1. Using (6.57)

$$\left[\frac{\partial f\left(\mathbf{x}', t'\right)}{\partial t'}\right]_{\text{ret}} = \frac{\partial \left[f\left(\mathbf{x}', t'\right)\right]_{\text{ret}}}{\partial t} \tag{1}$$

Jackson (6.56)

$$\mathbf{B}(\mathbf{x}, t) = \frac{\mu_0}{4\pi}\int \left\{[\mathbf{J}]_{\text{ret}} \times \left(\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\right) + \left[\frac{\partial \mathbf{J}}{\partial t'}\right]_{\text{ret}} \times \left(\frac{\mathbf{x}-\mathbf{x}'}{c\,|\mathbf{x}-\mathbf{x}'|^2}\right)\right\} d^3x' \tag{2}$$

can be written as

$$\mathbf{B}(\mathbf{x}, t) = \frac{\mu_0}{4\pi}\left\{\int [\mathbf{J}]_{\text{ret}} \times \left(\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\right) d^3x' + \frac{1}{c}\frac{\partial}{\partial t}\int [\mathbf{J}]_{\text{ret}} \times \left(\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^2}\right) d^3x'\right\} \tag{3}$$

Now expand $[\mathbf{J}]_{\text{ret}}$ around $t' = t - |\mathbf{x}|/c = t - r/c$, and keep up to the first order time derivative,

$$\begin{aligned}
\left[\mathbf{J}\left(\mathbf{x}', t'\right)\right]_{\text{ret}} &= \mathbf{J}\left(\mathbf{x}', t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right) \\
&\approx \mathbf{J}\left(\mathbf{x}', t - \frac{r}{c}\right) + \left(\frac{\partial \mathbf{J}}{\partial t'}\right)\bigg|_{(\mathbf{x}', t-r/c)} \left(-\frac{1}{c}\right)\left[\mathbf{x}' \cdot \left(\boldsymbol{\nabla}' \left|\mathbf{x}-\mathbf{x}'\right|\right)\bigg|_{\mathbf{x}'=0}\right] \\
&= \mathbf{J}\left(\mathbf{x}', t - \frac{r}{c}\right) + \frac{1}{c}\left(\frac{\partial \mathbf{J}}{\partial t'}\right)\bigg|_{(\mathbf{x}', t-r/c)} \left(\mathbf{x}' \cdot \hat{\mathbf{r}}\right)
\end{aligned} \tag{4}$$

When (4) is substituted into (3) and higher time derivatives are discarded, we have

$$\begin{aligned}
\mathbf{B}(\mathbf{x}, t) \approx \frac{\mu_0}{4\pi}\Bigg\{ &\int \mathbf{J}\left(\mathbf{x}', t - \frac{r}{c}\right) \times \left(\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\right) d^3x' + \\
&\frac{1}{c}\frac{\partial}{\partial t}\int \mathbf{J}\left(\mathbf{x}', t - \frac{r}{c}\right) \times \underbrace{\left[\left(\mathbf{x}' \cdot \hat{\mathbf{r}}\right)\left(\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\right) + \frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^2}\right]}_{A} d^3x' \Bigg\}
\end{aligned} \tag{5}$$

Notice up to the the dipole order $O(r'/r)$,

$$\begin{aligned}
A = \left(\mathbf{x}' \cdot \hat{\mathbf{r}}\right)\left(\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\right) + \frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^2} &= \frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\left(\mathbf{x}' \cdot \hat{\mathbf{r}} + \left|\mathbf{x}-\mathbf{x}'\right|\right) \\
&= \frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\left[\mathbf{x}' \cdot \hat{\mathbf{r}} + r\left(1 + \frac{r'^2}{r^2} - \frac{2\mathbf{x}\cdot\mathbf{x}'}{r^2}\right)^{1/2}\right] \\
&= \frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\left\{\mathbf{x}' \cdot \hat{\mathbf{r}} + r\left[1 - \frac{\mathbf{x}' \cdot \hat{\mathbf{r}}}{r} + O\left(\frac{r'^2}{r^2}\right)\right]\right\} \\
&\approx r \cdot \frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}
\end{aligned} \tag{6}$$

Inserting (6) back into (5) produces

$$\mathbf{B}(\mathbf{x}, t) \approx \frac{\mu_0}{4\pi}\left(1 + \frac{r}{c}\frac{\partial}{\partial t}\right)\underbrace{\int \mathbf{J}\left(\mathbf{x}', t - \frac{r}{c}\right) \times \left(\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\right) d^3x'}_{I} \tag{7}$$

Again, up to the dipole order,

$$
\begin{aligned}
\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3} &= \frac{\mathbf{x}-\mathbf{x}'}{r^3}\left(1+\frac{r'^2}{r^2}-\frac{2\mathbf{x}\cdot\mathbf{x}'}{r^2}\right)^{-3/2} \\
&= \frac{1}{r^2}\left(\hat{\mathbf{r}}-\frac{\mathbf{x}'}{r}\right)\left[1+\frac{3\hat{\mathbf{r}}\cdot\mathbf{x}'}{r}+O\left(\frac{r'^2}{r^2}\right)\right] \\
&= \frac{1}{r^2}\left[\hat{\mathbf{r}}+\frac{3\left(\hat{\mathbf{r}}\cdot\mathbf{x}'\right)\hat{\mathbf{r}}-\mathbf{x}'}{r}+O\left(\frac{r'^2}{r^2}\right)\right] \\
&\approx \frac{\mathbf{x}}{r^3}-\frac{\mathbf{x}'}{r^3}+\frac{3\left(\mathbf{x}\cdot\mathbf{x}'\right)\mathbf{x}}{r^5}
\end{aligned}
\tag{8}
$$

Then the integral

$$
\begin{aligned}
I = \int \mathbf{J}\left(\mathbf{x}',t-\frac{r}{c}\right)\times\left(\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}\right)d^3x' \approx & \frac{1}{r^3}\left[\overbrace{\int \mathbf{J}\left(\mathbf{x}',t-\frac{r}{c}\right)d^3x'}^{0}\right]\times\mathbf{x}-\frac{1}{r^3}\overbrace{\int \mathbf{J}\left(\mathbf{x}',t-\frac{r}{c}\right)\times\mathbf{x}'d^3x'}^{-2\mathbf{m}(t-r/c)}+ \\
& \frac{3}{r^5}\left[\underbrace{\int \mathbf{J}\left(\mathbf{x}',t-\frac{r}{c}\right)\left(\mathbf{x}\cdot\mathbf{x}'\right)d^3x'}_{-\mathbf{x}\times\mathbf{m}(t-r/c)}\right]\times\mathbf{x}
\end{aligned}
\tag{9}
$$

where we have used the relations derived after equation (5.52) on page 185.
This finally gives

$$
\begin{aligned}
I &\approx \frac{2\mathbf{m}(t-r/c)}{r^3}+\frac{3\mathbf{x}\times\left[\mathbf{x}\times\mathbf{m}(t-r/c)\right]}{r^5} \\
&= \frac{2\mathbf{m}(t-r/c)}{r^3}+\frac{3\left[\mathbf{x}\cdot\mathbf{m}(t-r/c)\right]\mathbf{x}-3r^2\mathbf{m}(t-r/c)}{r^5} \\
&= \frac{3\left[\hat{\mathbf{r}}\cdot\mathbf{m}(t-r/c)\right]\hat{\mathbf{r}}-\mathbf{m}(t-r/c)}{r^3}
\end{aligned}
\tag{10}
$$

and

$$
\mathbf{B}(\mathbf{x},t)\approx\frac{\mu_0}{4\pi r^3}\left(1+\frac{r}{c}\frac{\partial}{\partial t}\right)\left\{3\left[\hat{\mathbf{r}}\cdot\mathbf{m}(t-r/c)\right]\hat{\mathbf{r}}-\mathbf{m}(t-r/c)\right\}
\tag{11}
$$

From (6.55), if we ignore all the electric multipoles, we have

$$
\mathbf{E}(\mathbf{x},t) = -\frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\int\frac{[\mathbf{J}]_{\text{ret}}}{|\mathbf{x}-\mathbf{x}'|}d^3x'
\tag{12}
$$

where we can take the approximation up to first order time derivative, and ignore all space terms with $O\left(r'^2/r^2\right)$:

$$
[\mathbf{J}]_{\text{ret}}\approx\mathbf{J}\left(\mathbf{x}',t-\frac{r}{c}\right)
\tag{13}
$$

$$
\frac{1}{|\mathbf{x}-\mathbf{x}'|}=\frac{1}{r}\left(1+\frac{r'^2}{r^2}-\frac{2\mathbf{x}\cdot\mathbf{x}'}{r^2}\right)^{-1/2}\approx\frac{1}{r}+\frac{\mathbf{x}\cdot\mathbf{x}'}{r^3}
\tag{14}
$$

which gives

$$
\begin{aligned}
\mathbf{E}(\mathbf{x},t) &\approx -\frac{\mu_0}{4\pi}\frac{\partial}{\partial t}\left[\frac{1}{r}\overbrace{\int \mathbf{J}\left(\mathbf{x}',t-\frac{r}{c}\right)d^3x'}^{0}+\frac{1}{r^3}\overbrace{\int \mathbf{J}\left(\mathbf{x}',t-\frac{r}{c}\right)\left(\mathbf{x}\cdot\mathbf{x}'\right)d^3x'}^{-\mathbf{x}\times\mathbf{m}(t-r/c)}\right] \\
&= \frac{\mu_0}{4\pi r^2}\hat{\mathbf{r}}\times\frac{\partial\mathbf{m}(t-r/c)}{\partial t}
\end{aligned}
\tag{15}
$$

2. Without loss of generality, let the field point $\mathbf{x}$ be at $z = 0$. For the solenoid part between $z \to z + dz$, the differential magnetic induction at $\mathbf{x}$ is

$$d\mathbf{B}(\mathbf{x}, t) = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left(1 + \frac{r}{c} \frac{\partial}{\partial t}\right) N \pi a^2 I \left(t - \frac{r}{a}\right) \left[\left(3\cos^2\theta - 1\right)\hat{\mathbf{z}} - 3\sin\theta\cos\theta\,\hat{\boldsymbol{\rho}}\right] dz \tag{16}$$

When integrating dz from $-\infty \to \infty$, the $\sin\theta\cos\theta\,dz$ term drops since it is odd in z, thus,

$$\mathbf{B}(\mathbf{x}, t) \propto \hat{\mathbf{z}} \int_{-\infty}^{\infty} \frac{3z^2 - r^2}{r^5} \left(1 + \frac{r}{c}\frac{\partial}{\partial t}\right) I \left(t - \frac{r}{a}\right) dz \tag{17}$$

Expanding I around $t - \rho/c$ yields

$$I\left(t - \frac{r}{c}\right) = I\left(t - \frac{\rho}{c}\right) + \left.\frac{\partial I}{\partial t}\right|_{t-\rho/c} \cdot \left(\frac{\rho - r}{c}\right) + O\left(\frac{\partial^2 I}{\partial t^2}\right) \tag{18}$$

which implies

$$\begin{aligned}\left(1 + \frac{r}{c}\frac{\partial}{\partial t}\right) I\left(t - \frac{r}{a}\right) &= \left(1 + \frac{r}{c}\frac{\partial}{\partial t}\right)\left[I\left(t - \frac{\rho}{c}\right) + \left.\frac{\partial I}{\partial t}\right|_{t-\rho/c} \cdot \left(\frac{\rho - r}{c}\right) + O\left(\frac{\partial^2 I}{\partial t^2}\right)\right] \\ &= I\left(t - \frac{\rho}{c}\right) + \frac{\rho}{c}\left.\frac{\partial I}{\partial t}\right|_{t-\rho/c} + O\left(\frac{\partial^2 I}{\partial t^2}\right)\end{aligned} \tag{19}$$

which is the sum of something that has no z-dependence and I's second order time derivatives.
Thus (17) becomes

$$\mathbf{B}(\mathbf{x}, t) \propto \int_{-\infty}^{\infty} \frac{3z^2 - r^2}{r^5} dz + O\left(\frac{\partial^2 I}{\partial t^2}\right) = 0 + O\left(\frac{\partial^2 I}{\partial t^2}\right) \tag{20}$$

Similarly for the electric field,

$$\begin{aligned}\mathbf{E}(\mathbf{x}, t) &= \frac{\mu_0}{4\pi} \cdot N\pi a^2 \int_{-\infty}^{\infty} \frac{\partial I(t - r/c)}{\partial t} \frac{1}{r^2} \hat{\mathbf{r}} \times \hat{\mathbf{z}} dz \\ &= \frac{\mu_0}{4\pi} \cdot N\pi a^2 \int_{-\infty}^{\infty} \frac{\partial I(t - r/c)}{\partial t} \frac{\sin\theta}{r^2} \left(-\hat{\boldsymbol{\phi}}\right) dz\end{aligned} \tag{21}$$

Referring to (18) and using the assumption that $|dI/(I dt)| \ll \rho/c$, we can approximate (21) as

$$\begin{aligned}\mathbf{E}(\mathbf{x}, t) &\approx -\hat{\boldsymbol{\phi}} \frac{\mu_0}{4\pi} \cdot N\pi a^2 \left.\frac{\partial I}{\partial t}\right|_{t-\rho/c} \cdot \overbrace{\int_{-\infty}^{\infty} \frac{\rho}{r^3} dz}^{2/\rho} \\ &= -\hat{\boldsymbol{\phi}} \frac{\mu_0}{2} \frac{N a^2}{\rho} \left.\frac{\partial I}{\partial t}\right|_{t-\rho/c}\end{aligned} \tag{22}$$