Syntax and Semantics: Exercise Session 7

Recall that. *A CFG is in Chomsky normal form if every production is of the following form*

$$A \to BC \qquad\qquad or \qquad\qquad A \to a$$

where A, B, C are non-terminals, $a \in \Sigma$ and B, C are not the initial non-terminal. In addition $S \to \varepsilon$ is permitted only if S is the initial non-terminal.

Recall that. *A DFA $(Q, \Sigma, \gamma, q_0, F)$ can be encoded to a PDA $(Q, \Sigma, \Gamma, \delta, q_0, F)$ with* empty stack alphabet *(i.e., $\Gamma = \emptyset$) and transition function $\delta \colon Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to \wp(Q \times \Gamma_\varepsilon)$ defined, for arbitrary $s \in Q$ and $\sigma \in \Sigma_\varepsilon$ as*

$$\delta(q, \sigma, \varepsilon) = \{(q', \varepsilon) \mid q' \in \gamma(q, \sigma)\} .$$

This is an alternative way to prove that context-free languages are a superset of the regular languages (i.e., any regular language is a context-free language). It is worth to recall that the converse inclusion does not hold.

Exercise 1.

For each of the CFGs below find an equivalent CFG in Chomsky normal form. The grammar G_1 produces mathematical expressions with the alphabet $\Sigma = \{a, +, \times, (,)\}$.

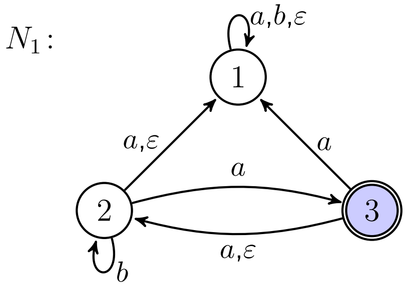
$$\begin{aligned} G_1 \colon E &\to E + T \mid T \\ T &\to T \times F \mid F \\ F &\to (E) \mid a \end{aligned} \qquad\qquad \begin{aligned} G_2 \colon R &\to XRX \mid S \\ S &\to aTb \mid bTa \\ T &\to XTX \mid X \mid \varepsilon \\ X &\to a \mid b \mid \varepsilon \end{aligned}$$

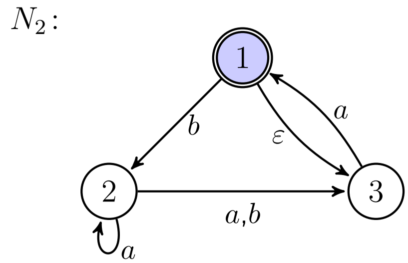
Exercise 2.
Provide an equivalent PDA for the languages generated by the grammars G_1 and G_2 from Exercise 1

Exercise 3.
Construct a PDA for each of the following languages.

$$L_1 = \{w \in \{0,1\}^* \mid w \text{ contains at least three 1s}\}$$
$$L_2 = \{w \in \{0,1\}^* \mid w \text{ starts and ends with the same symbol}\}$$
$$L_3 = \{w \in \{0,1\}^* \mid |w| \text{ is odd and 0 is its middle symbol}\}$$

Exercise 4.
Construct an equivalent PDA for each of the following NFAs.





Exercise 5.
Give context-free grammars in Chomsky normal form for the following languages

$$L_4 = \{w \in \{a,b\}^* \mid w \text{ has more } a\text{'s than } b\text{'s}\}$$
$$L_5 = \{w\#x \in \{0,1,\#\}^* \mid w, x \in \{0,1\}^* \text{ and } w^R \text{ is a prefix of } x\}$$