En el artículo se presentan las siguientes ecuaciones:

Predictor

$$r_{n+1} = r_n + h v_n + h^2 \sum_{p=1}^{q-1} b_p a_{n-q+1}$$

Corrector Posición

$$r_{n+1} = r_n + h v_n + h^2 \sum_{p=1}^{q-1} c_p a_{n-q+2}$$

$$h v_{n+1} = r_{n+1} - r_n + h^2 \sum_{p=1}^{q-1} d_p a_{n-q+2}$$

Sin embargo, las ecuaciones tienen un error ya que en las sumatorias, están términos a_{n-q+1} y a_{n-q+2} que dependen de q. y no p.

Se proponen las siguientes ecuaciones corregidas:

Predictor

$$r_{n+1} = r_n + h v_n + h^2 \sum_{p=1}^{q-1} b_p a_{n-p+1}$$

Corrector Posición

$$r_{n+1} = r_n + h v_n + h^2 \sum_{p=1}^{q-1} c_p a_{n-p+2}$$

$$h v_{n+1} = r_{n+1} - r_n + h^2 \sum_{p=1}^{q-1} d_p a_{n-p+2}$$

Cuando $q=3$

Predictor: $r_{n+1} = r_n + h v_n + h^2 (b_1 a_n + b_2 a_{n-1})$

Corrector: $r_{n+1} = r_n + h v_n + h^2 (c_1 a_{n+1} + c_2 a_n)$

Corrector: $h v_{n+1} = r_{n+1} - r_n + h^2 (d_1 a_{n+1} + d_2 a_n)$

Expandiendo en serie de Taylor con 3 terminos:

$$r_{n+1} = \frac{r_n}{0!} + \frac{v_n h}{1!} + \frac{h^2}{2!}(b_1 a_n + b_2 a_{n-1})$$

$$r_{n+1} = r_n + v_n h + \frac{h}{2!}(b_1 a_{n+1} + b_1 a_n)$$

$$r_{n+1} = \frac{r_n}{0!} + \frac{v_n h}{1!} + \frac{h^2}{2!}(c_1 a_{n+1} + c_2 a_n)$$

$$r_{n+1} = r_n + v_n h + \frac{h^2}{2}(c_1 a_{n+1} + c_2 a_n)$$

$$h v_{n+1} = \frac{h v_{n+1}}{0!} + \frac{v_n (-h)}{1!} + \frac{h^2}{2}(d_1 a_{n+1} + a_n)$$

Con eso se encuentran los coeficientes

... : $\frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = 0$

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$y' = f(t,y)$

$y(t_{n+1}) = y(t_n) + \int_{t_n}^{t_{n+1}} y'(t)\,dt$

entonces:

$$A = \int_{t_n}^{t_{n+1}} y'(t)\,dt = \int_{t_n}^{t_{n+1}} f(t, y(t))\,dt$$

interpolando:

$$P(t) = f(t_n, y_n)\frac{(t-t_{n-1})(t-t_{n-2})}{(t_n-t_{n-1})(t_n-t_{n-2})} + f(t_{n-1}, y_{n-1})\frac{(t-t_n)(t-t_{n-2})}{(t_{n-1}-t_n)(t_{n-1}-t_{n-2})} + f(t_{n-2}, y_{n-2})\frac{(t-t_n)(t-t_{n-1})}{(t_{n-2}-t_n)(t_{n-2}-t_{n-1})}$$

deducimos que $t_{n-1} - t_{n-2} = t_n - t_{n-1} = t_{n+1} - t_n = \underline{h}$

$$\frac{f(t_n, y_n)}{2h^2}\int_{t_n}^{t_{n+1}} (t-t_{n-1})(t-t_{n-2})\,dt = \frac{23}{12} h f(t_n, y_{n-1}).$$

$$-\frac{f(t_{n-1}, y_{n-1})}{h^2}\int_{t_n}^{t_{n+1}} (t-t_n)(t-t_{n-2})\,dt = -\frac{4}{3} h f(t_{n-1}, y_{n-1}).$$

$$\frac{f(t_{n-2}, y_{n-2})}{2h^2}\int_{t_n}^{t_{n+1}} (t-t_n)(t-t_{n-1})\,dt = \frac{5}{12} h f(t_{n-2}, y_{n-2})$$

entonces $A = \frac{23}{12} h f(t_n, y_n) - \frac{4}{3} h f(t_{n-1}, y_{n-1}) + \frac{5}{12} h f(t_{n-2}, y_{n-2})$

$$y(t_{n+1}) = y(t_n) + \frac{23}{12} h f(t_n, y_n) - \frac{4}{3} f(t_{n-1}, y_{n-1}) + \frac{5}{12} h f(t_{n-2}, y_{n-2})$$

$$y_{n+1} = y_n + \frac{h}{12}(23 f_n - 16 f_{n-1} + 5 f_{n-2}).$$

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$$y(n,y) = f_0 + n\nabla f_0 + \frac{n(n+1)}{2}\nabla^2 f_0 + \dots \to ①$$

$x = x_0 + nh$, $f_0 = f(x_0, y_0)$

$$y_1 = y_0 + \int_{x_0}^{x_0+h} f(x,y)\,dx = y_0 + \int_{x_0}^{x_1} \left(f_0 + n\nabla f_0 + \frac{n(n+1)}{2}\nabla^2 f_0 + \dots\right) dx$$

$$= y_0 + \int_0^1 \left(f_0 + n\nabla f_0 + \frac{n(n+1)}{2}\right) dx$$ utilizando esta vez $dx = h\,dn$

$$= y_0 + h\left[f_0 + \frac{n^2}{2}\nabla f_0 + \frac{\left(\frac{n^3}{3} + \frac{n^2}{2}\right)}{2}\nabla^2 f_0 + \dots\right]_0^1$$

$$= y_0 + h\left[f_0 + \frac{\nabla f_0}{2} + \frac{5}{12}\nabla^2 f_0 + \frac{3}{8}\nabla^3 f_0 + \dots\right]$$

$\frac{\nabla f_0}{2}$ $= (1 - E^{-1}) f_0 = f_0 - E^{-1} f_0 = f_0 - f_{-1}$

$\nabla^2 f_0 = \nabla(f_0 - f_{-1}) = \nabla f_0 - \nabla f_{-1} = f_0 - f_{-1} - (f_{-1} - f_{-2})$

$= f_0 - 2f_{-1} + f_{-2}$ $\qquad \nabla^3 f_0 = f_0 - 3f_{-1} + 3f_{-2} - f_{-3}$

Reemplazando:

$$\triangleright\ f_0 + \frac{(f_0 - f_{-1})}{2} + \frac{5}{12}(f_0 - 2f_{-1} + f_{-2}) + \frac{3}{8}(f_0 - 3f_{-1} + 3f_{-2} - f_{-3})$$

$$f_0 + \frac{f_0}{2} - \frac{f_{-1}}{2} + \frac{5f_0}{12} - \frac{10f_{-1}}{12} + \frac{5f_{-2}}{12} + \frac{3}{8}f_0 - \frac{9f_{-1}}{8} + \frac{9f_{-2}}{8} - \frac{3}{8}f_{-3}$$

Resolviendo

$$y_1 = y_0 + \frac{h}{24}\left[55f_0 - 59f_{-1} + 37f_{-2} - 9f_{-3}\right]$$

$$\frac{\partial f}{\partial x} - \frac{d}{dt}\left(\frac{\partial f}{\partial \dot{x}}\right) = 0$$

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$$P_4(\tau) = f_{n-2} L_{n-2}(\tau) + f_{n-1} L_{n-1}(\tau) + f_n L_n(\tau) + f_{n+1} L_{n+1}(\tau)$$

donde

$$L_{n-2}(\tau) = \frac{(\tau - t_{n-1})(\tau - t_n)(\tau - t_{n+1})}{(t_{n-2} - t_{n-1})(t_{n-2} - t_n)(t_{n-2} - t_{n+1})} = -\frac{1}{6h^3}(\tau - t_{n-1})(\tau - t_n)(\tau - t_{n+1})$$

$$L_{n-1}(\tau) = \frac{(\tau - t_{n-2})(\tau - t_n)(\tau - t_{n+1})}{(t_{n-1} - t_{n-2})(t_{n-1} - t_n)(t_{n-1} - t_{n+1})} = \frac{1}{2h^3}(\tau - t_{n-2})(\tau - t_n)(\tau - t_{n+1})$$

$$L_n(\tau) = \frac{(\tau - t_{n-2})(\tau - t_{n-1})(\tau - t_{n+1})}{(t_n - t_{n-2})(t_n - t_{n-1})(t_n - t_{n+1})} = -\frac{1}{2h^3}(\tau - t_{n-2})(\tau - t_{n-1})(\tau - t_{n+1})$$

$$L_{n+1}(\tau) = \frac{(\tau - t_{n-2})(\tau - t_{n-1})(\tau - t_n)}{(t_{n+1} - t_{n-2})(t_{n+1} - t_{n-1})(t_{n+1} - t_n)} = \frac{1}{6h^3}(\tau - t_{n-2})(\tau - t_{n-1})(\tau - t_n)$$

entonces:

$$y_{n+1} = y_n + \int_{t_n}^{t_{n+1}} P_4(\tau)\, d\tau =$$

$$y_n + f_{n-2}\int_{t_{n-1}}^{t_{n+1}} L_{n-2}(\tau)\, d\tau + f_{n-1}\int_{t_n}^{t_{n+1}} L_{n-1}(\tau)\, d\tau + f_n \int_{t_n}^{t_{n+1}} L_n(\tau)\, d\tau$$

$$+ f_{n+1}\int_{t_n} L_{n+1}(\tau)\, d\tau \qquad \tau = t_n + hu \quad d\tau = h\,du \quad u = \frac{\tau - t_n}{h}$$

sabiendo

$$l_{n-2}(u) = \frac{1}{6} u(1-u^2)$$

$$l_{n-1}(u) = -\frac{1}{2} u(u+2)(1-u)$$

$$l_n(u) = \frac{1}{2}(u+2)(1-u^2)$$

$$l_{n+1}(u) = \frac{1}{6} u(u+1)(u+2)$$

$f_{n-2} L_{n-2}(\tau) + f_{n-1} L_{n-1}(\tau) + f_n L_n(\tau) + f_{n+1} L_{n+1}(\tau)$

$(\tau) = \frac{(\tau - t_{n-1})(\tau - t_n)(\tau - t \ldots)}{(t_{n-2} - t \ldots) \ldots}$

$$\int_{t_n}^{t_{n+1}} L_{n-2}(\tau)\,d\tau = h \int_0^1 L_{n-2}(u)\,du = \frac{h}{6} \int_0^1 u(1-u^2)\,du = \frac{1}{24} h$$

$$\int_{t_n}^{t_{n+1}} L_{n-1}(\tau)\,d\tau = h \int_0^1 L_{n-1}(u)\,du = \frac{-h}{2} \int_0^1 u(u+2)(1-u)\,du = \frac{-5}{24} h$$

$$\int_{t_n}^{t_{n+1}} L_n(\tau)\,d\tau = h \int_0^1 L_n(u)\,du = \frac{h}{2} \int_0^1 (u+2)(1-u^2)\,du = \frac{19}{24} h$$

$$\int_{t_n}^{t_{n+1}} L_{n+1}(\tau)\,d\tau = h \int_0^1 L_{n+1}(u)\,du = \frac{h}{6} \int_0^1 u(u+1)(u+2)\,du = \frac{3}{8} h$$

entonces:

$$y_{n+1} = y_n + \frac{h}{24}\left(f_{n-2} - 5 f_{n-1} + 16 f_n + 9 f_{n+1}\right)$$

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interpolando:

$$\frac{(t-t_i)(t-t_{i-1})}{(t_{i+1}-t_i)(t_{i+1}-t_{i-1})} f_{(i+1)} + \frac{(t-t_{i+1})(t-t_{i-1})}{(t_{i+1}-t_{i+1})(t_{i+1}-t_{i-1})} f_{(i)}$$

$$+ \frac{(t-t_{i+1})(t-t_i)}{(t_{i-1}-t_{i+1})(t_{i-1}-t_i)} f_{i-1}\, dt$$

donde $f_{i+1} = f(t_{i+1}, w_{i+1})$

$f_i = f(t_i, w_i)$

$f_{i-1} = f(t_{i-1}, w_{i-1})$

Integrando: $t_{i+1} = t_i + h$

$f_i = t_i$

$f_{i-1} = t_i - h$

entonces

$$\int_{t_i}^{t_i+h} \frac{(t-t_i)(t-(t_i-h))}{(t_i+h-t_i)(t_i+h(t_i-h))} f_{i+1} + \frac{(t-(t_i+h))(t-(t_i-h))}{(t_i+h-(t_i+h))(t_i+h-(t_i-h))} f_i +$$

$$\frac{(t-(t_i+h))(t-t_i)}{(t_i-h-(t_i+h))(t_i-h-t_i)} f_{i-1}\, dt$$

Haciendo la sustitución $t_i+h = 1$ y $t_i = 0$, $h = 1$

$$\frac{f_{i+1}}{h^2}\int_0^1 (t-t_i)(t-(t_i-h))\,dt + \frac{f_i}{h^2}\int_0^1 (t-t_i-h)(t-t_i+h)$$

$$+ \frac{f_{i-1}}{2h^2}\int_0^1 (t-t_i-h)(t-t_i)$$

entonces

$$h\int_0^1 \frac{t(t+1)}{2} f_{i+1}\,dt + h\int_0^1 (t-1)(t+1) f_i\,dt + h\int_0^1 \frac{t(t-1)}{2} f_{i-1}\,dt$$

$$\frac{hf_{i+1}}{2}\left[\frac{t^3}{3}+\frac{t^2}{3}\right]_0^1 + hf_i\left[\frac{t^3}{3}-t\right]_0^1 + \frac{hf_{i-1}}{2}\left[\frac{t^3}{3}-\frac{t^2}{2}\right]_0^1$$

$h \frac{f_{i+1}}{2}\left(\frac{5}{6}\right) + h f_i \left(\frac{2}{3}\right) + h \frac{f_{i-1}}{2}\left(-\frac{1}{6}\right)$

entonces

$$y_{n+1} = y_n + \frac{h}{12}\left(5 f_{n+1} + 8 f_n - f_{n-1}\right)$$