

Lecture outline: Randomised and mixed strategies

Matching Pennies

		Player Odd	
		Heads	Tails
Player Even	Heads	1, -1	-1, 1
	Tails	-1, 1	1, -1

1. Solve the Matching Penny games: player Even plays Head with probability X and tails with $(1 - X)$, and Player Odd Y and $(1 - Y)$. What are X and Y ?

Expected utility of both players:

$$E_{Even}(H) = 1Y + -1(1 - Y) = 2Y - 1$$

$$E_{Even}(T) = -1Y + 1(1 - Y) = 1 - 2Y$$

$$E_{Odd}(H) = -1X + 1(1 - X) = 1 - 2X$$

$$E_{Odd}(T) = 1X + -1(1 - X) = 2X - 1$$

2. In equilibrium, a player should randomise their selection to make their opponent indifferent between their own strategy.

$$E_{Even}(H) = E_{Odd}(T)$$

$$\Rightarrow 2Y - 1 = 1 - 2Y$$

$$\Rightarrow 4Y = 2$$

$$\Rightarrow Y = \frac{1}{2}$$

$$\Rightarrow (1 - Y) = \frac{1}{2}$$

Similarly for X , therefore each player choose H and T with 50-50 probability.

Security games

		Adversary	
		Terminal 1	Terminal 2
Defender	Terminal 1	5, -3	-1, 1
	Terminal 2	-5, 5	2, -1

What are the strategies? They should not be just any random strategy – indifference rules!

1. For the Defender:

$$E(T1) = 5Y + -1(1 - Y) = 6Y - 1$$

$$E(T2) = -5Y + 2(1 - Y) = 2 - 7Y$$

$$E(T1) = E(T2)$$

$$\Rightarrow 6Y - 1 = 2 - 7Y$$

$$\Rightarrow 2 = 13Y$$

$$\Rightarrow Y = \frac{2}{13}$$

$$\Rightarrow (1 - Y) = \frac{11}{13}$$

2. For the attacker:

$$E(T1) = -3X + 5(1 - X) = 5 - 8X$$

$$E(T2) = 1X + -1(1 - X) = 2X - 1$$

$$E(T1) = E(T2)$$

$$\Rightarrow 5 - 8X = 2X - 1$$

$$\Rightarrow 6 = 10X$$

$$\Rightarrow X = \frac{3}{5}$$

$$\Rightarrow (1 - X) = \frac{2}{5}$$

Mixed and pure equilibria example

		Player 2	
		Left	Right
Player 1	Up	3, 1	0, 0
	Down	0, 0	1, 3

- There can be mixed strategy equilibria even if there are already pure strategy equilibria. See the example: 3,1 and 1,3 are pure strategy equilibria, BUT!:
 - Player 1: $E(U) = E(D) \rightarrow 3q = 1 - q \rightarrow 4q = 1 \rightarrow q = 1/4$
 - Player 2: $E(L) = E(R) \rightarrow p = 3(1 - p) \rightarrow 4p = 3 \rightarrow p = 3/4$
- So, Player 1 with $(3/4U + 1/4D)$ and Player 2 with $(1/4L + 3/4R)$.