$$U_i^{\lambda+1} = 2\left(1-\lambda^2\right)v_i^{\lambda} + \lambda^2\left(v_{i+1}^{\lambda} + v_{i-1}^{\lambda}\right) - v_i^{\lambda-1}$$

Fourier: $V_{i\pm1}^{\lambda} = e^{\pm jk\Delta x} \;\pi\; \omega k\Delta x$

$$U_i^{\lambda+1} = 2\left(1-\lambda^2\right)v_i^{\lambda} + \lambda^2\left(e^{\pm jk\Delta x} \;\pi\; \omega k\Delta x + e^{-jk\Delta x}\right)v_i^{\lambda}$$
$$- v_i^{\lambda-1}$$

$$U_i^{\lambda+1} = \left(2 + \lambda^2\left(-\frac{1}{2}\left(\frac{\sin^2(k\Delta x + \omega\Delta x)}{2}\right)\right)\right)v_i^{\lambda}$$
$$- u_i^{\lambda-1}$$

$$U_i^{\lambda+1} = \left(2 + \lambda^2\left(-\sin^2\left(\frac{k\Delta x + \omega\Delta x}{2}\right)\right)\right)v_i^{\lambda}$$
$$- v_i^{\lambda-1}$$

Para estabilidad:

$$1 + \lambda^2 \left[- \sin^2\left(\frac{k\Delta x}{2} + \frac{\omega \Delta x}{2}\right) \le 1\right.$$

$$\cancel{-} \lambda^2 \sin^2\left(\frac{k\Delta x + \omega \Delta x}{2}\right) \le \cancel{-} 1$$

En el peor caso el modo de Fourier es grande

$$\lambda^2 \le 1$$

$$\lambda \le 1$$

10.

a)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + u\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial x^2} + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{U_{t+1} - U_t}{\Delta p} + U\left(\frac{U_{x+h_1} - U_x}{2\Delta h_1}\right) + U\frac{\left(U_{y+h_2} - U_y\right)}{2\Delta h_2}$$

$$= \nu\left(\frac{U_{x_1+h_1} - 2U_x + U_{x_1-h}}{(\Delta h_1)^2} + \frac{U_{y_1+h_2} - 2U_y + U_{y_1-h_2}}{(\Delta h_2)^2}\right)$$

$$\Delta h_1 = \Delta h_2$$

$$\frac{U^{l+1} - U^{l}}{\Delta p} = -U\left(\frac{U_{x+h} - U_x + U_{y+h} - U_y}{2\Delta h}\right)$$

$$+ \nu\left(\frac{U_{x+h} - 2U_x + U_{x-h} + U_{y+h} - 2U_y + U_{y-h}}{(\Delta h)^2}\right)$$

$$v^{l+1} = v^{l} - \frac{\Delta p\, u}{2\Delta h}\left(u_{x+h} - u_{x} + u_{y+h} - u_{y}\right)$$

$$+ \nu \frac{\Delta p}{(\Delta h)^2}\left(u_{x+h} - 2u_{x} + u_{x-h} + u_{y+h} - 2u_{y} + u_{y-h}\right)$$