

1. From the view of duality transform, all particles in the universe may be considered to have only electric charge, only magnetic charge, or a constant proportion of both charges. If we view them to have only electric charge and zero magnetic charge, the force is given by the usual electromagnetic theory

$$\mathbf{F}_{\text{e-only-view}} = q_e \mathbf{E} + q_e \mathbf{v} \times \mathbf{B} \quad (1)$$

If we change our view so they all have a fixed proportion of electric and magnetic charge, a duality transform corresponding to ξ will be applied, i.e.,

$$Z_0 q_e = \cos \xi Z_0 q'_e + \sin \xi q'_m \quad \Rightarrow \quad q_e = \cos \xi q'_e + \sin \xi \frac{q'_m}{Z_0} \quad (2)$$

$$0 = q_m = -\sin \xi Z_0 q'_e + \cos \xi q'_m \quad \Rightarrow \quad \frac{q'_m}{Z_0} = q'_e \frac{\sin \xi}{\cos \xi} \quad (3)$$

$$\mathbf{E} = \cos \xi \mathbf{E}' + \sin \xi \frac{Z_0}{\mu_0} \mathbf{B}' \quad (4)$$

$$\frac{Z_0}{\mu_0} \mathbf{B} = -\sin \xi \mathbf{E}' + \cos \xi \frac{Z_0}{\mu} \mathbf{B}' \quad \Rightarrow \quad \mathbf{B} = -\sin \xi \frac{\mu_0}{Z_0} \mathbf{E}' + \cos \xi \mathbf{B}' \quad (5)$$

The same force should be produced in this mixed view, i.e.,

$$\begin{aligned} \mathbf{F}_{\text{mixed-view}} &= \mathbf{F}_{\text{e-only-view}} = q_e \mathbf{E} + q_e \mathbf{v} \times \mathbf{B} && \text{use (2),(4),(5)} \\ &= \left(\cos \xi q'_e + \sin \xi \frac{q'_m}{Z_0} \right) \left(\cos \xi \mathbf{E}' + \sin \xi \frac{Z_0}{\mu_0} \mathbf{B}' \right) + \\ &\quad \mathbf{v} \times \left(\cos \xi q'_e + \sin \xi \frac{q'_m}{Z_0} \right) \left(-\sin \xi \frac{\mu_0}{Z_0} \mathbf{E}' + \cos \xi \mathbf{B}' \right) \\ &= \cos^2 \xi q'_e \mathbf{E}' + \sin \xi \cos \xi \frac{q'_m}{Z_0} \mathbf{E}' + \sin \xi \cos \xi \frac{q'_e Z_0}{\mu_0} \mathbf{B}' + \sin^2 \xi \frac{q'_m}{\mu_0} \mathbf{B}' + \\ &\quad \mathbf{v} \times \left(-\sin \xi \cos \xi \frac{q'_e \mu_0}{Z_0} \mathbf{E}' + \cos^2 \xi q'_e \mathbf{B}' - \sin^2 \xi \frac{q'_m \mu_0}{Z_0^2} \mathbf{E}' + \sin \xi \cos \xi \frac{q'_m}{Z_0} \mathbf{B}' \right) && \text{use (3)} \\ &= q'_e \mathbf{E}' + \frac{q'_m \mathbf{B}'}{\mu_0} + q'_e \mathbf{v} \times \mathbf{B}' - q'_m \epsilon_0 \mathbf{v} \times \mathbf{E}' \end{aligned} \quad (6)$$

2. The invariance of \mathbf{F} under duality transform is nothing more than a statement of invariance of inner product between two 2D vectors under 2D rotation.

The duality transform equations (6.151), (6.152) allow us to treat

$$\mathbf{u}_1 = \begin{bmatrix} Z_0 q_e \\ q_m \end{bmatrix} \quad \mathbf{u}_2 = \begin{bmatrix} \mathbf{E} \\ \frac{Z_0}{\mu_0} \mathbf{B} \end{bmatrix} \quad \text{and} \quad \mathbf{u}_3 = \begin{bmatrix} \frac{Z_0}{\mu_0} \mathbf{B} \\ -\mathbf{E} \end{bmatrix} \quad (7)$$

as 2D vectors. Note the third vector is obtained by rotating the second by $\xi = \pi/2$, hence is a vector too. Under duality transform (i.e., 2D rotation by any ξ), the inner products between pairs of these vectors are preserved, i.e., the following quantities are invariant under duality transform

$$\langle \mathbf{u}_1, \mathbf{u}_2 \rangle = Z_0 q_e \mathbf{E} + \frac{Z_0}{\mu_0} q_m \mathbf{B} \quad (8)$$

$$\langle \mathbf{u}_1, \mathbf{u}_3 \rangle = \frac{Z_0^2}{\mu_0} q_e \mathbf{B} - q_m \mathbf{E} = \frac{q_e \mathbf{B}}{\epsilon_0} - q_m \mathbf{E} \quad (9)$$

$$\langle \mathbf{u}_2, \mathbf{u}_3 \rangle = 0 \quad (10)$$

Thus the claim of (6)'s invariance under duality can be deduced from the invariance of (8) and (9).

3. Consider the scattering process described in figure 6.6. Let (e_1, g_1) be the particle held still at the origin and let (e_2, g_2) be the particle that flies by. The pulse in the y direction must incorporate the full force (4) in this case where similar arguments leading to (6.155) yield

$$\Delta p_y = \frac{e_1 g_2 - e_2 g_1}{2\pi b} \quad \Rightarrow \quad \Delta L_z = \frac{e_1 g_2 - e_2 g_1}{2\pi} \quad (11)$$

Note the negative sign comes from the negative sign of the last term of (6).

By applying the postulate that any change in angular momentum must be integral multiple of \hbar , we obtain the generalized Dirac quantization condition

$$\frac{e_1 g_2 - e_2 g_1}{\hbar} = 2n\pi \quad (12)$$