1. The incident scalar wave can be represented as

$$\psi(\mathbf{x},t)=\sqrt{I_0}e^{ikz}e^{-i\omega t} \tag{1}$$

With $\mathbf{R}=\sqrt{Z^2+(x'-X)^2+y'^2}$, for large Z, we can write

$$R\approx Z+\frac{(x'-X)^2+y'^2}{2Z} \tag{2}$$

(which does not entirely make sense since both x' and y' can go to infinity, but this seems to be the approximation Jackson uses to arrive at the conclusion) hence

$$e^{ikR}\approx e^{ikZ}e^{ik(x'-X)^2/2Z}e^{iky'^2/2Z} \tag{3}$$

then in the 0-th order in kZ, we can use (10.85) to calculate the field

$$\begin{aligned}\psi(x)&=\frac{k}{2\pi i}\int_S\frac{e^{ikR}}{R}\left(1+\frac{i}{kR}\right)\frac{\hat{\mathbf{z}}\cdot\mathbf{R}}{R}\psi(\mathbf{x}')\,da'\\&\approx\frac{k}{2\pi i}\sqrt{I_0}\frac{e^{ikZ}}{Z}\underbrace{\int_{-\infty}^{\infty}e^{iky'^2/2Z}dy'}_{I_y}\underbrace{\int_0^{\infty}e^{ik(x'-X)^2/2Z}dx'}_{I_x}\end{aligned} \tag{4}$$

With the Fresnel integral

$$\int_{-\infty}^{\infty}e^{iax^2}dx=\sqrt{\frac{\pi}{a}}e^{i\pi/4} \tag{5}$$

we see that

$$I_y=\sqrt{\frac{2\pi Z}{k}}\left(\frac{1+i}{\sqrt{2}}\right)=\sqrt{\frac{\pi Z}{k}}(1+i) \qquad I_x=\sqrt{\frac{2Z}{k}}\int_{-X\sqrt{k/2Z}}^{\infty}e^{it^2}dt \tag{6}$$

giving

$$\psi(\mathbf{x})=\sqrt{I_0}e^{ikZ}\left(\frac{1+i}{2i}\right)\sqrt{\frac{2}{\pi}}\int_{-X\sqrt{k/2Z}}^{\infty}e^{it^2}dt \tag{7}$$

2. Note that

$$\int_{-\xi}^{\infty}e^{it^2}dt=\int_0^{\infty}+\int_{-\xi}^{0}=\int_0^{\infty}+\int_0^{\xi}=\sqrt{\frac{\pi}{2}}\left(\frac{1+i}{2}\right)+\sqrt{\frac{\pi}{2}}[C(\xi)+iS(\xi)] \tag{8}$$

where $C(\xi)$ and $S(\xi)$ are the normalized Fresnel integrals

$$C(x)=\sqrt{\frac{2}{\pi}}\int_0^x\cos(t^2)\,dt \qquad S(x)=\sqrt{\frac{2}{\pi}}\int_0^x\sin(t^2)\,dt \tag{9}$$

Then the intensity given by (7) is

$$I=|\psi|^2=\frac{I_0}{2}\left\{\left[C(\xi)+\frac{1}{2}\right]^2+\left[S(\xi)+\frac{1}{2}\right]^2\right\} \tag{10}$$

To see the asymptotic behavior of large $|\xi|$, we refer to the expansion of the Fresnel integrals for large arguments,

$$S(x)=\frac{1}{2}\operatorname{Sgn}x-\left[\sqrt{\frac{2}{\pi}}+O(x^{-4})\right]\left[\frac{\cos(x^2)}{2x}+\frac{\sin(x^2)}{4x^3}\right] \tag{11}$$

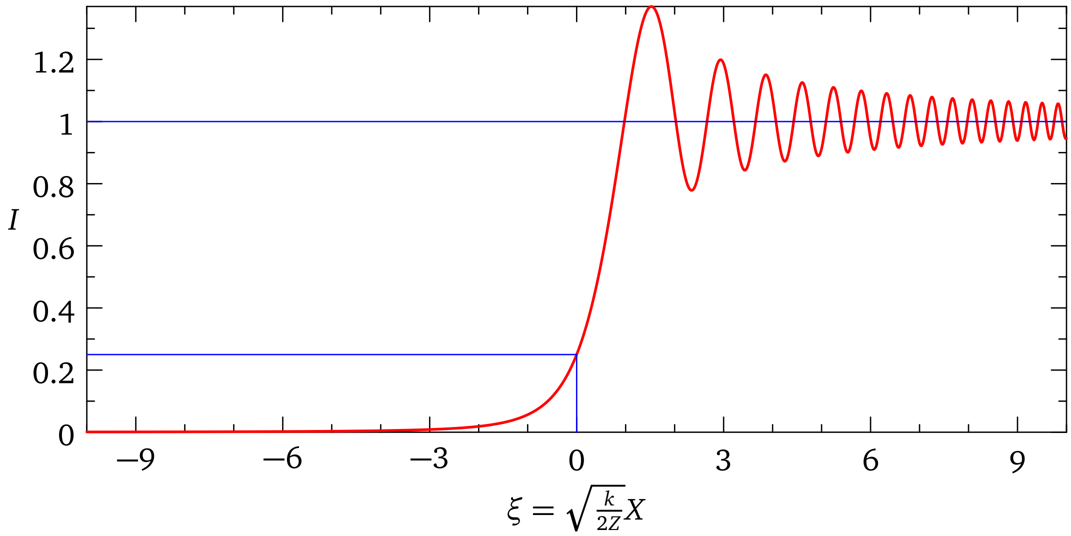
$$C(x)=\frac{1}{2}\operatorname{Sgn}x-\left[\sqrt{\frac{2}{\pi}}+O(x^{-4})\right]\left[\frac{\sin(x^2)}{2x}-\frac{\cos(x^2)}{4x^3}\right] \tag{12}$$

Thus

$$I \to \begin{cases} I_0 & \text{for } \xi \to +\infty \\ \dfrac{I_0}{4\pi\xi^2} & \text{for } \xi \to -\infty \end{cases} \tag{13}$$

In particular, when $X = 0$, $I = I_0/4$.

The plot of $I \sim \xi$ is shown below.



3. By (10.101),

$$\begin{aligned} \mathbf{E}(\mathbf{x}) &= \frac{1}{2\pi}\boldsymbol{\nabla}\times\int_{\text{aperture}}(\mathbf{n}\times\mathbf{E})\frac{e^{ikR}}{R}da' \\ &= \frac{1}{2\pi}\int_{\text{aperture}}\boldsymbol{\nabla}\left(\frac{e^{ikR}}{R}\right)\times(\mathbf{n}\times\mathbf{E})\,da' \\ &= \frac{1}{2\pi}\int_{\text{aperture}}\frac{e^{ikR}}{R}\left(ik-\frac{1}{R}\right)\left(\frac{\mathbf{x}-\mathbf{x}'}{R}\right)\times(\mathbf{n}\times\mathbf{E})\,da' \end{aligned} \tag{14}$$

If we take the $\mathbf{E}$ in the integrand the same as incident wave in the aperture,

$$\mathbf{E} = \left(E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}}\right)e^{ikz} \tag{15}$$

then

$$\begin{aligned} \left(\mathbf{x}-\mathbf{x}'\right)\times(\mathbf{n}\times\mathbf{E}) &= \left[\left(X-x'\right)\hat{\mathbf{x}} - y'\hat{\mathbf{y}} + Z\hat{\mathbf{z}}\right]\times\left(E_x\hat{\mathbf{y}} - E_y\hat{\mathbf{x}}\right) \\ &= -ZE_x\hat{\mathbf{x}} - ZE_y\hat{\mathbf{y}} + \left[\left(X-x'\right)E_x - y'E_y\right]\hat{\mathbf{z}} \end{aligned} \tag{16}$$

Plugging this back to (14), we would recover the scalar Kirchhoff integral (10.85) for each of the $\hat{\mathbf{x}}, \hat{\mathbf{y}}$ component. For the $\hat{\mathbf{z}}$ component, if we continue using the (somewhat dubious) approximation $|X - x'| \ll R, |y'| \ll R$, we can then completely ignore it.