

Syntax and Semantics

Written Exam, June the 7th 2022, 10:00-13:00

Please read the following before solving the exercises.

- This exam contains 5 exercises. Each exercise is compulsory and has an equal contribution to the final grade. The solution has to be composed in English. If you believe that the assignment wording is ambiguous or erroneous, then write down what additional assumption you are using and outline your reasons.
- Solutions have to be uploaded to Digital Exam in form of a single pdf file. Other submission formats will not be considered.
- The only allowed aid is an A4 piece of paper, i.e., two A4 pages. Anything else is rendered illegal, including, in particular, further notes, googling or asking other persons for help.
- In case of emergencies: Students can contact the instructor during the exam by approaching the study secretary.

Terminology applied in the exam:

- *Provide*: Give something without arguing why it is correct.
- *Prove*: Give a formal proof for the correctness of something.
- *Motivate*: Give an informal argument for the correctness or choice of something.

Last but not least, good luck!

PS: In your solution, state clearly which exercise is being solved.

Exercise 1.

1) Consider the following NFAs over the alphabet $\Sigma = \{a, b, c\}$.

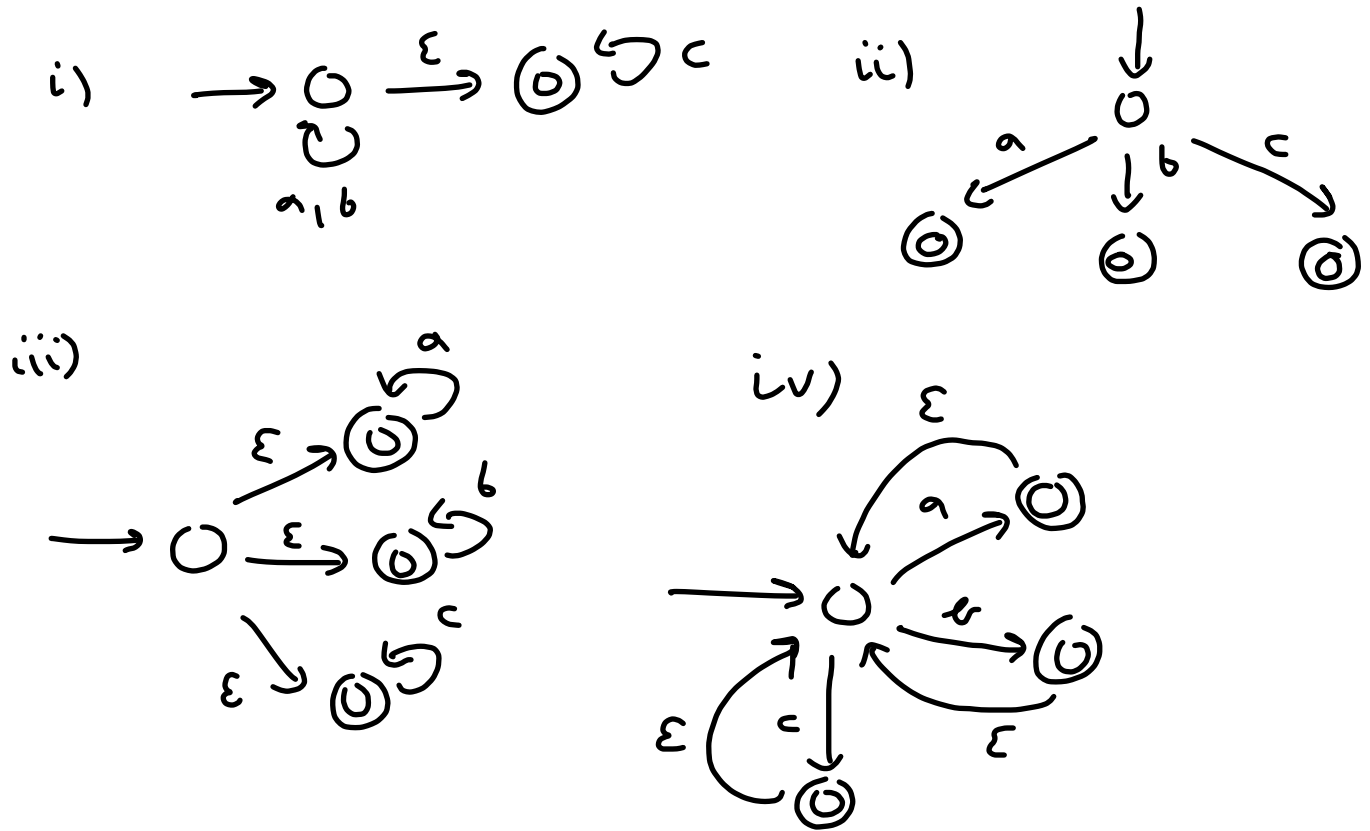


Figure 1: NFAs.

Associate each NFA to a regular expression that recognizes the same language.

A: $a \cup b \cup c$ ____ **B:** Σ^+ ____ **C:** $a^* \cup b^* \cup c^*$ ____ **D:** $(a \cup b)^* c^*$ ____

2) Answer ~~any~~ the following statements. Not answering is equivalent to giving a false answer.

2.1) The union of two regular languages is regular. Yes: ☐ No: ☐

2.2) The union of two context-free languages is context-free. Yes: ☐ No: ☐

2.3) Assuming that language L is recognized by the NFA $(Q, \Sigma, \delta, q_0, F)$, a possible pumping length for language L would be $p =$ ____

Exercise 2.

Consider the following PDA over the alphabet $\Sigma = \{a, b\}$, and stack alphabet

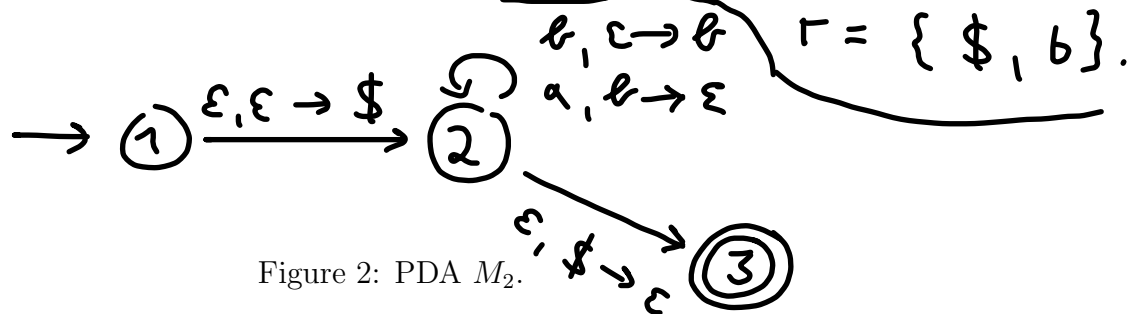


Figure 2: PDA M_2 .

- 1) Describe the language accepted by PDA M_2 . Motivate your answer.
- 2) Consider the CFG with variables $\{S\}$, terminals $\{a, b\}$, start variable S and rules

$$S ::= SS \mid bSa \mid bS \mid Sb \mid \varepsilon$$

Do the CFG and the PDA describe the same language? Motivate your answer.

Exercise 3.

Recall that the pumping lemma ensures that for any regular language L , there exists a pumping length $p \geq 1$ such that for every $w \in L$, there exists a decomposition $w = xyz$ such that:

- (i) $xy^iz \in L$ for any $i \geq 0$ and;
- (ii) $|y| > 0$ and;
- (iii) $|xy| \leq p$.

Using the pumping lemma for regular languages, prove that the following language is not regular.

$$L = \{w \in a^+ \mid w = a^{n^3} \text{ with } n \geq 1\} = \{a, a^8, a^{27}, a^{64}, \dots\}.$$

Exercise 4.

Consider the language $L' = \{a, b\}^+$ and the big step transitions $\rightarrow \subseteq L' \times \mathbb{N}$ given by

$$[r_1] \frac{}{a \rightarrow 1}, \quad [r_2] \frac{}{b \rightarrow 2}, \quad [r_3] \frac{w \rightarrow k}{aw \rightarrow k'} \quad k' = k + 1, \quad [r_4] \frac{w \rightarrow k}{bw \rightarrow k'} \quad k' = k + 2.$$

- 1) Using the big step semantics, prove $ab \rightarrow 3$ and $aab \rightarrow 4$.
- 2) Using the big step semantics and induction over the length of the word w , prove that $w \rightarrow v$ such that $v = |w|_a + 2|w|_b$. Here, $|w|_\sigma$ denotes the number of symbols $\sigma \in \{a, b\}$ in word w , e.g., $|aab|_a = 2$ and $|aab|_b = 1$.

Exercise 5.

Consider the following statement in **Bip**.

```
01 begin
02   var x:=-1;
03   var y:=2;
04   proc p is x:=x*y;
05   proc q is call p;
06   begin
07     var x:=3;
08     proc p is x:=x-y;
09     call q;
10     y:=x
11   end
12 end
```

- 1) What is the value of y after the statement is executed assuming fully static scope rules for both procedures and variables? ~~2~~
- 2) What is the value of y after the statement is executed assuming fully dynamic scope rules for both variables and procedures? ~~2~~
- 3) What is the value of y after the statement is executed assuming static scope rules for procedures and dynamic scope rules for variables? ~~2~~