

1. This is trivial by matching (10.64) and (8.11) and using the skin depth formula (8.8).
2. From (10.66), we have

$$\alpha_{\pm}(1) = -2 \left\{ \frac{j_1(x) - i \left( \frac{Z_s}{Z_0} \right) \left[ \frac{j_1(x)}{x} + j_1'(x) \right]}{h_1^{(1)}(x) - i \left( \frac{Z_s}{Z_0} \right) \left[ \frac{h_1^{(1)}(x)}{x} + h_1^{(1)'}(x) \right]} \right\} \quad (1)$$

$$\beta_{\pm}(1) = -2 \left\{ \frac{j_1(x) - i \left( \frac{Z_0}{Z_s} \right) \left[ \frac{j_1(x)}{x} + j_1'(x) \right]}{h_1^{(1)}(x) - i \left( \frac{Z_0}{Z_s} \right) \left[ \frac{h_1^{(1)}(x)}{x} + h_1^{(1)'}(x) \right]} \right\} \quad (2)$$

When  $x = ka \ll 1$ ,

$$j_1(x) \rightarrow \frac{x}{3} \quad h_1^{(1)}(x) \rightarrow -\frac{i}{x^2} \quad (3)$$

(1) and (2) turn into

$$\alpha_{\pm}(1) \rightarrow -2 \left[ \frac{\frac{x}{3} - i \frac{k\delta}{2} (1-i) \cdot \frac{2}{3}}{-\frac{i}{x^2} - i \frac{k\delta}{2} (1-i) \cdot \frac{i}{x^3}} \right] = -\frac{2i}{3} (ka)^3 \left[ \frac{\left(1 - \frac{\delta}{a}\right) - i \frac{\delta}{a}}{\left(1 + \frac{\delta}{2a}\right) + i \frac{\delta}{2a}} \right] \quad (4)$$

$$\beta_{\pm}(1) \rightarrow -2 \left[ \frac{\frac{x}{3} - i \frac{2}{k\delta(1-i)} \cdot \frac{2}{3}}{-\frac{i}{x^2} - i \frac{2}{k\delta(1-i)} \cdot \frac{i}{x^3}} \right] \approx \frac{4i}{3} (ka)^3 \quad (5)$$

where in  $\beta_{\pm}(1)$ , we dropped the  $x/3$  term in the numerator and the  $-i/x^2$  term in the denominator since they are negligible compared to the other term.

3. Keeping the  $l = 1$  order for the differential scattering cross section in (10.63) gives

$$\begin{aligned} \frac{d\sigma_{sc}}{d\Omega} = \frac{\pi}{2k^2} \cdot 3 \left\{ |\alpha_{\pm}(1)|^2 |\mathbf{X}_{1,\pm 1}|^2 + |\beta_{\pm}(1)|^2 |\mathbf{n} \times \mathbf{X}_{1,\pm 1}|^2 \right. \\ \left. \mp i\alpha_{\pm}(1)\beta_{\pm}^*(1) [\mathbf{X}_{1,\pm 1} \cdot (\mathbf{n} \times \mathbf{X}_{1,\pm 1}^*)] \pm i\alpha_{\pm}^*(1)\beta_{\pm}(1) [\mathbf{X}_{1,\pm 1}^* \cdot (\mathbf{n} \times \mathbf{X}_{1,\pm 1})] \right\} \end{aligned} \quad (6)$$

With the explicit form

$$\mathbf{X}_{1,1} = \sqrt{\frac{3}{16\pi}} e^{i\phi} (\hat{\theta} + i \cos \theta \hat{\phi}) \quad \mathbf{n} \times \mathbf{X}_{1,1}^* = \sqrt{\frac{3}{16\pi}} e^{-i\phi} (\hat{\phi} + i \cos \theta \hat{\theta}) \quad (7)$$

(6) becomes

$$\frac{d\sigma_{sc}}{d\Omega} = \frac{9}{32k^2} \{ [|\alpha_{\pm}(1)|^2 + |\beta_{\pm}(1)|^2] (1 + \cos^2 \theta) + 4 \operatorname{Re} [\alpha_{\pm}(1)\beta_{\pm}^*(1)] \cos \theta \} \quad (8)$$

Up to the desired order, we have

$$|\alpha_{\pm}(1)|^2 = \frac{4}{9} (ka)^6 \left[ \frac{\left(1 - \frac{\delta}{a}\right)^2 + \left(\frac{\delta}{a}\right)^2}{\left(1 + \frac{\delta}{2a}\right)^2 + \left(\frac{\delta}{2a}\right)^2} \right] \approx \frac{4}{9} (ka)^6 \left(1 - \frac{2\delta}{a}\right) \left(1 - \frac{\delta}{a}\right) \approx \frac{4}{9} (ka)^6 \left(1 - \frac{3\delta}{a}\right) \quad (9)$$

$$|\beta_{\pm}(1)|^2 = \frac{16}{9} (ka)^6 \quad (10)$$

$$\operatorname{Re} [\alpha_{\pm}(1)\beta_{\pm}^*(1)] = -\frac{8}{9} (ka)^6 \operatorname{Re} \left[ \frac{\left(1 - \frac{\delta}{a}\right) - i \frac{\delta}{a}}{\left(1 + \frac{\delta}{2a}\right) + i \frac{\delta}{2a}} \right] \approx -\frac{8}{9} (ka)^6 \left(1 - \frac{3\delta}{2a}\right) \quad (11)$$

By collecting all the terms in (8), we have

$$\frac{d\sigma_{sc}}{d\Omega} \approx \frac{k^4 a^6}{8} \left[ \left( 5 - \frac{3\delta}{a} \right) (1 + \cos^2 \theta) - 8 \left( 1 - \frac{3\delta}{2a} \right) \cos \theta \right] \quad (12)$$

4. By (10.61), the absorption cross section is

$$\sigma_{abs} = \frac{\pi}{2k^2} \sum_{l=1}^{\infty} (2l+1) [2 - |\alpha_{\pm}(1) + 1|^2 - |\beta_{\pm}(1) + 1|^2] \quad (13)$$

With only  $l = 1$  terms, it becomes

$$\sigma_{sc} \approx \frac{\pi}{2k^2} \cdot 3 [-|\alpha_{\pm}(1)|^2 - |\beta_{\pm}(1)|^2 - 2 \operatorname{Re} \alpha_{\pm}(1) - 2 \operatorname{Re} \beta_{\pm}(1)] \quad (14)$$

where by (4) and (5),

$$\operatorname{Re} \alpha_{\pm}(1) = \frac{2}{3} (ka)^3 \operatorname{Im} \left[ \frac{\left( 1 - \frac{\delta}{a} \right) - i \frac{\delta}{a}}{\left( 1 + \frac{\delta}{2a} \right) + i \frac{\delta}{2a}} \right] \approx -\frac{2}{3} (ka)^3 \frac{3\delta}{2a} \quad (15)$$

$$\operatorname{Re} \beta_{\pm}(1) = 0 \quad (16)$$

From (9) and (10), we see  $|\alpha_{\pm}(1)|^2, |\beta_{\pm}(1)|^2$  are of higher order  $(ka)^6$ , which are negligible in the presence of  $(ka)^3$  from  $\operatorname{Re} \alpha_{\pm}(1)$ . Thus (14) can be approximated as

$$\sigma_{abs} \approx 3\pi k a^2 \delta \quad (17)$$

5. If  $\delta = a$ ,  $\alpha_{\pm}(1)$  can be evaluated exactly from (4)

$$\alpha_{\pm}(1) = -\frac{2}{3} (ka)^3 \left( \frac{1}{\frac{3}{2} + \frac{i}{2}} \right) \implies \operatorname{Re} \alpha_{\pm}(1) = -\frac{2}{5} (ka)^3 \quad (18)$$

Comparing with (15), we see that for  $\delta = a$ , the absorption cross section is  $2/5$  of the approximation to the first order of  $\delta/a$ .