1. **Prob 12.20**

   Let the vector potential inside the superconductor have time dependence $e^{-i\omega t}$, then the differential equation of $\mathbf{A}$ inside (equation below 12.99)

$$\nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} - \mu^2\mathbf{A} = 0 \tag{1}$$

   becomes

$$\nabla^2\mathbf{A} + \left(\frac{\omega^2}{c^2} - \mu^2\right)\mathbf{A} = 0 \qquad \text{or} \qquad \nabla^2\mathbf{A} + \left[\left(\frac{2\pi}{\lambda}\right)^2 - \frac{1}{\lambda_L^2}\right]\mathbf{A} = 0 \tag{2}$$

   If we consider the circumstance in which $\lambda_L \ll \lambda$, (2) becomes

$$\nabla^2\mathbf{A} - \kappa^2\mathbf{A} = 0 \qquad \text{where} \qquad \kappa = \sqrt{\frac{1}{\lambda_L^2} - \frac{4\pi^2}{\lambda^2}} \approx \frac{1}{\lambda_L}\left[1 + O\left(\frac{\lambda_L^2}{\lambda^2}\right)\right] \tag{3}$$

   Since the incident wave has only y component, the solution thus has a general form

$$A_y = \left(a' e^{-\kappa x} + b' e^{\kappa x}\right) e^{-i\omega t} \tag{4}$$

   For physical solutions, we must choose $b' = 0$. The field inside the superconductor is thus

$$\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} = \frac{i\omega}{c}\left(a' e^{-\kappa x}\right) e^{-i\omega t}\hat{\mathbf{y}} \qquad \text{and} \qquad \mathbf{B} = \nabla\times\mathbf{A} = -\kappa a' e^{-\kappa x} e^{-i\omega t}\hat{\mathbf{z}} \tag{5}$$

   The boundary condition that tangential $\mathbf{E}$ be continuous across the interface yields

$$a' = a + b \tag{6}$$

   This gives the surface impedance to the first order of λ_L/λ

$$Z_s = \frac{4\pi}{c}\frac{\mathbf{E}_{\tan}}{\mathbf{n}\times\mathbf{B}_{\tan}} = \frac{4\pi}{c}\left[\frac{\frac{i\omega}{c}a'\hat{\mathbf{y}}}{(-\hat{\mathbf{x}})\times(-\kappa a')\hat{\mathbf{z}}}\right] = -\frac{4\pi i}{c}\frac{(\omega/c)}{\kappa} \approx -\frac{8\pi^2 i}{c}\frac{\lambda_L}{\lambda} \tag{7}$$

2. **Prob 12.21**

   (a) By (7.58) the conductivity due to normal and superconducting electron are

$$\sigma_N = \frac{n_N e^2}{m_e(\gamma_N - i\omega)} = \frac{n_N e^2(\gamma_N + i\omega)}{m_e\left(\gamma_N^2 + \omega^2\right)} \qquad \sigma_S = \frac{n_S e^2}{m_e(\gamma_S - i\omega)} = \frac{i n_S e^2}{\omega m_e} \tag{8}$$

   It is clear that at low frequency, the combined conductivity $\sigma_N + \sigma_N$ is dominated by the imaginary (inductive) part, with a small real part (resistive component).

   (b) By Ohm's law,

$$\mathbf{J} = \sigma\mathbf{E} = -\frac{\sigma}{c}\frac{\partial\mathbf{A}}{\partial t} = \frac{i\omega\sigma}{c}\mathbf{A} \tag{9}$$

   When only the superconducting electron is considered,

$$i\omega\sigma = -\frac{n_S e^2}{m_e} \qquad \Longrightarrow \qquad \mathbf{J} = -\frac{n_S e^2}{m_e c}\mathbf{A} \tag{10}$$

   which is just (12.99) with identification $Q \longleftrightarrow e, n_Q \longleftrightarrow n_S, m_Q \longleftrightarrow m_e$. Putting $\mathbf{J}$ into Proca equation gives the same London penetration depth λ_L as in (12.100).