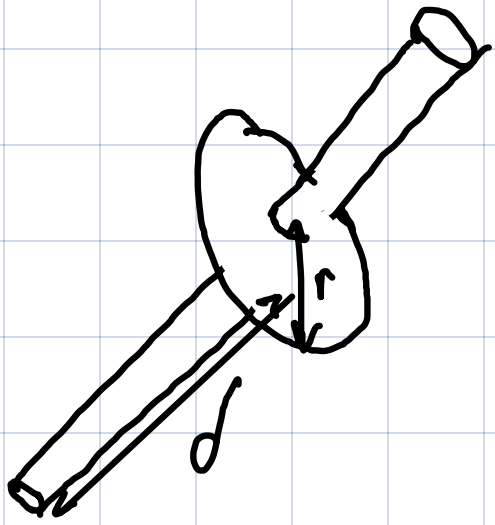
3.9



$$I_1 = \frac{1}{12} m (2d)^2 = \frac{1}{3} m d^2$$

$$I_2 = \frac{1}{2} m r^2 + m d^2$$

por el teorema de ejes paralelos

$$I = \frac{1}{3} m d^2 + m d^2 + \frac{1}{2} m r^2$$

$$= \frac{4}{3} m d^2 + \frac{1}{2} m r^2$$

$$= \frac{8 m d^2 + 6 m r^2}{6}$$

$$= \frac{1}{4} m r^2 + m d^2$$

b) $\sigma \rightarrow$ densidad masa

$$dI = \sigma r^2 dV$$

$$\sigma = \frac{m}{V} = \frac{M}{\pi r^2 h}$$

$$V = \pi r^2 h$$

$$dV = 2\pi r h \, dr$$

$$I = \frac{m}{\pi r^2 h} \int_0^r r^2 \, 2\pi r h \, dr$$

$$= \frac{m}{\pi r^2 h} 2\pi h \int_0^r r^3$$

$$= 2\frac{m}{r^2}\left(\frac{r^4}{4}\right)_0^r$$

$$= \frac{2m}{r^2}\left(\frac{r^4}{4}\right)$$

$$= \frac{1}{2} M r^2$$

c)

$$L = \frac{1}{2} I_0 \left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta \right) + \frac{1}{2} I_2 \left(\dot{\phi} \cos\theta + \dot{\psi} \right)^2 - M g \delta \delta \cos\theta$$

$$\frac{\partial L}{\partial \dot{\phi}} = \frac{1}{2} I_0 \left(2 \dot{\phi} \sin^2\theta \right) + \frac{1}{2} I_2 \left(2 \left(\dot{\phi} \cos\theta + \dot{\psi} \right) \cos\theta \right)$$

$$= I_0 \dot{\phi} \sin^2\theta + \frac{1}{2} I_2 \left(2 \dot{\phi} \cos^2\theta + 2 \dot{\psi} \cos\theta \right)$$

$$= I_0 \dot{\phi} \sin^2\theta + I_2 \dot{\phi} \cos^2\theta + I_2 \dot{\psi} \cos\theta$$

$$\frac{\partial L}{\partial \dot{\psi}} = \frac{1}{2} I_2 \left(2 \left(\dot{\phi} \cos\theta + \dot{\psi} \right) \right)$$

$$= I_2 \dot{\phi} \cos\theta + I_2 \dot{\psi}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{\partial L}{\partial \theta}$$

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{2} I_0 \cdot 2\dot{\theta} = I_0 \dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = I_0 \ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = mgd \sin\theta$$

$$I_0 \ddot{\theta} = mgd \sin\theta$$

$$\dot{\psi}^2 \sin\theta \cos\theta \left(I_\theta - I_z\right)$$

$$- \dot{\phi}\dot{\psi} I_z \sin\theta = 0$$

$$I_\theta \ddot{\theta} = \dot{\psi}^2 \sin\theta \cos\theta \left(I_\theta - I_z\right)$$

$$- \dot{\phi}\dot{\psi} I_z \sin\theta$$

$$+ mgd \sin\theta$$