1. **Steps leading from (5.174) to (5.176).**

From equation (5.174)

$$\begin{aligned}
H_x(z,t) &= \frac{2H_0}{\pi}\int_0^\infty e^{-\nu t\kappa^2}\frac{\sin\kappa}{\kappa}\cos\left[\left(\frac{z}{a}\right)\kappa\right]d\kappa \\
&= \frac{2H_0}{\pi}\int_0^\infty e^{-\nu t\kappa^2}\frac{\sin\kappa}{\kappa}\frac{1}{2}\left(e^{i\kappa z/a}+e^{-i\kappa z/a}\right)d\kappa \\
&= \frac{H_0}{\pi}\left(\underbrace{\int_0^\infty e^{-\nu t\kappa^2}e^{i\kappa z/a}\frac{\sin\kappa}{\kappa}d\kappa}_{I_+}+\underbrace{\int_0^\infty e^{-\nu t\kappa^2}e^{-i\kappa z/a}\frac{\sin\kappa}{\kappa}d\kappa}_{I_-}\right) \qquad (1)
\end{aligned}$$

Since

$$\frac{\sin\kappa}{\kappa}=\int_0^1\cos\kappa y\,dy \qquad (2)$$

we have

$$\begin{aligned}
I_\pm &= \int_0^1 dy\int_0^\infty d\kappa e^{-\nu t\kappa^2}e^{\pm i\kappa z/a}\cos\kappa y \\
&= \int_0^1 dy\,\mathrm{Re}\int_0^\infty d\kappa e^{-\nu t\kappa^2}e^{\pm i\kappa z/a}e^{i\kappa y} \\
&= \int_0^1 dy\,\mathrm{Re}\int_0^\infty d\kappa\exp\left[-\nu t\left(\kappa^2\mp\frac{iz/a}{\nu t}\kappa-\frac{iy}{\nu t}\kappa\right)\right] \\
&= \int_0^1 dy\,\mathrm{Re}\int_0^\infty d\kappa\exp\left\{-\nu t\left[\kappa\mp\frac{i(z/a\pm y)}{2\nu t}\right]^2-\frac{(z/a\pm y)^2}{4\nu t}\right\} \\
&= \int_0^1 dy\exp\left[-\frac{(z/a\pm y)^2}{4\nu t}\right]\cdot\mathrm{Re}\int_0^\infty d\kappa\exp\left\{-\nu t\left[\kappa\mp\frac{i(z/a\pm y)}{2\nu t}\right]^2\right\} \qquad (3)
\end{aligned}$$

where we recognize the inner interval as half of the Gaussian integral

$$\int_{-\infty}^\infty e^{-p(x+c)^2}dx=\sqrt{\frac{\pi}{p}} \qquad \text{for } p,c\in\mathbb{C},\mathrm{Re}\,p>0 \qquad (4)$$

so

$$\begin{aligned}
I_\pm &= \frac{1}{2}\sqrt{\frac{\pi}{\nu t}}\int_0^1\exp\left[-\frac{(z/a\pm y)^2}{4\nu t}\right]dy \qquad \text{define } u\equiv\frac{y\pm z/a}{2\sqrt{\nu t}} \\
&= \frac{1}{2}\sqrt{\frac{\pi}{\nu t}}\int_{\pm\frac{z/a}{2\sqrt{\nu t}}}^{\frac{1\pm z/a}{2\sqrt{\nu t}}}e^{-u^2}2\sqrt{\nu t}\,du \\
&= \frac{\pi}{2}\left[\mathrm{erf}\left(\frac{1\pm z/a}{2\sqrt{\nu t}}\right)-\mathrm{erf}\left(\pm\frac{z/a}{2\sqrt{\nu t}}\right)\right] \qquad (5)
\end{aligned}$$

Thus (1) becomes equation (5.176):

$$\begin{aligned}
H_x(z,t) &= \frac{H_0}{2}\left[\mathrm{erf}\left(\frac{1+z/a}{2\sqrt{\nu t}}\right)-\mathrm{erf}\left(\frac{z/a}{2\sqrt{\nu t}}\right)+\mathrm{erf}\left(\frac{1-z/a}{2\sqrt{\nu t}}\right)-\mathrm{erf}\left(-\frac{z/a}{2\sqrt{\nu t}}\right)\right] \\
&= \frac{H_0}{2}\left[\mathrm{erf}\left(\frac{1+z/a}{2\sqrt{\nu t}}\right)+\mathrm{erf}\left(\frac{1-z/a}{2\sqrt{\nu t}}\right)\right] \qquad (6)
\end{aligned}$$

where we used the fact that the erf function is odd.

2. **How to obtain (5.177).**

To see how (5.177) can be obtained, first recall the expansion of error function (see wolfram (10)):

$$\operatorname{erf}(x) = \frac{1}{\sqrt{\pi}} e^{-x^2} \sum_{n=0}^{\infty} \frac{(2x)^{2n+1}}{(2n+1)!!} = \frac{2}{\sqrt{\pi}} e^{-x^2} \left(x + \frac{2x^3}{3} + \frac{4x^5}{15} + \cdots \right) \tag{7}$$

Define

$$\beta \equiv \frac{z}{a} \qquad\qquad \eta \equiv \frac{1}{2\sqrt{\nu t}} \tag{8}$$

thus

$$\begin{aligned}
\operatorname{erf}\left(\frac{1 \pm z/a}{2\sqrt{\nu t}}\right) &= \operatorname{erf}[(1 \pm \beta)\eta] \\
&\approx \frac{2}{\sqrt{\pi}} e^{-\beta^2\eta^2} \cdot e^{-\eta^2(1\pm 2\beta)} \left[(1 \pm \beta)\eta + \frac{2(1\pm\beta)^3\eta^3}{3} + \frac{4(1\pm\beta)^5\eta^5}{15} \right] \\
&\approx \frac{2}{\sqrt{\pi}} e^{-\beta^2\eta^2} \underbrace{\left[1 - (1\pm 2\beta)\eta^2 + \frac{(1\pm 2\beta)^2\eta^4}{2} \right] \left[(1 \pm \beta)\eta + \frac{2(1\pm\beta)^3\eta^3}{3} + \frac{4(1\pm\beta)^5\eta^5}{15} \right]}_{K_\pm}
\end{aligned} \tag{9}$$

The coefficients for orders up to η^5 in $K_\pm$ are

$$\begin{aligned}
\eta^1 : &\qquad 1 \pm \beta \\
\eta^3 : &\qquad \frac{2(1\pm\beta)^3}{3} - (1\pm 2\beta)(1\pm\beta) \\
\eta^5 : &\qquad \frac{4(1\pm\beta)^5}{15} - \frac{2(1\pm\beta)^3(1\pm 2\beta)}{3} + \frac{(1\pm\beta)(1\pm 2\beta)^2}{2}
\end{aligned}$$

It follows that

$$K_+ + K_- = 2\eta - \frac{2}{3}\eta^3 + \left(\frac{4\beta^2}{3} + \frac{1}{5} \right) \eta^5 + \cdots \tag{10}$$

Now back to (6)

$$\begin{aligned}
H_x(z,t) &= \frac{H_0}{2} \cdot \frac{2}{\sqrt{\pi}} e^{-\beta^2\eta^2} 2\eta \left[1 - \frac{1}{3}\eta^2 + \left(\frac{2\beta^2}{3} + \frac{1}{10} \right) \eta^4 + \cdots \right] \\
&= \frac{H_0}{\sqrt{\pi \nu t}} e^{-z^2/4\nu t a^2} \left(1 - \frac{1}{12\nu t} + \frac{z^2}{24\nu^2 t^2 a^2} + \frac{1}{160\nu^2 t^2} + \cdots \right)
\end{aligned} \tag{11}$$

which agrees with (5.177).