Math 412 Summer 23 **Name:** ____________________

This is homework number 6 and it is **due on July 5**. You *must* use this template for your homework. You can either print it out and write on it and upload that, or you can use a tablet if you have one. Alternatively, I am providing you with a link the the LaTeXtemplate. If you type your homework then (1) you'll become accustomed to using LaTeXwhich is probably good and (2) you'll get a bonus of 2 points (each HW assignment is 10 points, so this means that your maximum is 12 if you type). Whichever you choose, you will turn it in on Gradescope.

Also, no matter which method you choose, your work must be neat, legible, and must flow clearly. This, of course, includes the requirement that you show you work appropriately. See examples in class of what I am looking for. In particular, there should be nothing that is scratched out (either use Whiteout or something similar or just re-write the whole thing). The work should more or less progress from left to right, top to bottom. In short, imagine that this is a history class or something and you're turning in your *final draft*. Part of what you are being graded on is your ability to communicate well which, at a minimum, means the reader can actually read what you have written.

Three of the problems will be graded for accuracy, the others will be graded for "completion". Here, "completion" means: is it clear the student made an honest attempt at the problem and wrote the solution/attempt up in a neat way?

1. Consider an infinitely long one–dimensional uniform rod. Let $e(x,t)$ denote the concentration of thermal energy per meter at location x and time t. By the diffusion equation, this satisfies $\partial_t e = \alpha \partial_{xx} e$. If $u(x,t)$ denotes the temperature of the rod at location x and time t, *using the diffusion equation* $\partial_t e = \alpha \partial_{xx} e$ derive a PDE that u satisfies. Note: the thermal energy in a slice of rod is the energy needed to raise the slice from a reference temperature of 0 degrees to its current temperature. The specific heat, $c(x,T)$, is the thermal energy needed to raise a unit mass of a substance on unit. So the thermal energy needed to raise a thin slice from 0 to $u(x,t)$ is $\int_{T=0}^{u(x,t)} c(x,T)\rho dT$ where ρ is the linear mass density of the rod. You may assume that $c(x,T)$ does not depend on T.

2. Solve the PDE $\partial_t u = k\partial_{xx} u - \gamma u$ for $-\infty < x < \infty$ if $u(x, 0) = f(x)$.

3. This is another way to solve the non–homogeneous problem: solve $\partial_t u(x,t) = \partial_{xx} u(x,t) + f(x,t)$ if $u(x,0) = 0$ by directly using Fourier transforms (that is, don't appeal to Duhamel's Principle).

4. Show that $\int u(x,t)dx = \int g(x)dx$. Show that $E(t) = \int(\partial_x u)^2 dx$ is decreasing. Here u solves $\partial_t u = \partial_{xx} u$ with $u(x,0) = g(x)$. There is a "hidden" assumption on u here. Ask me about it if you need a "hint" on what I mean.

5. Consider the non–linear diffusion equation $\partial_t u - \partial_{xx} u + 2(\partial_x u)^2 = 0$. Show that if u is a solution and if $v(x,t) = e^{-2u(x,t)}$ then v satisfies $\partial_t v - \partial_{xx} v = 0$. Use this to to solve $\partial_t u - \partial_{xx} u + 2(\partial_x u)^2 = 0$ if $u(x,0) = g(x)$.