1. From (10.125)

$$\boldsymbol{\epsilon}^* \cdot \mathbf{F}_{\text{sh}} \approx \frac{ik}{2\pi} E_0 \left(\boldsymbol{\epsilon}^* \cdot \boldsymbol{\epsilon}_0\right) \int_{\text{sh}} e^{-i\mathbf{k}_\perp \cdot \mathbf{x}_\perp} d^2 x_\perp \tag{1}$$

we can write the differential scattering cross section as

$$\frac{d\sigma_{\text{sc}}}{d\Omega}\left(\boldsymbol{\epsilon}, \boldsymbol{\epsilon}_0\right) = \frac{\left|\boldsymbol{\epsilon}^* \cdot \mathbf{F}_{\text{sh}}\right|^2}{E_0^2} \approx \frac{k^2}{4\pi^2} \left|\boldsymbol{\epsilon}^* \cdot \boldsymbol{\epsilon}_0\right|^2 \int_{\text{sh}} d^2 x_\perp \int_{\text{sh}} d^2 x'_\perp e^{i\mathbf{k}_\perp \cdot \left(\mathbf{x}_\perp - \mathbf{x}'_\perp\right)} \tag{2}$$

After summing over all outgoing polarizations and averaging over initial polarizations, we can replace $\left|\boldsymbol{\epsilon}^* \cdot \boldsymbol{\epsilon}_0\right|^2$ by $\left(1+\cos^2\theta\right)/2$ (see (10.10)), where θ is the angle between $\mathbf{k}$ and $\mathbf{k}_0$. Then the total scattering cross section can be obtained by integrating over all solid angles,

$$\sigma_{\text{sc}} \approx \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi \frac{k^2}{4\pi^2} \left(\frac{1+\cos^2\theta}{2}\right) \int_{\text{sh}} d^2 x_\perp \int_{\text{sh}} d^2 x'_\perp e^{i\mathbf{k}_\perp \cdot \left(\mathbf{x}_\perp - \mathbf{x}'_\perp\right)} \tag{3}$$

Let $\mathbf{k}_0$ be along the z direction, and the projection of $\mathbf{k}$ onto the x-y plane are

$$k_x = k \sin\theta \cos\phi \qquad\qquad k_y = k \sin\theta \sin\phi \tag{4}$$

thus the area element of the (k-space) x-y plane can be written in spherical coordinates

$$d^2 k_\perp = dk_x dk_y = \begin{vmatrix} \dfrac{\partial k_x}{\partial\theta} & \dfrac{\partial k_y}{\partial\theta} \\ \dfrac{\partial k_x}{\partial\phi} & \dfrac{\partial k_y}{\partial\phi} \end{vmatrix} d\theta d\phi = \begin{vmatrix} k\cos\theta\cos\phi & k\cos\theta\sin\phi \\ -k\sin\theta\sin\phi & k\sin\theta\cos\phi \end{vmatrix} d\theta d\phi = k^2 \sin\theta\cos\theta d\theta d\phi \tag{5}$$

At short-wavelength limit, the contribution to the cross section mainly comes from $\mathbf{k}$ with very small θ, so to the first order of θ, we can put $\cos\theta \approx 1$ and rewrite the total scattering cross section as

$$\sigma_{\text{sc}} \approx \int_{\text{sh}} d^2 x_\perp \int_{\text{sh}} d^2 x'_\perp \cdot \frac{1}{4\pi^2} \int_{|\mathbf{k}_\perp| \le k} e^{i\mathbf{k}_\perp \cdot \left(\mathbf{x}_\perp - \mathbf{x}'_\perp\right)} d^2 k_\perp \tag{6}$$

Again, at short-wavelength limit, the phase integrand oscillates rapidly, the dominant contribution comes near the stationary point where $\mathbf{k}_\perp$ is zero. This allows us to approximate the domain of the inner integral by the entire x-y plane, giving

$$\sigma_{\text{sc}} \approx \int_{\text{sh}} d^2 x_\perp \int_{\text{sh}} d^2 x'_\perp \delta\left(\mathbf{x}_\perp - \mathbf{x}'_\perp\right) = \int_{\text{sh}} d^2 x_\perp = \text{Projected Area} \tag{7}$$

2. If we ignore the illuminated side's contribution to the scattering cross section, by optical theorem, we have the total cross section (scattering plus absorption) as

$$\sigma_{\text{tot}} = \frac{4\pi}{k} \operatorname{Im}\left[\boldsymbol{\epsilon}_0^* \cdot \mathbf{f}(\mathbf{k} = \mathbf{k}_0)\right] = \frac{4\pi}{k} \operatorname{Im}\left[\frac{ik}{2\pi} \int_{\text{sh}} d^2 x_\perp\right] = 2 \times \text{Projected Area} \tag{8}$$

The difference between (8) and (7) is attributed to the absorption.