1. Why centrifugal compressors are not used in long-range aircraft as a component of the main engines?

Solution: Starting with the breguet's range equation:

$$S = \frac{L}{D} \eta_0 \frac{LHV}{g} \ln \left(\frac{m_0}{m_f} \right)$$

We will assume that the aircraft will have the same take-off weight and same dry weight and the only thing that we are changing is going to be the compressor geometry. Therefore the compressor will only effect the overall efficency η_0 , and $\eta_0 \propto \eta_c$.

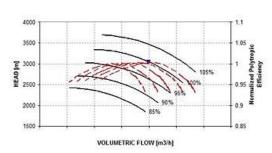
$$\to S \propto \eta_c$$

Axial Compressor

Actual Volume Flow (m3/h)

- 16 28 MW
- · up to 5 Kg/m3 inlet density
- · up to 300,000 m3/h inlet vol flow
- · high efficiency 90%
- · flexibility for operation and start up
- · fixed speed, VSV, reliability

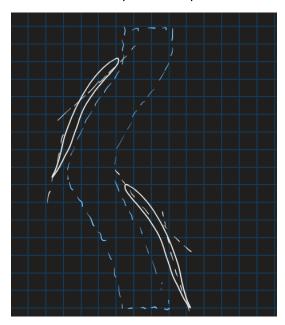
Centrifugal Compressor



- 16 44 MW
- · up to 60+ bar disch press
- · IGV&speed variation
- · good efficiency 86%
- up to 500,000 m3/h (double flow)
- · High reliability

because every little bit of efficiency matters over the lifetime of the engine flight time to save millons on fuel. That it makes sense that modern commerical jet would prefer for an axial compressor over a centrifugal one. 2. Explain, using mathematical expressions the source of pressure increases in a centrifugal compressor and make a reflection and describe what is the source/term that indicates the pressure increase in high performance centrifugal compressors

Solution: Assume: adiabatic, incompressible, quasi-1D flow, steady state



Starting with the conservation of mass:

$$\begin{split} \int_{\tau} \partial_{t} \rho \, d\tau + \int_{\sigma} \rho (\vec{\nu} \cdot \vec{n}) \, d\sigma &= 0 \\ -\rho_{1} \nu_{1} \sigma_{1} + \rho_{2} \nu_{2} \sigma_{2} &= 0 \\ \therefore \, \dot{m_{1}} &= \dot{m_{2}} \quad \text{or} \quad \nu_{1} \sigma_{1} = \nu_{2} \sigma_{2} \end{split}$$

After getting that relation we can use the momentum equation to produce the following

$$\begin{split} \int_{\tau} \vartheta_{t}(\rho \vec{\nu}) \; d\tau + \int_{\sigma} \rho \vec{\nu} (\vec{\nu} \cdot \vec{n}) \; d\sigma &= \vec{F}_{\nu} + \vec{F}_{s} \\ \begin{cases} \vec{F}_{\nu} = 0, \; \text{neglected} \\ \vec{F}_{s} = -\oint_{\sigma} P \cdot \vec{n} \; d\sigma &= \Delta P \sigma \\ \Delta P &= \frac{\dot{m}}{\sigma} \left(\nu_{2} - \nu_{1} \right) \end{split}$$

The changes in pressure are going to be related based off of the difference in velocity of the outlet versus the inlet with the current assumptions that I made.

3. Air enters the inducer blades of a centrifugal compressor at p01 = 1.02 bar, T01 = 335 K. The hub and tip diameters of the impeller eye are 10 and 25 cm respectively. If the compressor runs at 7200 rpm and delivers 5.0 kg/s of air, determine the air angle at the inducer blade entry and the relative Mach number.

Solution: Starting by finding the annulus area

$$\begin{split} \frac{d}{dr}\left(\pi r^2\right) &= 2\pi r\\ \sigma_1 &= 2\pi \int_{0.1}^{0.25} r\,dr \quad \left[m^2\right]\\ u_1 &= 7200\left[\text{rpm}\right]\left(\frac{2\pi\left[\text{rads}\right]}{60\left[\text{sec}\right]}\right) \end{split}$$

Then solving the continuity equation

$$\int_{\sigma} \partial_{\tau} \rho \, d\tau + \oint_{\sigma} \rho(\vec{v} \cdot \vec{n}) \, d\sigma = 0 \quad \text{(steady state)}$$

$$-\rho_1 \nu_1 \sigma_1 + \rho_2 \nu_2 \sigma_2 = 0 \quad \bigg| \nu_1 = w_1, \quad \sigma_1 = \sigma_2$$

$$\dot{m}_1 = \dot{m}_2$$

Using the dimensionaless mass flow rate equation [Cizmas equation 3.39] to find the mach number via Newton Raphson. My numerical scheme is the following

$$\begin{split} \zeta &= -\frac{\dot{m}\,\sqrt{RT_0}}{P_0\sigma} + \sqrt{\gamma}M\left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}} = 0 \\ &\frac{d\zeta}{dM} = \sqrt{\gamma}\left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}} \\ &-\sqrt{\gamma}\mathcal{M}\left(\frac{(\gamma+1)}{2(\gamma-1)}\right)\left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}-1} ([\gamma-1]M) \\ &M_{i+1} = M_i - \frac{\zeta}{\frac{d\zeta}{dM}} \\ &e_r = |\zeta(M_{i+1})| > 1e - 8 \end{split}$$

Which produces a M=3.5971, and using $w_1=M\sqrt{\gamma RT_0}$ we can get the relative velocity of $w_1=1319.7128\left[\frac{m}{s}\right]$. Using trig we can find that

$$\beta = \cos^- 1 \left(\frac{u_1}{w_1} \right)$$

therefore the final answers are the following

$$\begin{cases} M = 3.5971 \\ \beta_1 = 0.9627 \, [rads] \\ \beta_1 = 55.15747^{\circ} \end{cases}$$

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In [2]:
         ### Benjamin Tollison ###
         import matplotlib.pyplot as plt
         import numpy as np
         import pandas as pd
         import scipy
         import sympy as sp
         from IPython.display import Latex, Math, display
         from sympy import (
             Eq,
             Function,
             Matrix,
             cos,
             cosh,
             exp,
             integrate,
             lambdify,
             рi,
             sin,
             sinh,
             symbols,
         from decimal import Decimal
         from sympy.solvers.pde import pdsolve
         from sympy.solvers.solveset import linsolve
         def displayEquations(LHS,RHS):
             left = sp.latex(LHS)
             right = sp.latex(RHS)
             display(Math(left + '=' + right))
             np.set_printoptions(suppress=True)
         def displayVariable(variable:str,RHS):
             left = sp.latex(symbols(variable))
             right = sp.latex(RHS)
             display(Math(left + '=' + right))
         def displayVariableWithUnits(variable:str,RHS,units):
             left = sp.latex(symbols(variable))
             right = sp.latex(RHS)
             latexUnit = sp.latex(symbols(units))
             display(Math(left + '=' + right + '\\;' +'\\left['+ latexUnit + '\\right]'))
         def format scientific(number:float):
             a = '%E' % number
             return a.split('E')[0].rstrip('0').rstrip('.') + 'E' + a.split('E')[1]
         deg2rad = np.pi/180
         rad2deg = 180/np.pi
```

```
In [33]:
          inlet_area = np.pi*(.25**2-.1**2)
          rotation_speed = 7200*(2*np.pi/60)
          displayVariableWithUnits('\\sigma',inlet_area,'m^2')
          displayVariableWithUnits('u_1',rotation_speed,'\\frac{m}{s}')
          inlet_total_pressure = 1.02e5 # Pa
          inlet_total_temperature = 335 # K
          mass_flow_rate = 5 # kg/s
          demensionaless_mass_flow = (mass_flow_rate*(287*inlet_total_temperature)**0.5)/(inlet_total_temperature)**0.5)/
          def MachFlow(machnumber):
             kappa = demensionaless_mass_flow
             gamma = 1.4
            M = machnumber
            return (gamma)**0.5 * M * (1 + ((gamma-1)*M**2)/2)**((-gamma-1)/(2*gamma-2)) - kappa
          def MachFlowPrime(machnumber):
             gamma = 1.4
            M = machnumber
            first_part = (gamma)**0.5*(1 + ((gamma-1)*M**2)/2)**((-gamma-1)/(2*gamma-2))
```

```
second_part = (gamma)**0.5 *M*((-gamma-1)/(2*gamma-2))*(1 + ((gamma-1)*M**2)/2 )**((-
             return first_part + second_part
           intial_mach_guess = 1.2
           increment_cutoff = 100
           increment_count = 0
           while abs(MachFlow(intial_mach_guess))>1e-8:
             increment_count += 1
             if increment_count == increment_cutoff:
               print('scheme didn\'t converge')
               displayVariable('e_r',format_scientific(abs(MachFlow(intial_mach_guess))))
             intial_mach_guess = intial_mach_guess - MachFlow(intial_mach_guess)/MachFlowPrime(int
           final_mach = intial_mach_guess
           displayVariable('M_i',final_mach)
           displayVariable('e_r',format_scientific(abs(MachFlow(intial_mach_guess))))
           displayVariable('i',increment_count)
        \sigma = 0.164933614313464 \ [m^2]
        u_1 = 753.98223686155 \left[ \frac{m}{s} \right]
        M_i = 3.59709624357504
        e_r = \texttt{5.800749E-09}
        i = 5
In [32]:
           virtual_speed = final_mach*(1.4*287*inlet_total_temperature)**0.5
           displayVariableWithUnits('w_1', virtual_speed, '\\frac{m}{s}')
           inducer_angle = np.arccos(rotation_speed/virtual_speed)
           displayVariableWithUnits('\\beta_1',inducer_angle,'rad')
           displayVariableWithUnits('\\beta_1',inducer_angle*rad2deg,'deg')
        w_1 = 1319.71279595148 \left[ \frac{m}{s} \right]
        \beta_1 = 0.962679419542757 \ [rad]
        \beta_1 = 55.1574677639039 \ [deg]
```