From section 4.4, equation (4.57), we know under the influence of $E_0\hat{\mathbf{x}}$, there is a uniform polarization inside the sphere

$$\mathbf{P}\left(\mathbf{x}'\right) = 3\epsilon_0 \left(\frac{\epsilon/\epsilon_0 - 1}{\epsilon/\epsilon_0 + 2}\right) E_0\hat{\mathbf{x}} \equiv p_0\hat{\mathbf{x}} \tag{1}$$

When the sphere is in bulk motion, (6.100) says the polarization gives rise to an additional effective magnetization

$$\mathbf{M}_{\text{eff}}\left(\mathbf{x}'\right) = \mathbf{P}\times\mathbf{v} = p_0\hat{\mathbf{x}}\times\left(\omega a\sin\theta'\hat{\boldsymbol{\phi}}\right) = p_0\omega a\sin\theta'\hat{\mathbf{x}}\times\left(-\sin\phi'\hat{\mathbf{x}}+\cos\phi'\hat{\mathbf{y}}\right) = p_0\omega a\sin\theta'\cos\phi'\hat{\mathbf{z}} = p_0\omega x\hat{\mathbf{z}} \tag{2}$$

It's clear that this effective magnetization has vanishing divergence

$$\boldsymbol{\nabla}\cdot\mathbf{M}_{\text{eff}} = 0 \tag{3}$$

We now use (5.100) to find the scalar potential for the magnetic field using $\mathbf{M}_{\text{eff}}$

$$\begin{aligned}\Phi_M(\mathbf{x}) &= -\frac{1}{4\pi}\int_V \frac{\boldsymbol{\nabla}'\cdot\mathbf{M}_{\text{eff}}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d^3x' + \frac{1}{4\pi}\oint_S \frac{\mathbf{n}'\cdot\mathbf{M}_{\text{eff}}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}da' \\ &= \frac{1}{4\pi}\oint_S \frac{p_0\omega xz}{a\,|\mathbf{x}-\mathbf{x}'|}da' \\ &= \frac{p_0\omega a^3}{4\pi}\int \frac{\cos\theta'\sin\theta'\cos\phi'}{|\mathbf{x}-\mathbf{x}'|}d\Omega' \end{aligned} \tag{4}$$

The spherical harmonic

$$Y_{2,1}(\theta,\phi) = -\frac{1}{2}\sqrt{\frac{15}{2\pi}}\sin\theta\cos\theta e^{i\phi} \tag{5}$$

enables us to write

$$\cos\theta'\sin\theta'\cos\phi' = \left(-\frac{1}{2}\sqrt{\frac{15}{2\pi}}\right)^{-1}\text{Re}\left[Y_{2,1}\left(\theta',\phi'\right)\right] \tag{6}$$

With the expansion for inverse distance

$$\frac{1}{|\mathbf{x}-\mathbf{x}'|} = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{1}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y_{lm}^*\left(\theta',\phi'\right)Y_{lm}(\theta,\phi) \tag{7}$$

and the orthonormality of spherical harmonics, we can write (4) as

$$\begin{aligned}\Phi_M(\mathbf{x}) &= \frac{p_0\omega a^3}{4\pi}\cdot 4\pi\cdot\frac{1}{5}\frac{r_<^2}{r_>^3}\left(-\frac{1}{2}\sqrt{\frac{15}{2\pi}}\right)^{-1}\text{Re}\left[Y_{2,1}(\theta,\phi)\right] \\ &= \frac{p_0\omega a^3}{5}\frac{r_<^2}{r_>^3}\sin\theta\cos\theta\cos\phi \\ &= \frac{p_0\omega}{5}\left(\frac{a^3r_<^2}{r_>^3r^2}\right)\cdot xz \\ &= \frac{3}{5}\left(\frac{\epsilon-\epsilon_0}{\epsilon+2\epsilon_0}\right)\epsilon_0E_0\omega\left(\frac{a}{r_>}\right)^5\cdot xz\end{aligned} \tag{8}$$