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The Long Transmission Line

1. Introduction

Power transmission lines run for up to 1500 km in certain parts of the world where hydro-electric power stations are located far from city load centres. Lines longer than about 200 km need special "compensating" equipment to control the voltage along the line, and to ensure the stability of the power transmission. This is because the voltage along an uncompensated line deviates from the ideal "flat voltage profile", depending on the load and the length of the line.

2. Objectives

a. To demonstrate the *Ferranti effect* -voltage rise along a lightly-loaded transmission line;
b. To demonstrate the flat voltage profile and linear phase-shift along a long transmission line loaded at its *natural load* or *"surge -impedance load"* (SIL); and
c. To demonstrate the effect of line length.

3. Theory

A transmission line can be represented approximately by a ladder network of LC branches as shown in Fig. 1[1]. The inductance and capacitance are *distributed* along the line, but a ladder network with a large number of lumped elements can provide a fairly accurate model of the actual line. Note that the *resistance* of the cable is considered negligible: the electrical properties are dominated by the series inductance and shunt capacitance[2].

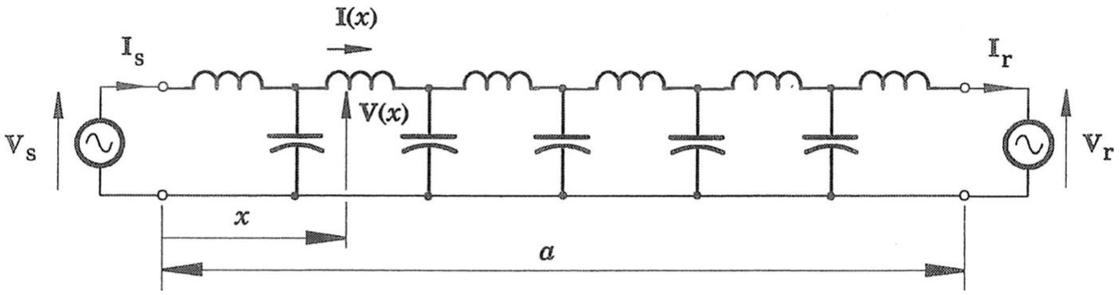

Fig. 1 Lumped-parameter model of long transmission line

The electrical properties of the line are dominated by two important parameters: the *surge impedance* and the *electrical length.*

[1] Throughout these instructions, boldface letters (**V**) refer to phasor values, and italic letters (*V*) to RMS values.

[2] The series inductance is just the inductance of the long cables. The "shunt" capacitance is the capacitance between lines and earth, or between lines of different phases. Fig. 1 represents only one phase of a three-phase transmission line, and all the shunt capacitances are combined together.

Surge impedance

The surge impedance Z_0 is given by $Z_0 = \sqrt{\frac{L}{C}}$ (1)

where L is the total line inductance [H] and C is the total line capacitance [F] (of course, in this case you could use ‘per-unit-length’ or ‘per-section’ values, due to cancellation in the formula). Even though L and C are basically reactive elements, Z_0 is a real number: in other words, it has the properties of *resistance.*

Electrical length

The actual length of a transmission line is measured in km, but the *electrical* length θ is measured in radians and is given by

$$\theta = \omega\sqrt{LC} \tag{2}$$

Note that here you must use the L and C values for the whole line. $\omega = 2\pi f$ is the radian frequency of the voltage and current (normally f = 50 or 60 Hz). A line for which $\theta = 2\pi$ is said to have a length of one *wavelength* at the operating frequency f, but such lines are impractical and θ rarely exceeds $\pi/6$ or 30°.

Key properties

When the receiving-end of a long line is open-circuited, the *voltage profile* along the line is given by

$$\mathbf{V}(x) = \mathbf{V_S}\frac{\cos\left(\theta\left(1 - x/a\right)\right)}{\cos\theta} \tag{3}$$

where x is the distance from the sending end, a is the actual line length, and $\mathbf{V_S}$ is the phasor voltage at the sending end. The voltage at the receiving end is given by setting $x = a$: thus

$$\mathbf{V_r} = \mathbf{V}(a) = \frac{\mathbf{V_S}}{\cos\theta} \tag{4}$$

This equation shows one problem with long lines: the receiving end voltage $\mathbf{V_r}$ exceeds the sending-end voltage $\mathbf{V_S}$ by the factor $1/\cos\theta$. For example if θ = 30°,
$\mathbf{V_r} = \mathbf{V_S} / \cos(30°) = 1.155\ \mathbf{V_S}$ —an excess voltage of 15% over the nominal or rated value. This is too far outside the acceptable range of voltage.
A line terminated in Z_0 has a flat voltage profile, i.e. $\mathbf{V}(x) = \mathbf{V_S} = \mathbf{V_r}$. The power corresponding to this load impedance is the *surge impedance load* (SIL) or *natural load.* If the load is greater than SIL, the voltage profile tends to sag. If it is less, the voltage profile tends to rise. [At SIL the reactive power requirements at the ends of the line are zero. Below SIL, the line generates excess reactive power at both ends, but above SIL, it absorbs reactive power at both ends.]

4. Experiments

The model transmission line has ten LC sections. In each section $L = 7.29$ mH and $C = 0.020$ µF. If the line is operated at 700 Hz, $\omega = 2\pi \times 700 = 4398\, rad/s$.

a. Use eqn. (1) to calculate the surge impedance Z_0 in ohms.
b. Use eqn. (2) to calculate the electrical length θ in degrees and radians. (Remember that L and C are for the whole line, not just one section).

Connect the model transmission line as shown in Fig. 2. Ensure that the function generator is producing a sine wave, and that the 50 Ω output is the one connected. Keep channel 1 of the oscilloscope connected to the sending-end voltage, and use channel 2 as a roving connection. The DVM / Multimeter can also be connected anywhere along the line.

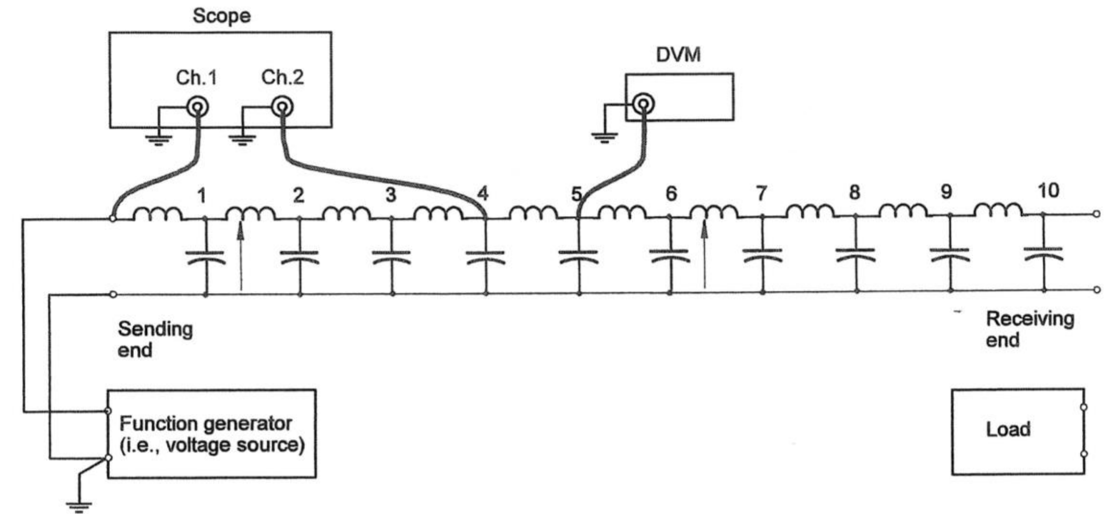

Fig. 2 Connection of model transmission line. The ground wires to the oscilloscope and the DVM are not shown in full.

c. Open-circuit test

Set the frequency to 700 Hz[3].4 Set the output voltage to about 10 V pk-pk. Set both oscilloscope channels to 2 V/division. Set the DVM to AC volts, 20 V range. The DVM measures the RMS voltage, i.e. $V_{pp}/2 \times 1/\sqrt{2} = 3.5\,V$ approximately.

Using the DVM, measure the voltage V at each of the 10 points along the line and calculate the ratio V/V_S for each point. Plot a graph showing the variation of *per-unit* voltage $v = V/V_S$ along the line, i.e. $v(x)$. Determine the ratio V/V_S and verify that it agrees with eqn. (4).

Use the oscilloscope to measure the phase angle of the voltage relative to $\mathbf{V}_S$ at the mid-point and at the receiving end. Use these phase angles together with the corresponding voltage values to draw a phasor diagram showing the relationship between $\mathbf{V}_S$, $\mathbf{V}_r$ and the mid-point voltage $\mathbf{V}_m$

d. Surge-impedance load test

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[3] Because this is a scale model, you can think of 700 Hz as the basic power frequency, i.e. 50 or 60 Hz.

Connect a resistive load equal to the surge impedance load Z_0 calculated at (a) above. Re-adjust the sending end voltage so that it has the same value as in experiment (c), or if you can't reach that voltage, as near to it as possible. Repeat all the measurements of test (c), including the phase angles of the voltages at the mid-point and the receiving end, relative to the sending end. Verify that the voltage profile is flat, and try to explain any deviations from true flatness. Compare the phase angle between $\mathbf{V_s}$ and $\mathbf{V_r}$ with the electrical length of the line, and comment on the result. Also comment on the phase angle between $\mathbf{V_s}$ and $\mathbf{V_m}$ and the phase angle between $\mathbf{V_r}$ and $\mathbf{V_m}$

e. Line loaded with less than the surge-impedance load

Connect a resistive load of resistance equal to half the surge impedance load Z_0 calculated in (a) above. Re-adjust the sending-end voltage so that it has the same value as in experiment (c), or if you can't reach that voltage, as near to it as possible. Repeat all the measurements of test (c), including the phase angles of the voltages at the mid-point and the receiving end, relative to the sending end. Comment on the voltage profile and the new values of the phase angles between $\mathbf{V_s}$ and $\mathbf{V_r}$, between $\mathbf{V_s}$ and $\mathbf{V_m}$ and between $\mathbf{V_r}$ and $\mathbf{V_m}$. How would you restore the receiving end voltage $\mathbf{V_r}$ to be equal to $\mathbf{V_s}$ without changing the real power transmitted to the load?[4]

f. Open-circuit test on double-length line

The line length can be doubled by connecting two model transmission lines in series. Alternatively, according to eqn. (2), we can simulate the same effect by doubling the frequency to 1.4 kHz. Do this, and repeat experiment (c). Comment on the differences between the single-length line and the double length line. How would you restore the receiving end voltage $\mathbf{V_r}$ to be equal to $\mathbf{V_s}$?

[4] Hint : connect a reactive impedance in parallel with the load, i.e. an inductor or a capacitor. Which should it be? Can you calculate its value? (See term 2 lecture notes)