

Syntax and Semantics:

Exercise Session 8

Exercise 1.

Consider the following three languages over the alphabet $\Sigma = \{a, b, c\}$

$$L_1 = \{a^i b^j c^k \mid i, j, k \geq 0, i = j = k\}$$

$$L_2 = \{a^i b^j c^k \mid i, j, k \geq 0, i = j\}$$

$$L_3 = \{a^i b^j c^k \mid i, j, k \geq 0, j = k\}$$

1. Find out which are the two context-free languages and prove that they are actually so.
2. Prove that one language is non context-free.
3. Conclude, using the results of the exercise, that the class of context-free languages is *not* closed under \cap .

Exercise 2.

Consider the language

$$L_4 = \{u\#w \mid u, w \in \{0, 1\}^*, u \text{ is a prefix of } w\}$$

1. Find two examples of strings which belong to L_4 and other two strings that are *not* elements of L_4 .
2. Is L_4 context-free? Justify your answer by means of a proof.

Exercise 3.

Here is a wrong attempt to show that a language is not context-free.

Consider the language

$$L_5 = \{w \mid \text{there exists } w_1 \in \{a, b\}^* \text{ such that } w = w_1 w_1\}.$$

We use the pumping lemma to show that L_5 is not context-free. Choose $s = aabbaabb$. So we can choose $u = aa$, $v = b$, $x = baa$, $y = bb$ and $z = \varepsilon$. But then we have that $uv^2xy^2z \notin L_5$.

1. Explain the main reasons why the above attempt of proof is wrong.
2. Prove that L_5 is not context-free. (It's a good idea not to try to fix the wrong proof!).

Exercise 4.

Here is an attempt to write a strategy for using the Pumping Lemma for context-free languages to prove that a language is *not* context-free. Underline places where the strategy does something wrong or where it is used a wrong terminology or notation, and then type the correct proposed strategy.

1. Let L be a grammar.
2. Set the pump length $p = 2$ and select a string $s \in L$ so that $|s| = 2$.
3. Select a single split of s of the form $uvxyz$ and show that this split does not comply the conditions 1–3 of the Pumping Lemma, i.e., that we do not have

(a) $uv^i xy^i$ for $i \geq 0$

(b) $vx > 0$

(c) $|vxy| > p$

4. Therefore we can conclude that L is not context-free.

Exercise 5.

Here is a version of the Pumping Lemma for context-free languages. Unfortunately, there are several errors in the formulation. Underline each error you may find. Then write the correct statement of the Pumping Lemma.

A language L is context-free if and only if there exists a $p > 0$ such that there exists $s \in L$ where $|s| > p$ such that for all divisions of s of the form $uvwxyz$ it holds that

1. $uv^i xy^i z$
2. $|vxy| \geq p$
3. $|vx| = 0$