AI Planning for Autonomy

5. Delete Relaxation Heuristics

It's a Long Way to the Goal, But How Long Exactly?
Part I: *Acting As If the World Can Only Get Better*

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Winter Term 2018

Agenda

Motivation

→ Delete relaxation is a method to relax planning tasks, and thus automatically compute heuristic functions h.

→ Every h yields good performance only in some domains! (Search reduction vs. computational overhead)
→ We must come up with as many alternative methods as possible!

We cover the 4 different methods currently known:

- Critical path heuristics:
- Delete relaxation. Soon to be Done.
- Abstractions.
- Landmarks.

→ Delete relaxation is very wide-spread, and highly successful for satisficing planning!

We introduce the method in STRIPS.

Reminder: Relaxing the World by Ignoring Delete Lists

"What was once true remains true forever."

Relaxed world: (after)

The Delete Relaxation

Definition (Delete Relaxation).

(i) *For a STRIPS action a, by a^+ we denote the corresponding delete relaxed action, or short relaxed action, defined by $pre_{a^+} := pre_a$, $add_{a^+} := add_a$, and $del_{a^+} := \emptyset$.*

(ii) *For a set A of STRIPS actions, by A^+ we denote the corresponding set of relaxed actions, $A^+ := \{a^+ \mid a \in A\}$; similarly, for a sequence $\vec{a} = \langle a_1, \dots, a_n \rangle$ of STRIPS actions, by $\vec{a}^+$ we denote the corresponding sequence of relaxed actions, $\vec{a}^+ := \langle a_1^+, \dots, a_n^+ \rangle$.*

(iii) *For a STRIPS planning task $\Pi = (F, A, c, I, G)$, by $\Pi^+ := (F, A^+, c, I, G)$ we denote the corresponding (delete) relaxed planning task.*

→ "+" super-script = delete relaxed. We'll also use this to denote states encountered within the relaxation. (For STRIPS, s^+ is a fact set just like s.)

Definition (Relaxed Plan). *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, and let s be a state. An (optimal) relaxed plan for s is an (optimal) plan for Π_s^+. A relaxed plan for I is also called a relaxed plan for Π.*

→ Anybody remember what Π_s is? $\Pi_s = (F, A, c, s, G)$

A Relaxed Plan for "TSP" in Australia

1. **Initial state:** $\{at(Sy), v(Sy)\}$.
2. **Apply** $drive(Sy, Br)^+$**:** $\{at(Br), v(Br), at(Sy), v(Sy)\}$.
3. **Apply** $drive(Sy, Ad)^+$**:** $\{at(Ad), v(Ad), at(Br), v(Br), at(Sy), v(Sy)\}$.
4. **Apply** $drive(Ad, Pe)^+$**:** $\{at(Pe), v(Pe), at(Ad), v(Ad), at(Br), v(Br), at(Sy), v(Sy)\}$.
5. **Apply** $drive(Ad, Da)^+$**:** $\{at(Da), v(Da), at(Pe), v(Pe), at(Ad), v(Ad), at(Br), v(Br), at(Sy), v(Sy)\}$.

State Dominance

Definition (Dominance). *Let $\Pi^+ = (F, A^+, c, I, G)$ be a STRIPS planning task, and let s^+, s'^+ be states. We say that s'^+ dominates s^+ if $s'^+ \supseteq s^+$.*

→ For example, on the previous slide, who dominates who? Each state along the relaxed plan dominates the previous one, simply because the actions don't delete any facts.

Proposition (Dominance). *Let $\Pi^+ = (F, A^+, c, I, G)$ be a STRIPS planning task, and let s^+, s'^+ be states where s'^+ dominates s^+. We have:*

(i) *If s^+ is a goal state, then s'^+ is a goal state as well.*

(ii) *If $\vec{a}^+$ is applicable in s^+, then $\vec{a}^+$ is applicable in s'^+ as well, and $appl(s'^+, \vec{a}^+)$ dominates $appl(s^+, \vec{a}^+)$.*

Proof. (i) is trivial. (ii) by induction over the length n of $\vec{a}^+$. Base case $n = 0$ is trivial. Inductive case $n \to n + 1$ follows directly from induction hypothesis and the definition of $appl(.,.)$.

→ It is always better to have more facts true.

The Delete Relaxation and State Dominance

Proposition. *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, let s be a state, and let $a \in A$. Then $appl(s, a^+)$ dominates both (i) s and (ii) $appl(s, a)$.*

Proof. Trivial from the definitions of $appl(s, a)$ and a^+.

$\Longrightarrow$ Optimal relaxed plans admissibly estimate the cost of optimal plans:

Proposition (Delete Relaxation is Admissible). *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, let s be a state, and let $\vec{a}$ be a plan for Π_s. Then $\vec{a}^+$ is a relaxed plan for s.*

Proof. Prove by induction over the length of $\vec{a}$ that $appl(s, \vec{a}^+)$ dominates $appl(s, \vec{a})$. Base case is trivial, inductive case follows from (ii) above.

$\Longrightarrow$ It is now clear how to find a relaxed plan:

- Applying a relaxed action can only ever make more facts true ((i) above).
- That can only be good, i.e., cannot render the task unsolvable (dominance proposition).

$\rightarrow$ So? Keep applying relaxed actions, stop if goal is true (see next slide).

Greedy Relaxed Planning

Greedy Relaxed Planning for Π_s^+

$s^+ := s$; $\vec{a}^+ := \langle\rangle$
while $G \not\subseteq s^+$ **do**:
  if $\exists a \in A$ s.t. $pre_a \subseteq s^+$ and $appl(s^+, a^+) \neq s^+$ **then**
    select one such a
    $s^+ := appl(s^+, a^+)$; $\vec{a}^+ := \vec{a}^+ \circ \langle a^+ \rangle$
  else return "Π_s^+ is unsolvable" **endif**
endwhile
return $\vec{a}^+$

Proposition. *Greedy relaxed planning is sound, complete, and terminates in time polynomial in the size of Π.*

Proof. Soundness: If $\vec{a}^+$ is returned then, by construction, $G \subseteq appl(s, \vec{a}^+)$. Completeness: If "$\Pi_s^+$ is unsolvable" is returned, then no relaxed plan exists for s^+ at that point; since s^+ dominates s, by the dominance proposition this implies that no relaxed plan can exist for s. Termination: Every $a \in A$ can be selected at most once because afterwards $appl(s^+, a^+) = s^+$.

$\Longrightarrow$ It is easy to decide whether a relaxed plan exists!

Greedy Relaxed Planning to Generate a Heuristic Function?

Using greedy relaxed planning to generate h

- In search state s during forward search, run greedy relaxed planning on Π_s^+.
- Set $h(s)$ to the cost of $\vec{a}^+$, or ∞ if "Π_s^+ is unsolvable" is returned.

$\rightarrow$ Is this heuristic safe? Yes: $h(s) = \infty$ only if no relaxed plan for s exists, which by admissibility of delete relaxation implies that no plan for s exists.

$\rightarrow$ Is this heuristic goal-aware? Yes, we'll have $G \subseteq s^+$ right at the start.

$\rightarrow$ Is this heuristic admissible? Would be if the relaxed plans were optimal; but they clearly aren't. So h isn't consistent either.

$\rightarrow$ To be informed (accurately estimate h^*), a heuristic needs to approximate the *minimum effort* needed to reach the goal. Greedy relaxed planning doesn't do this because it may select arbitrary actions that aren't relevant at all.

h^+: The Optimal Delete Relaxation Heuristic

Definition (h^+**).** *Let* $\Pi = (F, A, c, I, G)$ *be a STRIPS planning task with state space* $\Theta_\Pi = (S, A, c, T, I, G)$*. The* *optimal delete relaxation heuristic* h^+ *for* Π *is the function* $h^+ : S \mapsto \mathbb{R}^+_0 \cup \{\infty\}$ *where* $h^+(s)$ *is defined as the cost of an optimal relaxed plan for* s*.*

Corollary (h^+ **is Admissible).** *Let* $\Pi = (F, A, c, I, G)$ *be a STRIPS planning task. Then* h^+ *is admissible, and thus safe and goal-aware.* (By admissibility of delete relaxation.)

$\rightarrow$ To be informed (accurately estimate h^*), a heuristic needs to approximate the *minimum effort* needed to reach the goal. h^+ naturally does so by asking for the cheapest possible relaxed plans.

[$\rightarrow$ You might rightfully ask "But won't optimal relaxed plans usually under-estimate h^*?" Yes, but that's just the effect of considering a relaxed problem, and arbitrarily adding actions useless within the relaxation does not help to address it.]

h^+ in "TSP" in Australia

- P: $at(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$; $v(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$.
- A: $drive(x, y)$ where x, y have a road.

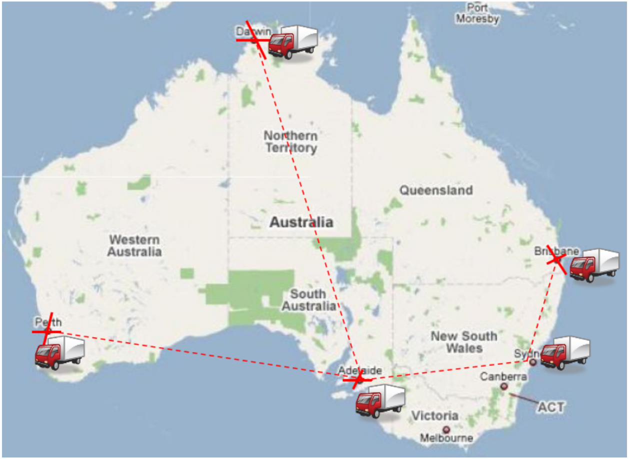
$$c(drive(x,y)) = \begin{cases} 1 & \{x,y\} = \{Sy, Br\} \\ 1.5 & \{x,y\} = \{Sy, Ad\} \\ 3.5 & \{x,y\} = \{Ad, Pe\} \\ 4 & \{x,y\} = \{Ad, Da\} \end{cases}$$

- I: $at(Sy), v(Sy)$; G: $at(Sy), v(x)$ for all x.

Planning vs. Relaxed Planning:

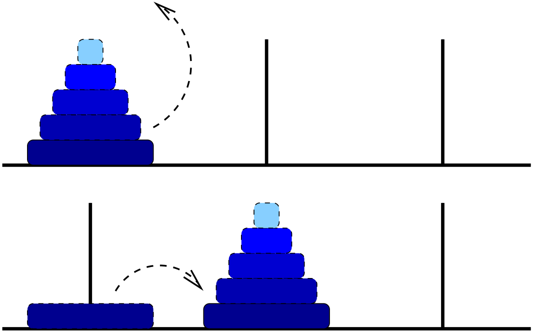
- Optimal plan: $\langle drive(Sy, Br), drive(Br, Sy), drive(Sy, Ad), drive(Ad, Pe), drive(Pe, Ad),$ $drive(Ad, Da), drive(Da, Ad), drive(Ad, Sy)\rangle$.
- Optimal relaxed plan: $\langle drive(Sy, Br), drive(Sy, Ad), drive(Ad, Pe), drive(Ad, Da)\rangle$.
- $h^*(I) = 20$; $h^+(I) = 10$.

Reminder: h^+ in (the real) TSP



$h^+(\text{TSP}) =$ Minimum Spanning Tree!

Reminder: h^+ in Hanoi



h^+(Hanoi) $= n$, not 2^n

But How to Compute h^+?

Definition (Optimal Relaxed Planning). *By PlanOpt$^+$, we denote the problem of deciding, given a STRIPS planning task $\Pi = (F, A, c, I, G)$ and $B \in \mathbb{R}_0^+$, whether there exists a relaxed plan for Π whose cost is at most B.*

$\rightarrow$ By computing h^+, we would solve PlanOpt$^+$.

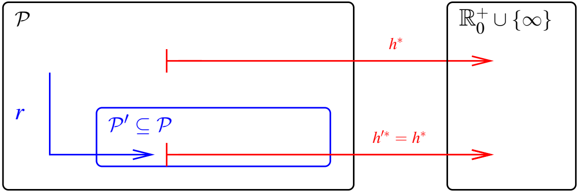
Theorem (Optimal Relaxed Planning is Hard). *PlanOpt$^+$ is* **NP***-complete.*

Proof. Membership: Guess action sequences of length $|A|$ – in a relaxed plan, each action is applied at most once!

Hardness: By reduction from SAT.

- For each variable $v_i \in \{v_1, \dots, v_m\}$ in the CNF, three facts v_i, $notv_i$, and $setv_i$; for each clause $c_j \in \{c_1, \dots, c_n\}$ in the CNF, one fact $satc_j$.
- Actions $setvtrue_i$: $(\emptyset, \{v_i, setv_i\}, \emptyset)$ and $setvfalse_i$: $(\emptyset, \{notv_i, setv_i\}, \emptyset)$.
- Actions $makesatc_j$: $(\{v_i\}, \{satc_j\}, \emptyset)$ where v_i appears positively in clause c_j; $(\{notv_i\}, \{satc_j\}, \emptyset)$ where v_i appears negatively in clause c_j.
- Initial state $\emptyset$, goal $\{setv_1, \dots, setv_m, satc_1, \dots, satc_n\}$; $B := m + n$.

h^+ as a Relaxation Heuristic



where, for all $\Pi \in \mathcal{P}$, $h^*(r(\Pi)) \leq h^*(\Pi)$.

For $h^+ = h^* \circ r$:

- Problem $\mathcal{P}$: All STRIPS planning tasks.
- Simpler problem $\mathcal{P}'$: All STRIPS planning tasks with empty deletes.
- Perfect heuristic h'^* for $\mathcal{P}'$: Optimal plan cost $= h^*$ on $\mathcal{P}'$.
- Transformation r: Drop the deletes.

$\rightarrow$ Is this a native relaxation? Yes.
$\rightarrow$ Is this relaxation efficiently constructible? Yes.
$\rightarrow$ Is this relaxation efficiently computable? No.

What shall we do with this relaxation?

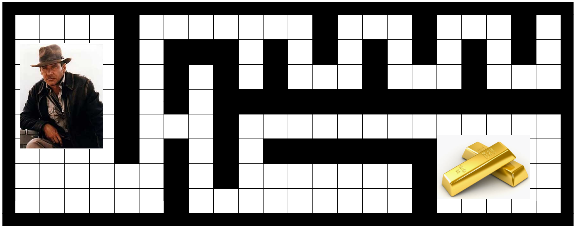
Reminder: $\rightarrow$ **Lecture 4**

What if $\mathcal{R}$ is not efficiently computable?

- Either (a) approximate h'^*, or (b) design h'^* in a way so that it will typically be feasible, or (c) just live with it and hope for the best.
- Many known relaxations (in planning) are efficiently computable, some aren't (like h^+). The latter use (a); (b) and (c) are not used anywhere right now.

$\rightarrow$ The delete relaxation heuristic we want is h^+. Unfortunately, this is hard to compute so the computational overhead is very likely to be prohibitive. All implemented systems using the delete relaxation approximate h^+ in one or the other way. We now look at the the most wide-spread approaches to do so.

Quizz@Speakup



Question!

In this domain, h^+ is equal to?

(A): Manhattan Distance.

(B): h^*.

(C): Horizontal distance.

(D): Vertical distance.

→ (A): No, relaxed plans can't walk through walls. (B): Yes, optimal plan = shortest path = relaxed plan (deletes do not matter because "shortest paths never walk back"). (C), (D): No, relaxed plans must move both horizontally and vertically.

The Additive and Max Heuristics

Definition (h^{add}). *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task. The additive heuristic h^{add} for Π is the function $h^{\text{add}}(s) := h^{\text{add}}(s, G)$ where $h^{\text{add}}(s, g)$ is the point-wise greatest function that satisfies $h^{\text{add}}(s, g) =$*

$$\begin{cases} 0 & g \subseteq s \\ min_{a \in A, g \in add_a} c(a) + h^{\text{add}}(s, pre_a) & |g| = 1 \\ \sum_{g' \in g} h^{\text{add}}(s, \{g'\}) & |g| > 1 \end{cases}$$

Definition (h^{max}). *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task. The max heuristic h^{max} for Π is the function $h^{\text{max}}(s) := h^{\text{max}}(s, G)$ where $h^{\text{max}}(s, g)$ is the point-wise greatest function that satisfies $h^{\text{max}}(s, g) =$*

$$\begin{cases} 0 & g \subseteq s \\ min_{a \in A, g \in add_a} c(a) + h^{\text{max}}(s, pre_a) & |g| = 1 \\ \max_{g' \in g} h^{\text{max}}(s, \{g'\}) & |g| > 1 \end{cases}$$

The Additive and Max Heuristics: Properties

Proposition (h^{max} is Optimistic). *$h^{\mathsf{max}} \leq h^+$, and thus $h^{\mathsf{max}} \leq h^*$.*

Proposition (h^{add} is Pessimistic). *For all STRIPS planning tasks Π, $h^{\mathsf{add}} \geq h^+$. There exist Π and s so that $h^{\mathsf{add}}(s) > h^*(s)$.*

$\rightarrow$ Both h^{max} and h^{add} approximate h^+ by assuming that singleton sub-goal facts are achieved independently. h^{max} estimates optimistically by the most costly singleton sub-goal, h^{add} estimates pessimistically by summing over all singleton sub-goals.

The Additive and Max Heuristics: Properties, ctd.

Proposition (h^{max} and h^{add} Agree with h^+ on ∞). *For all STRIPS planning tasks Π and states s in Π, $h^+(s) = \infty$ if and only if $h^{max}(s) = \infty$ if and only if $h^{add}(s) = \infty$.*

$\rightarrow$ States for which no relaxed plan exists are easy to recognize, and that is done by both h^{max} and h^{add}. Approximation is needed only for the cost of an optimal relaxed pan, if it exists.

Bellman-Ford for $h^{\max}$ and h^{add}

Bellman-Ford variant computing h^{add} for state s

new table $T_0^{\text{add}}(g)$, for $g \in F$

For all $g \in F$: $T_0^{\text{add}}(g) := \begin{cases} 0 & g \in s \\ \infty & \text{otherwise} \end{cases}$

fn $c_i(g) := \begin{cases} T_i^{\text{add}}(g) & |g| = 1 \\ \sum_{g' \in g} T_i^{\text{add}}(g') & |g| > 1 \end{cases}$

fn $f_i(g) := \min[c_i(g), \min_{a \in A, g \in add_a} c(a) + c_i(pre_a)]$

do forever:

  new table $T_{i+1}^{\text{add}}(g)$, for $g \in F$

  For all $g \in F$: $T_{i+1}^{\text{add}}(g) := f_i(g)$

  if $T_{i+1}^{\text{add}} = T_i^{\text{add}}$ **then** stop **endif**

  $i := i + 1$

enddo

$\rightarrow$ Basically the same algorithm works for $h^{\max}$, just change $\sum$ for $\max$

Proposition. *Let* $\Pi = (F, A, c, I, G)$ *be a STRIPS planning task. Then the series* $\{T_i^{\text{add}}(g)\}_{i=0,\ldots}$ *converges to* $h^{\text{add}}(s, g)$*, for all* g*.* (Proof omitted.)

Bellman-Ford for $h^{\max}$ in "TSP" in Australia

- F: $at(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$; $v(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$.
- A: $drive(x, y)$ where x, y have a road.

$$c(drive(x,y)) = \begin{cases} 1 & \{x,y\} = \{Sy, Br\} \\ 1.5 & \{x,y\} = \{Sy, Ad\} \\ 3.5 & \{x,y\} = \{Ad, Pe\} \\ 4 & \{x,y\} = \{Ad, Da\} \end{cases}$$

- I: $at(Sy), v(Sy)$; G: $at(Sy), v(x)$ for all x.

Content of Tables T_i^1:

i	$at(Sy)$	$at(Ad)$	$at(Br)$	$at(Pe)$	$at(Da)$	$v(Sy)$	$v(Ad)$	$v(Br)$	$v(Pe)$	$v(Da)$
0	0	∞	∞	∞	∞	0	∞	∞	∞	∞
1	0	1.5	1	∞	∞	0	1.5	1	∞	∞
2	0	1.5	1	5	5.5	0	1.5	1	5	5.5
3	0	1.5	1	5	5.5	0	1.5	1	5	5.5

$\rightarrow h^{\max}(I) = 5.5 << 20 = h^*(I)$.

Bellman-Ford for h^{add} in “TSP” in Australia

- F: $at(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$; $v(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$.
- A: $drive(x, y)$ where x, y have a road.

$$c(drive(x,y)) = \begin{cases} 1 & \{x,y\} = \{Sy, Br\} \\ 1.5 & \{x,y\} = \{Sy, Ad\} \\ 3.5 & \{x,y\} = \{Ad, Pe\} \\ 4 & \{x,y\} = \{Ad, Da\} \end{cases}$$

- I: $at(Sy), v(Sy)$; G: $at(Sy), v(x)$ for all x.

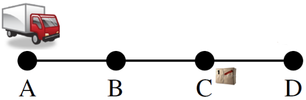
Content of Tables T_i^{add}:

i	$at(Sy)$	$at(Ad)$	$at(Br)$	$at(Pe)$	$at(Da)$	$v(Sy)$	$v(Ad)$	$v(Br)$	$v(Pe)$	$v(Da)$
0	0	∞	∞	∞	∞	0	∞	∞	∞	∞
1	0	1.5	1	∞	∞	0	1.5	1	∞	∞
2	0	1.5	1	5	5.5	0	1.5	1	5	5.5
3	0	1.5	1	5	5.5	0	1.5	1	5	5.5

$\rightarrow h^{\text{add}}(I) = 1.5 + 1 + 5 + 5.5 = 13 > 10 = h^{+}(I)$. But $< 20 = h^{*}(I)$.

$\rightarrow h^{\text{add}}(I) > h^{+}(I)$ because it counts the cost of $drive(Sy, Ad)$ 3 times:
As part of $h^{\text{add}}(I, \{v(Ad)\})$, $h^{\text{add}}(I, \{v(Pe)\})$, and $h^{\text{add}}(I, \{v(Da)\})$!

Bellman-Ford for h^{add} in "Logistics"



- Initial state I: $t(A), p(C)$.
- Goal G: $t(A), p(D)$.
- Actions A: $dr(X, Y), lo(X), ul(X)$.

Content of Tables T_i^{add}: (Table content T_i^1, where different, given in red)

i	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞
2	0	1	2	∞	∞	∞	∞	0	∞
3	0	1	2	3	3	∞	∞	0	∞
4	0	1	2	3	3	4	5 (4)	0	7 (4)
5	0	1	2	3	3	4	5 (4)	0	7 (4)

$\rightarrow h^{\text{add}}(I) = 7 > h^+(I) = 5$. But $< 8 = h^*(I)$.

$\rightarrow h^{\text{add}}(I) > h^+(I)$ because? It counts the cost of $dr(A, B), dr(B, C)$ 2 times, for the two preconditions $p(T)$ and $t(D)$ of achieving $p(D)$.

$\rightarrow$ So, what if $G = \{t(D), p(D)\}$? $h^{\text{add}}(I) = 10 > 5 = h^*(I) = h^+(I)$ because now $dr(A, B), dr(B, C), dr(C, D)$ is counted also as part of the goal $t(D)$.

The Additive and Max Heuristics: So What?

Summary of typical issues in practice with h^{add} and h^{max}:

- Both h^{add} and h^{max} can be computed reasonably quickly.
- h^{max} is admissible, but is typically far too optimistic.
- h^{add} is not admissible, but is typically a lot more informed than h^{max}.
- h^{add} is sometimes better informed than h^+, but for the "wrong reasons": rather than accounting for deletes, it overcounts by ignoring positive interactions, i.e., sub-plans shared between sub-goals.
- Such overcounting can result in dramatic over-estimates of h^*!!

$\rightarrow$ On slide 28 with goal $t(D)$, if we have 100 packages at C that need to go to D, what is $h^{\text{add}}(I)$? $703 >> 203 = h^*(I) = h^+(I)$: For every package, a count of 7 which includes $dr(A,B), dr(B,C)$ for getting the package into the truck, and $dr(A,B), dr(B,C), dr(C,D)$ for getting the truck to D.

$\rightarrow$ Relaxed plans (up next) are a means to reduce this kind of over-counting.

Relaxed Plans, Basic Idea

→ First compute a best-supporter function bs, which for every fact $p \in F$ returns an action that is deemed to be the cheapest achiever of p (within the relaxation). Then extract a relaxed plan from that function, by applying it to singleton sub-goals and collecting all the actions.

→ The best-supporter function can be based directly on $h^{\max}$ or h^{add}, simply selecting an action a achieving p that minimizes the sum of $c(a)$ and the cost estimate for pre_a.

And now for the details:

Relaxed Plan Extraction

Relaxed Plan Extraction for state s and best-supporter function bs

$Open := G \setminus s$; $Closed := \emptyset$; $RPlan := \emptyset$
while $Open \neq \emptyset$ **do**:
  select $g \in Open$
  $Open := Open \setminus \{g\}$; $Closed := Closed \cup \{g\}$;
  $RPlan := RPlan \cup \{bs(g)\}$; $Open := Open \cup (pre_{bs(g)} \setminus (s \cup Closed))$
endwhile
return $RPlan$

$\rightarrow$ Starting with the top-level goals, iteratively close open singleton sub-goals by selecting the best supporter.

This is fast! Number of iterations bounded by $|P|$, each near-constant time.

But is it correct?

$\rightarrow$ What if $g \notin add_{bs(g)}$? Doesn't make sense. $\rightarrow$ Prerequisite (A).
$\rightarrow$ What if $bs(g)$ is undefined? Runtime error. $\rightarrow$ Prerequisite (B).
$\rightarrow$ What if the support for g eventually requires g itself as a precondition? Then this does not actually yield a relaxed plan. $\rightarrow$ Prerequisite (C).

Best-Supporter Functions

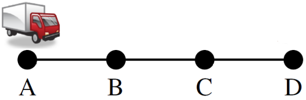
→ For relaxed plan extraction to make sense, it requires a *closed well-founded* best-supporter function:

Definition (Best-Supporter Function). *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, and let s be a state. A best-supporter function for s is a partial function $bs : (F \setminus s) \mapsto A$ such that $p \in add_a$ whenever $a = bs(p)$.*
The support graph of bs is the directed graph with vertices $F \cup A$ and arcs $\{(p, a) \mid p \in pre_a\} \cup \{(a, p) \mid a = bs(p)\}$. We say that bs is closed if $bs(p)$ is defined for every $p \in (F \setminus s)$ that has a path to a goal $g \in G$ in the support graph. We say that bs is well-founded if the support graph is acyclic.

- "$p \in add_a$ whenever $a = bs(p)$": Prerequisite (A).
- bs is closed: Prerequisite (B).
- bs is well-founded: Prerequisite (C).

→ Intuition for (C): Relaxed plan extraction starts at the goals, and chains backwards in the support graph. If there are cycles, then this backchaining may not reach the currently true state s, and thus not yield a relaxed plan.

Support Graphs and Prerequisite (C) in "Logistics"



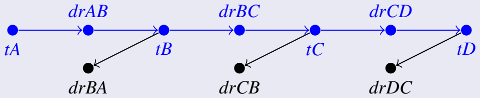
- Initial state: tA.
- Goal: tD.
- Actions: $drXY$.

How to do it (well-founded)

Best-supporter function:

p	$bs(p)$
$t(B)$	$dr(A, B)$
$t(C)$	$dr(B, C)$
$t(D)$	$dr(C, D)$

Yields support graph backchaining:

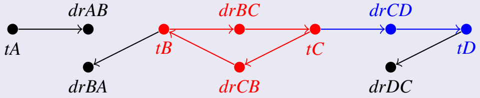


How to NOT do it (not well-founded)

Best-supporter function:

p	$bs(p)$
$t(B)$	$dr(C, B)$
$t(C)$	$dr(B, C)$
$t(D)$	$dr(C, D)$

Yields support graph backchaining:



How to obtain closed well-founded bs?

Definition (Best-Supporters from $h^{\max}$ and h^{add}). *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, and let s be a state.*
The $h^{\max}$ supporter function $bs_s^{\max} : \{p \in F \mid 0 < h^{\max}(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs_s^{\max}(p) := \arg\min_{a \in A, p \in add_a} c(a) + h^{\max}(s, pre_a)$.
The h^{add} supporter function $bs_s^{\text{add}} : \{p \in F \mid 0 < h^{\text{add}}(s, \{p\}) < \infty\} \mapsto A$ is defined by $bs_s^{\text{add}}(p) := \arg\min_{a \in A, p \in add_a} c(a) + h^{\text{add}}(s, pre_a)$.

Proposition. *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task such that, for all $a \in A$, $c(a) > 0$. Let s be a state where $h^+(s) < \infty$. Then both $bs_s^{\max}$ and bs_s^{add} are closed well-founded supporter functions for s.*

Proof. Since $h^+(s) < \infty$ implies $h^{\max}(s) < \infty$, it is easy to see that $bs_s^{\max}$ is closed (details omitted). If $a = bs_s^{\max}(p)$, then a is the action yielding $0 < h^{\max}(s, \{p\}) < \infty$ in the $h^{\max}$ equation. Since $c(a) > 0$, we have $h^{\max}(s, pre_a) < h^{\max}(s, \{p\})$ and thus, for all $q \in pre_a$, $h^{\max}(s, \{q\}) < h^{\max}(s, \{p\})$. Transitively, if the support graph contains a path from fact vertex r to fact vertex t, then $h^{\max}(s, \{r\}) < h^{\max}(s, \{t\})$. Thus there can't be cycles in the support graph and $bs_s^{\max}$ is well-founded. Similar for bs_s^{add}.

The Relaxed Plan Heuristic

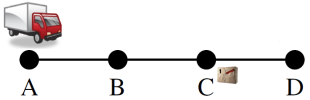
Proposition. *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, let s be a state, and let bs be a closed well-founded best-supporter function for s. Then the action set $RPlan$ returned by relaxed plan extraction can be sequenced into a relaxed plan $\vec{a}^+$ for s.*

Proof. Order a before a' whenever the support graph contains a path from a to a'. Since the support graph is acyclic, such a sequencing $\vec{a} := \langle a_1, \dots, a_n \rangle$ exists. We have $p \in s$ for all $p \in pre_{a_1}$, because otherwise $RPlan$ would contain the action $bs(p)$, necessarily ordered before a_1. We have $p \in s \cup add_{a_1}$ for all $p \in pre_{a_2}$, because otherwise $RPlan$ would contain the action $bs(p)$, necessarily ordered before a_2. Iterating the argument shows that $\vec{a}^+$ is a relaxed plan for s.

Definition (Relaxed Plan Heuristic). *A heuristic function is called a relaxed plan heuristic, denoted h^{FF}, if, given a state s, it returns ∞ if no relaxed plan exists, and otherwise returns $\sum_{a \in RPlan} c(a)$ where $RPlan$ is the action set returned by relaxed plan extraction on a closed well-founded best-supporter function for s.*

$\rightarrow$ Recall: If a relaxed plan exists, then there also exists a closed well-founded best-supporter function, see previous slide.

Relaxed Plan Extraction from h^{add} in "Logistics"



- Initial state I: $t(A), p(C)$.
- Goal G: $t(A), p(D)$.
- Actions A: $dr(X, Y), lo(X), ul(X)$.

	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
h^{add}	0	1	2	3	3	4	5	0	7

Extracting a relaxed plan:

1. $bs_s^{\text{add}}(p(D)) = ul(D)$; opens $t(D), p(T)$.
2. $bs_s^{\text{add}}(t(D)) = dr(C, D)$; opens $t(C)$.
3. $bs_s^{\text{add}}(t(C)) = dr(B, C)$; opens $t(B)$.
4. $bs_s^{\text{add}}(t(B)) = dr(A, B)$; opens nothing.
5. $bs_s^{\text{add}}(p(T)) = lo(C)$; opens nothing.
6. Anything more? No, open goals empty at this point.

$\rightarrow h^{\text{FF}}(I) = 5 = h^+(I) < 7 = h^{\text{add}}(I) < 8 = h^*(I)$.

$\rightarrow$ What if $G = \{t(D), p(D)\}$? $h^{\text{FF}}(I) = 5 = h^+(I) = h^*(I)$ because relaxed plan extraction selects the drive actions only once. By contrast, $h^{\text{add}}(I) = 10$ overcounts these actions, cf. slide 28.

The Relaxed Plan Heuristic, ctd.

Proposition (h^{FF} is Pessimistic and Agrees with h^* on ∞). *For all STRIPS planning tasks Π, $h^{\mathsf{FF}} \geq h^+$; for all states s, $h^+(s) = \infty$ if and only if $h^{\mathsf{FF}}(s) = \infty$. There exist Π and s so that $h^{\mathsf{FF}}(s) > h^*(s)$.*

Proof. $h^{\mathsf{FF}} \geq h^+$ follows directly from the previous proposition. Agrees with h^+ on ∞: direct from definition. Inadmissiblity: Whenever bs makes sub-optimal choices. $\rightarrow$ **Exercise, perhaps**

$\rightarrow$ Relaxed plan heuristics have the same theoretical properties as h^{add}.

So what's the point?

- Can h^{FF} over-count, i.e., count sub-plans shared between sub-goals more than once? No, due to the set union in "$RPlan := RPlan \cup \{bs(g)\}$".
- h^{FF} *may* be inadmissible, just like h^{add}, but for more subtle reasons.
- In practice, h^{FF} typically does not over-estimate h^* (or not by a large amount, anyway); cf. example on previous slide.

Questionnaire

Question!

How does ignoring delete lists simplify FreeCell?

(A): You can move all cards immediately to their goal.

(B): Free cells remain free.

→ (A): No, we don't get any new moves in the relaxation. (B): Yes, when putting a card into a free cell, it's still free for another card.

Question!

How does ignoring delete lists simplify Sokoban?

(A): Free positions remain free.

(B): You can walk through walls.

(C): You can push 2 stones to same position.

(D): Nothing ever becomes blocked.

→ (A), (C), (D): Yes (similar to above). (B): No, we don't get any new moves.

Summary

- The delete relaxation simplifies STRIPS by removing all delete effects of the actions.
- The cost of optimal relaxed plans yields the heuristic function h^+, which is admissible but hard to compute.
- We can approximate h^+ optimistically by h^{max}, and pessimistically by h^{add}. h^{max} is admissible, h^{add} is not. h^{add} is typically much more informative, but can suffer from over-counting.
- Either of h^{max} or h^{add} can be used to generate a closed well-founded best-supporter function, from which we can extract a relaxed plan. The resulting relaxed plan heuristic h^{FF} does not do over-counting, but otherwise has the same theoretical properties as h^{add}; it typically does not over-estimate h^*.

Example Systems

HSP [*Bonet and Geffner, AI-01*]

1. Search space: Progression (STRIPS-based).
2. Search algorithm: Greedy best-first search.
3. Search control: h^{add}.

FF [*Hoffmann and Nebel ,JAIR-01*]

1. Search space: Progression (STRIPS-based).
2. Search algorithm: Enforced hill-climbing.
3. Search control: h^{FF} extracted from $h^{\max}$ supporter function; helpful actions pruning (basically expand only those actions contained in the relaxed plan).

LAMA [*Richter and Westphal, JAIR-10*]

1. Search space: Progression (FDR-based).
2. Search algorithm: Multiple-queue greedy best-first search.
3. Search control: h^{FF} + a landmarks heuristic ($\rightarrow$ **next lecture**); for each, one search queue all actions, one search queue only helpful actions.

Remarks

- The delete relaxation is aka ignoring delete lists.
- HSP was competitive in the 1998 International Planning Competition (IPC'98); FF outclassed the competitors in IPC'00.
- The delete relaxation is still at large, most recently with the wins of LAMA in the satisficing planning tracks of IPC'08 and IPC'11.

Remarks, ctd.

- → More generally, the relaxation principle is very generic and potentially applicable in many different contexts, as are all relaxation principles covered in this course.
- While $h^{\max}$ is not informative in practice, other lower-bounding approximations of h^+ are very important for optimal planning: admissible landmarks heuristics [*Karpas and Domshlak, IJCAI-09*]; LM-cut heuristic [*Helmert and Domshlak, ICAPS-09*].
- It has always been a challenge to take *some* delete effects into account. Recent work done to interpolate smoothly between h^+ and h^*: explicitly represented fact conjunctions [*Keyder, Hoffmann, and Haslum ICAPS-12*].

Reading

- *Planning as Heuristic Search* [*Bonet and Geffner, AI-01*].

  Available at:

  `http://www.dtic.upf.edu/~hgeffner/html/reports/hsp-aij.ps`

  Content: This is "where it all started": the first paper[1] explicitly introducing the notion of heuristic search and automatically generated heuristic functions to planning. Introduces the additive and max heuristics h^{add} and h^{max}.

[1] Well, this is the first full journal paper treating the subject; the same authors published conference papers in AAAI'97 and ECP'99, which are subsumed by the present paper.

Reading, ctd.

- *The FF Planning System: Fast Plan Generation Through Heuristic Search* [*Hoffmann:nebel:jair-01*]. JAIR Best Paper Award 2005.

  Available at:

  `http://fai.cs.uni-saarland.de/hoffmann/papers/jair01.pdf`

  Content: The main reference for delete relaxation heuristics (cited > 1000 times). Introduces the relaxed plan heuristic, extracted from the h^{max} supporter function.[2] Also introduces helpful actions pruning, and enforced hill-climbing.

[2] Done in a uniform-cost setting presented in terms of relaxed planning graphs instead of h^{max}, and not identifying the more general idea of using a well-founded best-supporter function (I used the same simpler presentation in the AI'12 core course). The notion of best-supporter functions (handling non-uniform action costs) first appears in [*Keyder and Geffner, ECAI-08*].

Reading, ctd.

- *Semi-Relaxed Plan Heuristics* [*Keyder, Hoffmann, and Haslum ICAPS-12*]. Best Paper Award at ICAPS'12.

  Available at: `http://fai.cs.uni-saarland.de/hoffmann/papers/icaps12a.pdf`

  Content: Computes relaxed plan heuristics within a compiled planning task Π^C_{ce}, in which a subset C of all fact conjunctions in the task is represented explicitly as suggested by [*Haslum, ICAPS-12*]. C can in principle always be chosen so that $h^+_{\Pi^C_{ce}}$ is perfect (equals h^* in the original planning task), so the technique allows to interpolate between h^+ and h^*. In practice, small sets C sometimes suffice to obtain dramatically more informed relaxed plan heuristics.