1. With Kirchhoff approximation, let's plug the incident wave

$$\mathbf{E}_{\text{inc}} = E_0 \left(\sin\beta \boldsymbol{\epsilon}_1 + \cos\beta \boldsymbol{\epsilon}_2\right) e^{ikz} \tag{1}$$

into the integrand of (10.109)

$$\mathbf{E}(\mathbf{x}) = \frac{ie^{ikr}}{2\pi r}\mathbf{k}\times\int_{\text{slit}} \mathbf{n}\times\mathbf{E}\left(\mathbf{x}'\right)e^{-i\mathbf{k}\cdot\mathbf{x}'}da' \tag{2}$$

where

$$\mathbf{k} = \sin\theta\cos\phi\boldsymbol{\epsilon}_1 + \sin\theta\sin\phi\boldsymbol{\epsilon}_2 + \cos\theta\boldsymbol{\epsilon}_3 \qquad \left(\mathbf{n}\times\mathbf{E}\right)_{z=0} = E_0\left(\sin\beta\boldsymbol{\epsilon}_2 - \cos\beta\boldsymbol{\epsilon}_1\right) \tag{3}$$

Thus (2) is turned into

$$\mathbf{E}(\mathbf{x}) = \frac{ie^{ikr}}{2\pi r}E_0\left[\sin\beta\left(\mathbf{k}\times\boldsymbol{\epsilon}_2\right) - \cos\beta\left(\mathbf{k}\times\boldsymbol{\epsilon}_1\right)\right]\overbrace{\int_{-a/2}^{a/2} e^{-ik\sin\theta\cos\phi x'}dx'}^{I_x}\underbrace{\int_{-b/2}^{b/2} e^{-ik\sin\theta\sin\phi y'}dy'}_{I_y} \tag{4}$$

The two integrals are elementary:

$$I_x = \frac{2}{k\sin\theta\cos\phi}\sin\left(\frac{ka\sin\theta\cos\phi}{2}\right) \qquad I_y = \frac{2}{k\sin\theta\sin\phi}\sin\left(\frac{kb\sin\theta\sin\phi}{2}\right) \tag{5}$$

and the square bracket can also be evaluated explicitly

$$\left[\sin\beta\left(\mathbf{k}\times\boldsymbol{\epsilon}_2\right) - \cos\beta\left(\mathbf{k}\times\boldsymbol{\epsilon}_1\right)\right] = k\left[\sin\theta\sin\left(\beta+\phi\right)\boldsymbol{\epsilon}_3 - \cos\theta\sin\beta\boldsymbol{\epsilon}_1 - \cos\theta\cos\beta\boldsymbol{\epsilon}_2\right] \tag{6}$$

This gives the diffracted field

$$\begin{aligned}\mathbf{E}(\mathbf{x}) =& \frac{ie^{ikr}}{2\pi r}E_0\left[\sin\theta\sin\left(\beta+\phi\right)\boldsymbol{\epsilon}_3 - \cos\theta\sin\beta\boldsymbol{\epsilon}_1 - \cos\theta\cos\beta\boldsymbol{\epsilon}_2\right]\times \\ & \frac{4}{k\sin^2\theta\sin\phi\cos\phi}\sin\left(\frac{ka\sin\theta\cos\phi}{2}\right)\sin\left(\frac{kb\sin\theta\sin\phi}{2}\right)\end{aligned} \tag{7}$$

and the angular distribution of power

$$\frac{dP}{d\Omega} = P_i\cdot\frac{4}{\pi^2k^2ab}\left[\frac{\sin^2\theta\sin^2\left(\beta+\phi\right)+\cos^2\theta}{\sin^4\theta\sin^2\phi\cos^2\phi}\right]\left[\sin\left(\frac{ka\sin\theta\cos\phi}{2}\right)\sin\left(\frac{kb\sin\theta\sin\phi}{2}\right)\right]^2 \tag{8}$$

where

$$P_i = \frac{E_0^2}{2Z_0}ab \tag{9}$$

is the total power incident on the slit.

2. For scalar approximation, let's plug the scalar wave

$$\psi(\mathbf{x}) = E_0e^{ikz} \tag{10}$$

into (10.108)

$$\psi(\mathbf{x}) = -\frac{e^{ikr}}{4\pi r}\int_{\text{slit}} e^{-i\mathbf{k}\cdot\mathbf{x}'}\left[\mathbf{n}\cdot\boldsymbol{\nabla}'\psi\left(\mathbf{x}'\right) + i\mathbf{k}\cdot\mathbf{n}\psi\left(\mathbf{x}'\right)\right]da' \tag{11}$$

we get

$$\begin{aligned}\psi(\mathbf{x}) &= -\frac{e^{ikr}}{4\pi r}E_0\int_{\text{slit}} e^{-i\mathbf{k}\cdot\mathbf{x}'}\left(ik + ik\cos\theta\right)da' \\ &= -\frac{ie^{ikr}}{4\pi r}E_0\left(1+\cos\theta\right)\frac{4}{k\sin^2\theta\sin\phi\cos\phi}\sin\left(\frac{ka\sin\theta\cos\phi}{2}\right)\sin\left(\frac{kb\sin\theta\sin\phi}{2}\right)\end{aligned} \tag{12}$$

giving an angular distribution of power

$$\frac{dP}{d\Omega} = P_i \cdot \frac{1}{\pi^2 k^2 ab}\left[\frac{(1+\cos\theta)^2}{\sin^4\theta\,\sin^2\phi\,\cos^2\phi}\right]\left[\sin\left(\frac{ka\sin\theta\cos\phi}{2}\right)\sin\left(\frac{kb\sin\theta\sin\phi}{2}\right)\right]^2 \tag{13}$$

The difference between vector approximation (8) and scalar approximation (13) lies in the replacement

$$\sin^2\theta\,\sin^2(\beta+\phi)+\cos^2\theta \to (1+\cos\theta)^2 \tag{14}$$

where the scalar approximation has no dependence on the polarization β.

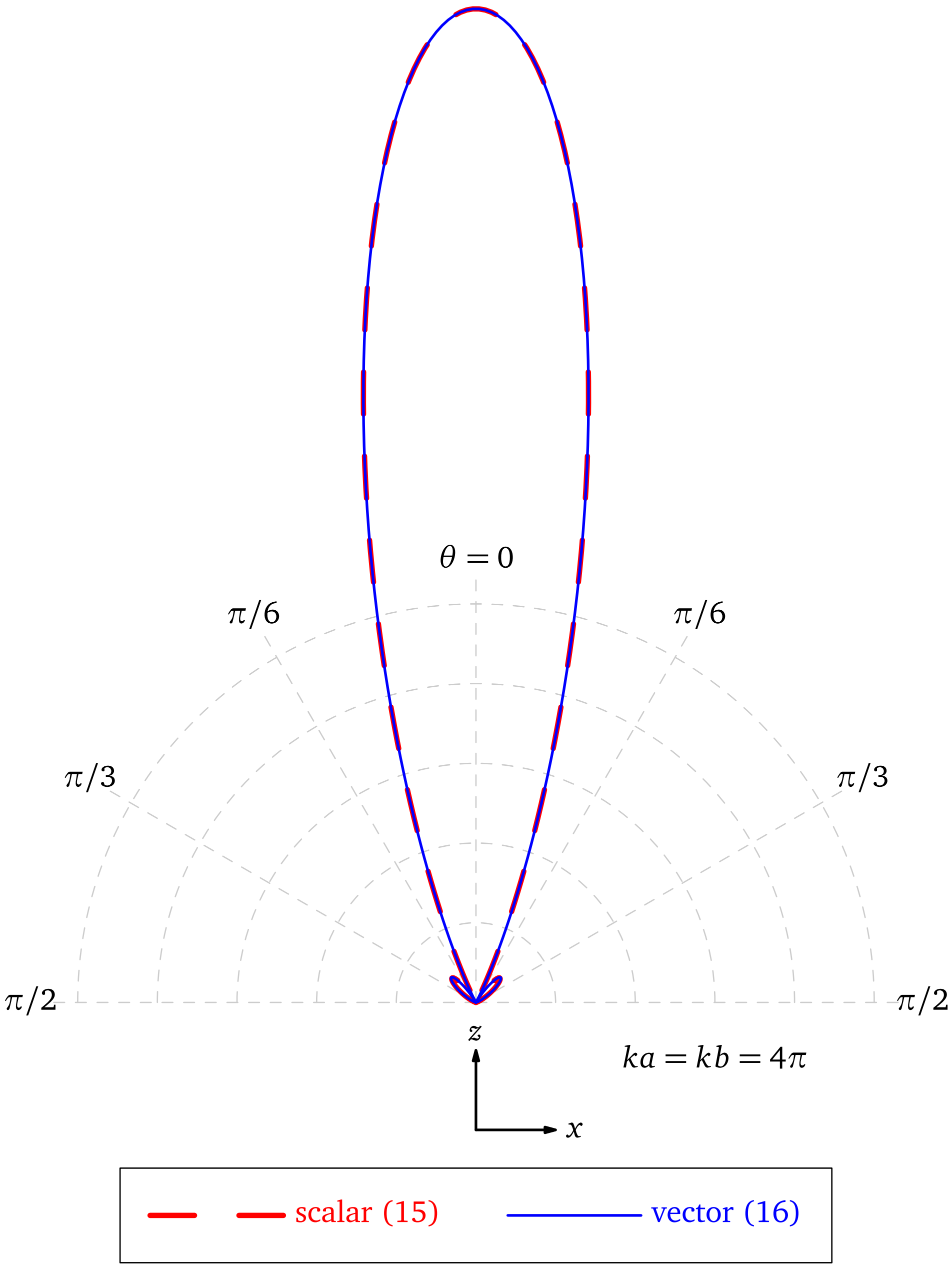
3. At the limit $\phi \to 0$, (8) and (13) become

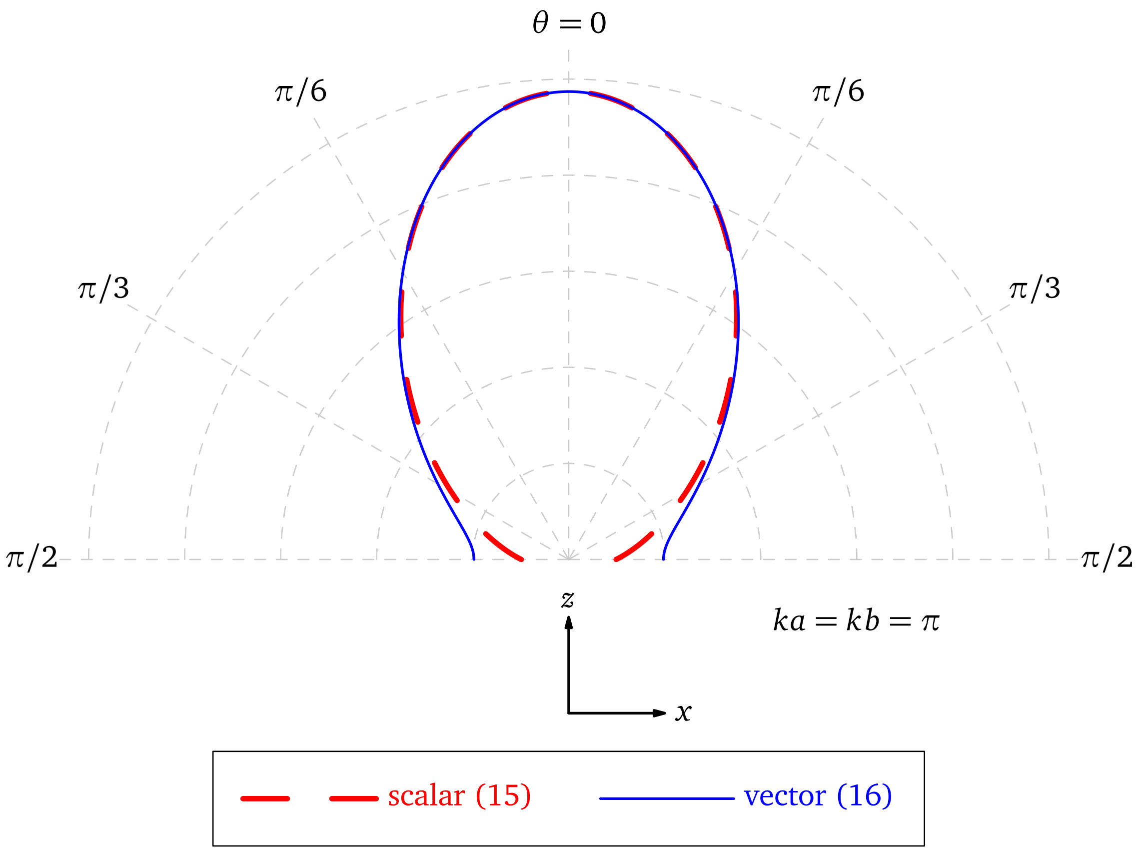
$$\left.\frac{dP}{d\Omega}\right|_{\text{vector}} = P_i \cdot \frac{4}{\pi^2 k^2 ab}\left(\frac{\sin^2\theta\,\sin^2\beta+\cos^2\theta}{\sin^4\theta}\right)\left[\sin\left(\frac{ka\sin\theta}{2}\right)\cdot\frac{kb\sin\theta}{2}\right]^2 \tag{15}$$

$$\left.\frac{dP}{d\Omega}\right|_{\text{scalar}} = P_i \cdot \frac{1}{\pi^2 k^2 ab}\,\frac{(1+\cos\theta)^2}{\sin^4\theta}\left[\sin\left(\frac{ka\sin\theta}{2}\right)\cdot\frac{kb\sin\theta}{2}\right]^2 \tag{16}$$

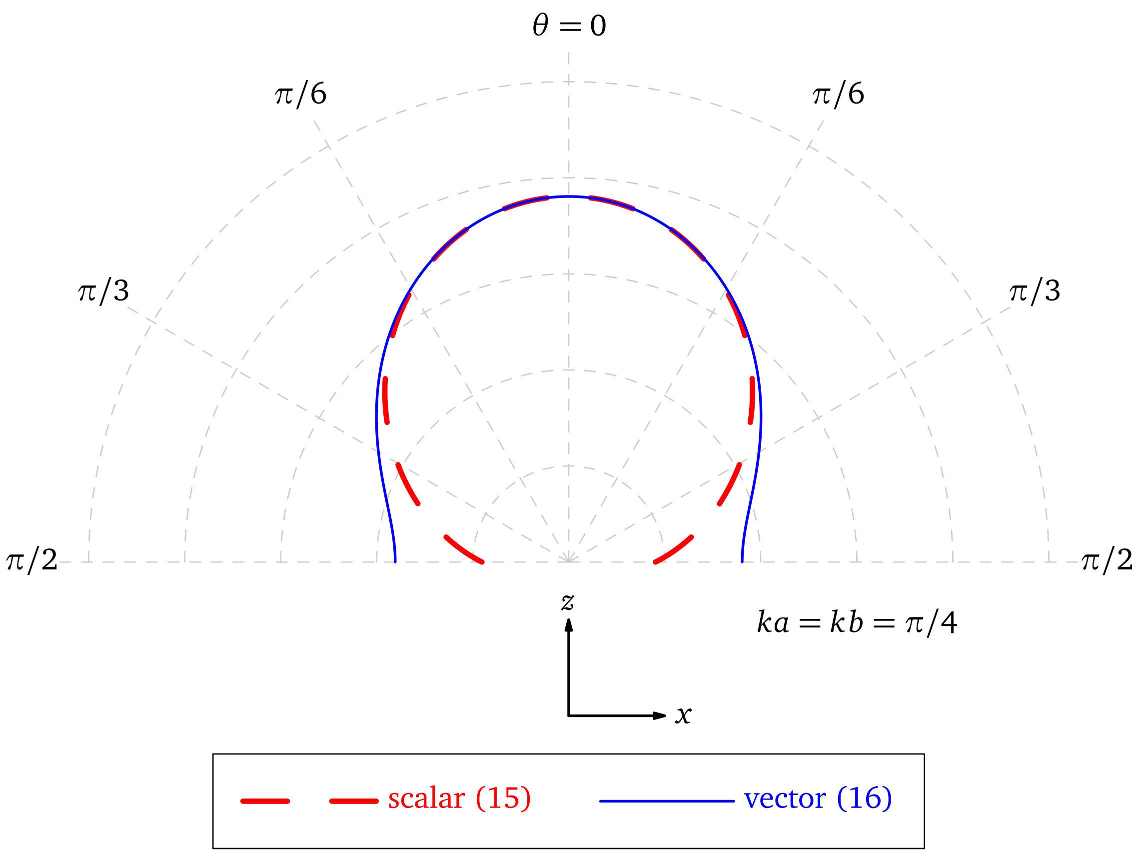
It is easy to see that (15) and (16) also apply to the limit $\phi \to \pi$.

Below we plot the angular distribution of power for polarization $\beta = \pi/4$ and the observation point on the x-z plane (i.e., $\phi = 0, \pi$). The first plot is for $ka = kb = 4\pi$, the second plot is for $ka = kb = \pi$ and the last is for $ka = kb = \pi/4$. They have been scaled to fit the size of the page.





$\theta = 0$
$\pi/6$
$\pi/6$
$\pi/3$
$\pi/3$
$\pi/2$
$\pi/2$
z
x
$ka = kb = \pi$
scalar (15)
vector (16)



$\theta = 0$
$\pi/6$
$\pi/6$
$\pi/3$
$\pi/3$
$\pi/2$
$\pi/2$
z
x
$ka = kb = \pi/4$
scalar (15)
vector (16)