1. We must emphasize that the acclaimed "exact" formula

$$b = \frac{ze^2}{pv}\cot\frac{\theta}{2} \tag{1}$$

is not exact in the relativistic regime after all. A detailed analysis is given in the previous notes.

From the change of momentum in the scattering process, we have

$$\begin{aligned} |\Delta \mathbf{p}| &= 2p\sin\frac{\theta}{2} \qquad \Longrightarrow \\ Q^2 = (\Delta p)^2 &= 4p^2\sin^2\frac{\theta}{2} = 4p^2\left(\frac{1}{1+\cot^2\theta/2}\right) = \frac{4p^2z^2e^4}{z^2e^4+b^2p^2v^2} \end{aligned} \tag{2}$$

giving the energy transfer

$$T = \frac{Q^2}{2m} = \frac{2z^2e^4}{mv^2}\left[\frac{1}{b^2+\left(\dfrac{ze^2}{pv}\right)^2}\right] = \frac{2z^2e^4}{mv^2}\left(\frac{1}{b^2+b_{\text{min}}^{(c)2}}\right) \tag{3}$$

2. Let K' be the rest frame of the heavy particle, and let $t = t' = 0$ be the time when the distance of the two particles are the closest (b_{min}). If the heavy particle is assumed to move in straight line, then b_{min} is the same as the impact parameter b. We can use (11.152) for the instantaneous transverse electric field

$$E_2 = \frac{\gamma zeb}{(b^2+\gamma^2v^2t^2)^{3/2}} \tag{4}$$

giving the transverse impulse

$$\begin{aligned} \Delta p_{\text{trans}} &= \int_{-\infty}^{\infty} eE_2 dt = \gamma ze^2 b\int_{-\infty}^{\infty}\frac{dt}{(b^2+\gamma^2v^2t^2)^{3/2}} \qquad \text{let } t = \frac{b}{\gamma v}\tan\xi \\ &= \gamma ze^2 b\int_{-\pi/2}^{\pi/2}\frac{\left(\dfrac{b}{\gamma v}\right)\dfrac{1}{\cos^2\xi}d\xi}{b^3\dfrac{1}{\cos^3\xi}} \\ &= \frac{2ze^2}{bv} \end{aligned} \tag{5}$$

So the approximate energy transfer is

$$T \approx \frac{(\Delta p)^2}{2m} = \frac{4z^2e^4}{mb^2v^2} \tag{6}$$

This agrees with (3) under the assumption of "large impact parameter" $b \gg b_{\text{min}}^{(c)}$.