This is a straightforward application of equation (1.44)

$$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0}\int_V \rho(\mathbf{x}')G(\mathbf{x},\mathbf{x}')d^3x' - \frac{1}{4\pi}\oint_S \Phi(\mathbf{x}')\frac{\partial G}{\partial n'}da' \tag{1}$$

where the second term vanishes because the potentials on the four "sides" are zero, while G on the two "ends" are zero.

In this 2D problem, ρ is to be replaced by the surface charge density (which is unit strength) with the understanding that V has unit length in z direction.

From problem 2.15

$$G(\mathbf{x},\mathbf{x}') = 8\sum_{n=1}^{\infty}\frac{1}{n\sinh(n\pi)}\sin(n\pi x)\sin\left(n\pi x'\right)\sinh(n\pi y_<)\sinh[n\pi(1-y_>)] \tag{2}$$

For volume integral, let's do the x' and y' integral separately

$$\int_0^1 \sin\left(n\pi x'\right)dx' = \frac{1}{n\pi}[1-\cos(n\pi)] = \frac{1}{n\pi}[1-(-1)^n] = \begin{cases}\dfrac{2}{n\pi} & n=2m+1\\ 0 & n=2m\end{cases} \tag{3}$$

$$\begin{aligned}\int_0^1 \sinh(n\pi y_<)\sinh[n\pi(1-y_>)]dy' &= \int_0^y \sinh\left(n\pi y'\right)\sinh[n\pi(1-y)]dy' + \int_y^1 \sinh(n\pi y)\sinh\left[n\pi\left(1-y'\right)\right]dy'\\ &= \underbrace{\sinh[n\pi(1-y)]\frac{1}{n\pi}\cosh\left(n\pi y'\right)\Big|_0^y}_{A} + \underbrace{\sinh(n\pi y)\left(\frac{-1}{n\pi}\right)\cosh\left[n\pi\left(1-y'\right)\right]\Big|_y^1}_{B}\end{aligned} \tag{4}$$

where

$$A = \frac{1}{n\pi}\sinh[n\pi(1-y)][\cosh(n\pi y)-1] \tag{5}$$

$$B = -\frac{1}{n\pi}\sinh(n\pi y)\{1-\cosh[n\pi(1-y)]\} \tag{6}$$

So

$$\begin{aligned}A+B &= \frac{1}{n\pi}\left\{\overbrace{\sinh[n\pi(1-y)]\cosh(n\pi y)+\sinh(n\pi y)\cosh[n\pi(1-y)]}^{\sinh(n\pi)} - \sinh[n\pi(1-y)] - \sinh(n\pi y)\right\}\\ &= \frac{\sinh(n\pi)-\sinh[n\pi(1-y)]-\sinh(n\pi y)}{n\pi}\end{aligned} \tag{7}$$

Therefore

$$\begin{aligned}\Phi(\mathbf{x}) &= \frac{8}{4\pi\epsilon_0}\sum_{m=0}^{\infty}\frac{\sin[(2m+1)\pi x]}{(2m+1)\sinh[(2m+1)\pi]}\cdot\frac{2}{(2m+1)\pi}(A+B)\\ &= \frac{4}{\pi^3\epsilon_0}\sum_{m=0}^{\infty}\frac{\sin[(2m+1)\pi x]}{(2m+1)^3}\left\{\frac{\sinh[(2m+1)\pi]-\sinh[(2m+1)\pi(1-y)]-\sinh[(2m+1)\pi y]}{\sinh[(2m+1)\pi]}\right\}\\ &= \frac{4}{\pi^3\epsilon_0}\sum_{m=0}^{\infty}\frac{\sin[(2m+1)\pi x]}{(2m+1)^3}\left\{1-\underbrace{\frac{\sinh[(2m+1)\pi(1-y)]+\sinh[(2m+1)\pi y]}{\sinh[(2m+1)\pi]}}_{C}\right\}\end{aligned} \tag{8}$$

With the identity

$$\sinh\eta + \sinh\xi = 2\sinh\left(\frac{\eta+\xi}{2}\right)\cosh\left(\frac{\eta-\xi}{2}\right) \tag{9}$$

C is reduced to

$$C = \frac{2\sinh[(m+1/2)\pi]\cdot\cosh[(2m+1)\pi(y-1/2)]}{2\sinh[(m+1/2)\pi]\cdot\cosh[(m+1/2)\pi]} = \frac{\cosh[(2m+1)\pi(y-1/2)]}{\cosh[(m+1/2)\pi]} \tag{10}$$

which finally gives

$$\Phi(\mathbf{x}) = \frac{4}{\pi^3\epsilon_0}\sum_{m=0}^{\infty}\frac{\sin[(2m+1)\pi x]}{(2m+1)^3}\left\{1-\frac{\cosh[(2m+1)\pi(y-1/2)]}{\cosh[(m+1/2)\pi]}\right\} \tag{11}$$