1. A small satellite has six thrusters, each capable of applying F units of force (if used). The thrust directions and thruster locations (in a centroidal body-fixed coordinate system) are provided in the table.

   Write a relationship between the thrust on-off selector vector and the net force and moment produced using a Ax = b system of equations.

Solution:
a)
The way that I approach this problem is that I wanted the first row in $\vec{b}$ to be $\sum F_x$ and the second and third to be $\sum F_y$ and $\sum F_z$. Therefore $a_{11} = F_{thruster1}\hat{x}$ and etc. Which ended up being the transpose of the thrust direction matrix given that we set the thrust equal to 1.
To find the 3 summations of the moments I was going to use the equation $M = \vec{r} \times \vec{F}$, but for each thruster and each force vector I ignored the appropriate columns when taking the cross product. Such as $M_x = (\cancelto{0}{r_x}, r_y, r_z) \times (\cancelto{0}{F_x}, F_y, F_z)$ and therefore building the following A matrix.

$$A = \begin{bmatrix} -1. & 0. & 0. & 0. & 0. & 1. \\ 0. & 1. & 0. & -1. & 0. & 0. \\ 0. & 0. & 1. & 0. & -1. & 0. \\ 0. & 8. & -8. & 8. & -8. & 0. \\ -8. & 0. & 8. & 0. & 8. & -8. \\ -8. & -8. & 0. & -8. & 0. & -8. \end{bmatrix}$$

b)
In order to find the sum of forces and moments when the 1,2,5 are all firing, we just have to set

$$\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

and do $A \cdot \vec{x}$ to find $\vec{b}$ Which is

$$\vec{b} = \begin{bmatrix} -1. \\ 1. \\ -1. \\ 0. \\ 0. \\ -16 \end{bmatrix}$$

c) To find all the 2 thruster combinations that only produce a moment, we can first define

$$\vec{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \neq 0 \\ \neq 0 \\ \neq 0 \end{bmatrix}$$

Then, I listed out all of the possible combinations of thrusters, which in total was 64, then I did $A \cdot \vec{x}$ and filtered out all the combinations that don't meet the requirements for $\vec{b}$. The $\vec{x}$ that met the requirements were (0, 0, 0, 0, 0, 0), (0, 0, 1, 0, 1, 0), (0, 1, 0, 1, 0, 0), (0, 1, 1, 1, 1, 0), (1, 0, 0, 0, 0, 1), (1, 0, 1, 0, 1, 1), (1, 1, 0, 1, 0, 1), (1, 1, 1, 1, 1, 1)

$\therefore$ the thruster pairs (3,5), (2,4), and (1,6)

2. A system is modeled by the ode: $\ddot{x} = -\dot{x} - x + \sin \omega t$; $x(0) = x_0$ and $\dot{x}(0) = \dot{x}_0$. The input frequency is ω (rad/sec) and the output is x(t)

Solution:

3. Find all eigenvalues and eigenfunctions for the equation: $\varphi'' + \lambda\varphi = 0$ with $\varphi(0) = 0$ and $\varphi'(\pi) = 0$.

Solution:

4. Solve

$$\begin{cases} \frac{\partial^2}{\partial t^2} u = \frac{\partial^2}{\partial x^2} u \\ u(0, t) = u(\pi, t) = 0 \\ u(x, 0) = f(x), \frac{\partial}{\partial t} u(x, 0) = 0, \end{cases}$$

where $f(x)$ is the "hat function": it's 0 and 0 and π and 1 and $\pi/2$ and linear in between.

Solution:

5. This is another perspective on the SOV method. Consider the problem

$$\begin{cases} \frac{\partial}{\partial t}u = \frac{\partial^2}{\partial x^2}u \\ u(0,t) = u(\pi,t) = 0 \\ u(x,0) = g(x) \end{cases} .$$

For each fixed x, assume that the solution can be written as $\sum_{k=1}^{\infty} B_k \sin(kx)$. Note that the B_k depend on t so a better way to write it is $\sum_{k=1}^{\infty} B_k(t)\sin(kx)$. Starting from this point, find the SOV solution. (Note: I had a typo that said $B_k(x)$ before; it should be $B_k(t)$ SORRY!!)

Solution: Plugging the function into the PDE we get:

$$\begin{aligned} \sum_{k=1}^{\infty} B_k'(t)\sin kx &= \sum_{k=1}^{\infty} B_k(t)\frac{d^2}{dx^2}\sin kx \\ &= \sum_{k=1}^{\infty} B_k(t)(-k^2\sin kx) \\ &= -\sum_{k=1}^{\infty} B_k(t)k^2\sin kx. \end{aligned}$$

Subtracting the left from the right:

$$0 = \sum_{k=1}^{\infty}(B_k'(t) + k^2B_k(t))\sin kx.$$

Since the $\sin kx$ functions are linearly independent, this implies that for all k, $B_k'(t) = -k^2B_k(t)$. So we get an ODE for B_k and the solution is $B_k(t) = B_k(0)e^{-k^2t}$. To find the values of the constants note that:

$$g(x) = u(x,0) = \sum_{k=1}^{\infty} B_k(0)\sin kx.$$

That is, as before, $B_k(0) = \frac{2}{\pi}\int_0^{\pi} g(y)\sin ky\,dy$.