









Relation Paths



Textual Descriptions



Logical Rules





- KG embedding with facts alone
  - 1. Translational Distance Models
  - 2. Semantic Matching Models
- Incorporating additional information





### Translational Distance Models

Method	Ent. embedding	Rel. embedding	Scoring function $f_r(h, t)$	Constraints/Regularization
TransE [14]	$\mathbf{h},\mathbf{t}\in\mathbb{R}^d$	$\mathbf{r} \in \mathbb{R}^d$	$-\ \mathbf{h}+\mathbf{r}-\mathbf{t}\ _{1/2}$	$\ \mathbf{h}\ _2 = 1, \ \mathbf{t}\ _2 = 1$
TransH [15]	$\mathbf{h},\mathbf{t}\in\mathbb{R}^d$	$\mathbf{r},\mathbf{w}_r \in \mathbb{R}^d$	$-\ (\mathbf{h} - \mathbf{w}_r^{\top} \mathbf{h} \mathbf{w}_r) + \mathbf{r} - (\mathbf{t} - \mathbf{w}_r^{\top} \mathbf{t} \mathbf{w}_r)\ _2^2$	$\ \mathbf{h}\ _{2} \leq 1, \ \mathbf{t}\ _{2} \leq 1$ $ \mathbf{w}_{r}^{\top}\mathbf{r} /\ \mathbf{r}\ _{2} \leq \epsilon, \ \mathbf{w}_{r}\ _{2} = 1$
TransR [16]	$\mathbf{h},\mathbf{t}\in\mathbb{R}^d$	$\mathbf{r} \in \mathbb{R}^k, \mathbf{M}_r \in \mathbb{R}^{k  imes d}$	$-\ \mathbf{M}_r\mathbf{h}+\mathbf{r}-\mathbf{M}_r\mathbf{t}\ _2^2$	$\ \mathbf{h}\ _{2} \le 1, \ \mathbf{t}\ _{2} \le 1, \ \mathbf{r}\ _{2} \le 1$ $\ \mathbf{M}_{r}\mathbf{h}\ _{2} \le 1, \ \mathbf{M}_{r}\mathbf{t}\ _{2} \le 1$
TransD [50]	$\mathbf{h}, \mathbf{w}_h \in \mathbb{R}^d \ \mathbf{t}, \mathbf{w}_t \in \mathbb{R}^d$	$\mathbf{r},\mathbf{w}_r \in \mathbb{R}^k$	$-\ (\mathbf{w}_r\mathbf{w}_h^\top + \mathbf{I})\mathbf{h} + \mathbf{r} - (\mathbf{w}_r\mathbf{w}_t^\top + \mathbf{I})\mathbf{t}\ _2^2$	$\ \mathbf{h}\ _{2} \leq 1, \ \mathbf{t}\ _{2} \leq 1, \ \mathbf{r}\ _{2} \leq 1$ $\ (\mathbf{w}_{r}\mathbf{w}_{h}^{\top} + \mathbf{I})\mathbf{h}\ _{2} \leq 1$ $\ (\mathbf{w}_{r}\mathbf{w}_{t}^{\top} + \mathbf{I})\mathbf{t}\ _{2} \leq 1$
TranSparse [51]	$\mathbf{h},\mathbf{t}\in\mathbb{R}^d$	$\mathbf{r} \in \mathbb{R}^k, \mathbf{M}_r(\theta_r) \in \mathbb{R}^{k \times d}$ $\mathbf{M}_r^1(\theta_r^1), \mathbf{M}_r^2(\theta_r^2) \in \mathbb{R}^{k \times d}$	$-\ \mathbf{M}_r(\theta_r)\mathbf{h} + \mathbf{r} - \mathbf{M}_r(\theta_r)\mathbf{t}\ _{1/2}^2$ $-\ \mathbf{M}_r^1(\theta_r^1)\mathbf{h} + \mathbf{r} - \mathbf{M}_r^2(\theta_r^2)\mathbf{t}\ _{1/2}^2$	$\ \mathbf{h}\ _{2} \leq 1, \ \mathbf{t}\ _{2} \leq 1, \ \mathbf{r}\ _{2} \leq 1$ $\ \mathbf{M}_{r}(\theta_{r})\mathbf{h}\ _{2} \leq 1, \ \mathbf{M}_{r}(\theta_{r})\mathbf{t}\ _{2} \leq 1$ $\ \mathbf{M}_{r}^{1}(\theta_{r}^{1})\mathbf{h}\ _{2} \leq 1, \ \mathbf{M}_{r}^{2}(\theta_{r}^{2})\mathbf{t}\ _{2} \leq 1$
TransM [52]	$ig  \mathbf{h}, \mathbf{t} \in \mathbb{R}^d$	$\mathbf{r} \in \mathbb{R}^d$	$-\theta_r \ \mathbf{h} + \mathbf{r} - \mathbf{t}\ _{1/2}$	$\ \mathbf{h}\ _2 = 1, \ \mathbf{t}\ _2 = 1$



# Semantic Matching Models

Method	Ent. embedding	Rel. embedding	Scoring function $f_r(h, t)$	Constraints/Regularization
RESCAL [13]	$\mathbf{h},\mathbf{t}\in\mathbb{R}^d$	$\mathbf{M}_r \in \mathbb{R}^{d  imes d}$	$\mathbf{h}^{\top}\mathbf{M}_{r}\mathbf{t}$	$\ \mathbf{h}\ _{2} \leq 1, \ \mathbf{t}\ _{2} \leq 1, \ \mathbf{M}_{r}\ _{F} \leq 1$ $\mathbf{M}_{r} = \sum_{i} \pi_{r}^{i} \mathbf{u}_{i} \mathbf{v}_{i}^{\top}$ (required in [17])
TATEC [64]	$\mathbf{h},\mathbf{t}\in\mathbb{R}^d$	$\mathbf{r} \in \mathbb{R}^d, \mathbf{M}_r \in \mathbb{R}^{d  imes d}$	$\mathbf{h}^{\top}\mathbf{M}_{r}\mathbf{t} + \mathbf{h}^{\top}\mathbf{r} + \mathbf{t}^{\top}\mathbf{r} + \mathbf{h}^{\top}\mathbf{D}\mathbf{t}$	$\ \mathbf{h}\ _{2} \le 1, \ \mathbf{t}\ _{2} \le 1, \ \mathbf{r}\ _{2} \le 1$ $\ \mathbf{M}_{r}\ _{F} \le 1$
DistMult [65]	$\mathbf{h},\mathbf{t}\in\mathbb{R}^d$	$\mathbf{r} \in \mathbb{R}^d$	$\mathbf{h}^{\top} diag(\mathbf{r})\mathbf{t}$	$\ \mathbf{h}\ _2 = 1, \ \mathbf{t}\ _2 = 1, \ \mathbf{r}\ _2 \le 1$
HolE [62]	$\mathbf{h},\mathbf{t}\in\mathbb{R}^d$	$\mathbf{r} \in \mathbb{R}^d$	$\mathbf{r}^{\top}(\mathbf{h} \star \mathbf{t})$	$\ \mathbf{h}\ _{2} \le 1, \ \mathbf{t}\ _{2} \le 1, \ \mathbf{r}\ _{2} \le 1$
ComplEx [66]	$\mathbf{h},\mathbf{t}\in\mathbb{C}^d$	$\mathbf{r} \in \mathbb{C}^d$	$\operatorname{Re}\left(\mathbf{h}^{\top}\operatorname{diag}(\mathbf{r})\bar{\mathbf{t}}\right)$	$\ \mathbf{h}\ _{2} \le 1, \ \mathbf{t}\ _{2} \le 1, \ \mathbf{r}\ _{2} \le 1$
ANALOGY [68]	$\mathbf{h},\mathbf{t} \in \mathbb{R}^d$	$\mathbf{M}_r \in \mathbb{R}^{d  imes d}$	$\mathbf{h}^{T}\mathbf{M}_{r}\mathbf{t}$	$\ \mathbf{h}\ _{2} \leq 1, \ \mathbf{t}\ _{2} \leq 1, \ \mathbf{M}_{r}\ _{F} \leq 1$ $\mathbf{M}_{r}\mathbf{M}_{r}^{\top} = \mathbf{M}_{r}^{\top}\mathbf{M}_{r}$ $\mathbf{M}_{r}\mathbf{M}_{r'} = \mathbf{M}_{r'}\mathbf{M}_{r}$
SME [18]	$\mathbf{h},\mathbf{t}\in\mathbb{R}^d$	$\mathbf{r} \in \mathbb{R}^d$	$ \begin{aligned} & (\mathbf{M}_u^1\mathbf{h} + \mathbf{M}_u^2\mathbf{r} + \mathbf{b}_u)^\top (\mathbf{M}_v^1\mathbf{t} + \mathbf{M}_v^2\mathbf{r} + \mathbf{b}_v) \\ & ((\mathbf{M}_u^1\mathbf{h}) \circ (\mathbf{M}_u^2\mathbf{r}) + \mathbf{b}_u)^\top ((\mathbf{M}_v^1\mathbf{t}) \circ (\mathbf{M}_v^2\mathbf{r}) + \mathbf{b}_v) \end{aligned} $	$\ \mathbf{h}\ _2 = 1, \ \mathbf{t}\ _2 = 1$
NTN [19]	$\mathbf{h},\mathbf{t}\in\mathbb{R}^d$	$\mathbf{r}, \mathbf{b}_r \in \mathbb{R}^k, \underline{\mathbf{M}}_r \in \mathbb{R}^{d \times d \times k}$ $\mathbf{M}_r^1, \mathbf{M}_r^2 \in \mathbb{R}^{k \times d}$	$\mathbf{r}^{\top} \tanh(\mathbf{h}^{\top} \underline{\mathbf{M}}_r \mathbf{t} + \mathbf{M}_r^1 \mathbf{h} + \mathbf{M}_r^2 \mathbf{t} + \mathbf{b}_r)$	$\ \mathbf{h}\ _{2} \leq 1, \ \mathbf{t}\ _{2} \leq 1, \ \mathbf{r}\ _{2} \leq 1$ $\ \mathbf{b}_{r}\ _{2} \leq 1, \ \underline{\mathbf{M}}_{r}^{[:,:,i]}\ _{F} \leq 1$ $\ \mathbf{M}_{r}^{1}\ _{F} \leq 1, \ \mathbf{M}_{r}^{2}\ _{F} \leq 1$

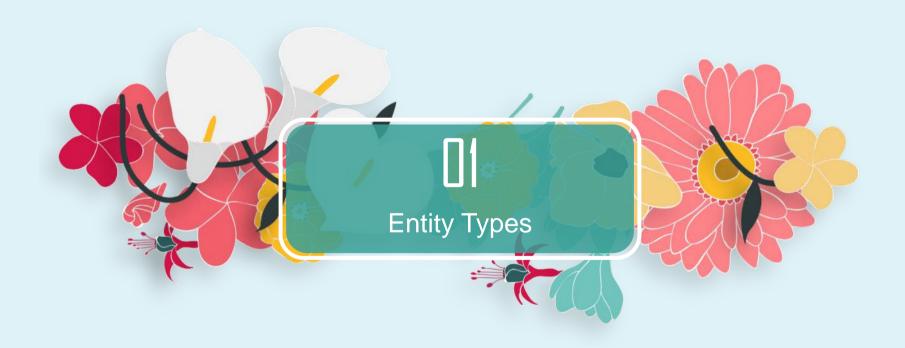


### **Model Comparison**

• First, models which represent entities and relations as vectors are more efficient.

- Second, models which represent relations as matrices or tensors usually have higher complexity in both space and time.
- Finally, models based on neural network architectures generally have higher complexity in time.







### **Entity Types**

- AlfredHitchcock has the type of Person
- *Psycho* the type of *CreativeWork*
- (Psycho, IsA, CreativeWork)





Entity Types ---- semantically smooth embedding (SSE)

**Definition:** Require entities of the same type to stay close to each other in the embedding space

• Laplacian eigenmaps: requires an entity to lie close to every other entity in the same category

$$\mathcal{R}_1 = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \|\mathbf{e}_i - \mathbf{e}_j\|_2^2 w_{ij}^1$$

• locally linear embedding: The latter represents an entity as a linear combination of its nearest neighbors, i.e., entities within the same category.

$$\mathcal{R}_2 = \sum_{i=1}^n \|\mathbf{e}_i - \sum_{e_j \in \mathbb{N}_{e_i}} w_{ij}^2 \mathbf{e}_j\|_2^2,$$





Entity Types ---- type-embodied knowledge representation learning (TKRL)

**Definition:** TKRL is a translational distance model with type-specific entity projections. Given a fact (h, r, t), it first projects h and t with type-specific projection matrices, and then models r as a translation between the two projected entities.

**Scoring Function:** 

$$f_r(h,t) = -\|\mathbf{M}_{rh}\mathbf{h} + \mathbf{r} - \mathbf{M}_{rt}\mathbf{t}\|_1,$$

$$\mathbf{M}_{rh} = \frac{\sum_{i=1}^{n_h} \alpha_i \mathbf{M}_{c_i}}{\sum_{i=1}^{n_h} \alpha_i}, \quad \alpha_i = \begin{cases} 1, \ c_i \in \mathbb{C}_{r_h}, \\ 0, \ c_i \notin \mathbb{C}_{r_h}, \end{cases}$$

addition:  $\mathbf{M}_{c_i} = \beta_1 \mathbf{M}_{c_i^{(1)}} + \cdots + \beta_\ell \mathbf{M}_{c_i^{(\ell)}};$ 

multiplication:  $\mathbf{M}_{c_i} = \mathbf{M}_{c_i^{(1)}} \circ \cdots \circ \mathbf{M}_{c_i^{(\ell)}}$ .







#### **Relation Paths**

**Definition:** A relation path is typically defined as a sequence of relations  $r1 \to \cdots \to r\ell$  through which two entities can be connected on the graph.

e.g.: AlfredHitchcock → BornIn → (Leytonstone) → LocatedIn → England

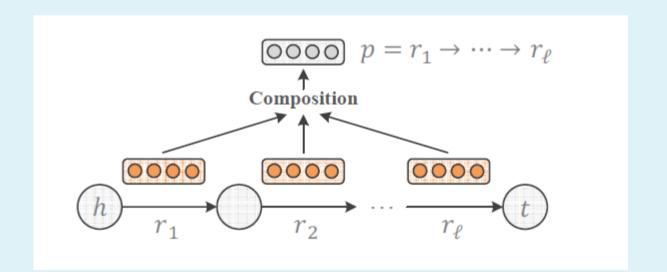






Relation Paths ---- path-based TransE (PTransE)

**Definition:** Given a path  $p = r1 \rightarrow \cdots \rightarrow r\ell$  linking two entities h and t, as well as the vector representations  $r1, \cdots, r\ell$  of the constituent relations



addition:  $\mathbf{p} = \mathbf{r}_1 + \cdots + \mathbf{r}_\ell$ ;

multiplication:  $\mathbf{p} = \mathbf{r}_1 \circ \cdots \circ \mathbf{r}_\ell$ ;

RNN:  $\mathbf{c}_i = f(\mathbf{W}[\mathbf{c}_{i-1}; \mathbf{r}_i])$ .





Relation Paths ---- path-based TransE (PTransE)

**Definition:** The path p is then required to be consistent with a direct relation r between the two entities, i.e., ||p-r|| tends to be small if (h, r, t) holds.

$$\mathcal{L}_{\text{path}} = \frac{1}{Z} \sum_{p \in \mathbb{P}(h,t)} R(p|h,t) \cdot \ell(p,r),$$

$$\ell(p,r) = \sum_{r'} \max(0, \gamma + ||\mathbf{p} - \mathbf{r}||_1 - ||\mathbf{p} - \mathbf{r}'||_1),$$





Relation Paths ---- extensions of both the TransE model and the RESCAL model

The score of (h, p, t):

$$f_p(h,t) = -\|\mathbf{h} + (\mathbf{r}_1 + \dots + \mathbf{r}_\ell) - \mathbf{t}\|_1,$$

$$f_p(h,t) = \mathbf{h}^{\top}(\mathbf{M}_1 \circ \cdots \circ \mathbf{M}_{\ell})\mathbf{t}.$$







#### **Textual Descriptions**

**Definition:** Actually, in most KGs there are concise descriptions for entities which contain rich semantic information about them.







**Definition:** The key idea is to align the given KG with an auxiliary text corpus, and then jointly conduct KG embedding and word embedding.

knowledge model, text model, and alignment model

$$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_T + \mathcal{L}_A.$$





#### • Knowledge Model:

score function:

$$z(\mathbf{h}, \mathbf{r}, \mathbf{t}) = b - \frac{1}{2} ||\mathbf{h} + \mathbf{r} - \mathbf{t}||^2$$

conditional probability:

$$\Pr(h|r,t) = \frac{\exp\{z(\mathbf{h}, \mathbf{r}, \mathbf{t})\}}{\sum_{\tilde{h} \in \mathcal{I}} \exp\{z(\tilde{\mathbf{h}}, \mathbf{r}, \mathbf{t})\}}$$

conditional likelihoods:

$$\mathcal{L}_f(h, r, t) = \log \Pr(h|r, t) + \log \Pr(t|h, r) + \log \Pr(r|h, t)$$
(2)





#### • Text Model:

**Relational Concurrence Assumption.** If two words w and v concur in some context, e.g., a window of text, then there is a relation  $r_{wv}$  between the two words. That is, we can state the triplet of  $(w, r_{wv}, v)$  is a fact.

$$z(\mathbf{w}, \mathbf{r}_{wv}, \mathbf{v}) \triangleq z(\mathbf{w}', \mathbf{v}) = b - \frac{1}{2} ||\mathbf{w}' - \mathbf{v}||^2$$
 (4)

and

$$\Pr(w|r_{wv}, v) \triangleq \Pr(w|v) = \frac{\exp\{z(\mathbf{w}', \mathbf{v})\}}{\sum_{\tilde{w} \in \mathcal{V}} \exp\{z(\tilde{\mathbf{w}}', \mathbf{v})\}}$$
(5)

$$\mathcal{L}_T = \sum_{(w,v)\in\mathcal{C}} n_{wv} \log \Pr(w|v).$$



- Alignment Model: align the two entity embedding space and word embedding space into the same one
- for most Wikipedia (English) pages, there is an unique corresponding entity  $e_v$  in Freebase

$$(w,v) \rightarrow (w,e_v)$$

$$\mathcal{L}_{AA} = \sum_{(w,v)\in\mathcal{C}, v\in\mathcal{A}} \log \Pr(w|e_v)$$





Textual Descriptions ---- description-embodied knowledge representation learning (DKRL)

**Definition:** The aim is to extend TransE so as to further handle entity descriptions.

$$f_r(h,t) = -\|\mathbf{h}_s + \mathbf{r} - \mathbf{t}_s\|_1 - \|\mathbf{h}_d + \mathbf{r} - \mathbf{t}_d\|_1$$
$$-\|\mathbf{h}_s + \mathbf{r} - \mathbf{t}_d\|_1 - \|\mathbf{h}_d + \mathbf{r} - \mathbf{t}_s\|_1,$$







Textual Descriptions ---- text-enhanced KG embedding model (TEKE)

**Definition:** Given a KG and a text corpus, TEKE first annotates entities in the corpus and constructs a co-occurrence network composed of entities and words. Then, for each entity e, TEKE defines its textual context n(e) as its neighbors in the co-occurrence network, i.e., words co-occurring frequently with the entity in the text corpus.

$$\hat{\mathbf{h}} = \mathbf{A}\mathbf{n}(h) + \mathbf{h},$$

$$\hat{\mathbf{t}} = \mathbf{A}\mathbf{n}(t) + \mathbf{t},$$

$$\hat{\mathbf{r}} = \mathbf{B}\mathbf{n}(h, t) + \mathbf{r}.$$







### Logical Rules

**Definition:** e.g., HasWife(x, y)  $\rightarrow$  HasSpouse(x, y) stating that any two entities linked by the relation HasWife should also be linked by the relation HasSpouse.







Logical Rules ---- A joint model which embeds KG facts and logical rules simultaneously

• A fact (h, r, t) is taken as a ground atom, with its truth value defined as:

$$I(h, r, t) = 1 - \frac{1}{3\sqrt{d}} \|\mathbf{h} + \mathbf{r} - \mathbf{t}\|_1,$$

Logical rules are first instantiated into ground rules, e.g., the universally quantified rule ∀x, y : HasWife(x, y) ⇒ HasSpouse(x, y) can be grounded into HasWife(AlfredHitchcock, AlmaReville)
 ⇒ HasSpouse(AlfredHitchcock, AlmaReville).



Logical Rules ----- A joint model which embeds KG facts and logical rules simultaneously

 Ground rules are then interpreted as complex formula constructed by combining ground atoms:

