Single period optimization

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1 Alpha Translation

- Variables
  - N: the number of stocks (including ETFs) in the universe.
  - N_e: the number of ETFs in the universe, note $N_e < N$.
  - $\Sigma \in \mathbb{R}^{N \times N}$: the asset covariance matrix.
  - $M \in \mathbb{R}^{N \times N_e}$: the ETF hedging binary matrix, $M_{ij} = 1$ iff stock i is hedged by ETF j.
  - $tg \in \mathbb{R}^N$: original target position of the instruments (in dollars).
  - $Sr \in \mathbb{R}^N$: portfolio Sharpe ratio.
  - $\alpha \in \mathbb{R}^N$: expected alpha of the instruments.
- Expression

$$\alpha = \frac{Sr\sqrt{tg^T \Sigma tg}}{tg(P^{-1}\Sigma tg)^T} P^{-1}\Sigma tg$$

where

$$P = I - M(M^T \Sigma^{-1} M)^{-1} M^T \Sigma^{-1}$$

please refer to alpha_conversion.pdf - Approach III for process details.

2 Optimizaton

- Variables
  - N: the number of stocks in the universe.
  - N_e: the number of ETFs in the universe, note $N_e < N$.
  - K: the number of barra factors.
  - K_g: the number of barra industry factors, where $K_g < K$.
  - $tg \in \mathbb{R}^N$: original target of the instruments.
  - $\alpha \in \mathbb{R}^N$: expected alpha of the instruments.
  - $b_k \in \mathbb{R}^N$: the exposure (to barra factors) of all of the instruments wrt barra factor k.
  - $b_{k_g} \in \mathbb{R}^N$: the exposure of all of the instruments wrt barra industry factor k_g.
  - $h \in \mathbb{R}^N$: the optimal position (in dollars).
  - $h_0 \in \mathbb{R}^N$: the starting (current) position (in dollars).
  - $z \in \mathbb{R}^N$: the trading mandate (in dollars), note that $h_0 + z = h$.
  - $\Sigma \in \mathbb{R}^{N\times N}$: the asset covariance matrix.
  - $\Omega \in \mathbb{R}^{K\times K}$: the barra factor covariance matrix.
  - $D \in \mathbb{R}^{N\times N}$: barra idiosyncratic volatility. $D = diag(\sigma_i^2)_{i=1:N}$, where $\sigma_i^2 = E(\epsilon_i^2)$.
  - $B \in \mathbb{R}^{N\times K}$: the barra exposure matrix, note that $B = col(b_k)_{b=1:K}$, $\Sigma = B\Omega B^T + D$.
  - $B_g \in \mathbb{R}^{N\times K_g}$: the barra industry exposure matrix, note that $B_g = col(b_g)_{b=1:K_g}$.
  - $M \in \mathbb{R}^{N\times N_e}$: the ETF hedging binary matrix, $M_{ij} = 1$ iff stock i is hedged by ETF j.
- Risk limits
  - $m_{b_{k_g}} \in \mathbb{R}^{K_g}$: maximum net exposure for barra industry factor k_g
  - $m_{|b_{k_g}|} \in \mathbb{R}^{K_g}$: maximum gross exposure for barra industry factor k_g
  - $m_\beta \in \mathbb{R}$: maximum beta (barra PredBeta) exposure.
  - $m_z \in \mathbb{R}^N$: maximum trade limit
  - $m_h \in \mathbb{R}^N$: maximum position limit
  - $m_{|z|} \in \mathbb{R}$: maximum turnover limit
- Parameters
  - $\lambda_\sigma \in \mathbb{R}$: risk aversion coefficient
  - $\theta_i \in \mathbb{R}^N$: market impact model
  - $\theta_{hc} \in \mathbb{R}^N$: holding cost model

- Problem

$$\max_{h,z} \langle \alpha, h \rangle - \lambda_\sigma \langle \Sigma h, h \rangle - \theta_i(z) - \theta_{hc}(h)$$

subject to

  - $\star$ $\langle B_g, |h| \rangle \leq m_{|b_{k_g}|}$: portfolio-wide industry gross limits.
  - $\star$ $|\langle B_g, h \rangle| \leq m_{b_{k_g}}$: portfolio-wide industry net limits.
  - $\star$ $|\langle \beta, h \rangle| \leq m_\beta$: portfolio-wide beta limit.
  - $\star$ $\left\langle M^T, h \right\rangle = 0$: portfolio-wide ETF dollar neutrality constraint.
  - $\star$ $h_i \cdot tg_i \geq 0$: alpha sign constraints for all non-ETF assets $i \in [1:N]$.
  - $\star$ $|z_i| \leq m_z$: max trade limit for all non-ETF assets $i \in [1:N]$.
  - $\star$ $|h_i| \leq m_h$: max position limit for all non-ETF assets $i \in [1:N]$.
  - $\star$ $\|z\|_1 \leq m_{|z|}$: max turnover limit for the portfolio.

3 JPM Market Impact Model

- Variables
  - $z \in \mathbb{R}^N$: the trading mandates (in dollars).
  - $V \in \mathbb{R}^N$: 30-day average daily volumes (in dollars).
  - ν: the pre-trade estimate of the constant trade rate of the order, note $\nu_i \cdot V_i = z_i$ for $i = 1 : N$.
  - T: duration of the order (in volume time), note $T = 1$ as we assume all day VWAP.
  - Δ: the pre-trade estimate of the bid-ask spread of the traded security over the trade period, expressed as the fraction of the price.
  - σ: volatility of the traded security
- Parameters
  - ⋆ $\alpha = 518$
  - ⋆ $\beta = 0.889$
  - ⋆ $\gamma = 0.663$
  - ⋆ $1 - \omega = 0.394$
  - ⋆ $\delta = 0.068$
- Expression

$$\theta_i(\cdot) = (10^{-4}\alpha) \times \sigma^{\beta} \times \nu^{\gamma} + \delta \times \Delta$$

Final result is in decimals and annualized.

4 Holding Cost Model

- Variables
  - $h \in \mathbb{R}^N$: the optimal position (in dollars).
  - $h_+ \in \mathbb{R}^N$: the positive part of the optimal position, note $(h_i)_+ = \max(h_i, 0)$ for $i = 1 : N$.
  - $h_- \in \mathbb{R}^N$: the negative part of the optimal position, note $(h_i)_- = -\min(h_i, 0)$ for $i = 1 : N$.
- Parameters
  - ⋆ $s_+ \in \mathbb{R}^N$: the financing fee, note $(s_i)_+ = 0.003$ for $i = 1 : N$.
  - ⋆ $s_- \in \mathbb{R}^N$: the borrowing fee, note $(s_i)_- = 0.003$ for $i = 1 : N$.
- Expression

$$\theta_{hc}(\cdot) = s_+^T \cdot h_+ + s_-^T \cdot h_-$$