

Ecuación de Onda 2D:

$$u_{tt}(x, y, t) = \alpha^2 u_{xx}(x, y, t) + \beta^2 u_{yy}(x, y, t)$$

En coordenadas cilíndricas

$$u_{tt}(\rho, \phi, t) = \alpha^2 \left(u_{\rho\rho} + \frac{1}{\rho} u_{\rho} \right) + \beta^2 \left(\frac{1}{\rho^2} u_{\phi\phi} \right)$$

Discretizando:

$$\frac{u_{ij}^{l+1} - 2u_{ij}^l + u_{ij}^{l-1}}{\Delta t^2} = \alpha^2 \left[\frac{u_{i+1,j}^l - 2u_{ij}^l + u_{i-1,j}^l}{(\Delta \rho)^2} + \frac{1}{\rho_{ij}^2} \left(\frac{u_{i,j+1}^l - u_{i,j-1}^l}{\Delta \phi} \right)^2 \right] + \beta^2 \left[\frac{u_{i,j+1}^l - 2u_{ij}^l + u_{i,j-1}^l}{(\Delta \phi)^2} \right]$$

$$\begin{aligned}
 U_{i,j}^{k+1} = & \frac{\Delta t^2}{\rho^2} \left[\cancel{U_{i+1,j}^1 - 2U_{i,j}^1 + U_{i-1,j}^1} \right] \\
 & + \frac{\Delta p}{\rho[i]} \left(v_{i+1,j}^1 - v_{i-1,j}^1 \right) \\
 & + \frac{\Delta t^2 \rho^2}{(\Delta \phi)^2} \left[U_{i,j+1}^1 - 2U_{i,j}^1 + U_{i,j-1}^1 \right]
 \end{aligned}$$

\therefore

$$\begin{aligned}
 U_{i,j}^{k+1} = & V^2 \left[\cancel{U_{i+1,j}^1 - 2U_{i,j}^1 + U_{i-1,j}^1} \right] \\
 & + \frac{\Delta p}{\rho[i]} \left(v_{i+1,j}^1 - v_{i-1,j}^1 \right)
 \end{aligned}$$

$$+ \lambda^2 \left[u_{i,j+1}^{(1)} - 2u_{i,j}^{(1)} + u_{i,j-1}^{(1)} \right]$$