. With this, consider the big step transitions $\to \subseteq L'' \times \mathbb{N}$ given by

$$[r_1]\ \frac{}{a \to 1} \qquad\qquad [r_2]\ \frac{w \to k}{aw \to k'}\ k' = k \cdot |aw|,$$

where $|aw|$ denotes the length of the word aw.

1. Using the big step semantics, prove $aa \to 2$ and $aaa \to 6$.
2. Using the big step semantics and induction, prove that $a^n \to v$ with $v = 1 \cdot 2 \cdot \ldots \cdot n$ for all $n \geq 1$. Note that a^n is the word consisting of n symbols a.

1) $\dfrac{r_2\ \dfrac{r_1\ \dfrac{}{a \to 1}}{aa \to 2}}{aaa \to 6}$ $\quad 2 = 1 \cdot |aa|$ $\quad 6 = 2 \cdot |aaa|$

2) $\underline{n = 1}$: $a^1 \to 1$ because $r_1\ \dfrac{}{a \to 1}$

$\underline{n \to n+1}$: By I.H., we have $a^n \to n!$.
With this, we obtain

$$r_1\ \frac{a^n \to n!}{aa^n \to k'}\ k' = n! \cdot |aa^n| = n!(n+1) = (n+1)!,$$

thus implying $a^{n+1} \to (n+1)!$

Exercise 5.
Consider the following statement in **Bip**.

```
01 begin
02    var x:=3;
03    var y:=5;
04    proc p is x:=x-y;
05    proc q is call p;
06    begin
07        var x:=7;
08        proc p is x:=x+y;
09        call q;
10        y:=x
11    end
12 end
```

1) What is the value of y after the statement is executed assuming static scope rules for procedures and dynamic scope rules for variables? Motivate your answer.

2) What is the value of y after the statement is executed assuming fully static scope rules for both procedures and variables? Motivate your answer.

3) What is the value of y after the statement is executed assuming fully dynamic scope rules for both variables and procedures? Motivate your answer.

1) 2 + explanation

2) 7 + explanation

3) 12 + explanation