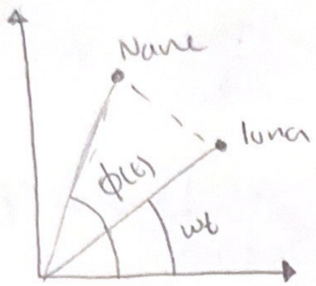


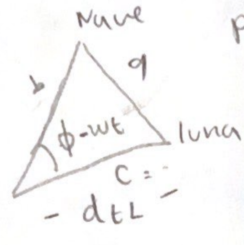
Por polares encontramos que ; para la nave :

$$x(t) = r(t)\cos\phi(t)$$
$$y(t) = r(t)\sin\phi(t)$$

la magnitud de la posición sería

$$r(t) = [x(t)^2 + y(t)^2]^{1/2}$$

asumiendo un triangulo entre las posiciones :



Por teorema del coseno :

$$a^2 = b^2 + c^2 - 2bc \cdot \cos(\alpha) .$$

$$r_L^2 = r(t)^2 + d_{tL}^2 - 2r(t)\, d_{TL}\cos(\phi - wt)$$

$$r_L = [r(t)^2 + d_{tL}^2 - 2r(t)\, d_{TL}\cos(\phi - wt)]^{1/2}$$

Sabiendo esto, el lagrangiano lo podemos definir como:
$L = T - V$, donde T es la energía cinetica y V es la energía Potencial.

donde: $$T = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2) \quad y \quad V = -\frac{G m M_T}{r} - \frac{G m M_L}{r_L(r, \phi, t)}$$

entonces, derivando para sacar los momentos conjugados

$$P_r = \frac{\partial L}{\partial \dot{r}} \quad P_\phi = \frac{\partial L}{\partial \dot{\phi}} \quad \text{donde } L = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2) - \frac{G m M_T}{r} - \frac{G m M_L}{r_L(r, \phi, t)}$$

como resultado :

$$p_r = m\dot{r} \ , \quad P_\phi = m r^2 \dot{\phi}$$

conociendo esto, el Hamiltoniano queda como:

$$H = p_r \dot{r} + p_\phi \dot{\phi} - L = \frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} - G\frac{mM_T}{r} - G\frac{mM_L}{r_L(r, \phi, t)}$$

entonces, conociendo H , se pueden hallar las ecuaciones de movimiento :

$$\dot{r} = \frac{\partial H}{\partial p_r}, \quad \dot{\phi} = \frac{\partial H}{\partial p_\phi}, \quad \dot{p}_r = -\frac{\partial H}{\partial r}, \quad \dot{p}_\phi = -\frac{\partial H}{\partial \phi}$$

$$\dot{r} = \frac{p_r}{m}, \quad \dot{\phi} = \frac{p_\phi}{mr^2}, \quad \dot{p}_r = \frac{p_\phi^2}{mr^3} - 6\frac{mM_T}{r^2} - \frac{6mM_L}{r_L(r,\phi,t)^3}\left[r - d\cos(\phi - \omega t)\right]$$

$$\dot{p}_\phi = -\frac{\partial H}{\partial \phi} = -\frac{6mM_L}{r_L(r,\phi,t)^3} rd\sin(\phi - \omega t).$$

tomando $\tilde{r} = \frac{p_r}{md}$ y $\tilde{p}_\phi = p_\phi/md^2$.

$$\dot{\tilde{r}} = \frac{\dot{r}}{d} = \frac{p_r}{md}$$

$$\dot{\phi} = \frac{p_\phi}{mr^2} = \frac{\cancel{m}d^2\tilde{p}_\phi}{d^2\tilde{r}^2\cancel{m}} = \frac{\tilde{p}_\phi}{\tilde{r}^2}$$

$$\dot{\tilde{p}}_r = \frac{\tilde{p}_\phi^2}{\tilde{r}^3} - \frac{6M_T}{d^3}\left(\frac{1}{\tilde{r}^2} + \frac{M}{\tilde{r}'^3}\left[\tilde{r} - \cos(\phi - \omega t)\right]\right]$$

$$\dot{\tilde{p}}_r = \frac{\dot{p}_r}{md} = \frac{-1}{md}\left(\frac{p_\phi^2}{mr^2} - \frac{6mM_T}{r^2} - \frac{6mM_L}{r_L^3}\left[r - d\cos(\phi - \omega t)\right]\right]$$

$$= \frac{-1}{md}\left(\frac{(md^2p_\phi)^2}{m(d\tilde{r})^3} - \frac{6mM_T}{(d\tilde{r})^2} - \frac{6mM_L}{(d\tilde{r}')^3}\left[d\tilde{r} - d\cos(\phi - \omega t)\right]\right)$$

$$= \frac{-1}{md}\left(\frac{(md^2\tilde{p}_\phi)^2}{m(d\tilde{r})^3} - \frac{6mM_T}{(d\tilde{r})^2} - \frac{6mM_L}{d^3(\tilde{r}')^3}\left[\tilde{r} - \cos(\phi - \omega t)\right]\right)$$

$$= -\frac{1}{d^2}\left(\frac{\tilde{p}_\phi^2}{\tilde{r}^3} - \Delta\left[\frac{1}{\tilde{r}^2} + \frac{M}{\tilde{r}'^3}\left[\tilde{r} - \cos(\phi - \omega t)\right]\right]\right)$$

$$\dot{p}_\phi = -\frac{1}{md^2} G\frac{mm_L}{r_L^3} rd\sin(\phi-\omega t)'$$

$$= -\frac{1}{md^2} G\frac{mm_L}{(d\tilde{r}')^3} dd\tilde{r}\sin(\phi-\omega t).$$

$$= \Delta\mu \frac{\tilde{r}}{\tilde{r}'^3}\sin(\phi-\omega t).$$

entonces

$$\dot{\tilde{r}} = \tilde{p}_r \cdot \quad \dot{\phi} = \frac{\tilde{p}_\phi}{\tilde{r}^2}, \quad \dot{\tilde{p}}_r = \frac{\tilde{p}_\phi^2}{\tilde{r}^3} - \Delta\left[\frac{1}{\tilde{r}^2} + \frac{\mu}{\tilde{r}'^3}[\tilde{r} - \cos(\phi-\omega t)]\right]$$

$$\dot{\tilde{p}}_\phi = -\Delta\mu\frac{\tilde{r}}{\tilde{r}'^3}\sin(\phi-\omega t).$$

donde $\Delta \equiv \frac{Gm_T}{d^3}$, $\mu = \frac{m_L}{m_T}$, $r' \equiv \sqrt{1+\tilde{r}^2-2\tilde{r}\cos(\phi-\omega t)}$