# MPP-E1180 Lecture 8: Statistical Modeling with R

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# Objectives for the week

- Assignment 3
- Review
- Introduction to the general syntax for statistical modelling in R.
- Specific examples using:
  - Normal Linear regression
  - Logistic regression

# Assignment 3

**Purpose**: Gather, clean, and analyse data

**Deadline**: 14 November 2015

You will submit a GitHub repo that:

- Gathers web-based data from at least **two sources**. Cleans and merges the data so that it is ready for statistical analyses.
- Conducts basic descriptive and inferential statistics with the data to address a relevant research question.
- Briefly describes the results including with dynamically generated tables and figures.
- Has a write up of 1,500 words maximum that describes the data gathering and analysis and uses literate programming.

# Assignment 3

This is ideally a **good first run** at the data gathering and analysis for your final project.

# Review

- What is web scraping? What are some of tools R has for web scraping?
- What are regular expressions (give at least two examples)? Why are they useful?
- What dplyr functions can you use to create a new variable in a data frame by running a command on values from groups in that data frame?

# Statistical Modelling in R

**Caveat**: We are definitely not going to cover anywhere near R's full capabilities for statistical modeling.

We are also not going to cover all of the modeling concerns/diagnostics you need to consider when using a given model.

What are we going to do?

- Discuss the basic syntax and capabilities in R for estimating normal linear and logistic regressions.
- Basic model checking.
- Discuss basic ways of interpreting results.

# The basic model

Most statistical models you will likely estimate are from a general class that has **two parts**:

**Stocastic Component** that generates the dependent variable $Y_i$ as a random draw from the probability density function:

$$Y_i \sim f(\theta_i, \alpha)$$

- $\theta_i$: parameter vector of the part of the function that varies between observations.
- $\alpha$: matrix of non-varying parameters.

Sometimes referred to as the 'error structure'.

# The basic model

The **systematic component** indicating how $\theta_i$ varies across observations depending on values of the explanatory variables and (often) some constant:

$$\theta_i = g(X_i,\ \beta)$$

- $X_i$: a $1 \text{ x } k$ vector of explanatory variables.
- $\beta$: a $1 \text{ x } k$ vector of parameters (i.e. coefficients).
- $g(.,.)$: the "link function", specifying how the explanatory variables and parameters are translated into $\theta_i$.

# Today

Today we will cover two variations of this general model:

- linear-normal regression (i.e. ordinary least squares)
- logit model

# Linear-normal regression

For continuous dependent variables assume that $Y_i$ is from the **normal distribution** $(N(.,.))$.

Set the main parameter vector $\theta_i$ to the scalar mean of: $\theta_i = E(y_i) = \mu_i$.

Assume the ancillary parameter matrix is the scalar homoskedastic variance: $\alpha = V(Y_i) = \sigma^2$.

- **Homoskedastic variance**: variance does not depend on the value of $x$. The standard deviation of the error terms is constant across values of $x$.

Set the systematic component to the linear form: $g(X_i, \beta) = X_i\beta = \beta_0 + X_{i1}\beta_1 + \ldots$.

# Linear-normal regression

So:

$$Y_i \sim N(\mu_i, \, \sigma^2), \quad \mu_i = X_i \beta$$

# Logit regression

For binary data (e.g. 0, 1) we can assume that the stochastic component has a Bernoulli distribution.

The main parameter is $\pi_i = \Pr(Y_i = 1)$.

The systematic component is set to a logistic form: $\pi_i = \frac{1}{1+e^{-X_i\beta}}$.

So:

$$Y_i \sim \text{Bernoulli}(\pi_i), \quad \pi_i = \frac{1}{1 + e^{-X_i\beta}}$$

# R syntax

The general syntax for estimating statistical models in are is:

```
response variable ~ explanatory variable(s)
```

Where ~ reads 'is modelled as a function of'.

# Model functions

We use model functions to specify the specific model structure.

Basic model functions include:

- `lm`: fits a linear model where $Y$ is assumed to be normally distributed and with homoskedastic variance.
- `glm`: allows the fitting of many Generalised Linear Models. Lets you specify the $Y$'s distribution `family` and the `link` function.

# Example of `lm`

Example data *Prestige* (example based on `http://www.princeton.edu/~otorres/Regression101R.pdf`).

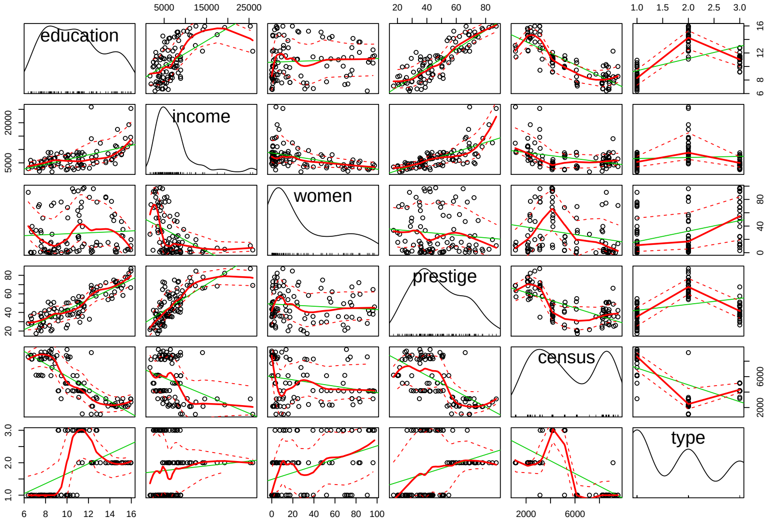
The observations are **occupations** and the dependent variable is a score of each occupations **prestige**.

# Example of `lm`

```
library(car)
data(Prestige)
```

# Examine correlation matrix

```
car::scatterplotMatrix(Prestige)
```



# Example of `lm`

Estimate simple model:

```
M1 <- lm(prestige ~ education, data = Prestige)
```

# Example of `lm`

```r
summary(M1)
```

```
##
## Call:
## lm(formula = prestige ~ education, data = Prestige)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -26.0397  -6.5228   0.6611   6.7430  18.1636
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -10.732      3.677  -2.919  0.00434 **
## education      5.361      0.332  16.148  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
##
## Residual standard error: 9.103 on 100 degrees of freedom
```

# Find confidence intervals of parameter estimates

Note: **Always prefer estimation intervals** over point estimates.

Deal with your **uncertainty**!

```r
confint(M1)
```

```
##                  2.5 %    97.5 %
## (Intercept) -18.027220 -3.436744
## education     4.702223  6.019533
```

# Example of `lm`

Estimate model with categorical variable:

```
M2 <- lm(prestige ~ education + type,
         data = Prestige)
```

# Example of `lm`

```r
summary(M2)
```

```
##
## Call:
## lm(formula = prestige ~ education + type, data = Prestig
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -19.410  -5.508   1.360   5.694  17.171
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -2.6982     5.7361  -0.470   0.6392
## education     4.5728     0.6716   6.809 9.16e-10 ***
## typeprof      6.1424     4.2590   1.442   0.1526
## typewc       -5.4585     2.6907  -2.029   0.0453 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.
```

# Create categorical variable from continuous variable

Use the `cut` function to create a categorical (factor) variable from a continuous variable.

```
Prestige$income_cat <- cut(Prestige$income,
                   breaks = c(0, 4999, 9999, 14999, 30000),
                   labels = c('< 5,000', '< 10,000', '< 15,00
                              '>= 15,000'))
summary(Prestige$income_cat)
```

```
##    < 5,000  < 10,000  < 15,000 >= 15,000
##         38        51         9         4
```

Note: `cut` excludes the left value and includes the right value, e.g. [0, 4999].

# Example of lm

```
M3 <- lm(prestige ~ education + income_cat,
         data = Prestige)
confint(M3)
```

```
##                           2.5 %    97.5 %
## (Intercept)          -10.914031  3.182500
## education              3.350201  4.778444
## income_cat< 10,000     6.085375 13.030546
## income_cat< 15,000     9.584532 23.391854
## income_cat>= 15,000   12.040936 29.902733
```

# Example of `lm`

Estimate models with polynomial transformations:

```r
# Cubic polynomial transformation
M4  <- lm(prestige ~ education + poly(income, 2),
          data = Prestige)
confint(M4)
```

```
##                         2.5 %    97.5 %
## (Intercept)         -1.470988  13.33552
## education            3.132827   4.48515
## poly(income, 2)1    45.174019  81.39445
## poly(income, 2)2   -43.150740 -12.87994
```

# Example of `lm`

Estimate models with (natural) logarithmic transformations:

```r
# Cubic polynomial transformation
M5  <- lm(prestige ~ education + log(income),
          data = Prestige)
```

# Example of lm

```r
summary(M5)
```

```
##
## Call:
## lm(formula = prestige ~ education + log(income), data =
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -17.0346  -4.5657  -0.1857   4.0577  18.1270
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -95.1940    10.9979  -8.656 9.27e-14 ***
## education     4.0020     0.3115  12.846  < 2e-16 ***
## log(income)  11.4375     1.4371   7.959 2.94e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.
##
```

# Example of `lm`

Estimate model with interactions:

```
M6 <- lm(prestige ~ education * type,
         data = Prestige)
```

# Example of `lm`

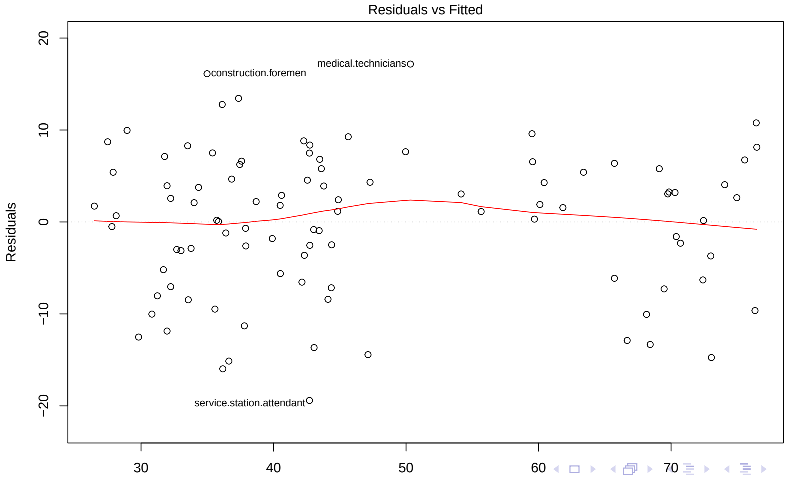
```r
summary(M6)
```

```
##
## Call:
## lm(formula = prestige ~ education * type, data = Prestig
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -19.7095  -5.3938   0.8125   5.3968  16.1411
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)          -4.2936     8.6470  -0.497    0.621
## education             4.7637     1.0247   4.649 1.11e-05
## typeprof             18.8637    16.8881   1.117    0.267
## typewc              -24.3833    21.7777  -1.120    0.266
## education:typeprof   -0.9808     1.4495  -0.677    0.500
## education:typewc      1.6709     2.0777   0.804    0.423
```

# Diagnose heteroscedasticity

Use `plot` on a model object to run visual diagnostics.

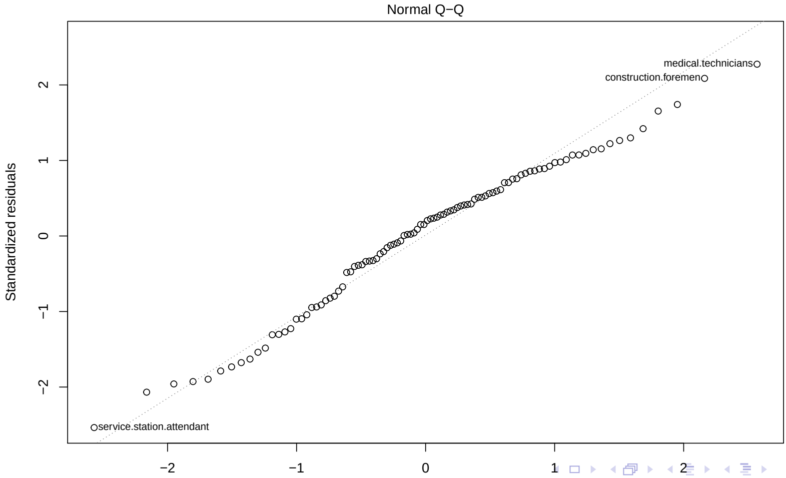
```
plot(M2, which = 1)
```



# Diagnose non-normality of errors

`plot` to see if a model's errors are normally distributed.

```
plot(M2, which = 2)
```



# Example of logistic regression with `glm`

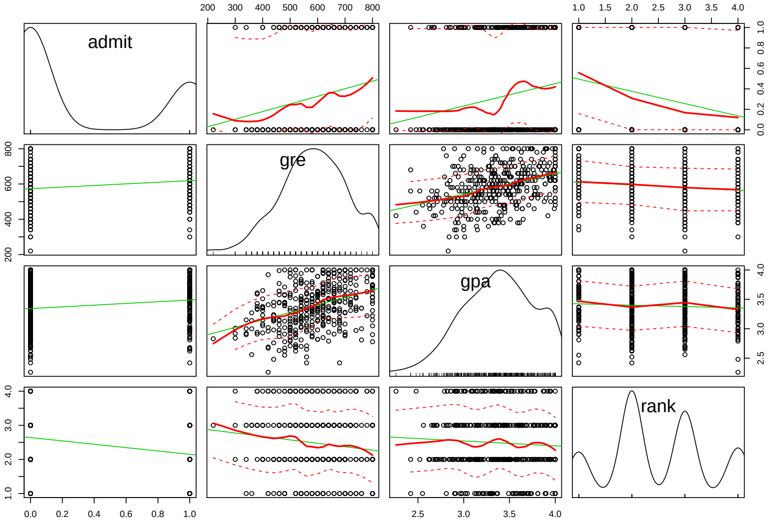
Example from UCLA IDRE.

Simulated data of admission to grad school.

```
# Load data
URL <- 'http://www.ats.ucla.edu/stat/data/binary.csv'
Admission <- read.csv(URL)
```

# Example of logistic regression with `glm`

```
car::scatterplotMatrix(Admission)
```



# Contingency table for school rank and admission

```
xtabs(~admit + rank, data = Admission)
```

```
##      rank
## admit  1  2  3  4
##     0 28 97 93 55
##     1 33 54 28 12
```

# Example of logistic regression with `glm`

```
Logit1 <- glm(admit ~ gre + gpa + as.factor(rank),
              data = Admission, family = 'binomial')
```

Note: Link function is assumed to be logit if `family = 'binomial'`.

# Example of logistic regression with `glm`

```r
confint(Logit1)
```

```
##                          2.5 %       97.5 %
## (Intercept)      -6.2716202334 -1.792547080
## gre               0.0001375921  0.004435874
## gpa               0.1602959439  1.464142727
## as.factor(rank)2 -1.3008888002 -0.056745722
## as.factor(rank)3 -2.0276713127 -0.670372346
## as.factor(rank)4 -2.4000265384 -0.753542605
```

# Interpreting logistic regression results

$\beta$'s in logistic regression are interpretable as **log odds**. These are weird.

If we exponentiate log odds we get **odds ratios**.

```
exp(cbind(OddsRatio = coef(Logit1), confint(Logit1)))
```

```
##                    OddsRatio       2.5 %    97.5 %
## (Intercept)        0.0185001 0.001889165 0.1665354
## gre                1.0022670 1.000137602 1.0044457
## gpa                2.2345448 1.173858216 4.3238349
## as.factor(rank)2 0.5089310 0.272289674 0.9448343
## as.factor(rank)3 0.2617923 0.131641717 0.5115181
## as.factor(rank)4 0.2119375 0.090715546 0.4706961
```

These are also weird.

# Interpreting logistic regression results

What we really want are **predicted probabilities**

**First** create a data frame of fitted values:

```
fitted <- with(Admission,
               data.frame(gre = mean(gre),
                          gpa = mean(gpa),
                          rank = factor(1:4)))
fitted
```

```
##     gre    gpa rank
## 1 587.7 3.3899    1
## 2 587.7 3.3899    2
## 3 587.7 3.3899    3
## 4 587.7 3.3899    4
```

# Interpreting logistic regression results

**Second** predict probability point estimates for each fitted value.

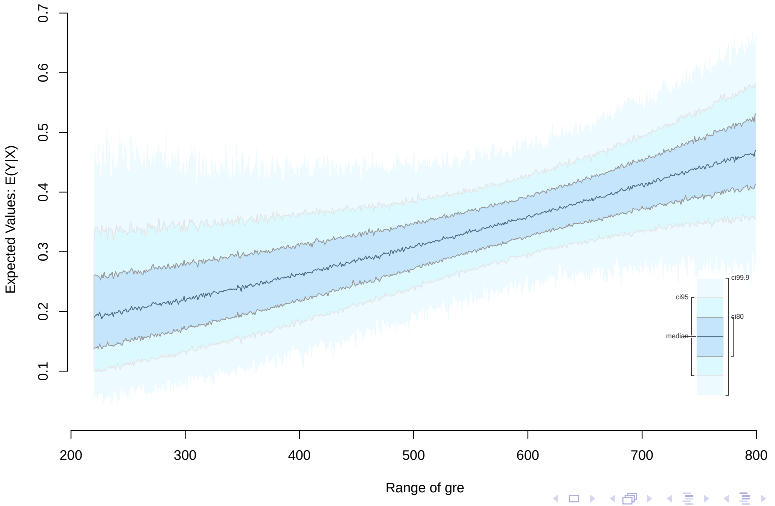
```
fitted$predicted <- predict(Logit1, newdata = fitted,
                            type = 'response')

fitted
```

```
##      gre    gpa rank predicted
## 1 587.7 3.3899    1 0.5166016
## 2 587.7 3.3899    2 0.3522846
## 3 587.7 3.3899    3 0.2186120
## 4 587.7 3.3899    4 0.1846684
```

# More interpretation

Next week we will explore other methods of interpreting results from regression models.



# Seminar: modeling

Begin working on the statistical models for **your project**.