1. The Proca Lagrangian is given in (12.91)

$$\mathscr{L}_{\text{Proca}} = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} + \frac{\mu^2}{8\pi}A_\lambda A^\lambda - \frac{1}{c}J_\lambda A^\lambda \tag{1}$$

We can follow the procedure (12.102) – (12.104) to define the canonical stress tensor

$$T^{\alpha\beta} = \frac{\partial \mathscr{L}_{\text{Proca}}}{\partial\left(\partial_\alpha A^\lambda\right)}\partial^\beta A^\lambda - g^{\alpha\beta}\mathscr{L}_{\text{Proca}} \tag{2}$$

Comparing with the free field Lagrangian

$$\mathscr{L}_{\text{em}} = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} \tag{3}$$

we see that the additional second and third term of the Proca Lagrangian (1) do not contribute to the partial derivative in (2), thus (12.104) still holds

$$\begin{aligned} T^{\alpha\beta} &= -\frac{1}{4\pi}g^{\alpha\mu}F_{\mu\lambda}\partial^\beta A^\lambda - g^{\alpha\beta}\mathscr{L}_{\text{Proca}} \\ &= -\frac{1}{4\pi}\left[g^{\alpha\mu}F_{\mu\lambda}\left(-F^{\lambda\beta} + \partial^\lambda A^\beta\right)\right] - g^{\alpha\beta}\left(-\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} + \frac{\mu^2}{8\pi}A_\lambda A^\lambda - \frac{1}{c}J_\lambda A^\lambda\right) \\ &= \frac{1}{4\pi}\left(g^{\alpha\mu}F_{\mu\lambda}F^{\lambda\beta} + \frac{1}{4}g^{\alpha\beta}F_{\mu\nu}F^{\mu\nu} - \frac{\mu^2}{2}g^{\alpha\beta}A_\lambda A^\lambda\right)\underbrace{-\frac{1}{4\pi}g^{\alpha\mu}F_{\mu\lambda}\partial^\lambda A^\beta}_{X} + \frac{1}{c}g^{\alpha\beta}J_\lambda A^\lambda \end{aligned} \tag{4}$$

where

$$\begin{aligned} X &= -\frac{1}{4\pi}g^{\alpha\mu}F_{\mu\lambda}\partial^\lambda A^\beta = \frac{1}{4\pi}F^{\lambda\alpha}\partial_\lambda A^\beta \\ &= \frac{1}{4\pi}\left[\partial_\lambda\left(F^{\lambda\alpha}A^\beta\right) - A^\beta\partial_\lambda F^{\lambda\alpha}\right] \qquad \text{use Proca equation (12.92) } \partial_\lambda F^{\lambda\alpha} = \frac{4\pi}{c}J^\alpha - \mu^2 A^\alpha \\ &= \underbrace{\frac{1}{4\pi}\partial_\lambda\left(F^{\lambda\alpha}A^\beta\right)}_{T_D^{\alpha\beta}} - \frac{1}{c}A^\beta J^\alpha + \frac{\mu^2}{4\pi}A^\alpha A^\beta \end{aligned} \tag{5}$$

Then (4) can be written as

$$T^{\alpha\beta} = \overbrace{\frac{1}{4\pi}\left[g^{\alpha\mu}F_{\mu\lambda}F^{\lambda\beta} + \frac{1}{4}g^{\alpha\beta}F_{\mu\nu}F^{\mu\nu} + \mu^2\left(A^\alpha A^\beta - \frac{1}{2}g^{\alpha\beta}A_\lambda A^\lambda\right)\right]}^{\Theta^{\alpha\beta}} + T_D^{\alpha\beta} + \frac{1}{c}\left(g^{\alpha\beta}J_\lambda A^\lambda - J^\alpha A^\beta\right) \tag{6}$$

where we have defined the symmetrized stress tensor

$$\Theta^{\alpha\beta} = \frac{1}{4\pi}\left[g^{\alpha\mu}F_{\mu\lambda}F^{\lambda\beta} + \frac{1}{4}g^{\alpha\beta}F_{\mu\nu}F^{\mu\nu} + \mu^2\left(A^\alpha A^\beta - \frac{1}{2}g^{\alpha\beta}A_\lambda A^\lambda\right)\right] \tag{7}$$

of which the conservation law is proved in the next part.

2. In proving (12.107), Jackson uses the general Lagrangian density $\mathscr{L}\left(\phi_k, \partial^\alpha\phi_k\right)$, so the derivation is valid until the second equation after (12.107)

$$\partial_\alpha T^{\alpha\beta} = \overbrace{\sum_k\left[\frac{\partial\mathscr{L}}{\partial\phi_k}\partial^\beta\phi_k + \frac{\partial\mathscr{L}}{\partial\left(\partial_\alpha\phi_k\right)}\partial^\beta\left(\partial_\alpha\phi_k\right)\right]}^{Y} - \partial^\beta\mathscr{L} \tag{8}$$

But when $\mathscr{L}$ now includes the external current term $-J_\lambda A^\lambda/c$, the derivative of $\mathscr{L}$ with respect to x_β is not fully described by Y above, but is instead

$$\partial^\beta\mathscr{L} = Y + \frac{\partial\mathscr{L}}{\partial J_\lambda}\partial^\beta J_\lambda = Y - \frac{1}{c}A^\lambda\partial^\beta J_\lambda \tag{9}$$

generalizing (12.107) to

$$\partial_\alpha T^{\alpha\beta} = \frac{1}{c} A^\lambda \partial^\beta J_\lambda \tag{10}$$

Putting (6) and (10) together and using the fact $\partial_\alpha T_D^{\alpha\beta} = 0$, we have

$$\begin{aligned}
\frac{1}{c} A^\lambda \partial^\beta J_\lambda = \partial_\alpha T^{\alpha\beta} &= \partial_\alpha \Theta^{\alpha\beta} + \frac{1}{c} \partial^\beta \left(J_\lambda A^\lambda\right) - \frac{1}{c} \partial_\alpha \left(J^\alpha A^\beta\right) \qquad \Longrightarrow \\
\partial_\alpha \Theta^{\alpha\beta} &= \frac{1}{c} \left(A^\lambda \partial^\beta J_\lambda - J_\lambda \partial^\beta A^\lambda - A^\lambda \partial^\beta J_\lambda + J^\lambda \partial_\lambda A^\beta\right) \\
&= \frac{1}{c} \left(J_\lambda \partial^\lambda A^\beta - J_\lambda \partial^\beta A^\lambda\right) = \frac{1}{c} J_\lambda F^{\lambda\beta}
\end{aligned} \tag{11}$$

3. Comparing (7) with (12.113), we only need to evaluate the additional components of the quadratic potential terms and add them on top of (12.114), giving

$$\Theta^{00} = \frac{1}{8\pi} \left[\mathbf{E}^2 + \mathbf{B}^2 + \mu^2 \left(A^0 A^0 + \mathbf{A} \cdot \mathbf{A}\right)\right] \tag{12}$$

$$\Theta^{i0} = \frac{1}{4\pi} \left[(\mathbf{E} \times \mathbf{B})_i + \mu^2 A^i A^0\right] \tag{13}$$