F-B phase plane ideas

Cóilín Minto

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Abstract

The aim of this brief document is to discuss some ideas for the analysis of the relationship between fishing mortality and biomass. The overarching goal is to present graphical and analytical methods to assess whether fishing is the dominant driver of the biomass dynamics for the population. In the process, probabilistic methods are explored for how likely a population is to recover from a depleted state. Methods for exploratory data analysis include: timeseries plots borrowed from meteorology, phase-diagrams, and rose-diagrams. These are accompanied by confirmatory data analysis including Markov transition equations and state space methods for the analysis of movement. These are compared and critiqued.

1 Markov transition probabilities

Figure 1 shows a hypothetically simplified trajectory in the fishing mortality-biomass phase plane. The plane is divided up into four states $\{1, 2, 3, 4\}$ denoted by i. At a given timestep t there are four possible state transitions for time $t+1$. There are a total of 4×4 possible transitions, denoted by X_t. Let $N_{x_{t+1}|x_t}$ denote the number of times a given transition was observed e.g. $N_{2|1}$ is the number of times the transition from state 1 to state 2 was observed. The row totals denote the sum of the number of movements to that state, as in

		time t				
		1	2	3	4	
	1	$N_{1\|1}$	$N_{1\|2}$	.	.	$\sum_{i=1}^{4} N_{1\|x_t=i}$
time $t+1$	2	$N_{2\|1}$	$N_{2\|2}$			$\sum_{i=1}^{4} N_{2\|x_t=i}$
	3	.		.		$\sum_{i=1}^{4} N_{3\|x_t=i}$
	4	.			.	$\sum_{i=1}^{4} N_{4\|x_t=i}$

A Markov property holds when

$$P(X_{t+1} = x | X_t = x_t, \ldots, X_1 = x_1) = P(X_{t+1} = x | X_t = x_t) \qquad (1)$$

that is, the probability of X being in a given state is solely dependent on the previous time step (WORK ON THIS!).

$$P(X_{t+1} = x | X_t = x_t) = N_{x_{t+1}|x_t} / \sum_{i=1}^{4} N_{x_{t+1}|x_t=i} \qquad (2)$$

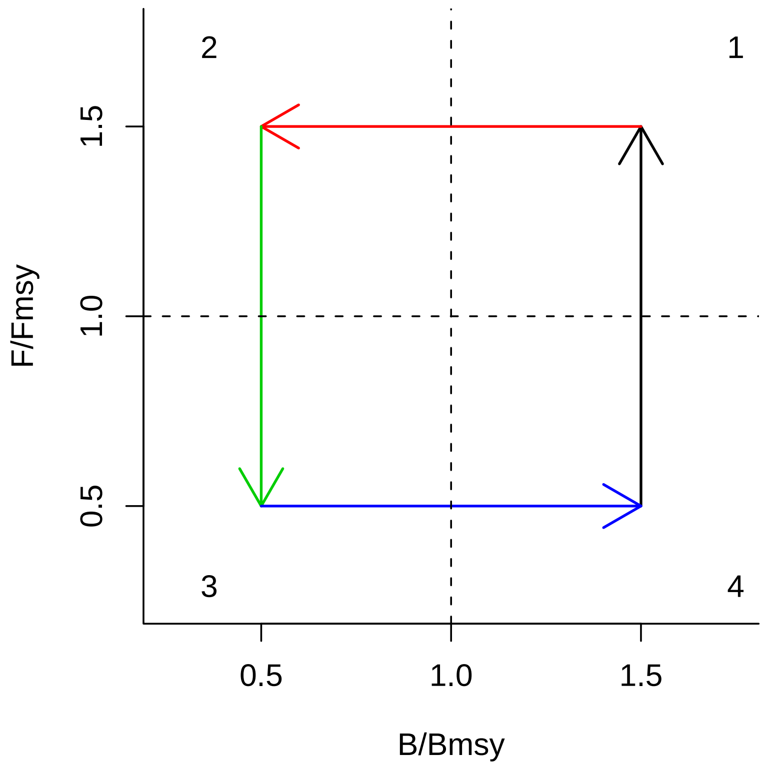

Figure 1: Hypothetical simplified phase-plane diagram