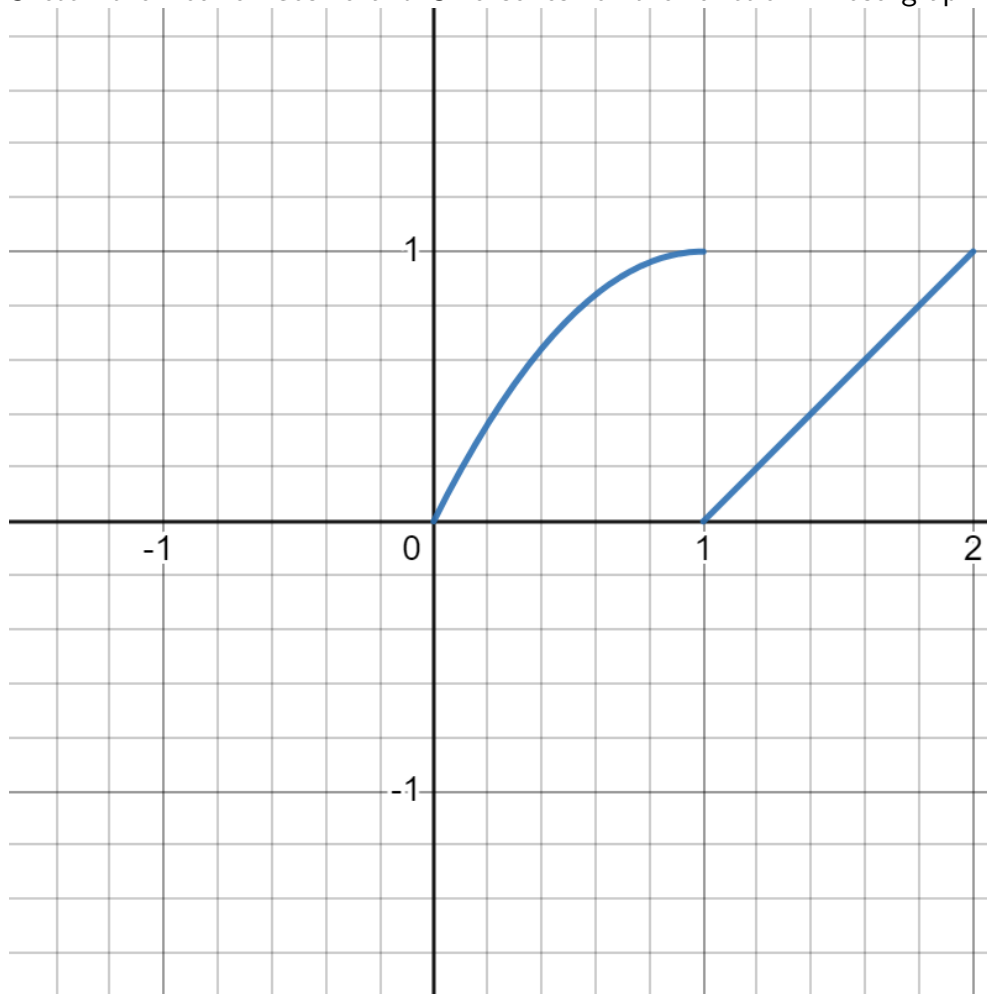


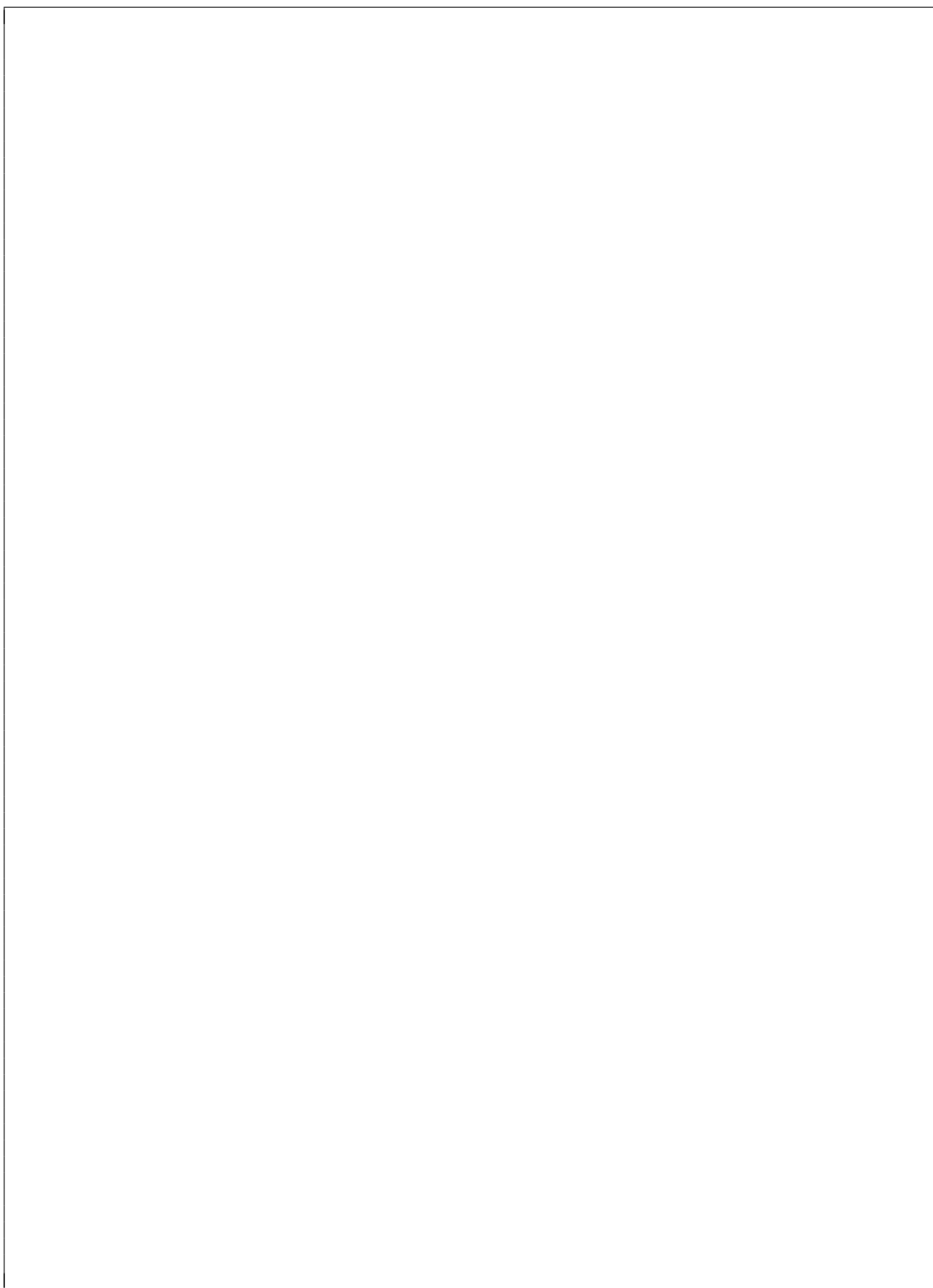
This is homework number 10 and it is **due on August 2**. You *must* use this template for your homework. You can either print it out and write on it and upload that, or you can use a tablet if you have one. Alternatively, I am providing you with a link to the \LaTeX template. If you type your homework then (1) you'll become accustomed to using \LaTeX which is probably good and (2) you'll get a bonus of 2 points (each HW assignment is 10 points, so this means that your maximum is 12 if you type). Whichever you choose, you will turn it in on Gradescope.

Also, no matter which method you choose, your work must be neat, legible, and must flow clearly. This, of course, includes the requirement that you show your work appropriately. See examples in class of what I am looking for. In particular, there should be nothing that is scratched out (either use Whiteout or something similar or just re-write the whole thing). The work should more or less progress from left to right, top to bottom. In short, imagine that this is a history class or something and you're turning in your *final draft*. Part of what you are being graded on is your ability to communicate well which, at a minimum, means the reader can actually read what you have written.

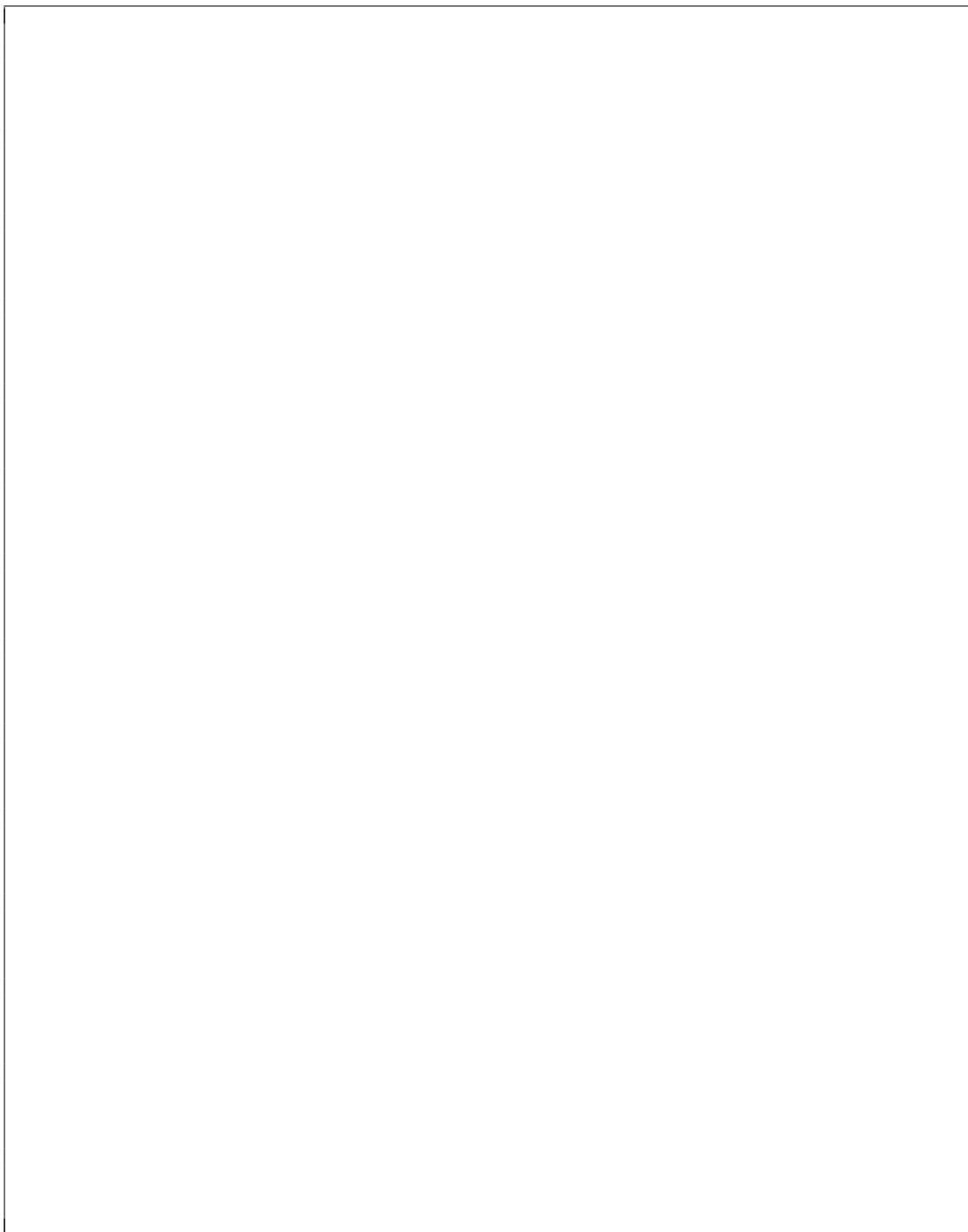
Three of the problems will be graded for accuracy, the others will be graded for "completion". Here, "completion" means: is it clear the student made an honest attempt at the problem and wrote the solution/attempt up in a neat way?

1. Sketch the Fourier Cosine and Sine series for the function whose graph is below:





2. Find a function, f , on $[0, 1]$ such that the fourier sine series converges to f on $[0, 1]$ with the exception of exactly one point in $(0, 1)$. Note: there are a gazillion different answers here. I'm not looking for a formula, just a graph will do.



3. Assume that if f and g satisfy:

$$-f''(x) + q(x)f(x) = \lambda f(x)$$

$$-g''(x) + q(x)g(x) = \gamma g(x),$$

with $f(0) = f(\pi) = g(0) = g(\pi) = 0$. Show that either $\gamma = \lambda$ or $\int_0^\pi f(x)g(x)dx = 0$. Here, $q(x)$ is a “given” but generic function.

4. For the boundary value problem $-u'' = \lambda u$ with $u(0) = 0$ and $u(\pi) + u'(\pi) = 0$. Show that the eigenvalues are non-negative.

5. For the boundary value problem above, you cannot find an explicit formula for the eigenvalues, but you can find an equation that the eigenvalues satisfy. What is that equation.