Syntax and Semantics

Digital Written Exam, August the 24th 2021, 10:00-13:00

Please read the following before solving the exercises.

- This exam contains 5 exercises. Each exercise is compulsory and has an equal contribution to the final grade. The solution has to be composed in English. If you believe that the assignment wording is ambiguous or erroneous, then write down what additional assumption you are using and outline your reasons.
- Each of the five exam exercises has to be solved on separate handwritten A4 pieces of paper, with at most 2 A4 pages per exercise. On each A4 page, state your name, your student number and the question(s) which is (are) solved.
- Solutions have to be uploaded to Digital Exam in form of a single pdf file consisting of at most ten digital pictures of at most ten handwritten A4 pages. Other submission formats will not be considered.
- The submitted digital pictures should be readable, i.e., of sufficient quality (high resolution, enough light, not blurry, etc.).
- Allowed aids are your own notes (made entirely by yourself or as an active participant of a group), lecture slides and exercise sheets, the books of Hans Hüttel and Michael Sipser used during the course. Anything else is rendered illegal, including, in particular, Googling or asking other persons for help.
- In case of emergencies: Students can contact the instructor during the exam by approaching the study secretary, as outlined in the guidelines for online exams. Keep an eye on your student mail for potential announcements during the exam.

Terminology applied in the exam:

- *Provide*: Give something without arguing why it is correct.
- *Prove*: Give a formal proof for the correctness of something.
- *Motivate*: Give an informal argument for the correctness or choice of something.

Last but not least, good luck!

Exercise 1.
Consider the following NFA:

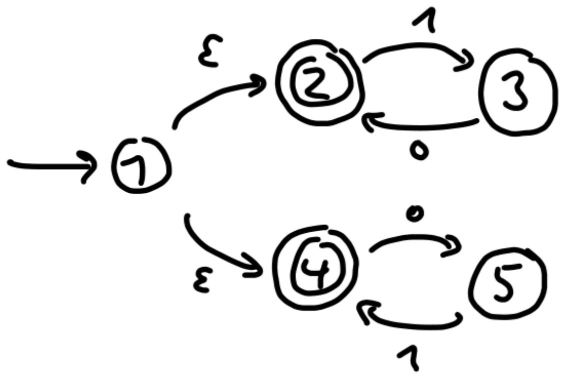


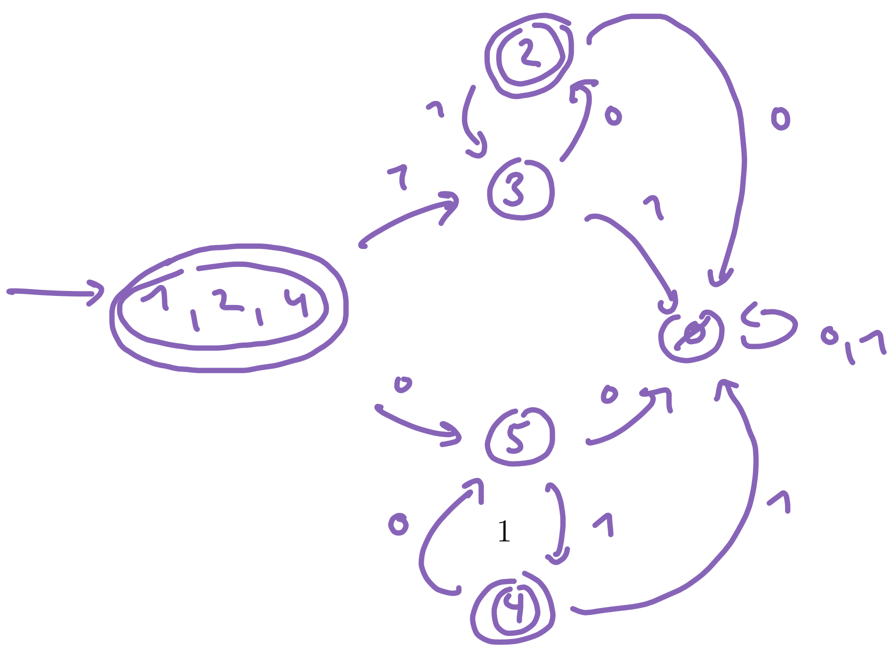
Figure 1: NFA M.

1) Provide two words that are accepted by M and two words that are rejected by M.
2) Provide a regular expression that describes the same language as M.
3) Convert the NFA M into a DFA that recognizes the same language by using the construction from the course (ad-hoc solutions will be not considered). DFA states that are not reachable from the initial state of the DFA can be omitted.

1) acc.: ε, 10, 01   rej.: 0, 1, 111

2) $(10)^* \cup (01)^*$

3)



Exercise 2.
Consider the context-free grammar G with variables $V = \{S\}$, terminals $\Sigma = \{a, b\}$, start variable S and rules

$$S ::= aSS \mid b$$

1) Provide the derivations of three words generated by G.

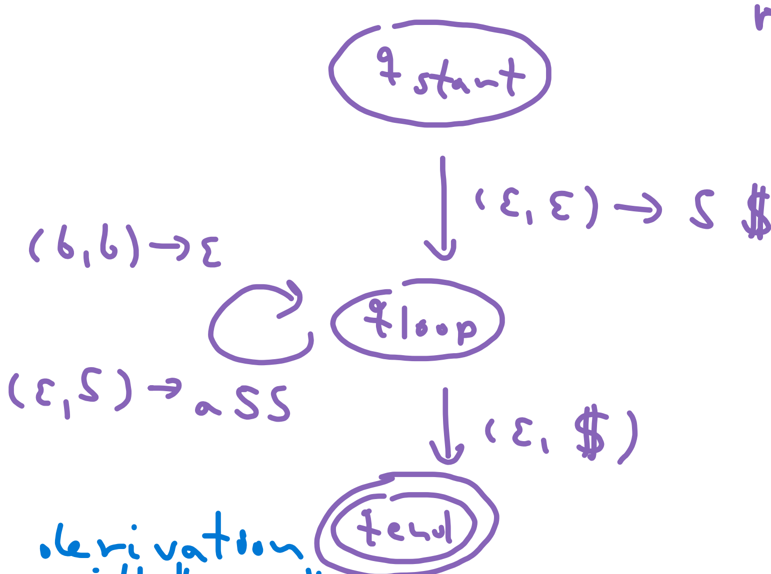
2) Describe the language generated by G, i.e., what is $\mathcal{L}(G)$? Motivate your answer.

3) Convert G into an equivalent PDA using the construction from the course (ad-hoc solutions will be not considered).

1) $S \to b$, $S \to aSS \to abS \to abb$,
$S \to aSS \to abS \to abaSS \to ababS \to ababb$

2) $w = b_1 b_2 \dots b_n \in \mathcal{L}(G)$ if
i) $b_n = b$ and
ii) $|b_1 b_2 \dots b_n|_b = |b_1 b_2 \dots b_n|_a + 1$ and
iii) $|b_1 b_2 \dots b_i|_b \leq |b_1 b_2 \dots b_i|_a$ for all $i \leq n-1$.

3)



Notation: $|w|_b$ denotes the number of $b \in \Sigma$ in $w \in \Sigma^*$.

i) any derivation ends with "$\dots \to b$"
iii) each "$\dots \to aSS$" puts an a before some b
ii) follows from i) & iii).

Exercise 3.
Using the pumping lemma for context-free languages, prove that the following language is not context-free.

$$L' = \{w \in \{a, b, c\}^* \mid w = a^i b^j c^k \text{ with } 0 < i < j < k\}.$$

Assume that L' is context-free. Then, by the PL, there is a $p \geq 1$ s.t. for $w = a^p b^{p+1} c^{p+2} \in L'$ there exist u, v, x, y, z satisfying $w = uvxyz$ and

i) $uv^ixy^iz \in L'$ for all $i \geq 0$

ii) $|vy| > 0$

iii) $|vxy| \leq p$

By ii) - iii), vy has to satisfy one of the cases below:

- $|vy|_a > 0 \wedge |vy|_c = 0$: With $w^{(i)} = uv^ixy^iz$, we obtain $|w^{(i)}|_a > |w^{(i)}|_c$ for some $i > 1$, hence $w^{(i)} \notin L'$ ↯
- $|vy|_a = 0 \wedge |vy|_b = 0 \wedge |vy|_c > 0$: Then $|w^{(0)}|_b \geq |w^{(0)}|_c$, hence $w^{(0)} \notin L'$ ↯
- $|vy|_a = 0 \wedge |vy|_b > 0 \wedge |vy|_c = 0$: Then $|w^{(i)}|_b > |w^{(i)}|_c$ for some $i > 1$, so $w^{(i)} \notin L'$ ↯

- $|vy|_a = 0 \wedge |vy|_b > 0 \wedge |vy|_c > 0$: Then $w^{(2)} \notin L'$ since some c's are followed by b's ↯

Exercise 4.

Consider the language $L'' = \{\underline{0}, \underline{1}\}^+$ of binary strings. With this, we consider the big step semantics $\rightarrow \subseteq L'' \times \mathbb{N}$ given by

$$[r_0]\ \frac{}{\underline{0} \rightarrow 0}, \qquad [r_1]\ \frac{}{\underline{1} \rightarrow 1}, \qquad [r_2]\ \frac{w \rightarrow k'}{\underline{0}w \rightarrow k}\ k = k', \qquad [r_3]\ \frac{w \rightarrow k'}{\underline{1}w \rightarrow k}\ k = 2^{|w|} + k'$$

As usual, $|w|$ denotes the length of w, while σw stands for a concatenation of $\sigma \in \{\underline{0}, \underline{1}\}$ and $w \in L''$. Essentially, $\rightarrow$ assigns each binary string its decimal value (i.e., the value represented by the string with 2 as the base of the numeral system).

1) Using the big step semantics, prove $\underline{10} \rightarrow 2$ and $\underline{110} \rightarrow 6$.

2) Consider the language of ternary strings $L''' = \{\underline{0}, \underline{1}, \underline{2}\}^+$. Provide the big step semantics $\rightarrow_3 \subseteq L''' \times \mathbb{N}$ that maps each ternary string to its decimal value (i.e., the value represented by the string with 3 as the base of the numeral system).

   [Hint: You may wish to double check your semantics by showing $\underline{12} \rightarrow_3 5$ and $\underline{212} \rightarrow_3 23$. This is only a suggestion and does not contribute to the score.]

1)

$$r_3\ \dfrac{r_0\ \dfrac{}{\underline{0} \rightarrow 0}}{\underline{10} \rightarrow 2}\ 2 = 2^{|\underline{0}|} + 0 \qquad r_3\ \dfrac{\dfrac{\dots}{\underline{10} \rightarrow 2}}{\underline{110} \rightarrow 6}\ 6 = 2^{|\underline{10}|} + 2$$

2)

$$t_0\ \frac{}{\underline{0} \rightarrow 0} \qquad t_1\ \frac{}{\underline{1} \rightarrow 1} \qquad t_2\ \frac{}{\underline{2} \rightarrow 2}$$

$$t_3\ \frac{w \rightarrow k'}{\underline{0}w \rightarrow k}\ k = k' \qquad t_4\ \frac{w \rightarrow k'}{\underline{1}w \rightarrow k}\ k = 3^{|w|} + k'$$

$$t_5\ \frac{w \rightarrow k'}{\underline{2}w \rightarrow k}\ k = 2 \cdot 3^{|w|} + k'$$

Exercise 5.

Consider the following statement in **Bip**.

```
begin
   var x:=8;
   var y:=2;
   proc p is x:=x*y;
   proc q is call p;
   begin
       var x:=3;
       proc p is x:=x+y;
       call q;
       y:=x
   end
end
```

1) What is the value of y after the statement is executed assuming fully static scope rules for both procedures and variables? Motivate your answer.

2) What is the value of y after the statement is executed assuming static scope rules for procedures and dynamic scope rules for variables? Motivate your answer.

3) What is the value of y after the statement is executed assuming fully dynamic scope rules for both variables and procedures? Motivate your answer.

1) 3 + explanation

2) 6 + explanation

3) 5 + explanation