Department of Informatics
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7CCSMQMF Quantitative Methods for Finance Course Work 2024.

Instructions: This option pricing Python course work should be completed using a Jupyter notebook. The .ipynb file should be uploaded to Keats no later than 16:00 on Tuesday 10 December 2024. All queries to riaz.ahmad@kcl.ac.uk

Introduction: A random variable $X \sim N(0,1)$ has cumulative distribution function (CDF)

$$N(x) = p(X < x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-s^2/2} ds,$$

for an observed value x. We can approximate this improper integral by using the polynomial based numerical scheme

$$N(x) = \begin{cases} 1 - n(x)\left(a_1 k + a_2 k^2 + a_3 k^3 + a_4 k^4 + a_5 k^5\right) & x \geq 0 \\ 1 - N(-x) & x < 0 \end{cases}$$

where

$$k = \frac{1}{1 + 0.2316419x}$$

and constants

$$a_1 = 0.319381530, a_2 = -0.356563782, a_3 = 1.781477937, a_4 = -1.821255978, a_5 = 1.330274429$$

and

$$n(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Task 1: CDF Function

- Write a function called **CDF()** taking a single value x which is a standardised normal random variable.
- Use CDF() in a loop from $x \in [-4, 4]$ and plot $N(x)$ on the vertical axis. You may use any plotting package. If anything other than matplotlib, make this clear.
- Compare the accuracy of your CDF with Python's built-in function norm. cdf() in SciPy

Task 2: Option Pricing using your CDF function

- This task involves producing an option pricing calculator using the Black-Scholes formulae given in the relevant lecture.
- Write functions to price calls and puts for Europeans and binaries. Input values should be entered using the keyboard by the user and an example set is

$$\begin{aligned} \text{Today's stock price } S_0 &= 100; \quad \text{Strike } E = 100 \\ \text{Today } t &= 0; \quad \text{Expiry } T = 1 \text{ year} \\ \text{volatility } \sigma &= 20\%; \quad \text{constant risk-free interest rate } r = 5\% \\ \text{continuous and constant dividend yield } D &= 0 \end{aligned}$$

- Focusing on vanilla calls and puts, change input values (one at a time) and experiment with how these affect option prices. You may include in a table and present a discussion of your observations.

Task 3: Plotting option prices

- Using the results in Task 2, plot the option price $V(S)$ against varying stock S. This should be for some time prior to expiry, $t < T$.
- On the same axis include the payoff function for the option.
- You are expected to present a total of four plots: One each for European call & put and binary call and put.