1. From Prob 3.26, we have the Green function

$$G\left(\mathbf{x},\mathbf{x}'\right)=\sum_{l=0}^{\infty}g_l\left(r,r'\right)P_l\left(\cos\gamma\right)\qquad\text{where}$$
$$g_l\left(r,r'\right)=\begin{cases}\dfrac{r_<^l}{r_>^{l+1}}+\dfrac{1}{b^{2l+1}-a^{2l+1}}\left[\dfrac{l+1}{l}(rr')^l+\dfrac{l}{l+1}\dfrac{(ab)^{2l+1}}{(rr')^{l+1}}+a^{2l+1}\left(\dfrac{r^l}{r'^{l+1}}+\dfrac{r'^l}{r^{l+1}}\right)\right] & \text{for } l>0\\ \dfrac{1}{r_>}-\left(\dfrac{a^2}{a^2+b^2}\right)\dfrac{1}{r'}+f(r) & \text{for } l=0\end{cases}\tag{1}$$

The potential with Neumann boundary condition is given by equation (1.46)

$$\Phi(\mathbf{x})=\langle\Phi\rangle_S+\frac{1}{4\pi\epsilon_0}\int_V\rho\left(\mathbf{x}'\right)G\left(\mathbf{x},\mathbf{x}'\right)d^3x'+\frac{1}{4\pi}\oint_S\frac{\partial\Phi}{\partial n'}G\left(\mathbf{x},\mathbf{x}'\right)da'\tag{2}$$

We can ignore the global constant $\langle\Phi\rangle_S$. Furthermore, for this problem, the volume integral is zero and the surface integral only has contribution from the outer sphere, where $\partial\Phi/\partial n'=-E_r=E_0\cos\theta'$. This gives

$$\begin{aligned}\Phi(\mathbf{x})&=\frac{1}{4\pi}\cdot b^2\int_0^{2\pi}d\phi'\int_0^{2\pi}\sin\theta'd\theta'E_0\cos\theta'\sum_{l=0}^{\infty}g_l(r,b)P_l(\cos\gamma)\\&=\frac{E_0b^2}{4\pi}\sum_{l=0}^{\infty}g_l(r,b)\underbrace{\int_0^{2\pi}d\phi'\int_0^{\pi}\sin\theta'\cos\theta'd\theta'P_l(\cos\gamma)}_{I}\end{aligned}\tag{3}$$

By addition theorem

$$\begin{aligned}I&=\int_0^{2\pi}d\phi'\int_0^{\pi}\sin\theta'\cos\theta'd\theta'\left[\frac{4\pi}{2l+1}\sum_{m=-l}^{l}Y_{lm}^*(\theta,\phi)Y_{lm}\left(\theta',\phi'\right)\right]\\&=\frac{4\pi}{2l+1}\sum_{m=-l}^{l}Y_{lm}^*(\theta,\phi)\overbrace{\int_0^{2\pi}d\phi'e^{im\phi'}}^{2\pi\delta_{m0}}\int_0^{\pi}\sin\theta'\cos\theta'd\theta'\sqrt{\frac{2l+1}{4\pi}}P_l^m\left(\cos\theta'\right)\\&=\sqrt{\frac{4\pi}{2l+1}}Y_{l0}^*(\theta,\phi)2\pi\underbrace{\int_0^{\pi}\sin\theta'\cos\theta'd\theta'P_l\left(\cos\theta'\right)}_{\int_{-1}^{1}xP_l(x)dx=\delta_{l1}\cdot 2/3}\\&=\frac{4\pi}{3}\cos\theta\cdot\delta_{l1}\end{aligned}\tag{4}$$

Plugging (4) back into (3) yields

$$\Phi(\mathbf{x})=\frac{E_0b^2}{4\pi}\cdot g_1(r,b)\cdot\frac{4\pi}{3}\cos\theta=\frac{E_0r\cos\theta}{1-p^3}\left(1+\frac{a^3}{2r^3}\right)\tag{5}$$

Simple calculation of spherical coordinate derivatives yields

$$E_r(r,\theta)=-\frac{\partial\Phi}{\partial r}=-\frac{E_0\cos\theta}{1-p^3}\left(1-\frac{a^3}{r^3}\right)\qquad\qquad E_\theta(r,\theta)=-\frac{1}{r}\frac{\partial\Phi}{\partial\theta}=\frac{E_0\sin\theta}{1-p^3}\left(1+\frac{a^3}{2r^3}\right)\tag{6}$$

2. Omitted.