1. $\boxed{f(x) = x^2}$ 1 función de prueba

$$f'(x) = \frac{-f(x+2h)+4f(x+h)-3f(x)}{2h}$$

$$\lim_{h\to 0} \frac{-(x+2h)^2+4(x+h)^2-3x^2}{2h}$$

$$=\lim_{h\to 0} \frac{-(x+2h)(x+2h)+4x^2+4h^2+8xh-3x^2}{2h}$$

$$=\lim_{h\to 0} \frac{-\cancel{(x^2}+4xh+\cancel{4h^2)}+\cancel{4x^2}+\cancel{4h^2}+8xh-\cancel{3x^2}}{2h}$$

$$=\lim_{h\to 0} 2\frac{\cancel{4}x\cancel{h}}{\cancel{2h}}$$

$$= \lim_{h\to 0} 2x = 2x$$

$$f'(x) = 2x$$ ✓ es consistente

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 2x^2 + (x-h)^2}{h^2}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + \cancel{2xh} + h^2 \cancel{- 2x^2} + \cancel{x^2} \cancel{- 2xh} + h^2}{h^2}$$

$$= \lim_{h \to 0} \frac{2\cancel{h^2}}{\cancel{h^2}} = \lim_{h \to 0} 2 = 2$$

$$f''(x) = 2$$

$$\boxed{f(x) = \sin(x)}$$ 2da funcion prueba

$$f'(x) = \frac{-f(x+2h)+4f(x+h)-3f(x)}{2h}$$

$$\lim_{h\to 0} \frac{-\sin(x+2h)+4\sin(x+h)-3\sin x}{2h}$$

$$= \lim_{h\to 0} \frac{-(\sin(x+2h))' + [4\sin(x+h)]' - 3[\sin x]'}{[2h]'}$$

$$= \lim_{h\to 0} \frac{-2\cos(x+2h)+4\cos(x+h)}{2}$$

$$= \lim_{h\to 0} -\cos(x+2h) + 2\cos(x+h)$$

$$= -\cos(x) + 2\cos(x)$$

$$= \cos(x)$$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

$$\lim_{h\to 0} \frac{\sin(x+h) - 2\sin x + \sin(x-h)}{h^2}$$

$$= \lim_{h\to 0} \frac{\cos(x+h) - \cos(x-h)}{2h}$$

$$= \lim_{h \to 0} \frac{-\sin(x+h) - (-\sin(x-h))}{2}$$

$$= \lim_{h \to 0} \frac{-\sin(x+h) - \sin(x-h)}{2}$$

$$\frac{-\sin(x) - \sin(x)}{2}$$

$$-\frac{\cancel{2}\sin(x)}{\cancel{2}}$$

$$-\sin(x)$$

$f''(x) = -\sin x$ ✓

es consistente

3. $c = 3 \times 10^8$ m/s

$1\,au = 1.496 \times 10^{11}$ m

1 día = 86400 s

1 año = 365 días

$$c = 3 \times 10^8 \frac{m}{s} \left(\frac{1\,au}{1.496 \times 10^{11}\,m} \right) \left(\frac{86400\,s}{1\,\text{día}} \right) \left(\frac{365\,\text{días}}{\text{año}} \right)$$

$$c = 63{,}240 \text{ ua/año}$$

5. $v_0(0) = v_0$

$$\frac{\delta v}{\delta t} = \alpha v$$

$$\delta u = \alpha v \, dt$$

$$\Delta v = \alpha v_0 \Delta t$$

$$v_1 = v_0 + \Delta v$$

$$v_1 = v_0 + \alpha v_0 \Delta t$$

$$v_1 = v_0 (1 + \alpha \Delta t)$$

$$v_k = v_{k-1} (1 + \alpha \Delta t)$$

$$U_k = \left(U_{k-2}(1+\alpha\Delta t)\right)(1+\alpha\Delta t)$$

$$U_k = U_{k-k}\left[\prod_{i=1}^{k}(1+\alpha\Delta t)\right]$$

$$U_k = U_0(1+\alpha\Delta t)^k$$

Si $\alpha < 0$:

$$1+\alpha\Delta t = 1-|\alpha|\Delta t$$

Si $\Delta t > -\frac{1}{\alpha}$

$$\Delta t > \frac{1}{|\alpha|}$$

$$|\alpha| \Delta t > 1$$

$$0 > 1 - |\alpha| \Delta t$$

$$1 - |\alpha| \Delta t < 0$$

$$1 + \alpha \Delta t < 0$$

$$\rightarrow \quad \forall_{n \in \mathbb{N}} :$$

$$k = 2n \Leftrightarrow (1 + \alpha \Delta t)^k > 0$$

$$k = 2n+1 \Leftrightarrow (1 + \alpha \Delta t)^k < 0$$

$$\therefore \; v_k = v_0 (1 + \alpha \Delta t)^k$$

oscila de positivo a negativo en cada punto