The force on a moving charge is

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right) \tag{1}$$

thus the torque on the same charge is given by $\mathbf{x}\times\mathbf{F}$, which is equal to the rate of change in the particle's angular momentum, i.e.,

$$\frac{\partial \mathscr{L}_{\text{mech}}}{\partial t} = \mathbf{x}\times\left(\rho\mathbf{E} + \mathbf{J}\times\mathbf{B}\right) \tag{2}$$

Using the equation above 6.116 in Jackson

$$\rho\mathbf{E} + \mathbf{J}\times\mathbf{B} = \epsilon\left[\mathbf{E}\left(\boldsymbol{\nabla}\cdot\mathbf{E}\right) + \frac{1}{\mu\epsilon}\mathbf{B}\left(\boldsymbol{\nabla}\cdot\mathbf{B}\right) - \mathbf{E}\times\left(\boldsymbol{\nabla}\times\mathbf{E}\right) - \frac{1}{\mu\epsilon}\mathbf{B}\times\left(\boldsymbol{\nabla}\times\mathbf{B}\right)\right] - \epsilon\frac{\partial}{\partial t}\left(\mathbf{E}\times\mathbf{B}\right) \tag{3}$$

we can rewrite (2) as

$$\frac{d}{dt}\int_V \mathscr{L}_{\text{mech}} d^3x + \frac{d}{dt}\int_V \epsilon\mathbf{x}\times\left(\mathbf{E}\times\mathbf{B}\right)d^3x = \int_V \epsilon\mathbf{x}\times\left[\mathbf{E}\left(\boldsymbol{\nabla}\cdot\mathbf{E}\right) + \frac{1}{\mu\epsilon}\mathbf{B}\left(\boldsymbol{\nabla}\cdot\mathbf{B}\right) - \mathbf{E}\times\left(\boldsymbol{\nabla}\times\mathbf{E}\right) - \frac{1}{\mu\epsilon}\mathbf{B}\times\left(\boldsymbol{\nabla}\times\mathbf{B}\right)\right]d^3x \tag{4}$$

We shall identify

$$\mathscr{L}_{\text{field}} \equiv \epsilon\mathbf{x}\times\left(\mathbf{E}\times\mathbf{B}\right) = \mu\epsilon\mathbf{x}\times\left(\mathbf{E}\times\mathbf{H}\right) \tag{5}$$

as the density of angular momentum of the field.

Moreover, from (6.119) and (6.120), we can write

$$\epsilon\left[\mathbf{E}\left(\boldsymbol{\nabla}\cdot\mathbf{E}\right) + \frac{1}{\mu\epsilon}\mathbf{B}\left(\boldsymbol{\nabla}\cdot\mathbf{B}\right) - \mathbf{E}\times\left(\boldsymbol{\nabla}\times\mathbf{E}\right) - \frac{1}{\mu\epsilon}\mathbf{B}\times\left(\boldsymbol{\nabla}\times\mathbf{B}\right)\right] = \sum_j \frac{\partial}{\partial x_j}T_{ij}\hat{\mathbf{x}}_i = \boldsymbol{\nabla}\cdot\overleftrightarrow{\mathbf{T}} \tag{6}$$

then

$$\left[\mathbf{x}\times\left(\boldsymbol{\nabla}\cdot\overleftrightarrow{\mathbf{T}}\right)\right]_k = \sum_{i,j}\epsilon_{ijk}x_i\left(\sum_l \frac{\partial}{\partial x_l}T_{jl}\right) = \sum_l \frac{\partial}{\partial x_l}\left(\sum_{i,j}\epsilon_{ijk}x_iT_{jl}\right) \tag{7}$$

Per the note of the problem, the third-rank antisymmetric tensor $\mathbf{M}^{(3)}$ is defined by

$$M_{ijk} = T_{ij}x_k - T_{ik}x_j \tag{8}$$

This can be identified with the il component of the second-rank tensor $\mathbf{M}^{(2)}$ via

$$M_{il} \equiv \sum_{j,k}\epsilon_{jkl}T_{ij}x_k \tag{9}$$

Compare (9) with (7) we see

$$\left[\mathbf{x}\times\left(\boldsymbol{\nabla}\cdot\overleftrightarrow{\mathbf{T}}\right)\right]_k = \sum_l \frac{\partial}{\partial x_l}\left(-M_{lk}\right) = -\boldsymbol{\nabla}\cdot\overleftrightarrow{\mathbf{M}} \tag{10}$$

This makes the equivalent form of (4)

$$\begin{aligned}
\frac{d}{dt}\int_V\left(\mathscr{L}_{\text{mech}} + \mathscr{L}_{\text{field}}\right)d^3x + \int_V \boldsymbol{\nabla}\cdot\overleftrightarrow{\mathbf{M}}d^3x = 0 &\qquad \Longrightarrow \\
\frac{d}{dt}\int_V\left(\mathscr{L}_{\text{mech}} + \mathscr{L}_{\text{field}}\right)d^3x + \oint_S \mathbf{n}\cdot\overleftrightarrow{\mathbf{M}}\,da = 0 &\qquad \Longrightarrow \\
\frac{\partial}{\partial t}\left(\mathscr{L}_{\text{mech}} + \mathscr{L}_{\text{field}}\right) + \boldsymbol{\nabla}\cdot\overleftrightarrow{\mathbf{M}} = 0 &
\end{aligned} \tag{11}$$