

The emission angle is given in (13.50)

$$\theta_c = \cos^{-1}\left(\frac{1}{\beta\sqrt{\epsilon}}\right) = \cos^{-1}\left(\frac{1}{\beta n}\right) \quad (1)$$

We can express β in terms of kinetic energy T ,

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{\left(\frac{T}{mc^2} + 1\right)^2}} = \frac{\sqrt{T(T + 2mc^2)}}{T + mc^2} \quad (2)$$

Plugging the numbers, we get

electron with $T = 1\text{MeV}$:	$\beta = 0.94$	$\theta_c = 44.9^\circ$
proton with $T = 500\text{MeV}$:	$\beta = 0.76$	$\theta_c = 28.4^\circ$
proton with $T = 5\text{GeV}$:	$\beta = 0.99$	$\theta_c = 47.5^\circ$

From Frank-Tamm formula (13.48), we have

$$\frac{dW}{dx} = \frac{q^2}{c^2} \int_{\epsilon(\omega) > 1/\beta^2} \omega \left[1 - \frac{1}{\beta^2 \epsilon(\omega)}\right] d\omega \quad \Rightarrow \quad \frac{d}{dx} \left(\frac{dW}{d\omega} \right) = \frac{q^2}{c^2} \left[1 - \frac{1}{\beta^2 \epsilon(\omega)}\right] \omega \quad \text{for } \epsilon(\omega) > 1/\beta^2$$

Each quanta carries energy $\hbar\omega$, the above becomes

$$\frac{d}{dx} \left(\frac{dW/\hbar\omega}{d\omega} \right) = \frac{d}{dx} \left(\frac{dN}{d\omega} \right) = \frac{q^2}{\hbar c^2} \left[1 - \frac{1}{\beta^2 \epsilon(\omega)}\right] \quad (3)$$

Converting the derivative to be with respect to the wavelength, via $\omega = 2\pi c/\lambda$, we have

$$\frac{d}{dx} \left(\frac{dN}{d\lambda} \right) = \frac{d}{dx} \left(\frac{dN/d\lambda}{d\omega/d\lambda} \right) \quad \Rightarrow \quad \frac{d}{dx} \left(\frac{dN}{d\lambda} \right) = -\frac{2\pi c}{\lambda^2} \frac{q^2}{\hbar c^2} \left[1 - \frac{1}{\beta^2 \epsilon(\omega)}\right] \quad (4)$$

Integrating in the range of wavelength gives emitted photons per unit length

$$\frac{dN}{dx} = 2\pi \left(\frac{q^2}{\hbar c} \right) \left[1 - \frac{1}{\beta^2 \epsilon(\omega)}\right] \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) \approx \frac{2\pi}{137} \left(1 - \frac{1}{\beta^2 1.5^2}\right) \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) \quad (5)$$

The numerical results are shown below.

electron with $T = 1\text{MeV}$:	$\frac{dN}{dx} \approx 190/\text{cm}$
proton with $T = 500\text{MeV}$:	$\frac{dN}{dx} \approx 86/\text{cm}$
proton with $T = 5\text{GeV}$:	$\frac{dN}{dx} \approx 208/\text{cm}$