Examen de Control de Sistemas Lineales

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$$\frac{d^3}{dt^3}y(t) - \frac{dy(t)}{dt} = \left(1+\frac{2d}{dt}\right)\left(1-\frac{2d}{dt}\right)u(t) \qquad (1)$$

→ Factorización Coprima obtenga una realización polinomial coprima

$$M\left(\frac{d}{dt}\right)y(t) = N\left(\frac{d}{dt}\right)u(t)$$

de la EDO (1)

$$\frac{d^3}{dt^3}y(t) - \frac{dy(t)}{dt} = \left(1+\frac{2d}{dt}\right)\left(1-\frac{2d}{dt}\right)u(t)$$

$$\left(\frac{d^3}{dt} - \frac{dy}{dt}\right)y(t) = \left(1+\frac{2d}{dt}\right)\left(1-\frac{2d}{dt}\right)u(t)$$

$$\underbrace{(s^3-s)}_{M(s)}y(t) = (1+2s)(1-2s)u(t)$$

$$= 1-\cancel{2s}+\cancel{2s}-4s^2$$

$$= 1-4s^2$$

$$= (\underbrace{-4s^2+1}_{N(s)})u(t)$$

$M(s) = (s^3-s)$

$N(s) = (-4s^2+1)$

Algoritmo de división de Euclides Diseñe una ley de control lineal propia tal que la función de transferencia en lazo cerrado sea

$$F_{lc}(s) = \frac{1}{(s+1)^3}$$

¿Es razonable aplicar esta Ley de Control?

Sean $M(s) = (s^3 - s)$ y $N(s) = (-4s^2 + 1)$

se desea

$$Q(s) = (s+1)^3$$

MATLAB

```
clear
clc
syms s a e
N= -4*s^2+1
M = collect(s^3-s, s)
Q= collect((s+1)^3, s)
[S, R] = quorem(Q, M, s)
factor(R)
```

```
N= 1-4*s^2
M = s^3 - s
Q= s^3+3*s^3+3s+1
S= 1
R= 3*s^3 + 4*s + 1
   (s+1)(3*s+1)
```

$S = 1$

$R = (s+1)(3s+1)$

$M(s) = s^3 - s$

$N(s) = -4s^2 + 1$

Ley de control

$S(d/dt)\,N(d/dt)\,u(t) = -R(d/dt)\,y(t) + u_{ref}$

$$(1)\left(-4\frac{d^2}{dt^2}+1\right)u(t) = -\left(\frac{d}{dt}+1\right)\left(3\frac{d}{dt}+1\right)y(t) + u_{ref}$$

¿Es razonable aplicar esta ley de control?

Si es razonable por el grado de $Q(s)$ que es mayor a 2

Matriz de Silvester Obtenga la matriz de silvester de la realizacion coprima obtenida en el inciso 1 Verifique su invertibilidad

$$M(s) = s^3 - s \rightarrow \overset{(1)}{s^3} + \overset{a_1}{0s^2} - \overset{a_2}{s} + \overset{a_3}{0}$$

$$N(s) = -4s^2 + 1 \rightarrow \underset{b_0}{0s^3} - \underset{b_1}{4s^2} + \underset{b_2}{0s} + \underset{b_3}{1}$$

$$M(s) = s^n + a_1 s^{n-1} + \dots + a_{n-1}s + a_n$$

$$N(s) = b_0 s^n + b_1 s^{n-1} + \dots + b_{n-1}s + b_n$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -4 & 0 & 0 \\ -1 & 0 & 1 & 0 & -4 & 0 \\ \hdashline 0 & -1 & 0 & 1 & 0 & -4 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

MATLAB

```
S=[1,0,0,0,0,0; 0 1 0 -4 0 0,-1 0 1 0 -4 0;
0 -1 0 1 0 -4; 0 0 -1 0 1 0; 0 0 0 0 0 1]
```

la matriz es invertible

El $MCD(M(s), N(s)) = 1 \rightarrow \det S_{(M,N)} \neq 0$

Ecuación Diofantina Diseñe una ley de control lineal tal que la función de transferencia en lazo cerrado sea:

$$F_{lc}(s) = \frac{X(0)\ (1-2s)}{X(s)(s+1)^2}$$

donde $X(s)$ es un polinomio con una dinamica cinco veces mas rapida que la dinamica dominante.

$X(0) \to X(s)$

$X(s) = (s+2.5)^2$

$X(0) = (2.5)^2 = 6.25$

$X(s) = (s+2.5)^2$

MATLAB

```
syms s
Q = collect((s+1)^2*(2*s+1), s)
N = -4*s^2+1
M = collect(s^3-s, s)
a1=0; a2=-1; a3=0;
b0=0; b1=-4; b2=0; b3= 1;
x = [1  0  0  b0 0  0
     a1 1  0  b1 b0 0
     a2 a1 1  b2 b1 b0
     a3 a2 a1 b3 b2 b1;
     0  a3 a2 0  b3 b2;
     0  0  a3 0  0  b3;
factor(det(x))
Xinv = inv(x)
pLC = simplify(collect((s+2.5)^2*Q, s))
q = (sym2poly(pLC))'
coef = inv(X)*q
Sd = collect(coef(1)*s^2 + coef(2)*s + coef(3), s)
Rd = collect(coef(4)*s^2 + coef(5)*s + coef(6), s)
```

```
Qob= simplify(M*sd+N*Rd)
Qob-pLC

FT=simplify((1/sd)*((N/M)/(1+(Rd/Sd)*(N/M)))
gg= double(subs(FT,s,0))
roots([1 -54 -109/4])

ps=[coef(1) coef(2) coef(3)]
pR=[coef(4) coef(5) coef(6)]

A=[0 -pS(3);
   1 -pS(2)]

pSR=[(pR(2)-pR(1)*pS(2)) (pR(3)-pR(1)*pS(3))]

B=[1 -psR(2);
   1 -psR(1);]

C=[0 1]
D=[0 -pR(1)]

N=[0 -4 0 1]
M=sym2poly(collect(s^3-s,s))
```

$Q = 2s^3 + 5s^2 + 4s + 1$

$N = 1 - 4s^2$

$M = s^3 - s$

$$X = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -4 & 0 & 0 \\ -1 & 0 & 1 & 0 & -4 & 0 \\ \hline 0 & -1 & 0 & 1 & 0 & -4 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$X_{inv} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.33 & 0 & -1.33 & 0 & -5.33 \\ -0.33 & 0 & -0.33 & 0 & -1.33 & 0 \\ 0 & -0.33 & 0 & -0.33 & 0 & -1.33 \\ -0.33 & 0 & -0.33 & 0 & -0.33 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$pLC = (2s+1)(2s^2+7s+5)^2/4$

$$q = \begin{bmatrix} 2.0 \\ 15.0 \\ 41.5 \\ 52.25 \\ 30.0 \\ 6.25 \end{bmatrix} \qquad coef = \begin{bmatrix} 2.0 \\ -108.0 \\ -54.5 \\ -30.75 \\ -24.5 \\ 6.25 \end{bmatrix}$$

$Sd = 2s^2 - 108s - 109/2$

$Rd = 25/4 - 49s/2 - \frac{123s^2}{2}$

$Qob = (2s+1)(2s^2+7s+5)^2/4$

$$FT = \frac{-(8s-4)}{(2s^2+7s+5)^2}$$

$pS = [2.0 \ \ -108 \ \ -54.5]$

$pR = [-30.75 \ \ -24.5 \ \ 6.25]$

$gg = 0.16$

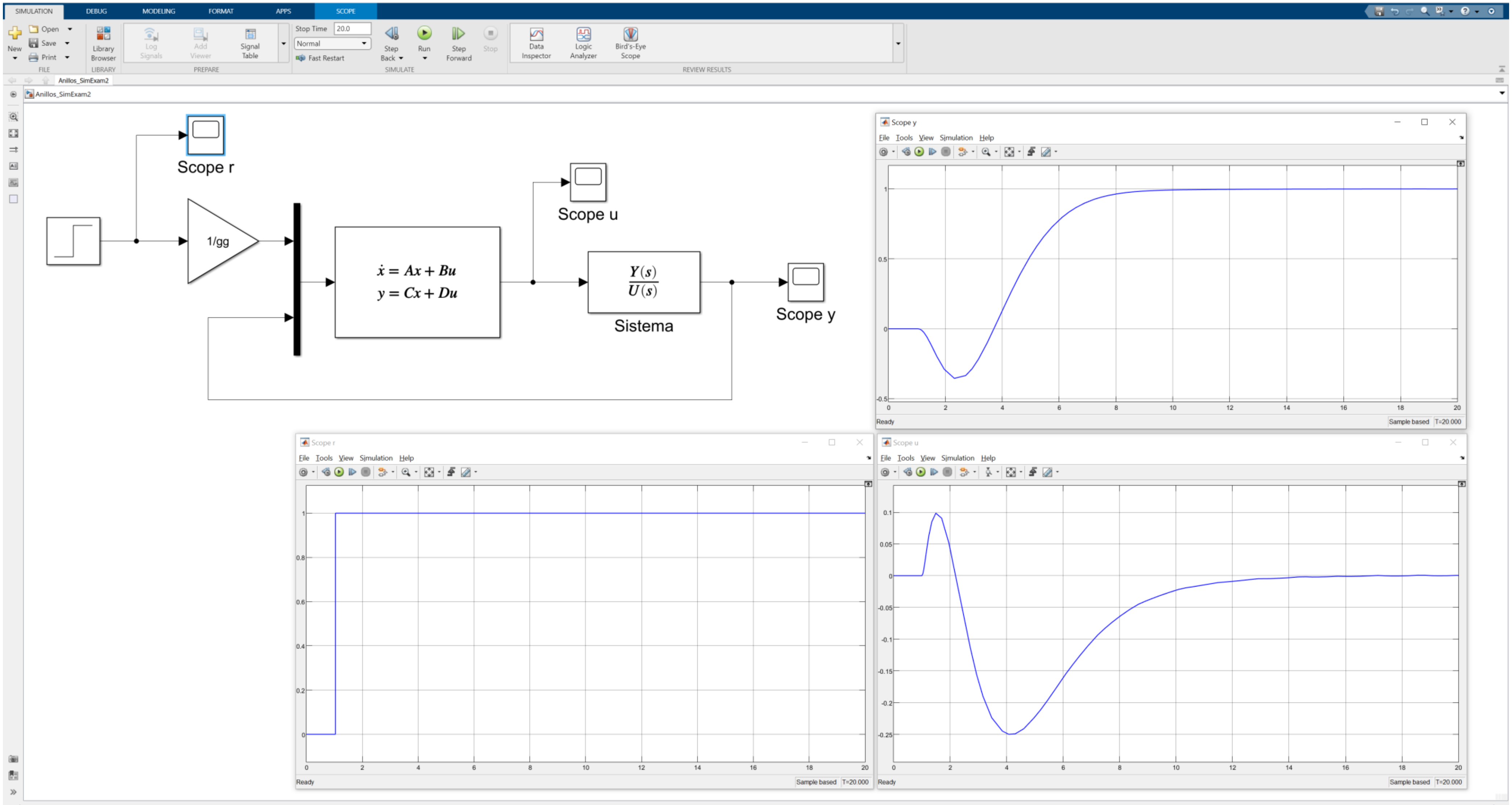
$$A = \begin{bmatrix} 0 & 54.5 \\ 1.0 & 108.0 \end{bmatrix} \qquad pSR = [-3.3455 \quad -1.66]\, 1\times10^{3}$$

$$B = 1\times10^{3} \begin{bmatrix} 0.001 & 1.0696 \\ 0 & 3.3453 \end{bmatrix}$$

$$C = [0 \quad 1]$$

$$N = [0 \quad -4 \quad 0 \quad 1]$$

$$M = [1 \quad 0 \quad -1 \quad 0]$$


SIMULATION
DEBUG
MODELING
FORMAT
APPS
SCOPE
Stop Time 20.0
Normal
Fast Restart
Data Inspector
Logic Analyzer
Bird's-Eye Scope
Anillos_SimExam2
Scope r
1/gg
$\dot{x} = Ax + Bu$
$y = Cx + Du$
Scope u
$\frac{Y(s)}{U(s)}$
Sistema
Scope y
Ready
Sample based
T=20.000
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