Lecture 2

Consider a right-travelling transverse wave:

[Image]

A particle on the string only has transverse velocity. So $\frac{\mathrm{d}y}{\mathrm{d}t}$ is not the speed of the wave. The same is true for longitudinal waves such as sound; the movement of the air molecules is vibrational.

Phase and Phase Difference

$$u(t) = A\cos(kx - \omega t + \eta)$$

where η is the **phase of the wave**. η is important for the superposition of waves, which leads to interference effects.

[Image]

Principle of Superposition

Waves of the same frequency

$$\begin{aligned} u_1 &= A_1 \cos(kx - \omega t) \\ u_2 &= A_2 \cos(kx - \omega t + \eta) \end{aligned}$$

Using the following trigonometric identity:

$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

If $\eta = 0$, $u_{total} = 2u_1 \leftarrow$ **constructive interference**
If $\eta = \pi$, $u_{total} = 0 \leftarrow$ **destructive interference**

Waves of a similar frequency

$$\begin{array}{rll} \text{Wave } A & k_1 = k + \Delta k & \omega_1 = \omega + \Delta\omega \\ B & k_2 = k - \Delta k & \omega_2 = \omega - \Delta\omega \end{array}$$

From this, we get:

$$u = 2\cos(\Delta kx + \Delta\omega t)\cos(kx + \omega t)$$

The first cosine in the above equation represents the 'beat wave', and the second represents the 'actual' wave.

Pianos have three strings per note, which are tuned to slightly different frequencies. Overall, when a note is played, we hear a note of frequency ω (an average of the different waves' frequencies), and a beat frequency $\Delta\omega$. If a piano had only two strings per note, and they were tuned to 254Hz and 258Hz, we would hear a note at 256Hz

$$\text{beat frequency } = 2\Delta\omega$$

Group Velocity

There are two components to the total wave, with different speeds:

$$\begin{aligned} \text{the wave} \quad v &= \frac{\omega}{k} \\ \text{the beat wave} \quad v &= \frac{\Delta\omega}{\Delta k} \end{aligned}$$

As $\Delta \to 0$,

$$v_g = \frac{\partial \omega}{\partial k}$$