periodic signals

$$x(n) = x(n-\tau)$$

for some τ

$$s(n) = \sum_{l=1}^{L} h_l(n) = \sum_{l=1}^{L} A_l \cos(l\omega_0 n + \phi_l)$$

ω_0
Ł
A_l
ϕ_l

$$\begin{aligned} s(n) &= \sum_{l=1}^{L} [a_l \cos(l\omega_0 n) - b_l \sin(l\omega_0 n)] \\ &= \sum_{l=1}^{L} [\cos(l\omega_0 n) - \sin(l\omega_0 n)] \begin{bmatrix} a_l \\ b_l \end{bmatrix} \end{aligned}$$

where $a_l = A_l \cos(\phi_l)$ and $b_l = A_l \sin(\phi_l)$

$$\omega_0 = 2\pi/\tau$$

comb filter

$$e(n) = x(n) - ax(n-\tau)$$

a

$$E(\omega) = X(\omega) - aX(\omega)\mathrm{e}^{-j\omega\tau} = X(\omega)(1 - a\mathrm{e}^{-j\omega\tau})$$

$$J(a,\tau) = \frac{1}{N-\tau} \sum_{n=\tau}^{N-1} |e(n)|^2 = \frac{1}{n-\tau} \sum_{n=\tau}^{N-1} |x(n) - ax(n-\tau)|^2$$

$$E(\omega) = X(\omega) - X(\omega)\mathrm{e}^{-j\omega\tau} = X(\omega)(1 - \mathrm{e}^{-j\omega\tau})$$

$$J(\tau) = \frac{1}{N-\tau} \sum_{n=\tau}^{N-1} |e(n)|^2 = \frac{1}{n-\tau} \sum_{n=\tau}^{N-1} |x(n) - x(n-\tau)|^2$$

$$\hat{\tau} = \underset{\tau \in [\tau_{MIN}, \tau_{MAX}]}{\operatorname{argmin}} J(\tau)$$

autocorrelation

$$\begin{aligned} G(a,\tau) = \mathbb{E}[|e(n)|^2] = \mathbb{E}[|x(n) - ax(n-\tau)|^2] \\ = \mathbb{E}[|x(n)|^2] + a^2\mathbb{E}[|x(n-\tau)|^2] - 2a\mathbb{E}[x(n)x(n-\tau] \\ = \sigma_x^2 + a^2\sigma_x^2 - 2ar_x(\tau) \end{aligned}$$

$$J(a,\tau) = \frac{1}{N-\tau}\sum_{n=\tau}^{N-1}|e(n)|^2 = \frac{1}{n-\tau}\sum_{n=\tau}^{N-1}x(n)ax(n-\tau)$$

$$J(\tau) = \frac{1}{N-\tau}\sum_{n=\tau}^{N-1}|e(n)|^2 = \frac{1}{n-\tau}\sum_{n=\tau}^{N-1}x(n)x(n-\tau)$$

$$R(\tau) = \frac{1}{N-\tau}\sum_{n=\tau}^{N-1}x_n x_{n-\tau}$$

$$\hat{\tau} = \underset{\tau\in[\tau_{MIN},\tau_{MAX}]}{\text{argmax}} R_x(\tau)$$

NLS

$$s(n,\theta) = \sum_{l=1}^{L} h_l(n) = \sum_{l=1}^{L} A_l\cos(l\omega_0 n + \phi_l)$$

$$\theta = [A_1 ... A_L \phi_1 ... \phi_L \omega_0]^T$$

solving

$$\hat{\phi} = \underset{\phi}{\text{argmin}}\, J(\phi)$$

the error is defined as

$$e(n) = x(n) - s(n,\theta)$$

and minimise

$$J_l(\theta) = \sum_{n=0}^{N-1}|e(n)|^2 = \sum_{n=0}^{N-1}|x(n) - s(n-\theta)|^2$$

approximate NLS and the estimator is the pitch maximising

$$G_l(\omega_0) = \sum_{l=1}^{L}|X(l\omega_0)|^2$$

$\{x(n)\}_{n=0}^{N-1}$

calculating a

$$\hat{a} = \frac{\sum_{n=\tau}^{N-1} x(n)x(n-\tau)}{\sum_{n=\tau}^{N-1} x(n-\tau)}$$

$$\hat{\tau} = \underset{\tau \in [\tau_{MIN}, \tau_{MAX}]}{\operatorname{argmin}} J(\tau) \frac{1}{n-\tau} \left[\sum_{n=\tau}^{N-1} x^2(n) - \frac{[\sum_{n=\tau}^{N-1} x(n)x(n-\tau)]^2}{\sum_{n=\tau}^{N-1} x(n-\tau)} \right]$$

$$\hat{\tau} = \underset{\tau \in [\tau_{MIN}, \tau_{MAX}]}{\operatorname{argmax}} J(\tau) \frac{1}{n-\tau} \left[\sum_{n=\tau}^{N-1} x^2(n) \frac{[\sum_{n=\tau}^{N-1} x(n)x(n-\tau)]^2}{\sum_{n=\tau}^{N-1} x(n-\tau)} \right]$$

$$J(a, \tau)$$