On page pp543, the relation

$$\frac{\partial}{\partial x'^\alpha} = \frac{\partial x^\beta}{\partial x'^\alpha}\frac{\partial}{\partial x^\beta}, \tag{1}$$

is obtained straightforwardly by applying the partial derivative chain rule with respect to the coordinate transformation $x \longleftrightarrow x'$, and this has the form given in (11.62) which indicates that $\partial/\partial x^\alpha$ is a covariant vector operator.

But the proof of the dual relationship

$$\frac{\partial}{\partial x'_\alpha} = \frac{\partial x'^\alpha}{\partial x^\beta}\frac{\partial}{\partial x_\beta} \tag{2}$$

which would have matched (11.61) and indicated that the operator $\partial/\partial x_\alpha$ is a *contravariant* vector operator, is not given in the text. We shall prove this here.

First observe that chain rule implies

$$\frac{\partial}{\partial x'_\alpha} = \frac{\partial x'^\beta}{\partial x'_\alpha}\frac{\partial}{\partial x'^\beta} \tag{3}$$

but from (11.73)

$$x'^\beta = g'^{\beta\alpha}x'_\alpha = g'^{\alpha\beta}x'_\alpha \tag{4}$$

so (3) can be written as

$$\frac{\partial}{\partial x'_\alpha} = g'^{\alpha\beta}\frac{\partial}{\partial x'^\beta}, \tag{5}$$

Since $g'^{\alpha\beta}$ is a rank-2 contravariant tensor, we can express it in terms of the unprimed counterpart (see (11.63))

$$g'^{\alpha\beta} = \frac{\partial x'^\alpha}{\partial x^\gamma}\frac{\partial x'^\beta}{\partial x^\delta}g^{\gamma\delta} \tag{6}$$

Also, we use the fact that $\partial/\partial x'^\beta$ is a contravariant vector operator to write it as (following (1))

$$\frac{\partial}{\partial x'^\beta} = \frac{\partial x^\mu}{\partial x'^\beta}\frac{\partial}{\partial x^\mu} \tag{7}$$

Plugging (6) and (7) into (5) proves (2)

$$\begin{aligned}
\frac{\partial}{\partial x'_\alpha} &= \frac{\partial x'^\alpha}{\partial x^\gamma}\frac{\partial x'^\beta}{\partial x^\delta}g^{\gamma\delta}\frac{\partial x^\mu}{\partial x'^\beta}\frac{\partial}{\partial x^\mu} \\
&= \frac{\partial x'^\alpha}{\partial x^\gamma}g^{\gamma\delta}\underbrace{\overbrace{\left(\frac{\partial x'^\beta}{\partial x^\delta}\frac{\partial x^\mu}{\partial x'^\beta}\right)}^{\partial x^\mu/\partial x^\delta}\frac{\partial}{\partial x^\mu}}_{\partial/\partial x^\delta} \\
&= \frac{\partial x'^\alpha}{\partial x^\gamma}g^{\gamma\delta}\frac{\partial}{\partial x^\delta} && \text{apply (5) to unprimed coordinates} \\
&= \frac{\partial x'^\alpha}{\partial x^\gamma}\frac{\partial}{\partial x_\gamma}
\end{aligned} \tag{8}$$