STAT2006 Basic Concepts in Statistics and Probability II

Tutorial 6 – Midterm Revision: Point Estimation Exercises

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Abstract

It is an extension of Tutorial 5: Point Estimation. More exercises are provided for students to practice. Note that the concept of method of moments estimator is not covered in tutorials but exercises are provided here. Some are extracted from textbooks "Statistical Inference" (Casella and Berger) used in STAT4003.

1 Exercises

Exercise 1. Let $X_1, \ldots, X_n$ be a random sample from the pdf

$$f(x;\theta) = \theta x^{-2}, \ \ 0 < \theta \leq x < \infty.$$

Find the maximum likelihood estimator (MLE) of θ.[1]

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[1] "Statistical Inference" 2nd ed. (Casella and Berger) Ch7 p.355 Q7.6

Exercise 2. [15-16 Final Q2(a), 6%]
Assume that a random sample of $X_1, \ldots, X_n$ is following a common distribution

$$P(X_i \leq x|\alpha, \beta) = \begin{cases} 0, & \text{if } x < 0; \\ (\frac{x}{\alpha})^\beta, & \text{if } 0 \leq x \leq \alpha; \\ 1, & \text{if } x > \alpha; \end{cases}$$

where the parameters α and β are positive. Assume α and β are both unknown, find the MLEs of α and β.

Exercise 3. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a uniform$[\theta, 2\theta]$ distribution, where $\theta > 0$.[2]

(a) Find the method of moments estimator, $\tilde{\theta}$, and find a constant c such that $E(c\tilde{\theta}) = \theta$.

(b) Find the MLE, $\hat{\theta}$, and find a constant k such that $E(k\hat{\theta}) = \theta$.

[2] "Statistical Inference" 2nd ed. (Casella and Berger) Ch7 p.364 Q7.46

(Optional) Exercise 4. [Related to Simple Linear Regression]
Suppose that the random variables $Y_1, \ldots, Y_n$ satisfy

$$Y_i = \beta x_i + \epsilon_i, \quad i = 1, \ldots, n,$$

where $x_1, \ldots, x_n$ are fixed constants, and $\epsilon_1, \ldots, \epsilon_n$ are iid $N(0, \sigma^2)$, σ^2 unknown. Find the MLE of β, and show that it is an unbiased estimator of β. Also, find the distribution of the MLE of β.[3]

[3] "Statistical Inference" 2nd ed. (Casella and Berger) Ch7 p.358 Q7.19