1. The current density of problem 8.19 is

$$\mathbf{J}(\mathbf{x}) = \hat{\mathbf{y}} I_0 \sin\left[\frac{\omega}{c}(h-y)\right]\delta(z)\,\delta(x-X) \qquad \text{for } 0 \le y \le h \tag{1}$$

The charge density is then

$$\rho(\mathbf{x}) = \frac{1}{i\omega}\nabla\cdot\mathbf{J}(\mathbf{x}) = \frac{I_0}{i\omega}\left(-\frac{\omega}{c}\right)\cos\left[\frac{\omega}{c}(h-y)\right]\delta(z)\,\delta(x-X) \qquad \text{for } 0 \le y \le h \tag{2}$$

Denote $k = \omega/c$, the electric dipole mopment is then

$$\mathbf{p} = \int \rho(\mathbf{x})\mathbf{x} d^3x = \hat{\mathbf{y}}\int_0^h dy \frac{iI_0}{c}\cos[k(h-y)]\,y = \hat{\mathbf{y}}\frac{iI_0}{ck^2}[1-\cos(kh)] \tag{3}$$

Since $\mathbf{x}$ and $\mathbf{J}(\mathbf{x})$ are colinear for locations where $\mathbf{J}$ is non-zero, then $\mathscr{M} = \mathbf{x}\times\mathbf{J}(\mathbf{x})/2 = 0$, from which we can conclude that $\rho^M = -\nabla\cdot\mathscr{M} = 0$, therefore all orders of magnetic multipole moments vanish. The quadrupole component is given by

$$Q_{\alpha\beta} = \int \rho(\mathbf{x})\left(3x_\alpha x_\beta - r^2\delta_{\alpha\beta}\right)d^3x \tag{4}$$

Because of the $\delta(z)$ factor in $\rho(\mathbf{x})$, we can quickly determine that $Q_{13} = Q_{31} = Q_{23} = Q_{32} = 0$. For the remaining components, note

$$\int \rho(\mathbf{x})\,d^3x = 0 \tag{5}$$

so

$$Q_{22} = \int \rho(\mathbf{x})\left(2y^2 - x^2 - z^2\right)d^3x = 2\int_0^h \rho(\mathbf{x})\,y^2 dy = \frac{4iI_0}{ck^2}\left[h - \frac{1}{k}\sin(kh)\right] \tag{6}$$

$$Q_{11} = \int \rho(\mathbf{x})\left(2x^2 - y^2 - z^2\right)d^3x = -\int_0^h \rho(\mathbf{x})\,y^2 dy = -\frac{Q_{22}}{2} \tag{7}$$

$$Q_{33} = \int \rho(\mathbf{x})\left(2z^2 - x^2 - y^2\right)d^3x = -\int_0^h \rho(\mathbf{x})\,y^2 dy = -\frac{Q_{22}}{2} \tag{8}$$

2. We now refer back to the solution of problem 8.19, where we found the TE_{10} mode field to be

$$\mathbf{E}_{10}^{\text{TE}} = \hat{\mathbf{y}}\frac{\sqrt{2}\pi}{\gamma_{10}a\sqrt{ab}}\sin\left(\frac{\pi x}{a}\right) = \hat{\mathbf{y}}\frac{\sqrt{2}}{\sqrt{ab}}\sin\left(\frac{\pi x}{a}\right) \qquad \text{recall } \gamma_{10} = \frac{\pi}{a} \tag{9}$$

In particular since $E_{z10}^{\text{TE}} = 0$, then by (8.129) we will have $\mathbf{E}_{10}^{\text{TE}(\pm)} = \mathbf{E}_{10}^{\text{TE}}$.
Taking the origin to be the root of the antenna $\mathbf{X} = (X, 0, 0)$ and truncating (9.69) up to the quadrupole term, we get the amplitude of the TE_{10} mode as

$$A_{10}^{\text{TE}(\pm)} = \frac{i\omega Z_{10}^{\text{TE}}}{2}\left[\mathbf{p}\cdot\mathbf{E}_{10}^{\text{TE}}(\mathbf{X}) + \frac{1}{6}\sum_{\alpha,\beta} Q_{\alpha\beta}\frac{\partial E_{\alpha 10}^{\text{TE}}}{\partial x_\beta}(\mathbf{X}) + \cdots\right] \tag{10}$$

where we have dropped magnetic terms since they are all zero. But on a closer inspection, the quadrupole term also vanishes, since from part (a), for $Q_{\alpha\beta}$ to be non-zero, we need $\alpha = \beta$, in which case $\partial E_{\alpha 10}^{\text{TE}}/\partial x_\beta$ will also vanish.
This turns (10) into

$$A_{10}^{\text{TE}(\pm)} = \frac{i\omega Z_{10}^{\text{TE}}}{2}\frac{iI_0}{ck^2}[1-\cos(kh)]\frac{\sqrt{2}}{\sqrt{ab}}\sin\left(\frac{\pi X}{a}\right) = -\frac{\sqrt{2}Z_{10}^{\text{TE}}I_0}{\sqrt{ab}k}\sin\left(\frac{\pi X}{a}\right)\sin^2\left(\frac{kh}{2}\right) \tag{11}$$

which gives the exact amplitude for the TE_{10} mode we have obtained in problem 8.19. Of course, the power flow calculated with only dipole contribution will agree with the exact power flow obtained in problem 8.19 too:

$$P^{(\pm)} = \frac{\mu c^2 I_0^2}{\omega kab}\sin^2\left(\frac{\pi X}{a}\right)\sin^4\left(\frac{kh}{2}\right) \tag{12}$$

The reason why the dipole coupling alone gives the exact result is because for TE_{10} mode (actually this is true for any TE_{m0} mode), the electric field is uniform in the $\hat{\mathbf{y}}$ direction, but our source only distributes along the $\hat{\mathbf{y}}$ direction, so all the other coupling terms in the multipole expansion (9.69) don't contribute.