1. Let $\mathbf{p}(\tau)$ be the time-dependent dipole source, and let $\mathbf{p}(\omega)$ be its Fourier transform, i.e.,

$$\mathbf{p}(\omega)=\frac{1}{(2\pi)^3}\int \mathbf{p}(\tau)e^{i\omega\tau}d\tau \qquad\qquad \mathbf{p}(\tau)=\int \mathbf{p}(\omega)e^{-i\omega\tau}d\omega \tag{1}$$

For a single frequency ω, the corresponding electric and magnetic field in the far zone are given by (9.19), which we write with explicit time dependence at t:

$$\mathbf{H}(\mathbf{x},\omega,t)=\frac{ck^2}{4\pi}\frac{e^{ikr}}{r}e^{-i\omega t}\mathbf{n}\times\mathbf{p}(\omega)=\frac{\omega^2}{4\pi c}\frac{e^{-i\omega(t-r/c)}}{r}\mathbf{n}\times\mathbf{p}(\omega) \tag{2}$$

$$\mathbf{E}(\mathbf{x},\omega,t)=Z_0\mathbf{H}(\mathbf{x},\omega,t)\times\mathbf{n}=\frac{Z_0\omega^2}{4\pi c}\frac{e^{-i\omega(t-r/c)}}{r}[\mathbf{n}\times\mathbf{p}(\omega)]\times\mathbf{n} \tag{3}$$

The instantaneous power radiated per unit solid angle at $(\mathbf{x},t)$ can be obtained by integrating over all ω,ω',

$$\frac{dP}{d\Omega}(\mathbf{x},t)=\int d\omega\int d\omega'\,\mathrm{Re}\left\{r^2\mathbf{n}\cdot\left[\mathbf{E}(\mathbf{x},\omega,t)\times\mathbf{H}^*\left(\mathbf{x},\omega',t\right)\right]\right\} \tag{4}$$

Immediately we can see that (4) will be a sum of the following integral forms with various scalar functions $f(\omega), g(\omega')$:

$$I=\frac{Z_0}{16\pi^2c^2}\mathrm{Re}\left[\int\omega^2e^{-i\omega t'}f(\omega)\,d\omega\cdot\int\omega'^2e^{i\omega' t'}g^*\left(\omega'\right)d\omega'\right]\qquad\text{where}\qquad t'=t-\frac{r}{c} \tag{5}$$

Let $f(\tau)\longleftrightarrow f(\omega)$ be a Fourier pair, then the first integral in (5) can be written as

$$\int\omega^2e^{-i\omega t'}f(\omega)\,d\omega=-\left[\frac{d^2}{d\tau^2}\int e^{-i\omega\tau}f(\omega)\,d\omega\right]\Bigg|_{\tau=t'}=-\frac{d^2f(\tau)}{d\tau^2}\Bigg|_{\tau=t'}\equiv-\ddot{f}\left(t'\right) \tag{6}$$

Similarly for the second integral involving ω' and g. This simplifies the integral I to

$$I=\frac{Z_0}{16\pi^2c^2}\mathrm{Re}\left[\ddot{f}\left(t'\right)\ddot{g}^*\left(t'\right)\right] \tag{7}$$

Back to (4), with the vector identities

- $(\mathbf{a}\times\mathbf{b})\times\mathbf{c}=(\mathbf{a}\cdot\mathbf{c})\mathbf{b}-(\mathbf{b}\cdot\mathbf{c})\mathbf{a}$
- $(\mathbf{a}\times\mathbf{b})\cdot(\mathbf{c}\times\mathbf{d})=(\mathbf{a}\cdot\mathbf{c})(\mathbf{b}\cdot\mathbf{d})-(\mathbf{a}\cdot\mathbf{d})(\mathbf{b}\cdot\mathbf{c})$

the complicated cross product $\mathbf{E}(\mathbf{x},\omega,t)\times\mathbf{H}^*(\mathbf{x},\omega,t)$ can be rewritten as

$$\begin{aligned}\{[\mathbf{n}\times\mathbf{p}(\omega)]\times\mathbf{n}\}\times\left[\mathbf{n}\times\mathbf{p}^*\left(\omega'\right)\right]&=\mathbf{n}\left\{[\mathbf{n}\times\mathbf{p}(\omega)]\cdot\left[\mathbf{n}\times\mathbf{p}^*\left(\omega'\right)\right]\right\}\\&=\mathbf{n}\left\{\mathbf{p}(\omega)\cdot\mathbf{p}^*\left(\omega'\right)-[\mathbf{n}\cdot\mathbf{p}(\omega)]\left[\mathbf{n}\cdot\mathbf{p}^*\left(\omega'\right)\right]\right\}\end{aligned} \tag{8}$$

After dotting with $\mathbf{n}$ and invoking (7) for all the components, we obtain the desired form of (4)

$$\frac{dP}{d\Omega}(\mathbf{x},t)=\frac{Z_0}{16\pi^2c^2}\mathrm{Re}\left\{\ddot{\mathbf{p}}\left(t'\right)\cdot\ddot{\mathbf{p}}^*\left(t'\right)-\left[\mathbf{n}\cdot\ddot{\mathbf{p}}\left(t'\right)\right]\left[\mathbf{n}\cdot\ddot{\mathbf{p}}^*\left(t'\right)\right]\right\}=\frac{Z_0}{16\pi^2c^2}\left|\left[\mathbf{n}\times\ddot{\mathbf{p}}\left(t'\right)\right]\times\mathbf{n}\right|^2 \tag{9}$$

For radiation due to magnetic dipole $\mathbf{m}(\tau)$, the far-zone fields of (9.35) and (9.36) are

$$\mathbf{H}(\mathbf{x},\omega,t)=\frac{k^2}{4\pi}\frac{e^{ikr}}{r}e^{-i\omega t}[\mathbf{n}\times\mathbf{m}(\omega)]\times\mathbf{n} \tag{10}$$

$$\mathbf{E}(\mathbf{x},\omega,t)=-\frac{Z_0k^2}{4\pi}\mathbf{n}\times\mathbf{m}(\omega)\frac{e^{ikr}}{r}e^{-i\omega t} \tag{11}$$

If we formally substitute $\mathbf{m}(\omega)\times\mathbf{n}$ with $c[\mathbf{n}\times\mathbf{p}(\omega)]\times\mathbf{n}$, we end up with the same expression as (2) and (3), hence substituting $[\mathbf{n}\times\ddot{\mathbf{p}}(t')]\times\mathbf{n}$ with $\ddot{\mathbf{m}}(t')\times\mathbf{n}/c$ in (9) gives the equivalent for a magnetic dipole.

2. For quadrupole moments, we see that (9.44) is equivalent to

$$\mathbf{H}(\mathbf{x},\omega,t)=-\frac{i\omega^3}{24\pi c^2}\frac{e^{ikr}}{r}e^{-i\omega t}\mathbf{n}\times\mathbf{Q}(\mathbf{n},\omega) \tag{12}$$

Comparing this form with (2), we see that the same approach (except with $\omega^2\to-i\omega^3$ and numerical constants change) as in the previous part gives the radiation power per unit solid angle

$$\frac{dP}{d\Omega}(\mathbf{x},t)=\frac{Z_0}{576\pi^2c^4}\left|\left[\mathbf{n}\times\dddot{\mathbf{Q}}\left(\mathbf{n},t'\right)\right]\times\mathbf{n}\right|^2 \tag{13}$$