1. Equation (8.123) implies

$$V\xi = \tan^{-1}\sqrt{\frac{1}{\xi^2}-1}+\frac{p\pi}{2} \qquad \Longrightarrow \qquad \frac{n_1 a}{c}\sqrt{2\Delta}\omega\xi = \tan^{-1}\sqrt{\frac{1}{\xi^2}-1}+\frac{p\pi}{2} \tag{1}$$

Taking the derivative with respect to ξ gives

$$\frac{n_1 a}{c}\sqrt{2\Delta}\left(\omega+\frac{d\omega}{d\xi}\xi\right) = \left[\frac{1}{1+\left(\frac{1}{\xi^2}-1\right)}\right]\cdot\frac{1}{2}\frac{1}{\sqrt{\frac{1}{\xi^2}-1}}\left(\frac{-2}{\xi^3}\right) = -\frac{1}{\sqrt{1-\xi^2}} \tag{2}$$

With $\xi = \sin\theta/\sqrt{2\Delta}$, we have

$$\begin{aligned}
\frac{n_1 a}{c}\sqrt{2\Delta}\left(\omega+\frac{d\omega}{d\theta\cdot\cos\theta/\sqrt{2\Delta}}\frac{\sin\theta}{\sqrt{2\Delta}}\right) &= -\frac{\sqrt{2\Delta}}{\sqrt{2\Delta-\sin^2\theta}} && \Longrightarrow\\
\frac{n_1 a}{c}\left(\omega+\frac{d\omega}{d\theta}\tan\theta\right) &= -\frac{1}{\sqrt{2\Delta-\sin^2\theta}} && \Longrightarrow\\
\frac{d\omega}{d\theta} &= -\cot\theta\left(\frac{c}{n_1 a}\frac{1}{\sqrt{2\Delta-\sin^2\theta}}+\omega\right) && \text{by (8.124)}\\
&= -\cot\theta\left(\frac{c}{n_1 a}\frac{k}{\beta}+\omega\right)\\
&= -\cot\theta\,\omega\left(1+\frac{1}{\beta a}\right)
\end{aligned} \tag{3}$$

Since $k_z = k\cos\theta = n_1\omega\cos\theta/c$, then

$$\begin{aligned}
\frac{dk_z}{d\omega} &= \frac{n_1}{c}\left(\cos\theta-\omega\frac{d\omega}{d\theta}\sin\theta\right)\\
&= \frac{n_1}{c}\left[\cos\theta+\tan\theta\left(\frac{\beta a}{\beta a+1}\right)\sin\theta\right]\\
&= \frac{n_1}{c}\left[\frac{\cos\theta\,(\beta a+1)+\beta a\tan\theta\sin\theta}{\beta a+1}\right]\\
&= \frac{n_1}{c}\left[\frac{\cos^2\theta+\beta a}{\cos\theta\,(\beta a+1)}\right]
\end{aligned} \tag{4}$$

which gives

$$v_g = \frac{d\omega}{dk_z} = \frac{c\cos\theta}{n_1}\left(\frac{1+\beta a}{\cos^2\theta+\beta a}\right) \tag{5}$$

This is slightly greater than $c\cos\theta/n_1$ and is consistent with the Goos Hänchen effect in which the totally reflected ray shifts forward along the propagation direction, which increases the group velocity.

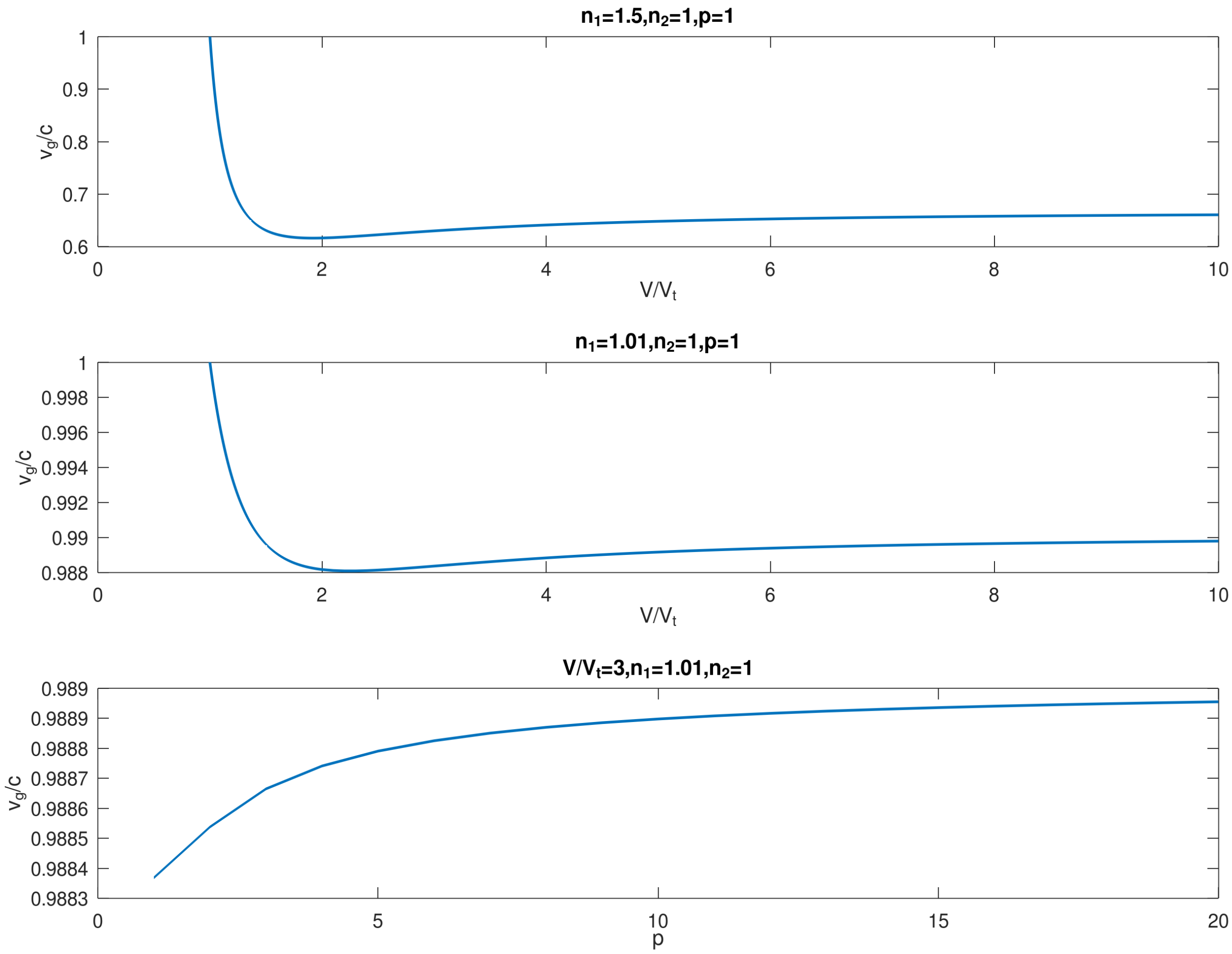
2. With $\cos\theta = \sqrt{1-2\Delta\xi^2}$, (5) implies

$$\frac{v_g}{c} = \frac{\sqrt{1-2\Delta\xi^2}}{n_1}\left(\frac{1+V\sqrt{1-\xi^2}}{1-2\Delta\xi^2+V\sqrt{1-\xi^2}}\right) \tag{6}$$

We can write a program to find the zero of (8.123) and feed ξ to (6) to get v_g/c.

It is not hard to see that when $V/V_t < 1$, (8.123) has no zero. When $V = p\pi/2$ or $V/V_t = 1$, the zero of (8.123) is $\xi = 1$, in which case $v_g/c = 1/n_2$. On the other hand, when $V \to \infty$, $v_g/c \to n_2/n_1$.

The plots for $p = 1, n_1 = 1.5, n_2 = 1$, as well as $p = 1, n_1 = 1.01, n_2 = 1$ are shown below.



3. From the middle plot, we see that the group velocities for different V's (hence ω's) are very close to each other for $n_1 = 1.01, n_2 = 1$. This qualitatively agrees with figure (8.12b), but the difference of optical path lengths in the latter is due to the ray's different launch angles.

   The third plot shows the effect of different eigenangle θ_p's. Let's fix $V/V_t = 3$ (hence the same frequency), and let p vary from 1 to 20. This can be related to figure (8.12b) better.