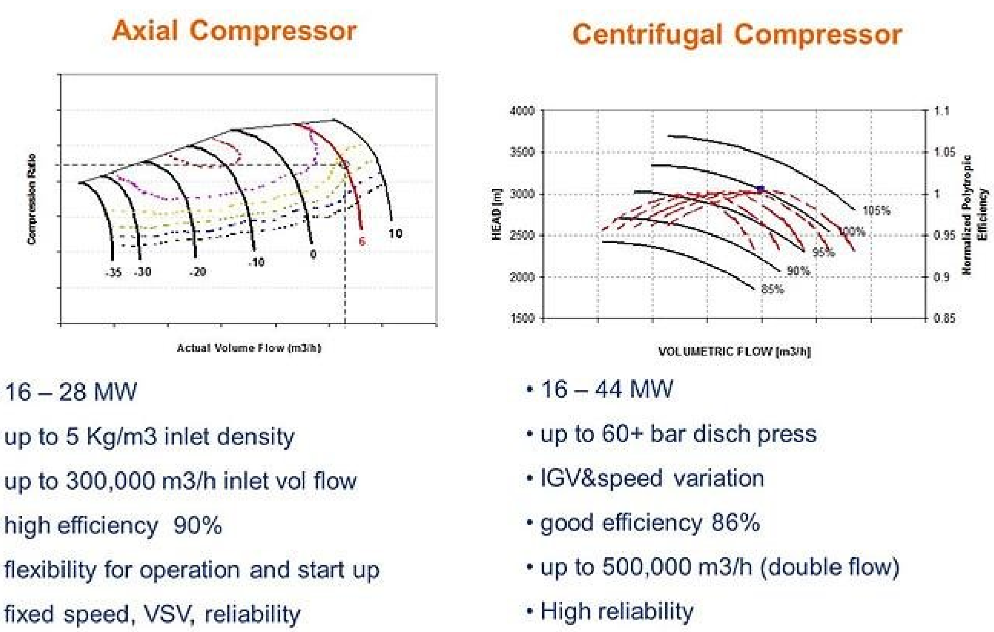
1. Why centrifugal compressors are not used in long-range aircraft as a component of the main engines?

**Solution:** Starting with the breguet's range equation:

$$S = \frac{L}{D}\eta_0 \frac{\text{LHV}}{g} \ln\left(\frac{m_0}{m_f}\right)$$

We will assume that the aircraft will have the same take-off weight and same dry weight and the only thing that we are changing is going to be the compressor geometry. Therefore the compressor will only effect the overall efficency $\eta_0$, and $\eta_0 \propto \eta_c$.

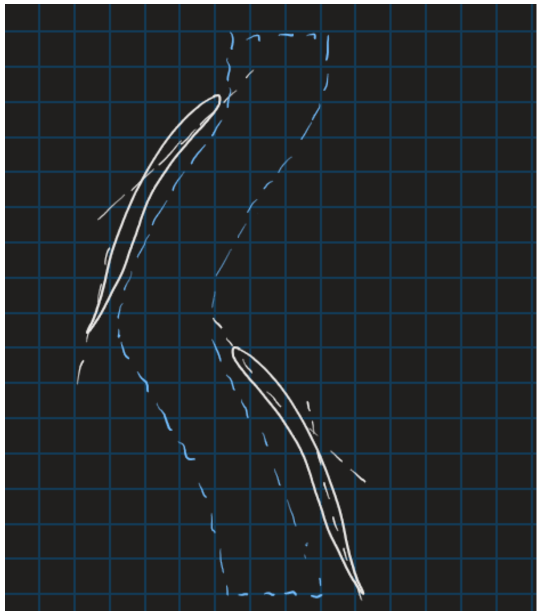
$$\rightarrow S \propto \eta_c$$




because every little bit of efficency matters over the lifetime of the engine flight time to save millons on fuel. That it makes sense that modern commerical jet would prefer for an axial compressor over a centrifugal one.

2. Explain, using mathematical expressions the source of pressure increases in a centrifugal compressor and make a reflection and describe what is the source/term that indicates the pressure increase in high performance centrifugal compressors

**Solution:** Assume: adiabatic, incompressible, quasi-1D flow, steady state



Starting with the conservation of mass:

$$\int_\tau \partial_t \rho \, d\tau + \int_\sigma \rho(\vec{v} \cdot \vec{n}) \, d\sigma = 0$$

$$-\rho_1 v_1 \sigma_1 + \rho_2 v_2 \sigma_2 = 0$$

$$\therefore \dot{m}_1 = \dot{m}_2 \quad \text{or} \quad v_1\sigma_1 = v_2\sigma_2$$

After getting that relation we can use the momentum equation to produce the following

$$\int_\tau \partial_t(\rho\vec{v}) \, d\tau + \int_\sigma \rho\vec{v}(\vec{v} \cdot \vec{n}) \, d\sigma = \vec{F}_v + \vec{F}_s$$

$$\begin{cases} \vec{F}_v = 0, \text{ neglected} \\ \vec{F}_s = -\oint_\sigma P \cdot \vec{n} \, d\sigma = \Delta P \sigma \end{cases}$$

$$\Delta P = \frac{\dot{m}}{\sigma}(v_2 - v_1)$$

The changes in pressure are going to be related based off of the difference in velocity of the outlet versus the inlet with the current assumptions that I made.

3. Air enters the inducer blades of a centrifugal compressor at p01 = 1.02 bar, T01 = 335 K. The hub and tip diameters of the impeller eye are 10 and 25 cm respectively. If the compressor runs at 7200 rpm and delivers 5.0 kg/s of air, determine the air angle at the inducer blade entry and the relative Mach number.

**Solution:** Starting by finding the annulus area

$$\frac{d}{dr}\left(\pi r^2\right) = 2\pi r$$

$$\sigma_1 = 2\pi \int_{0.1}^{0.25} r\,dr \quad \left[m^2\right]$$

$$u_1 = 7200\,[\text{rpm}]\left(\frac{2\pi\,[\text{rads}]}{60\,[\text{sec}]}\right)$$

Then solving the continuity equation

$$\int_\tau \cancelto{0}{\partial_t \rho\, d\tau} + \oint_\sigma \rho(\vec{v}\cdot\vec{n})\,d\sigma = 0 \quad \text{(steady state)}$$

$$-\rho_1 v_1 \sigma_1 + \rho_2 v_2 \sigma_2 = 0 \quad \Big|\, v_1 = w_1, \quad \sigma_1 = \sigma_2$$

$$\dot{m}_1 = \dot{m}_2$$

Using the dimensionaless mass flow rate equation to find the mach number via Newton Raphson. My numerical scheme is the following

$$\zeta = -\frac{\dot{m}\sqrt{RT_0}}{P_0\sigma} + \sqrt{\gamma}M\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{(\delta+1)}{2(\delta-1)}} = 0$$

$$\frac{d\zeta}{dM} = -\sqrt{\gamma}\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}}\frac{(\gamma+1)}{2(\gamma-1)}M$$

$$+\sqrt{\gamma}\mathcal{M}\left(\frac{(\gamma+1)}{2(\gamma-1)}\right)\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}-1}([\gamma-1]M)$$

$$M_{i+1} = M_i - \frac{\zeta}{\frac{d\zeta}{dM}}$$

$$e_r = |\zeta(M_{i+1})| > 1e-8$$

Which produces a $M = 3.5971$, and using $w_1 = M\sqrt{\gamma R T_0}$ we can get the relative velocity of $w_1 = 1319.7128\left[\frac{m}{s}\right]$. Using trig we can find that

$$\beta = \cos^{-}1\left(\frac{u_1}{w_1}\right)$$

therefore the final answers are the following

$$\begin{cases} M = 3.5971 \\ \beta_1 = 0.9627\,[rads] \\ \beta_1 = 55.15747^\circ \end{cases}$$