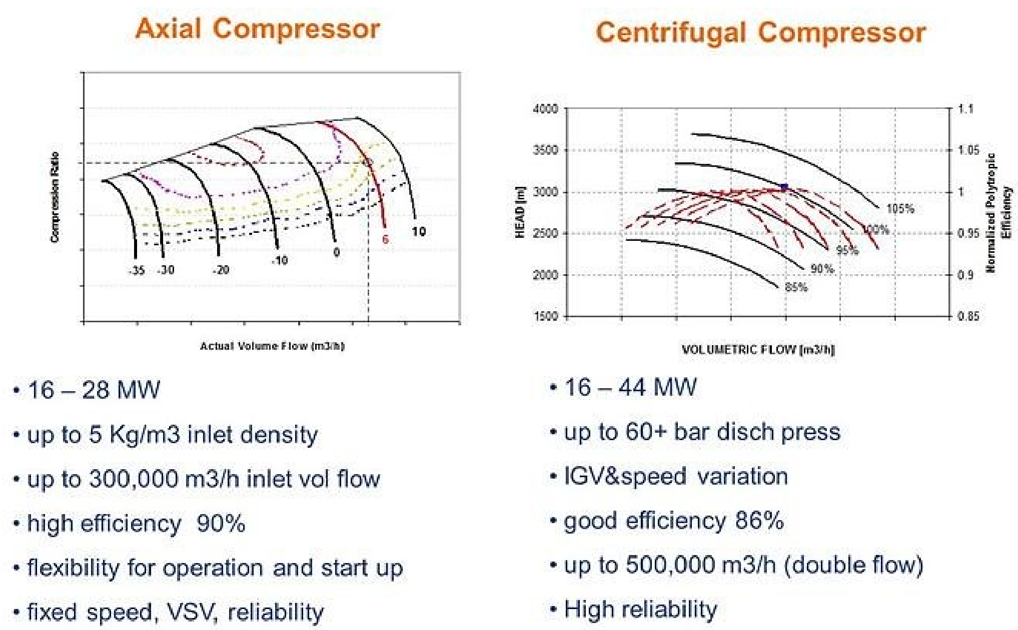
1. Why centrifugal compressors are not used in long-range aircraft as a component of the main engines?

Solution: Starting with the breguet's range equation:

$$S = \frac{L}{D}\eta_0\frac{LHV}{g}\ln\left(\frac{m_0}{m_f}\right)$$

We will assume that the aircraft will have the same take-off weight and same dry weight and the only thing that we are changing is going to be the compressor geometry. Therefore the compressor will only effect the overall efficency η_0, and $\eta_0 \propto \eta_c$.

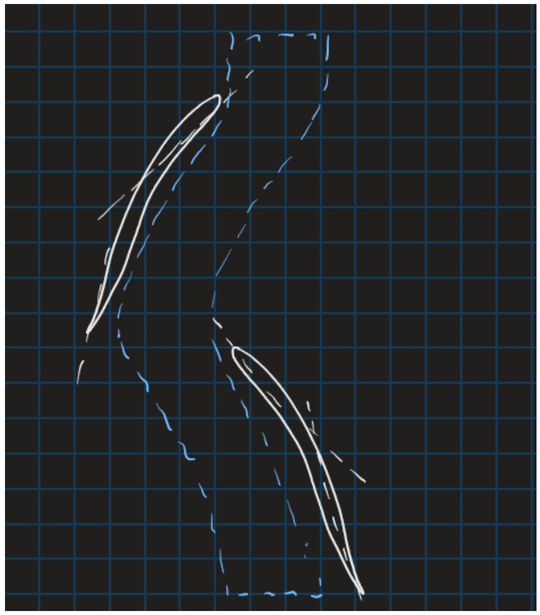
$$\rightarrow S \propto \eta_c$$



because every little bit of efficency matters over the lifetime of the engine flight time to save millons on fuel. That it makes sense that modern commerical jet would prefer for an axial compressor over a centrifugal one.

2. Explain, using mathematical expressions the source of pressure increases in a centrifugal compressor and make a reflection and describe what is the source/term that indicates the pressure increase in high performance centrifugal compressors

Solution: Assume: adiabatic, incompressible, quasi-1D flow, steady state



Starting with the conservation of mass:

$$\int_\tau \partial_t \rho \, d\tau + \int_\sigma \rho(\vec{v} \cdot \vec{n}) \, d\sigma = 0$$

$$-\rho_1 v_1 \sigma_1 + \rho_2 v_2 \sigma_2 = 0$$

$$\therefore \dot{m}_1 = \dot{m}_2 \quad \text{or} \quad v_1 \sigma_1 = v_2 \sigma_2$$

After getting that relation we can use the momentum equation to produce the following

$$\int_\tau \partial_t(\rho \vec{v}) \, d\tau + \int_\sigma \rho \vec{v}(\vec{v} \cdot \vec{n}) \, d\sigma = \vec{F}_v + \vec{F}_s$$

$$\begin{cases} \vec{F}_v = 0, \text{ neglected} \\ \vec{F}_s = -\oint_\sigma P \cdot \vec{n} \, d\sigma = \Delta P \sigma \end{cases}$$

$$\Delta P = \frac{\dot{m}}{\sigma}(v_2 - v_1)$$

The changes in pressure are going to be related based off of the difference in velocity of the outlet versus the inlet with the current assumptions that I made.

3. Air enters the inducer blades of a centrifugal compressor at p01 = 1.02 bar, T01 = 335 K. The hub and tip diameters of the impeller eye are 10 and 25 cm respectively. If the compressor runs at 7200 rpm and delivers 5.0 kg/s of air, determine the air angle at the inducer blade entry and the relative Mach number.

Solution: Starting by finding the annulus area

$$\frac{d}{dr}\left(\pi r^2\right) = 2\pi r$$

$$\sigma_1 = 2\pi \int_{0.1}^{0.25} r\, dr \quad \left[m^2\right]$$

$$u_1 = 7200\,[\text{rpm}]\left(\frac{2\pi\,[\text{rads}]}{60\,[\text{sec}]}\right)$$

Then solving the continuity equation

$$\int_\tau \cancelto{0}{\partial_t \rho\, d\tau} + \oint_\sigma \rho(\vec{v}\cdot\vec{n})\, d\sigma = 0 \quad \text{(steady state)}$$

$$-\rho_1 v_1 \sigma_1 + \rho_2 v_2 \sigma_2 = 0 \quad \Big| \; v_1 = w_1, \quad \sigma_1 = \sigma_2$$

$$\dot{m}_1 = \dot{m}_2$$

Using the dimensionaless mass flow rate equation [Cizmas equation 3.39] to find the mach number via Newton Raphson. My numerical scheme is the following

$$\zeta = -\frac{\dot{m}\sqrt{RT_0}}{P_0\sigma} + \sqrt{\gamma}M\left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{(\delta+1)}{2(\delta-1)}} = 0$$

$$\frac{d\zeta}{dM} = \sqrt{\gamma}\left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}}$$

$$-\sqrt{\gamma}\mathcal{M}\left(\frac{(\gamma+1)}{2(\gamma-1)}\right)\left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}-1}([\gamma-1]M)$$

$$M_{i+1} = M_i - \frac{\zeta}{\frac{d\zeta}{dM}}$$

$$e_r = |\zeta(M_{i+1})| > 1e-8$$

Which produces a $M = 3.5971$, and using $w_1 = M\sqrt{\gamma R T_0}$ we can get the relative velocity of $w_1 = 1319.7128\left[\frac{m}{s}\right]$. Using trig we can find that

$$\beta = \cos^{-}1\left(\frac{u_1}{w_1}\right)$$

therefore the final answers are the following

$$\begin{cases} M = 3.5971 \\ \beta_1 = 0.9627\,[rads] \\ \beta_1 = 55.15747^\circ \end{cases}$$

In [2]:

```
### Benjamin Tollison ###
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import scipy
import sympy as sp
from IPython.display import Latex, Math, display
from sympy import (
    Eq,
    Function,
    Matrix,
    cos,
    cosh,
    exp,
    integrate,
    lambdify,
    pi,
    sin,
    sinh,
    symbols,
)
from decimal import Decimal
from sympy.solvers.pde import pdsolve
from sympy.solvers.solveset import linsolve
def displayEquations(LHS,RHS):
    left = sp.latex(LHS)
    right = sp.latex(RHS)
    display(Math(left + '=' + right))
    np.set_printoptions(suppress=True)
def displayVariable(variable:str,RHS):
    left = sp.latex(symbols(variable))
    right = sp.latex(RHS)
    display(Math(left + '=' + right))
def displayVariableWithUnits(variable:str,RHS,units):
    left = sp.latex(symbols(variable))
    right = sp.latex(RHS)
    latexUnit = sp.latex(symbols(units))
    display(Math(left + '=' + right + '\\;' +'\\left['+ latexUnit + '\\right]'))
def format_scientific(number:float):
    a = '%E' % number
    return a.split('E')[0].rstrip('0').rstrip('.') + 'E' + a.split('E')[1]
deg2rad = np.pi/180
rad2deg = 180/np.pi
```

In [33]:

```
inlet_area = np.pi*(.25**2-.1**2)
rotation_speed = 7200*(2*np.pi/60)
displayVariableWithUnits('\\sigma',inlet_area,'m^2')
displayVariableWithUnits('u_1',rotation_speed,'\\frac{m}{s}')
inlet_total_pressure = 1.02e5 # Pa
inlet_total_temperature = 335 # K
mass_flow_rate = 5 # kg/s
demensionaless_mass_flow = (mass_flow_rate*(287*inlet_total_temperature)**0.5)/(inlet_t
def MachFlow(machnumber):
  kappa = demensionaless_mass_flow
  gamma = 1.4
  M = machnumber
  return (gamma)**0.5 * M * (1 + ((gamma-1)*M**2)/2 )**((-gamma-1)/(2*gamma-2)) - kappa
def MachFlowPrime(machnumber):
  gamma = 1.4
  M = machnumber
  first_part = (gamma)**0.5*(1 + ((gamma-1)*M**2)/2 )**((-gamma-1)/(2*gamma-2))
```

```
    second_part = (gamma)**0.5 *M*((-gamma-1)/(2*gamma-2))*(1 + ((gamma-1)*M**2)/2 )**((-
    return first_part + second_part
  intial_mach_guess = 1.2
  increment_cutoff = 100
  increment_count = 0
  while abs(MachFlow(intial_mach_guess))>1e-8:
    increment_count += 1
    if increment_count == increment_cutoff:
      print('scheme didn\'t converge')
      displayVariable('e_r',format_scientific(abs(MachFlow(intial_mach_guess))))
      break
    intial_mach_guess = intial_mach_guess - MachFlow(intial_mach_guess)/MachFlowPrime(int
  final_mach = intial_mach_guess
  displayVariable('M_i',final_mach)
  displayVariable('e_r',format_scientific(abs(MachFlow(intial_mach_guess))))
  displayVariable('i',increment_count)
```

$\sigma = 0.164933614313464 \ \left[m^2\right]$

$u_1 = 753.98223686155 \ \left[\frac{m}{s}\right]$

$M_i = 3.59709624357504$

$e_r = \texttt{5.800749E-09}$

$i = 5$

In [32]:

```
virtual_speed = final_mach*(1.4*287*inlet_total_temperature)**0.5
displayVariableWithUnits('w_1',virtual_speed,'\\frac{m}{s}')
inducer_angle = np.arccos(rotation_speed/virtual_speed)
displayVariableWithUnits('\\beta_1',inducer_angle,'rad')
displayVariableWithUnits('\\beta_1',inducer_angle*rad2deg,'deg')
```

$w_1 = 1319.71279595148 \ \left[\frac{m}{s}\right]$

$\beta_1 = 0.962679419542757 \ [rad]$

$\beta_1 = 55.1574677639039 \ [deg]$