1. **Prob 11.19**

   (a) In the rest frame of the decaying particle, the invariance of the norm of the 4-momentum for the original particle gives

$$(E_1+E_2)^2-|\mathbf{P}|^2=M^2 \quad \Longrightarrow \quad E_1+E_2=M \tag{1}$$

   For the two resulting particles, momentum conservation requires $\mathbf{p}_1+\mathbf{p}_2=0$, and the invariance requires

$$E_1^2-p_1^2=m_1^2 \tag{2}$$
$$E_2^2-p_2^2=m_2^2 \tag{3}$$

   Subtracting (3) from (2) and substitute E_2 using (1) gives

$$E_1^2-(M-E_1)^2=m_1^2-m_2^2 \quad \Longrightarrow \quad E_1=\frac{M^2+m_1^2-m_2^2}{2M} \tag{4}$$

   (b) It is trivial algebra to show that

$$T_i=E_i-m_i=\Delta M\left(1-\frac{m_i}{M}-\frac{\Delta M}{2M}\right) \tag{5}$$

   (c) Putting in the numbers, we have $\Delta M=33.9\text{MeV}$, hence

$$T_\mu=4.1\text{MeV} \qquad T_\nu=29.8\text{MeV} \tag{6}$$

2. **Prob 11.20**

   (a) In the lab frame, using the invariance of the norm of the 4-momentum applied to the original particle and the resulting particles, as well as conservation of energy and momentum, we have

$$E_1^2-p_1^2=m_1^2 \tag{7}$$
$$E_2^2-p_2^2=m_2^2 \tag{8}$$
$$(E_1+E_2)^2-|\mathbf{p}_1+\mathbf{p}_2|^2=M^2 \tag{9}$$

   This gives

$$M^2=m_1^2+m_2^2+2E_1E_2-2\mathbf{p}_1\cdot\mathbf{p}_2=m_1^2+m_2^2+2E_1E_2-2p_1p_2\cos\theta \tag{10}$$

   (b) For the particle Λ to have a total energy of 10GeV, its Lorentz factor γ is

$$\gamma=\frac{M}{M_{\text{rest}}}=\frac{10\text{GeV}}{1.115\text{GeV}}=8.97 \tag{11}$$

   hence

$$\beta=\sqrt{1-\frac{1}{\gamma^2}}=0.994 \tag{12}$$

   so its track length in the lab frame is obtained by its speed multiplied by the dilated time

$$l=\gamma\tau\cdot\beta c\approx 78\text{cm} \tag{13}$$

   In the rest frame of the Λ particle, let the resulting particles' 3-mometa be $\mathbf{p}_1'=-\mathbf{p}_2'=\mathbf{p}'$, where $\mathbf{p}'$ is at angle ϕ with respect to the x axis. Equation (4) of the previous problem gives the energy of the resulting particles as measured by the rest frame of Λ:

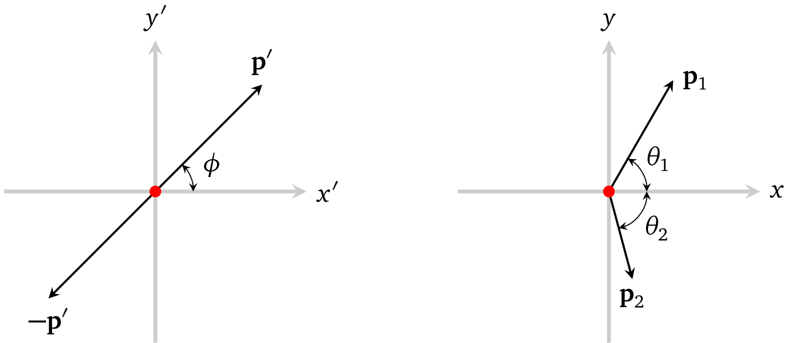
$$E_1'=\frac{M^2+m_1^2-m_2^2}{2M} \qquad E_2'=\frac{M^2+m_2^2-m_1^2}{2M} \tag{14}$$

   Using (2) or (3), we can obtain

$$p'=\sqrt{E_1'^2-m_1^2} \tag{15}$$

   Referencing the figure below, and by using the Lorentz transformation, we can calculate the perpendicular and parallel component of the resulting particles' 3-momenta in the lab frame

$$p_{1\perp}=p_1\sin\theta_1=p'\sin\phi \qquad p_{1\parallel}=p_1\cos\theta_1=\gamma\left(p'\cos\phi+\beta E_1'\right) \tag{16}$$
$$p_{2\perp}=p_2\sin\theta_2=p'\sin\phi \qquad p_{2\parallel}=p_2\cos\theta_2=\gamma\left(-p'\cos\phi+\beta E_2'\right) \tag{17}$$



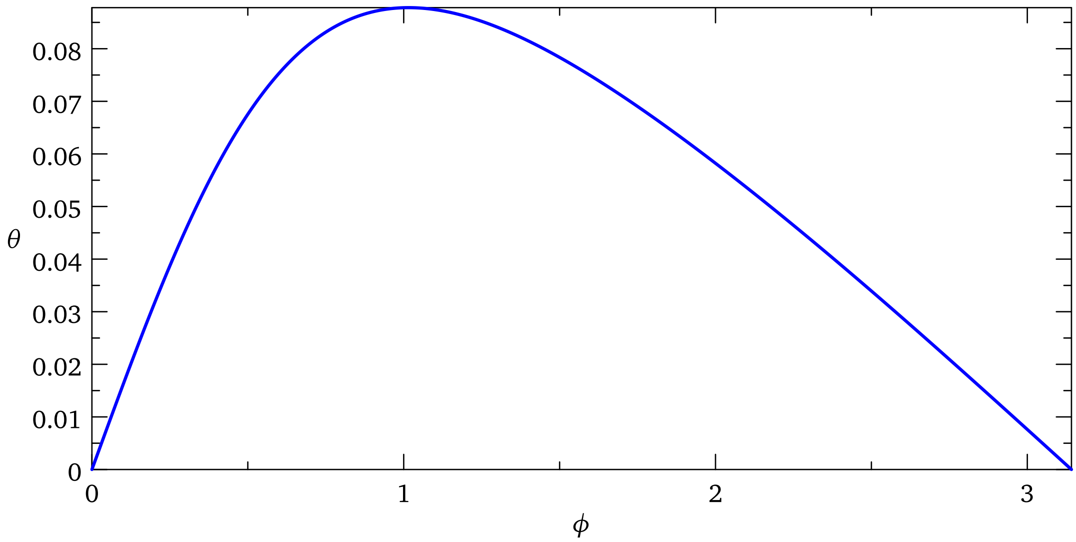
From these we can obtain the opening angle $\theta = \theta_1 + \theta_2$ of the two resulting particles measured in the lab frame

$$\theta = \theta_1 + \theta_2 = \tan^{-1}\left[\frac{p' \sin\phi}{\gamma\left(p' \cos\phi + \beta E_1'\right)}\right] + \tan^{-1}\left[\frac{p' \sin\phi}{\gamma\left(-p' \cos\phi + \beta E_2'\right)}\right] \tag{18}$$

If ϕ can take any value between $[0, \pi]$, it will be tedious routine to calculate the maximum value of θ as a function of ϕ. For this problem, we can plug in the numbers and get the numerical value of the maximum θ, which is

$$\theta_{\max} \approx 0.088 \approx 5^\circ \tag{19}$$

The $\theta \sim \phi$ graph is plotted below.



3. **Prob 11.21**

Let $P_i^\alpha = (E_i, \mathbf{p}_i)$ be the 4-momentum of the i-th particle after the reaction. In the rest frame of the original system, their sum will be

$$Q^\alpha = \sum_{i=1}^{n} P_i^\alpha = \left(\sum_{i=1}^{n} E_i, \sum_{i=1}^{n} \mathbf{p}_i\right) = (M, 0) \tag{20}$$

Pick an i and define the 4-vector

$$\bar{P}_i^\alpha \equiv Q^\alpha - P_i^\alpha = (M - E_i, -\mathbf{p}_i) \tag{21}$$

On the one hand, its norm is

$$\bar{P}_{i\alpha} \bar{P}_i^\alpha = (M - E_i)^2 - p_i^2 = M^2 - 2ME_i + \overbrace{\left(E_i^2 - p_i^2\right)}^{m_i^2} = M^2 - 2ME_i + m_i^2 \tag{22}$$

On the other hand, since

$$\bar{P}_i^\alpha = \left(\sum_{j \neq i} E_i, \sum_{j \neq i} \mathbf{p}_j\right) \tag{23}$$

we can write

$$\bar{P}_{i\alpha}\bar{P}_i^\alpha = \left(\sum_{j\neq i} E_j\right)^2 - \left|\sum_{j\neq i} \mathbf{p}_j\right|^2 \tag{24}$$

As a Lorentz scalar, $\bar{P}_{i\alpha}\bar{P}_i^\alpha$ is invariant for all inertial frames. Let's choose a frame $\bar{K}_i$ in which $\sum_{j\neq i} \mathbf{p}_j = 0$. In such a frame, (24) implies

$$\bar{P}_{i\alpha}\bar{P}_i^\alpha = \left(\sum_{j\neq i} E_j\right)^2 \geq \left(\sum_{j\neq i} m_j\right)^2 \tag{25}$$

since the j-th particle's energy in $\bar{K}_i$ is always no less that its rest mass.
Putting the inequality (25) into (22) gives

$$E_i = \frac{M^2 + m_i^2 - \bar{P}_{i\alpha}\bar{P}_i^\alpha}{2M} \leq \frac{M^2 + m_i^2 - \left(\sum_{j\neq i} m_j\right)^2}{2M} \tag{26}$$

or, in terms of the kinetic energy

$$\begin{aligned} T_i = E_i - m_i &\leq \frac{M^2 + m_i^2 - 2Mm_i - \left(\sum_{j\neq i} m_j\right)^2}{2M} \\ &= \frac{(M - m_i)^2 - \left(\sum_{j\neq i} m_j\right)^2}{2M} \\ &= \frac{\left(M - \sum_{i=1}^n m_i\right)\left(M - m_i + \sum_{j\neq i} m_j\right)}{2M} \\ &= \Delta M \left(1 - \frac{m_i}{M} - \frac{\Delta M}{2M}\right) \end{aligned} \tag{27}$$