1. By problem 5.10(b), the vector potential from one wire is

$$A_{2,\phi}(\rho,z)=\frac{\mu_0 I_2 a}{2}\int_0^\infty dk e^{-k|z|}J_1(ka)J_1(k\rho) \tag{1}$$

hence

$$W_{12}=\int \mathbf{J}_1\cdot\mathbf{A}_2 d^3x=2\pi a\frac{\mu_0 I_1 I_2 a}{2}\int_0^\infty dk e^{-kR}J_1^2(ka) \quad\Longrightarrow\quad M_{12}=\frac{W_{12}}{I_1 I_2}=\pi\mu_0 a^2\int_0^\infty dk e^{-kR}J_1^2(ka) \tag{2}$$

2. By equation 10.22.66 on dlmf.nist.gov

$$\int_0^\infty e^{-at}J_\nu(bt)J_\nu(ct)=\frac{1}{\pi\sqrt{bc}}Q_{\nu-1/2}\left(\frac{a^2+b^2+c^2}{2bc}\right) \tag{3}$$

where Q_ν^μ is the associated Legendre function of the second kind (see section 14.3 on dlmf.nist.gov):

$$Q_\nu^\mu(x)=e^{\mu\pi i}\Gamma(\nu+\mu+1)\mathbf{Q}_\nu^\mu(x) \tag{14.3.10}$$

$$\mathbf{Q}_\nu^\mu(x)=\frac{2^\nu\Gamma(\nu+1)(x+1)^{\mu/2}}{(x-1)^{\mu/2+\nu+1}}\mathbf{F}\left(\nu+1,\nu+\mu+1;2\nu+2;\frac{2}{1-x}\right) \tag{14.3.19}$$

$$\mathbf{F}(a,b;c;x)=\frac{1}{\Gamma(c)}F(a,b;c;x)=\frac{1}{\Gamma(c)}\left[1+\frac{ab}{c}x+\frac{1}{2!}\frac{a(a+1)b(b+1)}{c(c+1)}x^2+\cdots\right] \tag{14.3.3}$$

Plugging into (2)

$$\begin{aligned}
M_{12}&=\pi\mu_0 a^2\int_0^\infty dk e^{-kR}J_1^2(ka) &(4)\\
&=\pi\mu_0 a^2\cdot\frac{1}{\pi a}\cdot Q_{1/2}\left(\frac{R^2+2a^2}{2a^2}\right)\\
&=\mu_0 a\frac{\sqrt{2}[\Gamma(3/2)]^2}{\left(\frac{R^2}{2a^2}\right)^{3/2}}\mathbf{F}\left(\frac{3}{2},\frac{3}{2};3;-\frac{4a^2}{R^2}\right)\\
&=\frac{\mu_0\pi a}{2}\cdot\left(\frac{a^3}{R^3}\right)\left[1+\frac{\frac{3}{2}\cdot\frac{3}{2}}{3}\left(-\frac{4a^2}{R^2}\right)+\frac{1}{2!}\frac{\frac{3}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}}{3\cdot 4}\left(-\frac{4a^2}{R^2}\right)^2+\cdots\right]\\
&=\frac{\mu_0\pi a}{2}\left[\left(\frac{a}{R}\right)^3-3\left(\frac{a}{R}\right)^5+\frac{75}{8}\left(\frac{a}{R}\right)^7+\cdots\right] &(5)
\end{aligned}$$

where we see that the condition $R>2a$ is so the hypergeometric function F can converge.

3. From problem 5.33(b), M_{12} satisfies the Laplace equation with respect to the relative position of the loops' centers. So we can apply the argument in section 3.3, i.e., if

$$M_{12}(r\hat{\mathbf{z}})=\sum_{l=0}^\infty\left[A_l r^l+B_l r^{-(l+1)}\right] \tag{6}$$

then for point $\mathbf{x}=(r,\theta,\phi)$,

$$M_{12}(r,\theta,\phi)=\sum_{l=0}^\infty\left[A_l r^l+B_l r^{-(l+1)}\right]P_l(\cos\theta) \tag{7}$$

In this case, $\theta=\pi/2$, and

$$P_{2n}(0)=\frac{(-1)^n(2n)!}{2^{2n}(n!)^2} \tag{8}$$

we have

$$
\begin{aligned}
M_{12}\left(R, \frac{\pi}{2}, \phi\right) &= \frac{\mu_0 \pi a}{2}\left[\left(\frac{a}{R}\right)^3 P_2(0) - 3\left(\frac{a}{R}\right)^5 P_4(0) + \frac{75}{8}\left(\frac{a}{R}\right)^7 P_6(0) + \cdots\right] \\
&= \frac{\mu_0 \pi a}{2}\left[\left(\frac{a}{R}\right)^3\left(-\frac{1}{2}\right) - 3\left(\frac{a}{R}\right)^5\left(\frac{3}{8}\right) + \frac{75}{8}\left(\frac{a}{R}\right)^7\left(-\frac{5}{16}\right) + \cdots\right] \\
&= -\frac{\mu_0 \pi a}{4}\left[\left(\frac{a}{R}\right)^3 + \frac{9}{4}\left(\frac{a}{R}\right)^5 + \frac{375}{64}\left(\frac{a}{R}\right)^7 + \cdots\right] \qquad (9)
\end{aligned}
$$

4. By problem 5.33, the force is given by

$$
\mathbf{F}_{12} = I_1 I_2 \boldsymbol{\nabla}_{\mathbf{R}} M_{12}(\mathbf{R}) \qquad (10)
$$

When the two wires are concentric, the leading order of the force is thus

$$
\mathbf{F}_{12} = I_1 I_2 \boldsymbol{\nabla}_{\mathbf{R}}\left[\frac{\mu_0 \pi a}{2}\left(\frac{a}{R}\right)^3\right] = -\frac{3\mu_0 \pi I_1 I_2 a^4}{2R^4}\hat{\mathbf{z}} \qquad (11)
$$

which agrees with problem 5.18(c), while we take the relative permeability μ to be infinity, and also $R = 2d$.

Similarly when the two wires are coplanar, the leading order's force contribution has half of the magnitude of (11), which also agrees with problem 5.18(c). Note the additional sign compared with 5.18(c) is due to the orientation of the coplanar image current.