# Team notebook

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# 1 ContestPrep

# 1.1 Template

```
#include <bits/stdc++.h>
using namespace std;
using ul = unsigned long long;
using 11 = long long;
using db = double;
using pi = pair<int, int>;
using pl = pair<ll, ll>;
using pd = pair<db, db>;
using vi = vector<int>;
using vl = vector<11>;
using vd = vector<db>;
#define rep(i, n) for (int i = 0; i < (int)n; i++)
#define repx(i, a, b) for (int i = (int)a; i < (int)b; i++)
#define all(x) begin(x), end(x)
#define eb emplace_back
#define pb push_back
#define mp make_pair
#define ff first
#define ss second
```

## 2 DP

# 2.1 DivideAndConquer

```
1l cost(int i, int j) { return ; }
vl last, now;
void compute(int 1, int r, int optl, int optr) {
   if (1 > r) return;
   int mid = (1 + r) / 2;
   pair<1l, int> best = {cost(0, mid), -1};
   repx(k, max(1, optl), min(mid, optr) + 1)
```

```
best = min(best, {last[k - 1] + cost(k, mid), k});
now[mid] = best.ff;
compute(1, mid - 1, optl, best.ss);
compute(mid + 1, r, best.ss, optr);
}
```

# 2.2 KnuthOptimization

```
int N; vi A;
vector<vi>DP, OPT;

int main()
{
    DP.assign(N + 1, vi(N + 1));
    OPT.assign(N + 1, vi(N + 1));

    rep(i, N) DP[i][i + 1] = A[i + 1] - A[i], OPT[i][i + 1] = i;

    repx(d, 2, N + 1) rep(1, N + 1 - d)
    {
        int r = 1 + d, l_ = OPT[l][r - 1], r_ = OPT[l + 1][r];
        DP[l][r] = 1e9;
        repx(i, l_, r_ + 1)
        {
            int aux = DP[l][i] + DP[i][r] + A[r] - A[l];
            if (aux < DP[l][r]) DP[l][r] = aux, OPT[l][r] = i;
        }
    }
}</pre>
```

#### 2.3 LIS

```
int LIS(vi &v)
{
      vi L; int S = 0;
      for(int x : v)
      {
          int i = upper_bound(all(L), x) - L.begin();
          if(i == S) L.pb(x), S++;
          else L[i] = x;
      }
      return S;
}
```

# 3 Geometry

#### 3.1 2D

```
const db PI = acos(-1.0L);
const db EPS = 1e-12;
// POINT 2D
typedef db T; struct P
   T x, y;
   P() \{ \} P(T x, T y) : x(x), y(y) \{ \}
   P operator+(const P &p) const { return P(x + p.x, v + p.v); }
   P operator-(const P &p) const { return P(x - p.x, y - p.y); }
   P operator*(const db &c) const { return P(x * c, y * c); }
   P operator/(const db &c) const { return P(x / c, y / c); }
   T operator^(const P &p) const { return x * p.y - y * p.x; }
   T operator*(const P &p) const { return x * p.x + v * p.v: }
   bool operator == (const P &p) const
       return abs(x - p.x) + abs(v - p.v) < EPS:
   bool operator<(const P &p) const</pre>
       return abs(x - p.x) > EPS ? p.x - x > EPS : p.y - y > EPS;
   T norm2() const { return x * x + y * y; }
   db norm() const { return sqrt(norm2()); }
   db ang()
       db = atan2(v, x);
       if (a < 0) a += 2. * PI:
       return a:
   P unit() { return (*this) / norm(): }
   P perp() { return P(-y, x); }
   P rot(P r) { return P((*this) ^ r, (*this) * r); }
   P rot(db a) { return rot(P(sin(a), cos(a))); }
}:
P polar(db r, db a) { return P(r * cos(a), r * sin(a)); }
istream & operator >> (istream &s, P &p) { return s >> p.x >> p.y; }
ostream &operator<<(ostream &s, const P &p)
   return s << '(' << p.x << ", " << p.y << ')';
}
db ang(db a)
   while (a \ge 2. * PI) a = 2. * PI;
   while (a < 0) a += 2. * PI;
```

```
return a;
}
T turn(P &a, P &b, P &c) { return (b - a) ^ (c - a); }
bool isConvex(vector<P> p)
   int n = p.size();
   bool hasPos = false, hasNeg = false, hasCol = false;
   rep(i, n)
   {
       int o = turn(p[i], p[(i + 1) \% n], p[(i + 2) \% n]);
       if (o > 0) hasPos = true:
       if (o < 0) hasNeg = true;</pre>
       if (o == 0) hasCol = true:
   return !(hasPos && hasNeg) && !hasCol;
}
bool half(P &p) { return p.y > 0 || (p.y == 0 && p.x > 0); }
void polarSort(vector<P> &v)
   sort(v.begin(), v.end(), [](P &p1, P &p2)
       int h1 = half(p1), h2 = half(p2);
       return h1 != h2 ? h1 > h2 : (p1 ^ p2) > 0;
}
// LINE
struct L
   P v: T c:
   L() {} L(P v, T c) : v(v), c(c) {}
   L(T a, T b, T c) : v(P(b, -a)), c(c) {}
   L(P p, P q) : v(q - p), c(v ^ p) {}
   T side(P p) { return (v ^ p) - c; }
   db dist(P p) { return abs(side(p) / v.norm()); }
   db dist2(P p) { return side(p) * side(p) / (db)v.norm2(); }
   L perp(P p) { return L(p, p + v.perp()); }
   L translate(P t) { return L(v, c + (v ^ t)); }
   P proj(P p) { return p - v.perp() * side(p) / v.norm2(); }
   P refl(P p) { return p - v.perp() * 2 * side(p) / v.norm2(); }
};
bool parallel(L 11, L 12) {return abs(11.v ^ 12.v) < EPS; }</pre>
// only if not parallel
P inter(L 11, L 12) { return (12.v * 11.c - 11.v * 12.c) / (11.v ^ 12.v); }
L bisector(L 11, L 12, bool in)
```

```
db sign = in ? 1 : -1;
   return L(12.v / 12.v.norm() + 11.v / 11.v.norm() * sign,
            12.c / 12.v.norm() + 11.c / 11.v.norm() * sign);
}
struct HASH // Hashing for integer coordinates lines
   ll a, b, c;
   HASH(const P &p1, const P &p2)
       a = p1.y - p2.y, b = p2.x - p1.x;
       c = p1.x * (p2.y - p1.y) - p1.y * (p2.x - p1.x);
       ll sgn = (a < 0 \text{ or } (a == 0 \text{ and } b < 0)) ? -1 : 1:
       ll g = \_gcd(abs(a), \_gcd(abs(b), abs(c))) * sgn;
       a /= g, b /= g, c /= g;
   bool operator<(const HASH &h) const
       return a < h.a or (a == h.a and (b < h.b or (b == h.b and c < h.c)));
};
// SEGMENT
bool inDisk(P &a, P &b, P &p) { return (a - p) * (b - p) <= 0; }
bool onSegment(P &a, P &b, P &p)
{
   return abs(turn(a, b, p)) < EPS && inDisk(a, b, p);</pre>
bool properInter(P &a, P &b, P &c, P &d, P &out)
   T ta = turn(c, d, a), tb = turn(c, d, b).
     tc = turn(a, b, c), td = turn(a, b, d);
   out = (a * tb - b * ta) / (tb - ta);
   return (ta * tb < 0 && tc * td < 0):
}
set<P> inter(P &a, P &b, P &c, P &d)
{
   P out;
   if (properInter(a, b, c, d, out)) return {out};
   set<P> ans;
   if (onSegment(c, d, a)) ans.insert(a);
   if (onSegment(c, d, b)) ans.insert(b);
   if (onSegment(a, b, c)) ans.insert(c);
   if (onSegment(a, b, d)) ans.insert(d);
   return ans;
}
db segPoint(P &a, P &b, P &p)
   if ((p - a) * (b - a) >= 0 && (p - b) * (a - b) >= 0)
       return abs(((b - a) ^ (p - a)) / (b - a).norm());
```

```
return min((p - a).norm(), (b - a).norm());
}
db segSeg(P &a, P &b, P &c, P &d)
   if (properInter(a, b, c, d, aux)) return 0;
   return min({segPoint(a, b, c), segPoint(a, b, d),
              segPoint(c, d, a), segPoint(c, d, b)});
}
// POLYGONS
db areaTriangle(P &a, P &b, P &c)
   return abs((b - a) ^ (c - a)) / 2.:
db areaPolygon(vector<P> &p)
   db ans = 0; int n = p.size();
   rep(i, n) ans += p[i] ^ p[(i + 1) % n];
   return abs(ans) / 2.;
bool above(P &a, P &p) { return p.y >= a.y; }
bool crossesRay(P &a, P &p, P &q)
   return (above(a, q) - above(a, p)) * turn(a, p, q) > 0;
// if strict, returns false when a is on the boundary
bool inPolygon(vector<P> &p. P &a. bool strict = true)
   int c = 0, n = p.size();
   rep(i, n)
       if (onSegment(p[i], p[(i + 1) % n], a)) return !strict;
       c += crossesRay(a, p[i], p[(i + 1) % n]);
   return c & 1;
db areaPolygonUnion(vector<vector<P>> &pol) // Slow O((NE)^2log(NE))
   db area = 0:
   rep(i, pol.size()) rep(j, pol[i].size())
       int m = pol[i].size();
       P p1 = pol[i][j], p2 = pol[i][(j + 1) % m];
       vector<pair<db, int>> s; s.emplace_back(1., 0);
       rep(ii, pol.size()) if (ii != i) rep(jj, pol[ii].size())
```

```
int mm = pol[ii].size();
           P p3 = pol[ii][jj], p4 = pol[ii][(jj + 1) % mm];
           db t1 = turn(p1, p2, p3), t2 = turn(p1, p2, p4),
                 t3 = turn(p3, p4, p1), t4 = turn(p3, p4, p2);
           if (!t1 && !t2 && (p2 - p1) * (p4 - p3) > 0 && i > ii)
              s.emplace_back((p3 - p1) * (p2 - p1).unit(), 1);
              s.emplace_back((p4 - p1) * (p2 - p1).unit(), -1);
           if (t1 \ge 0 \&\& t2 < 0) s.emplace_back(t3 / (t3 - t4), 1);
           if (t1 < 0 \&\& t2 >= 0) s.emplace_back(t3 / (t3 - t4), -1);
       }
       sort(s.begin(), s.end()):
       int c = 0:
       db last = 0, f = 0;
       for (auto e : s)
           db now = min(1., max(0., e.ff));
           if (c == 0) f += now - last;
           c += e.ss, last = now;
       }
       area += (p1 ^p2) * f;
   return area;
}
vector<P> convexHull(vector<P> &p)
   int n = p.size(), k = 0;
   vector<P> H(2 * n); sort(p.begin(), p.end());
   rep(i, n)
       while (k \ge 2 \&\& turn(H[k - 2], H[k - 1], p[i]) \le 0) k--;
       H[k++] = p[i];
   for (int i = n - 2, t = k + 1; i \ge 0; i--)
       while (k \ge t \&\& turn(H[k - 2], H[k - 1], p[i]) \le 0) k--;
       H[k++] = p[i];
   H.resize(k - 1);
   return H;
// MISCELLANEOUS
// Smallest Enclosing cicle
P bary(P &A, P &B, P &C, db a, db b, db c)
```

```
return (A * a + B * b + C * c) / (a + b + c);
P circum(P &A, P &B, P &C)
   db = (B - C).norm2(), b = (C - A).norm2(), c = (A - B).norm2();
   return bary(A, B, C, a * (b + c - a), b * (c + a - b), c * (a + b - c));
pair<P, db> smallestEnclosingCircle(vector<P> &p)
   random_shuffle(all(p));
   P c = p[0]; db r = 0; int N = p.size();
   rep(i, N) if (i \&\& (p[i] - c).norm() > r + EPS)
       c = p[i]; r = 0;
       rep(j, i) if ((p[j] - c).norm() > r + EPS)
           c = (p[i] + p[j]) * 0.5;
           r = (p[i] - c).norm();
           rep(k, j) if ((p[k] - c).norm() > r + EPS)
              c = circum(p[i], p[j], p[k]);
              r = (p[k] - c).norm();
       }
   }
   return {c, r};
// Closest pair of points from array "a" (mindist: squared mindist)
const int MAXN = 1000010;
int n: T mindist:
pair<P, P> best;
Pa[MAXN], t[MAXN];
T sq(T x) \{ return x * x; \}
bool cmpY(P & a, P & b) { return a.v < b.v; }</pre>
void update(P &p1, P &p2)
   T = (p1 - p2).norm2();
   if (aux < mindist) { mindist = aux; best = {p1, p2}; }</pre>
// sort "a" before usage (P must have default operator<)
void closest(int 1, int r)
   if (r - 1 \le 3)
```

#### 3.2 3D

```
const db PI = acos(-1.0L);
const db EPS = 1e-12;
// POINT 3D
struct P
{
   db x, y, z;
   P() {}
   P(db x, db y, db z) : x(x), y(y), z(z) \{\}
   P operator+(const P &p) const { return P(x + p.x, y + p.y, z + p.z); }
   P operator-(const P &p) const { return P(x - p.x, y - p.y, z - p.z); }
   P operator*(const db &c) const { return P(x * c, y * c, z * c); }
   P operator/(const db &c) const { return P(x / c, y / c, z / c); }
   P operator (const P &p) const
       return P(y * p.z - z * p.y,
               z * p.x - x * p.z,
               x * p.y - y * p.x);
   db operator*(const P &p) const { return x * p.x + y * p.y + z * p.z; }
   db operator%(const P &p) const
   {
       return acos((*this) * p) / (norm() * p.norm());
   bool operator==(const P &p) const
       return abs(x - p.x) + abs(y - p.y) + abs(z - p.z) < EPS;
```

```
}
   db norm() const { return sqrt(norm2()); }
   db norm2() const { return \bar{x} * x + y * y + z * z; }
   P unit() { return (*this) / norm(); }
P polar(db r, db a, db b)
   return P(r * cos(a) * cos(b), r * cos(a) * sin(b), r * sin(a)):
istream &operator>>(istream &s, P &p) { return s >> p.x >> p.y >> p.z; }
ostream & operator << (ostream &s, const P &p)
   return s << '(' << p.x << ", " << p.y << ", " << p.z << ')';
}
// ARCS
bool in_arc(P &a, P &b, P &n, P &p)
   db ab = a \% b;
   db ap = a \% p;
   P c = (a * cos(ap) + (n ^a) * sin(ap));
   return ab > ap && p == c;
bool find_intersection(P &a1, P &b1, P &a2, P &b2, db &angle)
   P n1 = (a1 ^b1).unit(), n2 = (a2 ^b2).unit(), i = (n1 ^n2);
   if (i.norm() < EPS) return false;</pre>
   i = i.unit() * a1.norm():
   if (in_arc(a1, b1, n1, i) and in_arc(a2, b2, n2, i))
       angle = a1 % i:
       return true;
   i = i * -1.:
   if (in_arc(a1, b1, n1, i) and in_arc(a2, b2, n2, i))
       angle = a1 % i:
       return true;
   return false;
```

6

## 4 Graphs

#### 4.1 BellmanFord

```
const int INF = 1e9;
```

```
struct Edge
{
    int v, w;
    Edge(int v, int w) : v(v), w(w) {}
};
int n; vi D;
vector<vector<Edge>> G;

bool bellmanFord(int s)
{
    D.assign(n, INF); D[s] = 0;
    rep(i, n - 1) rep(j, n) for (Edge e : G[j])
        D[e.v] = min(D[e.v], D[j] + e.w);

    bool neg = false;
    rep(i, n) for (Edge e : G[i]) if (D[e.v] > D[i] + e.w) neg = true;
    return neg;
}
```

## 4.2 Dijkstra

```
const int INF = 1e9:
struct Edge
{
   int v. w: // CHECK FOR OVERFLOW
   Edge(int v, int w) : v(v), w(w) {}
   bool operator<(const Edge &e) const { return w > e.w; }
};
vector<vector<Edge>> G;
int dijkstra(int s, int t)
   vi C(n, INF); C[s] = 0; // CHECK FOR OVERFLOW
   priority_queue<Edge> q; q.emplace(s, 0);
   while (!q.empty())
       int u = q.top().v, w = q.top().w; q.pop(); // CHECK FOR OVERFLOW
       if (C[u] < w) continue;</pre>
       for (auto e : G[u]) if (C[e.v] > e.w + w)
           C[e.v] = e.w + w, q.emplace(e.v, C[e.v]);
   return C[t];
```

## 4.3 Dinic

```
class Dinic
   struct Edge { int to, rev; ll f, c; };
   int n, t_; vector<vector<Edge>> G;
   vl D; vi q, W;
   bool bfs(int s, int t)
       W.assign(n, 0); D.assign(n, -1); D[s] = 0;
       int f = 0, l = 0; q[l++] = s;
       while (f < 1)</pre>
           int u = q[f++];
           for (const Edge &e : G[u]) if (D[e.to] == -1 && e.f < e.c)
              D[e.to] = D[u] + 1, q[1++] = e.to;
       return D[t] != -1;
   ll dfs(int u, ll f)
       if (u == t_) return f;
       for (int &i = W[u]: i < (int)G[u].size(): ++i)</pre>
           Edge &e = G[u][i]; int v = e.to;
           if (e.c <= e.f || D[v] != D[u] + 1) continue:
          ll df = dfs(v, min(f, e.c - e.f));
           if (df > 0) { e.f += df, G[v][e.rev].f -= df; return df; }
       return 0;
   }
   Dinic(int N) : n(N), G(N), D(N), q(N) {}
   void addEdge(int u, int v, ll cap)
       G[u].push_back({v, (int)G[v].size(), 0, cap});
       G[v].push_back({u, (int)}G[u].size() - 1, 0, 0}); // cap if bidirectional
   11 maxFlow(int s, int t)
       t_{-} = t; 11 ans = 0;
       while (bfs(s, t)) while (ll dl = dfs(s, LLONG_MAX)) ans += dl;
       return ans:
   }
};
```

# 4.4 FloydWarshall

```
const int INF = 1e9;
int n;
vector<vi>D;

//D is an adjacency matrix at the beginning
void floydWarshall ()
{
   rep(k, n) rep(i, n) rep(j, n) if (D[i][k] < INF && D[k][j] < INF)
        D[i][j] = min(D[i][j], D[i][k] + D[k][j]);

   // opcional, si hay ciclos negativos
   rep(k, n) rep(i, n) rep(j, n)
        if (D[i][k] < INF && D[k][j] < INF && D[k][k] < 0) D[i][j] = -INF;
}</pre>
```

## 4.5 HeavyLightDecomposition

```
template <class ST, class node>
class HLD
{
   ST st;
   vi A, H, D, R, P;
   int dfs(vector<vi> &G, int u)
       int ans = 1, M = 0, s;
       for (int v : G[u]) if (v != A[u])
           A[v] = u, D[v] = D[u] + 1;
          s = dfs(G, v), ans += s;
          if (s > M) H[u] = v, M = s;
       }
       return ans:
   template <class OP>
   void path(int u, int v, OP op)
       for (; R[u] != R[v]; v = A[R[v]])
           if (D[R[u]] > D[R[v]]) swap(u, v);
           op(P[R[v]], P[v] + 1);
       if (D[u] > D[v]) swap(u, v);
       op(P[u], P[v] + 1);
                                       // VALUES ON VERTEX
       // op(P[u] + 1, P[v] + 1);
                                       // VALUES ON EDGE
   }
public:
   HLD(vector<vi> &G, int n) : A(n), st(n), D(n), R(n), P(n)
   {
```

```
H.assign(n, -1); A[0] = -1, D[0] = 0; dfs(G, 0); int p = 0;
       rep(i, n) if (A[i] == -1 || H[A[i]] != i)
          for (int j = i; j != -1; j = H[j]) R[j] = i, P[j] = p++;
   void set(int v, const node &x) { st.set(P[v], x); } // VALUES ON VERTEX
   // void set(int u, int v, const node &x)
                                                  // VALUES ON EDGE
   // {
          if (D[u] > D[v]) swap(u, v);
         st.set(P[v]. x):
   // }
   void update(int u, int v, const node& x)
                                                        // OPTIONAL FOR RANGE
   { path(u, v, [this, &x](int 1, int r) { st.update(1, r, x); }); }
   node query(int u, int v)
       node ans = node():
       path(u, v, [this, &ans](int 1, int r) { ans = node(ans, st.query(1, r));
           }):
       return ans;
};
// USAGE: HLD<ST<Node>, Node> hld(G, N);
//// NON COMMUTATIVE QUERIES :
template <class ST, class node>
class HLD
   ST st:
   vi A, H, D, R, P;
   int dfs(vector<vi> &G, int u)
       int ans = 1, M = 0, s;
       for (int v : G[u]) if (v != A[u])
          A[v] = u, D[v] = D[u] + 1;
           s = dfs(G, v), ans += s:
          if (s > M) H[u] = v. M = s:
       return ans;
   }
public:
   node path(int u, int v)
       node ans1, ans2; bool d = 0;
       for (; R[u] != R[v]; v = A[R[v]])
           if (D[R[u]] > D[R[v]]) swap(u, v), d = !d;
           if (d) ans1 = node(st.query(P[R[v]], P[v] + 1), ans1);
           else ans2 = node(st.query(P[R[v]], P[v] + 1), ans2);
       if (D[u] > D[v]) swap(u, v), d = !d;
```

```
if (d) ans1 = node(st.query(P[u], P[v] + 1), ans1);
    else ans2 = node(st.query(P[u], P[v] + 1), ans2);
    ans1.sw(); return node(ans1, ans2);
}
HLD(vector<vi> &G, int n) : A(n), st(n), D(n), R(n), P(n)
{
    H.assign(n, -1); A[0] = -1, D[0] = 0; dfs(G, 0); int p = 0;
    rep(i, n) if (A[i] == -1 || H[A[i]] != i)
        for (int j = i; j != -1; j = H[j]) R[j] = i, P[j] = p++;
}
void set(int v, const node &x) { st.set(P[v], x); }
};
```

# 4.6 HopcroftKarp

```
class Hopcroft
   vi U. dist:
   vector<vi> g;
   int inf = 1e9;
   bool bfs() {
       queue<int> q;
       for (int u : U) if (match[u] == nil) dist[u] = 0, q.push(u);
       else dist[u] = inf;
       dist[nil] = inf;
       while (!q.empty())
          int u = q.front(); q.pop();
          if (u != nil) for (int v : g[u]) if (dist[match[v]] == inf)
              dist[match[v]] = dist[u] + 1;
              q.push(match[v]);
       }
       return (dist[nil] != inf):
   bool dfs(int u) {
       if (u == nil)
          return true:
       for (int v : g[u]) if (dist[match[v]] == dist[u]+1 and dfs(match[v]))
          match[v] = u, match[u] = v;
          return true;
       }
       dist[u] = inf;
       return false;
public:
```

```
vi match;
int nil, isPerfect, matchSize = 0;

// gg is a bidirectional graph, UU has the nodes in the left partition
Hopcroft(vector<vi>&gg, vi &UU)
{
    g = gg; U = UU; nil = g.size();
    match.assign(g.size() + 1, nil);
    dist.assign(g.size() + 1, inf);
    while (bfs()) for (int u : U) if (match[u] == nil and dfs(u))
        matchSize++;
    isPerfect = (matchSize == U.size() and g.size() == U.size() * 2);
};
```

## 4.7 Hungarian

```
template<class T>
class Hungarian
   T inf = numeric limits<T>::max() / 2:
   bool maxi, swapped = false;
   vector<vector<T>> cost;
   vector<T> u, v;
   vi p, way;
   int 1, r;
public:
   // left/right == partition sizes
   Hungarian(int left, int right, bool maximizing)
       1 = left, r = right, maxi = maximizing;
       if (swapped = 1 > r) swap(1, r);
       cost.assign(1 + 1, vector < T > (r + 1, 0)):
       u.assign(1 + 1, 0); v.assign(r + 1, 0);
       p.assign(r + 1, 0); way.assign(r + 1, 0);
   }
   void add edge(int 1, int r, T w)
       assert(1 and r): // indices start from 1 !!
       if (swapped) swap(1, r);
       cost[1][r] = maxi ? -w : w;
   // execute after all edges were added
   void calculate()
       repx(i, 1, 1 + 1)
          vector<bool> used(r+1, false);
```

```
vector<T> minv(r+1, inf);
           int j0 = 0, p[0] = i;
           while (p[j0])
              int j1, i0 = p[j0], used[j0] = true;
              T delta = inf;
              repx(j, 1, r + 1) if (not used[j])
                  T cur = cost[i0][j] - u[i0] - v[j];
                  if (cur < minv[j]) minv[j] = cur, way[j] = j0;</pre>
                  if(minv[j] < delta) delta = minv[j], j1 = j;</pre>
              }
              rep(j, r + 1)
                  if (used[j]) u[p[j]] += delta, v[j] -= delta;
                  else minv[j] -= delta;
              }
              j0 = j1;
           while (j0) p[j0] = p[way[j0]], j0 = way[j0];
       }
   }
   // execute after executing calculate()
   T answer() { return maxi ? v[0] : -v[0]; }
   bool are_matched(int 1, int r)
       if (swapped) swap(1, r);
       return p[r] == 1;
   }
};
```

#### 4.8 LCA

```
struct LCA
{
    vector<vi>G;
    int N, LOG; vi A, D;
    int &anc(int u, int l) { return A[1 * N + u]; }
    LCA(vector<vi>&G, int N, int root) : G(G), N(N)
{
        D.assign(N, -1); A.resize(N * (LOG + 1));
        dfs(root, -1, 0), LOG = 31 - __builtin_clz(N);
        rep(1, LOG + 1) if (1) rep(u, N)
        {
            int a = anc(u, l - 1);
            anc(u, l) = (a == -1 ? -1 : anc(a, l - 1));
        }
}
```

```
void dfs(int u, int p, int depth)
       anc(u, 0) = p, D[u] = depth;
       for (int v : G[u]) if (D[v] == -1) dfs(v, u, depth + 1);
   int raise(int u, int k)
       for (int l = 0; k; l++, k >>= 1) if (k & 1) u = anc(u, 1);
       return u:
   int lca(int u, int v)
       if (D[u] < D[v]) swap(u, v);
       u = raise(u, D[u] - D[v]);
       if (u == v) return u:
       for (int 1 = LOG; 1 >= 0; 1--) if (anc(u, 1) != anc(v, 1))
           u = anc(u, 1), v = anc(v, 1);
       return anc(u,0);
   int dist(int u, int v) { return D[u] + D[v] - 2 * D[lca(u, v)]; }
   int raise_in_path(int u, int v, int k)
       if (D[u] - D[lca(u, v)] >= k) return raise(u, k);
       return raise(v, dist(u, v) - k);
   int add_child(int p, int u)
       G[p].pb(u);
       D[u] = D[p] + 1, anc(u, 0) = p;
       rep(1, LOG) if (1)
           p = anc(p, 1 - 1);
           if (p == -1) break:
           anc(u, 1) = p;
   }
};
```

#### 4.9 MST

```
// PRIM //
struct Edge
{
   int u, v, w;
   Edge(int u, int v, int w) : u(u), v(v), w(w) {}
   bool operator>(const Edge &e) const { return w > e.w; }
};
int find_mst(vector<vector<Edge>> &g, vector<vector<Edge>> &T)
```

```
int n = g.size();
   vector<bool> V(n, 0);
   T.assign(n, {});
   int ans = 0, c = 1; V[0] = 1;
   priority_queue<Edge, vector<Edge>, greater<Edge>> q;
   for (Edge &p : g[0]) q.emplace(0, p.v, p.w);
   while (!q.empty())
       Edge e = q.top(); q.pop();
       if (V[e.v]) continue;
       int u = e.u. v = e.v. w = e.w:
       V[v] = true, ans += w:
       T[u].eb(u, v, w);
       T[v].eb(v, u, w);
       if (++c == n) break;
       for (Edge &p : g[v]) if (!V[p.v]) q.emplace(v, p.v, p.w);
   return ans;
}
```

#### 4.10 MinCostMaxFlow

```
template <class T>
class MCMF
   typedef pair<T, T> pTT;
   T INF = numeric_limits<T>::max();
   struct Edge
   {
       int v: T c. w:
       Edge(int v, T c, T w) : v(v), c(c), w(w) {}
   }:
   int n: vector<vi> E:
   vector<Edge> L; vi F; vector<T> D, P; vector<bool> V;
   bool dij(int s, int t)
      D.assign(n, INF); F.assign(n, -1); V.assign(n, false);
      D[s] = 0;
       rep(_, n)
       ł
          int best = -1;
```

```
rep(i, n) if (!V[i] && (best == -1 || D[best] > D[i])) best = i;
           if (D[best] >= INF) break;
           V[best] = true;
           for (int e : E[best])
              Edge ed = L[e];
              if (ed.c == 0) continue;
              T toD = D[best] + ed.w + P[best] - P[ed.v];
              if (toD < D[ed.v]) D[ed.v] = toD, F[ed.v] = e;</pre>
       }
       return D[t] < INF;</pre>
   }
   pTT augment(int s, int t)
       pTT flow(L[F[t]].c. 0):
       for (int v = t; v != s; v = L[F[v] ^ 1].v)
          flow.ff = min(flow.ff, L[F[v]].c), flow.ss += L[F[v]].w;
       for (int v = t; v != s; v = L[F[v] ^1].v)
          L[F[v]].c = flow.ff, L[F[v] ^ 1].c += flow.ff;
       return flow;
   }
public:
   MCMF(int n) : n(n), E(n), D(n), P(n, 0), V(n, 0) {}
   pTT mcmf(int s, int t)
       pTT ans(0, 0);
       if (!dij(s, t)) return ans;
       rep(i, n) if (D[i] < INF) P[i] += D[i];
       while (dij(s, t))
           auto flow = augment(s, t);
           ans.ff += flow.ff, ans.ss += flow.ff * flow.ss:
           rep(i, n) if (D[i] < INF) P[i] += D[i];
       return ans:
   void addEdge(int u, int v, T c, T w)
       E[u].pb(L.size()); L.eb(v, c, w);
       E[v].pb(L.size()); L.eb(u, 0, -w);
   }
};
```

#### 4.11 SCC

```
int N, id;
vector<vi>> G; // Directed Graph
vi D, L, I; stack<int> S;
```

```
void dfs(int u)
{
    D[u] = L[u] = id++, I[u] = 1; S.push(u);
    for (int v : G[u])
    {
        if (D[v] == -1) { dfs(v); L[u] = min(L[v], L[u]); }
        else if (I[v]) L[u] = min(L[v], L[u]);
    }
    if (L[u] == D[u]) while (1) // SCC FOUND
    {
        int x = S.top(); S.pop(); I[x] = 0;
        if (x == u) break;
    }
}

void find_sccs()
{
    D.assign(N, -1); L.resize(N); I.assign(N, 0);
    id = 0; rep(u, N) if (D[u] == -1) dfs(u);
}
```

# 4.12 Tarjan

```
vector<vi> G;
vi D, L;
void dfs(int u, int p, int d)
{
   D[u] = L[u] = d;
   for(int v : G[u]) if (v != p)
       if (D[v] == -1)
           dfs(v, u, d + 1);
           if (L[v] > D[u]) {} // (u - v) cut edge
           L[u] = min(L[u], L[v]);
       }
       else L[u] = min(L[u], D[v]);
int rc = 0;
void dfs(int u, int p, int d)
   D[u] = L[u] = d;
   for(int v : G[u]) if (v != p)
       if (D[v] == -1)
       {
           dfs(v, u, d + 1);
           if ((p == -1 \&\& ++rc == 2) \mid | (p != -1 \&\& L[v] >= d)) {} // u is AP
```

```
L[u] = min(L[u], L[v]);
       }
       else if (D[v] < d) L[u] = min(L[u], D[v]);
}
stack<pi> S;
void dfs(int u, int p, int d)
   D[u] = L[u] = d;
   for(int v : G[u]) if (v != p)
       if (D[v] == -1)
           S.emplace(u, v); dfs(v, u, d + 1);
           if (p == -1 \text{ or } L[v] >= d) while (1) // BCC found
              pi e = S.top(); S.pop();
              if (e == make_pair(u, v)) break;
          L[u] = min(L[u], L[v]);
       else if (D[v] < d) { S.emplace(u, v); L[u] = min(L[u], D[v]); }
}
```

# 4.13 Toposort

```
int N; vi V, S;
vector<vi> G;

void dfs(int u)
{
    V[u] = 1;
    for (int v : G[u]) if (!V[v]) dfs(v);
    S.pb(u);
}

void topo_sort()
{
    V.assign(N, 0); S.clear();
    rep(i, N) if (!V[i]) dfs(i);
}
```

## 5 Math

## 5.1 Binomial

```
#include <Euclid.cpp>
#include <Modular.cpp>
const 11 MOD = 1e9 + 7;
const int MAXN = 1000;
// DP bottom-up
11 C[MAXN + 1][MAXN + 1];
void init C()
   rep(m, MAXN + 1) if (m)
       C[m][0] = C[m][m] = 1;
       rep(k, m) if (k) C[m][k] = add(C[m-1][k], C[m-1][k-1]);
}
// By definition and euclid
11 F[MAXN];
11 choose(int n, int k)
{
   return mul(F[n], minv(mul(F[k], F[n - k]), MOD));
// Multinomial Coefficient
11 multinomial(vector<int> K)
   11 n = 0, ans = 1;
   for (int k : K) n += k, ans = mul(ans, choose(n, k));
   return ans:
}
```

#### 5.2 Euclid

```
// find (x, y) such that Ax + By = gcd(A, B), and |Ax|, |By| <= AB/gcd(A, B)
pl euclid(11 A, 11 B)
{
    if (!B) return {1, 0};
    pl p = euclid(B, A % B);
    return {p.ss, p.ff - (A / B) * p.ss};
}

// find x in [0, M) such that Ax = 1 mod M
ll minv(ll A, ll M)
{
    pl p = euclid(A, M);
    assert(p.ff * A + p.ss * M == 1);
        return p.ff + (p.ff < 0) * M;
}

// find (x, y)'s such that Ax + By = R where R is multiple of gcd(A, B);</pre>
```

#### 5.3 FFT

```
#define PI acos(-1.0L)
typedef complex<double> C:
void fft(vector<C> &a)
   int n = a.size(), L = 31 - __builtin_clz(n);
   static vector<complex<long double>> R(2, 1);
   static vector<C> rt(2, 1):
   for (static int k = 2; k < n; k *= 2)
       R.resize(n): rt.resize(n):
       auto x = polar(1.0L, PI / k);
       repx(i, k, 2 * k) rt[i] = R[i] = i & 1 ? R[i / 2] * x : R[i / 2];
   vector<int> rev(n);
   rep(i, n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
   rep(i, n) if (i < rev[i]) swap(a[i], a[rev[i]]);</pre>
   for (int k = 1; k < n; k *= 2) for (int i = 0; i < n; i += 2 * k) rep(j, k)
       auto x = (double *) &rt[j + k], y = (double *) &a[i + j + k];
       C z(x[0] * y[0] - x[1] * y[1], x[0] * y[1] + x[1] * y[0]);
       a[i + j + k] = a[i + j] - z, a[i + j] += z;
}
vd conv(const vd &a, const vd &b)
   if (a.empty() || b.empty()) return {};
   vd res(a.size() + b.size() - 1):
   int L = 32 - __builtin_clz(res.size()), n = 1 << L;</pre>
   vector<C> in(n), out(n);
   copy(a.begin(), a.end(), in.begin());
   rep(i, b.size()) in[i].imag(b[i]);
   fft(in); for (auto &x : in) x *= x;
   rep(i, n) out[i] = in[-i & (n - 1)] - conj(in[i]);
   fft(out); rep(i, res.size()) res[i] = imag(out[i]) / (4 * n);
   return res;
}
vl convMod(const vl &a, const vl &b, int M)
```

```
if (a.empty() || b.empty()) return {};
vl res(a.size() + b.size() - 1);
int B = 32 - __builtin_clz(res.size()), n = 1 << B, cut = int(sqrt(M));</pre>
vector<C> L(n), R(n), outs(n), outl(n);
rep(i, a.size()) L[i] = C((int)a[i] / cut, (int)a[i] % cut);
rep(i, b.size()) R[i] = C((int)b[i] / cut, (int)b[i] % cut);
fft(L), fft(R);
rep(i, n)
{
   int j = -i & (n - 1);
   outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
   outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
fft(outl), fft(outs);
rep(i, res.size())
   11 av = 11(real(outl[i]) + .5), cv = 11(imag(outs[i]) + .5);
   11 bv = ll(imag(outl[i]) + .5) + ll(real(outs[i]) + .5);
   res[i] = ((av \% M * cut + bv) \% M * cut + cv) \% M;
return res;
```

## 5.4 MillerRabin

```
#include <Modular.cpp>
bool prime(ul n)
{
    if (n < 2 || n % 6 % 4 != 1) return n - 2 < 2;
    ul A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
    ul s = __builtin_ctzll(n - 1), d = n >> s;
        for (int a : A)
    {
        ul p = modPow(a, d, n), i = s;
        while (p != 1 && p != n - 1 && a % n && i--) p = modMul(p, p, n);
        if (p != n - 1 && i != s) return 0;
    }
    return 1;
}
```

#### 5.5 Modular

```
const 11 MOD = 1e9 + 7;
inline ll add(ll x, ll y, ll M = MOD) { return (x + y) % M; }
inline ll mul(ll x, ll y, ll M = MOD) { return (x * y) % M; }
ul binPow(ll a, ll b, const ll M)
```

```
{
    if (b == 0) return 1;
    ul r = binPow(a, b >> 1, M); r = mul(r, r, M);
    return (b & 1) ? mul(r, a, M) : r;
}

// If multiplication goes over ll

ul modMul(ul a, ul b, const ul M)
{
    ll r = a * b - M * (ul)((db)a * b / M);
    return r + ((r < 0) - (r >= (ll)M)) * M;
}

ul modPow(ul a, ul b, const ul M)
{
    if (b == 0) return 1;
    ul r = modPow(a, b >> 1, M); r = modMul(r, r, M);
    return b & 1 ? modMul(r, a, M) : r;
}
```

#### 5.6 PrimeFactorization

```
// Naive with precomputed primes
vi P;
vi factor(int x)
{
   vi ans;
   for (int d : P)
       if (d * d > x) break;
       while (x \% d == 0)
          ans.pb(d);
          if ((x /= d) == 1) return ans:
   if (x > 1) ans.pb(x);
   return ans:
// Pollard Rho ( O(n^{1/4}) )
#include <MillerRabin.cpp>
ul pollard(ul n) // return some nontrivial factor of n
{
       auto f = [n](ul x) { return modMul(x, x, n) + 1; };
       ul x = 0, y = 0, t = 30, prd = 2, i = 1, q;
       while (t++ \% 40 || _-gcd(prd, n) == 1)
```

```
15
```

```
{
            if (x == y) x = ++i, y = f(x);
            if ((q = modMul(prd, max(x, y) - min(x, y), n))) prd = q;
            x = f(x), y = f(f(y));
      }
      return __gcd(prd, n);
}

void rec(ul n, map<ul, int> &C)
{
      if (n == 1) return;
      if (prime(n)) { C[n]++; return; }
      ul u = pollard(n); rec(u, C), rec(n / u, C);
}

vector<pair<ul, int>> factor(ul n)
{
      map<ul, int> C; rec(n, C);
      return vector<pair<ul, int>>(all(C));
}
```

## 5.7 Sieve

```
template<int SZ> struct Sieve
{
    bitset<SZ> I; vi P;
    Sieve()
    {
        I.set();
        for (int i = 2; i * i < SZ; i++) if (I[i])
            for (int j = i * i; j < SZ; j += i) I[j] = 0;
        rep(i, SZ) if (i > 1 && I[i]) P.pb(i);
    }
};
Sieve<320000> S;
```

# 6 Strings

## 6.1 AhoCorasick

```
struct AC
{
    static const int MAX = 1e4, ASZ = 26;
    int N[MAX][ASZ] = {0}, L[MAX] = {0}, E[MAX] = {0}, c = 0;
    void add(string s)
    {
```

```
int p = 0;
       for (char 1 : s)
           int t = 1 - 'a';
          if (!N[p][t]) N[p][t] = ++c;
          p = N[p][t];
       E[p] = 1;
   void init()
       queue<int> q; q.push(0); L[0] = -1;
       while (!q.empty())
           int p = q.front(); q.pop();
          rep(c, ASZ)
              int u = N[p][c]; if (!u) continue;
              L[u] = L[p] == -1 ? 0 : N[L[p]][c], q.push(u);
           if (p) rep(c, ASZ) if (!N[p][c]) N[p][c] = N[L[p]][c];
   }
};
```

#### 6.2 Hash

```
struct RH
   int B = 1777771, M[2] = \{999727999, 1070777777\}, P[2] = \{325255434, 10018302\};
   vi H[2], I[2];
   RH(string &s)
   {
       int N = s.size(); rep(k, 2)
          H[k].resize(N + 1), I[k].resize(N + 1);
          H[k][0] = 0, I[k][0] = 1; 11 b = 1;
          rep(i, N + 1) if (i)
              H[k][i] = (H[k][i-1] + b * s[i-1]) % M[k];
              I[k][i] = (1LL * I[k][i - 1] * P[k]) % M[k];
              b = (b * B) % M[k];
          }
      }
   11 get(int 1, int r) // inclusive - exclusive
      11 h0 = (H[0][r] - H[0][1] + M[0]) \% M[0];
      h0 = (1LL * h0 * I[0][1]) % M[0];
      11 h1 = (H[1][r] - H[1][r] + M[1]) % M[1];
      h1 = (1LL * h1 * I[1][1]) % M[1];
       return (h0 << 32) | h1;
```

```
};
```

#### 6.3 KMP

```
int match(string &p, string &t)
{
    int n = p.size(), m = t.size(), L[n]; L[0] = 0;
    rep(j, n - 1)
    {
        int i = L[j]; while (p[i] != p[j + 1] && i) i = L[i - 1];
        L[j + 1] = (p[i] == p[j + 1] ? i + 1 : 0);
    }
    int ans = 0, i = 0; rep(j, m)
    {
        while (p[i] != t[j] && i) i = L[i - 1];
        if (p[i] == t[j] && ++i == n) i = L[n - 1], ans++;
    }
    return ans;
}
```

#### 6.4 Manacher

```
int n;
string s;
int main()
   vi d1(n); // odd sized palindromes
   for (int i = 0, l = 0, r = -1; i < n; i++) {
       int k = (i > r) ? 1 : min(d1[1 + r - i], r - i + 1);
       while (0 \le i - k \&\& i + k \le n \&\& s[i - k] == s[i + k]) k++:
       d1[i] = k--:
       if (i + k > r) l = i - k, r = i + k;
   vi d2(n); // even sized palindromes (center to the right)
   for (int i = 0, l = 0, r = -1; i < n; i++) {
       int k = (i > r) ? 0 : min(d2[1 + r - i + 1], r - i + 1):
       while (0 \le i - k - 1 \&\& i + k \le n \&\& s[i - k - 1] == s[i + k]) k++;
       d2[i] = k--;
       if (i + k > r) l = i - k - 1, r = i + k;
   }
}
```

# 6.5 SuffixArray

```
struct SA
   int n; vi C, R, R_, sa, sa_, lcp;
   inline int gr(int i) { return i < n ? R[i] : 0; }</pre>
   void csort(int maxv, int k)
       C.assign(maxv + 1, 0); rep(i, n) C[gr(i + k)]++;
       repx(i, 1, maxv + 1) C[i] += C[i - 1];
       for (int i = (int)n - 1; i \ge 0; i--) sa_{-}[-C[gr(sa[i] + k)]] = sa[i];
       sa.swap(sa_);
   void getSA(vi& s)
       R = R_{-} = sa = sa_{-} = vi(n); rep(i, n) sa[i] = i;
       sort(all(sa), [&s](int i, int j) { return s[i] < s[j]; });</pre>
       int r = R[sa[0]] = 1:
       repx(i, 1, n) R[sa[i]] = (s[sa[i]] != s[sa[i - 1]]) ? ++r : r;
       for (int h = 1: h < n && r < n: h <<= 1)
           csort(r, h): csort(r, 0): r = R [sa[0]] = 1:
          repx(i, 1, n)
          {
              if (R[sa[i]] != R[sa[i-1]] || gr(sa[i] + h) != gr(sa[i-1] + h))
              R [sa[i]] = r:
          } R.swap(R_);
       }
   }
   void getLCP(vi &s)
       lcp.assign(n, 0); int k = 0;
       rep(i, n)
           int r = R[i] - 1;
           if (r == n - 1) { k = 0; continue; }
           int j = sa[r + 1];
           while (i + k < n \&\& j + k < n \text{ and } s[i + k] == s[j + k]) k++;
          lcp[r] = k: if (k) k--:
   SA(vi &s) { n = s.size(); getSA(s); getLCP(s); }
};
```

#### 6.6 SuffixAutomaton

```
// vector implementation
struct SA
{
   int sz, l; vi L, Lk, S, C, T;
   vector<vi> N, Ilk;
```

```
SA(string s, int n) : L(2 * n), Lk(2 * n), C(2 * n), N(2 * n, vi(26, -1))
       1 = L[0] = 0, Lk[0] = -1, sz = 1; int p;
       for (char c : s) p = extend(c - 'A');
       Ilk.resize(sz); S.assign(sz, -1);
       rep(i, sz) if (i) Ilk[Lk[i]].pb(i);
       T.assign(sz, 0); while (p != -1) T[p] = 1, p = Lk[p];
   int extend(char c)
   {
       int cur = sz++, p = 1; C[cur] = 0, L[cur] = L[1] + 1;
       while (p != -1 \&\& N[p][c] == -1) N[p][c] = cur, p = Lk[p];
       if (p == -1) { Lk[cur] = 0, 1 = cur; return cur; }
       int q = N[p][c];
       if (L[p] + 1 == L[q]) { Lk[cur] = q, 1 = cur; return cur; }
       int w = sz++; C[w] = 1, L[w] = L[p] + 1, Lk[w] = Lk[q], N[w] = N[q];
       while (p != -1 \&\& N[p][c] == q) N[p][c] = w, p = Lk[p];
       Lk[q] = Lk[cur] = w, 1 = cur; return cur;
   int size(int p)
       if (S[p] != -1) return S[p];
       for (int i : Ilk[p]) S[p] += size(i);
       return S[p] += (1 - C[p]) + 1;
};
// 101 vector implementation
struct SA
{
   int sz, 1; vi L, Lk;
   vector<vi> N. Ilk:
   SA(string s, int n) : L(2 * n), Lk(2 * n), N(2 * n, vi(26, -1))
       1 = L[0] = 0, Lk[0] = -1, sz = 1; int p;
       for (char c : s) p = extend(c - 'A');
   int extend(char c)
       int cur = sz++, p = 1; L[cur] = L[1] + 1;
       while (p != -1 \&\& N[p][c] == -1) N[p][c] = cur, p = Lk[p];
       if (p == -1) { Lk[cur] = 0, l = cur; return cur; }
       int q = N[p][c];
       if (L[p] + 1 == L[q]) { Lk[cur] = q, 1 = cur; return cur; }
       int w = sz++; L[w] = L[p] + 1, Lk[w] = Lk[q], N[w] = N[q];
       while (p != -1 \&\& N[p][c] == q) N[p][c] = w, p = Lk[p];
       Lk[q] = Lk[cur] = w, 1 = cur; return cur;
};
// 101 map implementation
struct SA
```

```
int sz, 1; vi L, Lk;
   vector<map<char, int>> N;
   SA(string s, int n) : L(2 * n), Lk(2 * n), N(2 * n)
       1 = L[0] = 0, Lk[0] = -1, sz = 1;
       for (char c : s) extend(c);
   void extend(char c)
       int cur = sz++, p = 1; L[cur] = L[1] + 1;
       while (p != -1 \&\& !N[p].count(c)) N[p][c] = cur, p = Lk[p];
       if (p == -1) { Lk[cur] = 0, 1 = cur; return; }
       int q = N[p][c];
       if (L[p] + 1 == L[q]) { Lk[cur] = q, 1 = cur; return; }
       int w = sz++: L[w] = L[p] + 1. Lk[w] = Lk[a]. N[w] = N[a]:
       while (p != -1 \&\& N[p][c] == q) N[p][c] = w, p = Lk[p];
       Lk[q] = Lk[cur] = w, 1 = cur;
   }
};
```

#### 6.7 Trie

```
struct Trie
    static const int MAX = 1e6;
   int N[MAX][26] = \{0\}, S[MAX] = \{0\}, c = 0;
    void add(string s, int a = 1)
       int p = 0; S[p] += a;
       for (char 1 : s)
           int t = 1 - 'a';
           if (!N[p][t]) N[p][t] = ++c;
           S[p = N[p][t]] += a;
   }
};
struct TrieXOR
    static const int MAX = 1e6;
   int N[MAX][2] = \{0\}, S[MAX] = \{0\}, c = 0;
   void add(int x, int a = 1)
       int p = 0; S[p] += a;
       rep(i, 31)
           int t = (x >> (30 - i)) & 1;
           if (!N[p][t]) N[p][t] = ++c;
           S[p = N[p][t]] += a;
```

```
}
int get(int x)
{
    if (!S[0]) return -1;
    int p = 0; rep(i, 31)
    {
        int t = ((x >> (30 - i)) & 1) ^ 1;
        if (!N[p][t] || !S[N[p][t]]) t ^ = 1;
        p = N[p][t]; if (t) x ^ = (1 << (30 - i));
    }
    return x;
}
</pre>
```

## 7 Structures

## 7.1 FenwickTree

```
// 1 - indexed / inclusive - inclusive
struct FT
{
    vi t;
    FT(int N) { t.resize(N + 1, 0); }
    int query(int i)
    {
        int ans = 0;
        for (; i; i -= i & (-i)) ans += t[i];
        return ans;
    }
    int query(int i, int j) { return query(j) - query(i - 1); }
    void update(int i, int v)
    {
        int s = query(i, i); // Sets
        for (; i < t.size(); i += i & (-i)) t[i] += v - s;
    }
    void update(int i, int j, int v)
    {
        update(i, v); update(j + 1, -v);
    }
};</pre>
```

#### 7.2 FenwickTree2D

```
// 0 - indexed / inclusive - inclusive
template <class T>
class FT2D
```

```
vector<vector<T>> t;
   int n, m;
public:
   FT2D(int n, int m) : n(n), m(m) { t.assign(n, vector<T>(m, 0)); }
   void add(int r, int c, T value)
       for (int i = r; i < n; i | = i + 1)
          for (int j = c; j < m; j |= j + 1)</pre>
              t[i][j] += value;
   T sum(int r. int c)
       T res = 0:
       for (int i = r; i \ge 0; i = (i & (i + 1)) - 1)
          for (int j = c; j >= 0; j = (j & (j + 1)) - 1)
              res += t[i][i];
       return res;
   T sum(int r1, int c1, int r2, int c2)
       return sum(r2, c2) - sum(r1 - 1, c2) - sum(r2, c1 - 1) +
             sum(r1 - 1, c1 - 1);
   T get(int r, int c) { return sum(r, c, r, c); }
   void set(int r, int c, T value) { add(r, c, -get(r, c) + value); }
};
```

## 7.3 LineContainer

```
struct Line
{
    mutable ll k, m, p;
    bool operator<(const Line &o) const { return k < o.k; }
    bool operator<(ll x) const { return p < x; }
};

// (for doubles, use inf = 1/.0, div(a,b) = a/b)
struct LineContainer : multiset<Line, less<>>
{
    const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y)
    {</pre>
```

```
19
```

```
if (y == end()) { x->p = inf; return false; }
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
}

void add(ll k, ll m)
{
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p) isect(x, erase(y));
}
ll query(ll x)
{
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
}
};
```

## 7.4 MoQueries

```
int S; // Size of buckets (generally sqrt(N))
struct Query
{
   int 1, r, id;
   Query(int 1, int r, int id) : 1(1), r(r), id(id) {}
   bool operator<(Query &q)
   {
      return 1 / S < q.1 / S or (1 / S == q.1 / S and r < q.r);
   }
};</pre>
```

## 7.5 PersistentSegmentTree

```
struct Node
{
    int v, 1, r;
    Node() : v(0), 1(0), r(0) {}
    Node(int v) : v(v) {}
    Node(const Node &a, const Node &b, int 1, int r) : 1(1), r(r)
    { v = a.v + b.v; }
};

// 0 - indexed / inclusive - inclusive
template <class node>
struct PST
```

```
int cnt = 0, n, rc = 0;
   vector<node> st; vi rt;
   node query(int u, int a, int b, int i, int j)
       if (j < a || b < i) return node();</pre>
       if (a <= i && j <= b) return st[u];</pre>
       int m = (i + j) / 2, l = st[u].l, r = st[u].r;
       return node(query(1, a, b, i, m), query(r, a, b, m + 1, j), l, r);
   int update(int u, int p, node v, int i, int i)
       if (j 
       st[cnt] = st[u]; int x = cnt++, m = (i + j) / 2;
       if (i == j) { st[x] = v: return x: }
       int 1 = st[x].1 = update(st[x].1, p, v, i, m);
       int r = st[x].r = update(st[x].r, p, v, m + 1, j);
       st[x] = node(st[1], st[r], 1, r); return x;
   int build(vector<node> &arr, int i, int j)
       int u = cnt++, m = (i + j) / 2;
       if (i == j) { st[u] = arr[i]; return u; }
       int 1 = st[u].1 = build(arr, i, m);
       int r = st[u].r = build(arr, m + 1, j);
       st[u] = node(st[1], st[r], 1, r); return u;
   PST(vector<node> &arr) : st(1e7), rt(1e5)
   \{ n = arr.size(): rt[rc++] = build(arr. 0, n - 1): \}
   void update(int t, int p, node v) { rt[rc++] = update(rt[t], p, v, 0, n - 1);
   node query(int t, int a, int b) { return query(rt[t], a, b, 0, n - 1); }
}:
// Init with null's (no build) (requires l = r = 0 as default)
template <class node>
struct PST
   int cnt = 1, n, rc = 1;
   vector<node> st; vi rt;
   node query(int u, int a, int b, int i, int j)
       if (j < a || b < i) return node();</pre>
       if (a <= i && j <= b) return st[u];</pre>
       int m = (i + j) / 2, l = st[u].l, r = st[u].r;
       return node(query(1, a, b, i, m), query(r, a, b, m + 1, j), l, r);
   int update(int u, int p, node v, int i, int j)
       if (j 
       st[cnt] = st[u]; int x = cnt++, m = (i + j) / 2;
       if (i == i) { st[x] = v: return x: }
```

```
int 1 = st[x].1 = update(st[x].1, p, v, i, m);
   int r = st[x].r = update(st[x].r, p, v, m + 1, j);
   st[x] = node(st[1], st[r], 1, r); return x;
}

PST(int N) : st(1e7), rt(1e5), n(N) {}
void update(int t, int p, node v) { rt[rc++] = update(rt[t], p, v, 0, n - 1);
   }
node query(int t, int a, int b) { return query(rt[t], a, b, 0, n - 1); }
};
```

## 7.6 PersistentSegmentTreeLazy

```
struct Node
   int v, 1 = 0, r = 0, 1zv = 0;
   bool lz = false:
   Node(): v(0) {}
   Node(int x) : v(x) \{ \}
   Node (const Node &a, const Node &b, int 1, int r): v(a.v + b.v), l(1), r(r) {}
   Node(int x, int i, int i, const Node &b): 1(b.1), r(b.r)
   \{ v = b.v + (j - i + 1) * x; \}
}:
// 0 - indexed / inclusive - inclusive
template <class node>
struct PSTL
   int cnt = 0, n, rc = 0;
   vector<node> st; vi rt;
   void push(int u, int v, int i, int j)
       st[u] = node(v, i, j, st[u]);
       if (i == j) return;
       st[cnt] = st[st[u].1]: int 1 = cnt++:
       st[cnt] = st[st[u].r]: int r = cnt++:
       st[u].1 = 1, st[1].1z = 1, st[1].1zv += v;
              st[u].r = r. st[r].lz = 1. st[r].lzv += v:
   node query(int u, int a, int b, int i, int j)
       if (j < a || b < i) return node();</pre>
       if (st[u].lz) push(u, st[u].lzv, i, j);
       if (a <= i && j <= b) return st[u];</pre>
       int m = (i + j) / 2, l = st[u].l, r = st[u].r;
       return node(query(1, a, b, i, m), query(r, a, b, m + 1, j), l, r);
   int update(int u, int a, int b, int v, int i, int j)
       if (st[u].lz) push(u, st[u].lzv, i, j);
```

```
if (j < a \mid | b < i) return u;
       st[cnt] = st[u]; int x = cnt++, m = (i + j) / 2;
       if (a <= i && j <= b) { push(x, v, i, j); return x; }
       int l = st[x].l = update(st[x].l, a, b, v, i, m);
       int r = st[x].r = update(st[x].r, a, b, v, m + 1, j);
       st[x] = node(st[1], st[r], 1, r); return x;
   int build(vector<node> &arr, int i, int j)
       int u = cnt++, m = (i + i) / 2:
       if (i == j) { st[u] = arr[i]; return u; }
       int l = st[u].l = build(arr, i, m):
       int r = st[u].r = build(arr, m + 1, i):
       st[u] = node(st[1], st[r], 1, r); return u:
   PSTL(vector<node> &arr) : st(1e7), rt(1e5)
   \{ n = arr.size(): rt[rc++] = build(arr. 0, n - 1): \}
   void update(int t, int a, int b, int v)
   { rt[rc++] = update(rt[t], a, b, v, 0, n - 1); }
   node query(int t, int a, int b) { return query(rt[t], a, b, 0, n - 1); }
}:
// Direct accumulate (No Push) Faster and shorter
struct Node
   int v = 0, 1 = 0, r = 0, 1zv = 0;
   bool lz = false;
   Node() {}
   Node(int x) : v(x) {}
   Node(const Node &a, const Node &b, int 1, int r): v(a.v + b.v), 1(1), r(r) {}
   Node(int x, int i, int i, const Node &b)
   \{ *this = b; v += (j - i + 1) * x; \} // *this = b needed in this variant
        (keeps lazv)
};
template <class node>
struct PSTL
   int cnt = 0, n, rc = 0:
   vector<node> st; vi rt;
   node query(int u, int a, int b, int i, int j, ll acc)
       if (j < a || b < i) return node();</pre>
       if (st[u].lz) acc += st[u].lzv;
       int m = (i + j) / 2, l = st[u].l, r = st[u].r;
       if (a <= i && j <= b) return node(acc, i, j, st[u]);</pre>
       return node(query(1, a, b, i, m, acc), query(r, a, b, m + 1, j, acc), l,
            r):
   }
   int update(int u, int a, int b, int v, int i, int j)
       if (j < a || b < i) return u;</pre>
       st[cnt] = st[u]: int x = cnt++, m = (i + i) / 2:
```

```
if (a <= i && j <= b) { st[x].lz = 1, st[x].lzv += v; return x; }</pre>
       int 1 = st[x].1 = update(st[x].1, a, b, v, i, m);
       int r = st[x].r = update(st[x].r, a, b, v, m + 1, j);
       st[x] = node(v, max(i, a), min(j, b), st[x]); return x;
   int build(vector<node> &arr, int i, int j)
       int u = cnt++, m = (i + j) / 2;
       if (i == j) { st[u] = arr[i]: return u: }
       int 1 = st[u].1 = build(arr, i, m);
       int r = st[u].r = build(arr, m + 1, j);
       st[u] = node(st[1], st[r], 1, r); return u;
   PSTL(vector<node> &arr) : st(5e6), rt(2e5)
   \{ n = arr.size(): rt[rc++] = build(arr. 0, n - 1): \}
   int update(int t, int a, int b, int v)
   { rt[rc] = update(rt[t], a, b, v, 0, n - 1); return rc++; }
   node query(int t, int a, int b) { return query(rt[t], a, b, 0, n - 1, 0); }
};
```

# 7.7 PolicyBased

# 7.8 SegmentTree

```
struct Node
{
    int v;
    Node() { v = 0; } // neutro
    Node(int v) : v(v) {}
    Node(const Node &a, const Node &b) { v = a.v + b.v; }
};
// 0 - indexed / inclusive - exclusive
```

```
template <class node>
struct ST
   vector<node> t; int n;
   ST(vector<node> &arr, int N) : n(N), t(N * 2)
       copy(arr.begin(), arr.end(), t.begin() + n);
       for (int i = n - 1; i > 0; --i) t[i] = node(t[i << 1], t[i << 1 | 1]);
   void set(int p, const node &value)
       for (t[p += n] = value: p >>= 1:)
          t[p] = node(t[p << 1], t[p << 1 | 1]);
   node querv(int 1. int r)
       node ansl. ansr:
       for (1 += n, r += n; 1 < r; 1 >>= 1, r >>= 1)
          if (l \& 1) ansl = node(ansl, t[l++]);
          if (r \& 1) ansr = node(t[--r], ansr);
       return node(ansl, ansr);
};
```

# 7.9 SegmentTreeLazy

```
struct Node
{
    int v, lzv = 0;
    bool lz = false;
    Node() : v(0) {}
    Node(int x) : v(x) {}
    Node(const Node &a, const Node &b) : v(a.v + b.v) {}
    Node(int x, int i, int j, const Node &b)
    {
        v = b.v + (j - i + 1) * x;
    }
};

// 0 - indexed / inclusive - inclusive
template <class node>
struct STL
{
    vector<node> st; int n;

    void build(int u, int i, int j, vector<node> &arr)
    {
        if (i == j) { st[u] = arr[i]; return; }
}
```

```
int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
       build(l, i, m, arr), build(r, m + 1, j, arr);
       st[u] = node(st[1], st[r]);
   void push(int u, int i, int j, int x)
       st[u] = node(x, i, j, st[u]);
       if (i == j) return;
       st[u * 2 + 1].lz = 1, st[u * 2 + 1].lzv += x:
       st[u * 2 + 2].lz = 1, st[u * 2 + 2].lzv += x:
   }
   node query(int a, int b, int u, int i, int j)
       if (j < a || b < i) return node();</pre>
       int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
       if (st[u].lz) push(u, i, i, st[u].lzv):
       if (a <= i && j <= b) return st[u];</pre>
       return node(query(a, b, 1, i, m), query(a, b, r, m + 1, j));
   void update(int a, int b, int v, int u, int i, int j)
       if (st[u].lz) push(u, i, j, st[u].lzv);
       if (j < a || b < i) return;</pre>
       int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
       if (a <= i && j <= b) { push(u, i, j, v); return; }</pre>
       update(a, b, v, 1, i, m); update(a, b, v, r, m + 1, j);
       st[u] = node(st[1], st[r]);
   }
   STL(vector < node > &v, int N) : n(N), st(N * 4 + 5) { build(0, 0, n - 1, v); }
   node query(int a, int b) { return query(a, b, 0, 0, n - 1); }
   void update(int a, int b, int v) { update(a, b, v, 0, 0, n - 1); }
};
```

# 7.10 SparseTable

```
template <class t>
class ST
{
   int n;
   vi memo, *arr;

public:
   ST(vi &_arr)
   {
      arr = &_arr; n = arr->size();
      int maxlog = 31 - __builtin_clz(n);
      memo.assign(n * (maxlog + 1), -1);
   }
   int dp(int i, int e)
   {
```

```
int & ans = memo[e * n + i];
       if (ans != -1) return ans;
       if (e == 0) return ans = (*arr)[i];
       return ans = t::merge(dp(i, e - 1), dp(i + (1 << (e - 1)), e - 1));</pre>
   }
   int query_01(int 1, int r)
       int e = 31 - builtin clz(r - 1 + 1):
       return t::merge(dp(1, e), dp(r - (1 << e) + 1, e));
   int query_Ologn(int 1, int r)
       int ans = t::neutro;
       int d = r - 1 + 1;
       for (int e = 0; d; e++, d >>= 1) if (d & 1)
           ans = t::merge(ans, dp(1, e));
           1 += 1 << e;
       return ans;
};
struct OP
{
    static const int neutro = 0;
    static int merge(int x, int y) { return x + y; }
};
```

# **7.11** Treap

```
mt19937 gen(chrono::high_resolution_clock::now().time_since_epoch().count());
typedef pair<int, int> ii;

// 101 Treap //
struct Node
{
   int p, sz = 0, v, acc, l = -1, r = -1;
   Node() : v(0), acc(0) {}
   Node(int x): p(gen()), sz(1), v(x), acc(x) {}
   void recalc(const Node &a, const Node &b)
   {
      sz = a.sz + b.sz + 1;
      acc = v + a.acc + b.acc;
   }
};
template <class node>
```

```
struct Treap
₹
   vector\langle node \rangle t; int n, r = -1;
   node get(int u) { return u != -1 ? t[u] : node(); }
   void recalc(int u) { t[u].recalc(get(t[u].l), get(t[u].r)); }
   int merge(int 1, int r)
       if (min(1, r) == -1) return 1 != -1 ? 1 : r:
       int ans = (t[1].p < t[r].p) ? 1 : r;
       if (ans == 1) t[1].r = merge(t[1].r, r), recalc(1);
       if (ans == r) t[r].l = merge(l, t[r].l), recalc(r):
       return ans:
   ii split(int u. int id)
       if (u == -1) return {-1, -1};
       int szl = get(t[u].1).sz:
       if (szl >= id)
           ii ans = split(t[u].1, id);
           t[u].1 = ans.ss; recalc(u);
           return {ans.ff, u};
       }
       ii ans = split(t[u].r, id - szl - 1);
       t[u].r = ans.ff; recalc(u);
       return {u, ans.ss};
   Treap(vector<int> &v) : n(v.size())
   { for (int i = 0; i < n; i++) t.eb(v[i]), r = merge(r, i); }
}:
// Complete Treap with Lazy propagation //
struct Node
   int p, sz = 0, v, acc, l = -1, r = -1, par = -1, lzv = 0;
   bool lz = false, f = false:
   Node(): v(0), acc(0) {}
   Node(int x): p(gen()), sz(1), v(x), acc(x) {}
   void recalc(const Node &a, const Node &b)
       sz = a.sz + b.sz + 1;
       acc = v + a.acc + b.acc;
   void upd_lazy(int x) { lz = 1, lzv += x; }
   void lazy() { v += lzv, acc += sz * lzv, lz = 0, lzv = 0; }
   void flip() { swap(1, r), f = 0; }
};
template <class node>
struct Treap
   vector<node> t: int n. r = -1:
```

```
node get(int u) { return u != -1 ? t[u] : node(); }
void recalc(int u)
   int 1 = t[u].1, r = t[u].r;
   push(1); push(r); flip(1); flip(r);
   t[u].recalc(get(1), get(r));
void push(int u)
   if (u == -1 || !t[u].lz) return;
   int l = t[u].l. r = t[u].r:
   if (1 != -1) t[1].upd lazv(t[u].lzv):
   if (r != -1) t[r].upd_lazy(t[u].lzv);
   t[u].lazy();
}
void flip(int u)
   if (u == -1 || !t[u].f) return;
   int 1 = t[u].1, r = t[u].r;
   if (1 != -1) t[1].f ^= 1;
   if (r != -1) t[r].f ^= 1;
   t[u].flip();
}
int merge(int 1, int r)
   if (min(1, r) == -1) return 1 != -1 ? 1 : r;
   push(1); push(r); flip(1); flip(r);
   int ans = (t[1].p < t[r].p) ? 1 : r;
   if (ans == 1) t[1].r = merge(t[1].r, r), recalc(1);
   if (ans == r) t[r].l = merge(l, t[r].l), recalc(r);
   if (t[ans].l != -1) t[t[ans].l].par = ans; // only if parent needed
   if (t[ans].r != -1) t[t[ans].r].par = ans; // only if parent needed
   return ans:
ii split(int u. int id)
   if (u == -1) return {-1, -1};
   push(u): flip(u):
   int szl = get(t[u].1).sz:
   if (szl >= id)
       ii ans = split(t[u].1, id);
       if (ans.ss != -1) t[ans.ss].par = u; // only if parent needed
       if (ans.ff != -1) t[ans.ff].par = -1; // only if parent needed
       t[u].1 = ans.ss; recalc(u);
       return {ans.ff, u};
   ii ans = split(t[u].r, id - szl - 1);
   if (ans.ff != -1) t[ans.ff].par = u; // only if parent needed
   if (ans.ss != -1) t[ans.ss].par = -1; // only if parent needed
   t[u].r = ans.ff; recalc(u);
   return {u. ans.ss}:
int update(int u. int l. int r. int v)
```

```
{
    ii a = split(u, 1), b = split(a.ss, r - 1 + 1);
        t[b.ff].upd_lazy(v);
    return merge(a.ff, merge(b.ff, b.ss));
}
void print(int u)
{
    if (u == -1) return;
    push(u); flip(u);
    print(t[u].1);
    cout << t[u].v << ' ';
    print(t[u].r);
}
Treap(vector<int> &v) : n(v.size())
{    for (int i = 0; i < n; i++) t.eb(v[i]), r = merge(r, i); }
};</pre>
```

## 7.12 UnionFind

```
struct DSU
{
    vi p;
    DSU(int N) : p(N, -1) {}
    int get(int x) { return p[x] < 0 ? x : p[x] = get(p[x]); }
    bool sameSet(int a, int b) { return get(a) == get(b); }
    int size(int x) { return -p[get(x)]; }
    void unite(int x, int y)
    {
        if ((x = get(x)) == (y = get(y))) return;
          if (p[x] > p[y]) swap(x,y);
        p[x] += p[y], p[y] = x;
    }
};
```

#### 7.13 WaveletTree

```
class WT
{
    typedef vi::iterator iter;
    vector<vi> r0;
    vi arrCopy;
    int n, s;

    void build(iter b, iter e, int 1, int r, int u)
    {
        if (1 == r) return;
        int m = (1 + r) / 2;
    }
}
```

```
r0[u].reserve(e - b + 1); r0[u].pb(0);
       for (iter it = b; it != e; ++it)
           r0[u].pb(r0[u].back() + (*it <= m));
       iter p = stable_partition(b, e, [=](int i) { return i <= m; });</pre>
       build(b, p, 1, m, u * 2); build(p, e, m + 1, r, u * 2 + 1);
   int range(int a, int b, int 1, int r, int u)
       if (r < q \text{ or } w < 1) \text{ return } 0;
       if (q <= 1 && r <= w) return b - a:
       int m = (1 + r) / 2, za = r0[u][a], zb = r0[u][b];
       return range(za, zb, 1, m, u * 2) +
              range(a - za, b - zb, m + 1, r, u * 2 + 1);
   }
public:
   // arr[i] in [0,sigma)
   WT(vi arr, int sigma)
       n = arr.size(); s = sigma; r0.resize(s * 2);
       arrCopy = arr;
       build(all(arr), 0, s - 1, 1);
   }
   // k in [1,n], [a,b) is 0-indexed, -1 if error
   int quantile(int k, int a, int b)
       // extra conditions disabled
       if (/*a < 0 \text{ or } b > n \text{ or } */ k < 1 \text{ or } k > b - a) return -1:
       int 1 = 0, r = s - 1, u = 1, m, za, zb;
       while (1 != r)
           m = (1 + r) / 2:
           za = r0[u][a], zb = r0[u][b], u *= 2;
           if (k \le zb - za) a = za, b = zb, r = m:
           else k = zb - za, a = za, b = zb, l = m + 1, ++u;
       return r;
   // counts numbers in [x,y] in positions [a,b)
   int range(int x, int y, int a, int b)
       if (y < x \text{ or } b \le a) \text{ return } 0;
       q = x, w = y;
       return range(a, b, 0, s - 1, 1);
   // count occurrences of x in positions [0,k)
   int rank(int x, int k)
       int 1 = 0, r = s - 1, u = 1, m, z;
       while (1 != r)
```

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```
{
       m = (1 + r) / 2;
       z = r0[u][k], u *= 2;
       if (x \le m) k = z, r = m;
       else k -= z, l = m + 1, ++u;
   }
   return k;
}
// x in [0,sigma)
void pb(int x)
   int 1 = 0, r = s - 1, u = 1, m, p;
   ++n;
   while (1 != r)
       m = (1 + r) / 2;
       p = (x \le m);
       r0[u].pb(r0[u].back() + p);
       u *= 2;
      if (p) r = m;
       else 1 = m + 1, ++u;
   }
}
// doesn't check if empty
void pop_back()
{
   int 1 = 0, r = s - 1, u = 1, m, p, k;
   --n:
   while (1 != r)
```

```
m = (1 + r) / 2;
          k = r0[u].size(), p = r0[u][k - 1] - r0[u][k - 2];
          r0[u].pop_back();
          u *= 2;
          if (p) r = m;
           else l = m + 1, ++u;
      }
   }
   // swap arr[i] with arr[i+1], i in [0,n-1)
   void swap_adj(int i)
       int &x = arrCopy[i], &y = arrCopy[i + 1];
       int 1 = 0, r = s - 1, u = 1;
       while (1 != r)
       {
          int m = (1 + r) / 2, p = (x \le m), q = (y \le m);
          if (p != q)
          {
              r0[u][i + 1] ^= r0[u][i] ^ r0[u][i + 2];
          }
          u *= 2;
          if (p) r = m;
          else 1 = m + 1, ++u;
       }
       swap(x, y);
};
```