

Syntax and Semantics:

Exercise Session 1

1. Let us assume that we are given the sets $A = \{1, 2, 3\}$ and $B = \{2, 3\}$.
 - (a) Does $A \subseteq B$ hold true?
 - (b) Does $B \subseteq A$ hold true?
 - (c) In what relation stands B with respect to A ?
 - (d) Find the set $A \cup B$.
 - (e) How does one call $A \cup B$?
 - (f) Find the set $A \cap B$.
 - (g) How does one call $A \cap B$?
 - (h) Find the set $A \times B$.
 - (i) How does one call $A \times B$?
 - (j) Find $\mathcal{P}(B)$, the power set of B .
 - (k) If E is some finite set, what is the cardinality of $\mathcal{P}(E)$? That is, how many elements does $\mathcal{P}(E)$ have?
2. Formalize the following informal statements by using predicate logic.
 - (a) There exists a real number x such that $x + x$ is greater than 8.
 - (b) Every real number x is the double of some real number y .
 - (c) All natural numbers are positive.
3. Consider the sets $C = \{1, 2, 17, 484\}$ and $D = \{x, y, z\}$ and consider the set of pairs $R = \{(1, x), (2, y), (17, y), (17, z)\}$.
 - (a) Does R encode a function (from C to D)?
 - (b) Does R encode a relation (among C and D)?
 - (c) Does $R \cup \{484\}$ encode a relation (among C and D)?
 - (d) Let $X \setminus Y$ denote the elements of X not present in Y . With this, does $R \setminus \{(17, z)\}$ encode a function (from C to D)? If so, is the function total or partial?

4. Prove by induction on k that

$$1 + \sum_{i=1}^k (2i + 1) = (k + 1)^2$$

for all $k \geq 1$.

5. Using the formula from 1 (k), compute first the cardinality of $\mathcal{P}(\mathcal{P}(B))$, where $B = \{2, 3\}$ as in Exercise 1. Afterwards, compute $\mathcal{P}(\mathcal{P}(B))$ itself and check whether the cardinality of your answer matches the cardinality obtained in the first part of the exercise.