- **1.12**

  This is actually pretty straightforward despite the complicated look. With the "unprimed" distribution ρ and σ, the potential is given by

$$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0}\left[\int_V \rho(\mathbf{x}')d^3x' + \oint_S \sigma(\mathbf{x}')da'\right] \tag{1}$$

  Similarly for the "primed" distribution,

$$\Phi'(\mathbf{x}) = \frac{1}{4\pi\epsilon_0}\left[\int_V \rho'(\mathbf{x}')d^3x' + \oint_S \sigma'(\mathbf{x}')da'\right] \tag{2}$$

  Green's reciprocation theorem claims

$$\int_V \rho\Phi' d^3x + \oint_S \sigma\Phi' da = \int_V \rho'\Phi d^3x + \oint_S \sigma'\Phi da \tag{3}$$

  Indeed, plugging (2) into the LHS of (3) gives

$$\frac{1}{4\pi\epsilon_0}\cdot\text{LHS} = \int_V \rho(\mathbf{x})d^3x\left[\int_V \rho'(\mathbf{x}')d^3x' + \oint_S \sigma'(\mathbf{x}')da'\right] + \oint_S \sigma(\mathbf{x})da\left[\int_V \rho'(\mathbf{x}')d^3x' + \oint_S \sigma'(\mathbf{x}')da'\right] \tag{4}$$

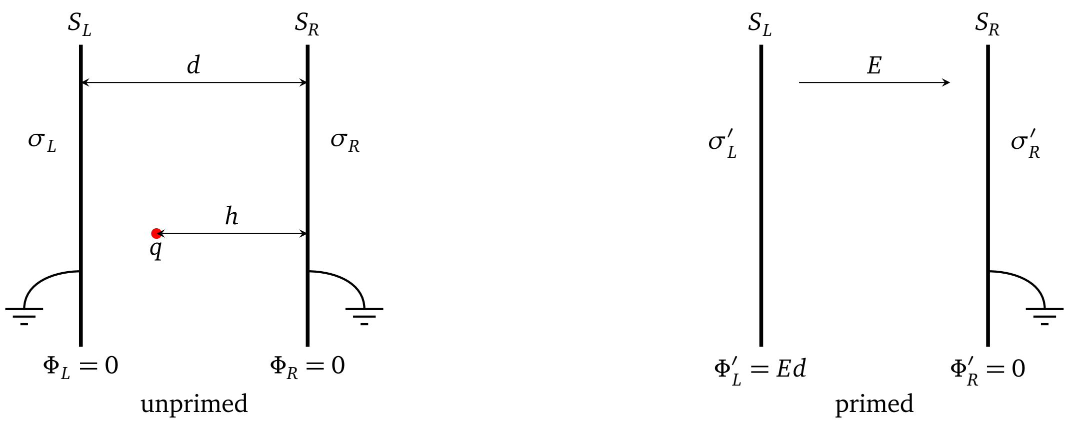
  and plugging (1) into the RHS gives

$$\frac{1}{4\pi\epsilon_0}\cdot\text{RHS} = \int_V \rho'(\mathbf{x})d^3x\left[\int_V \rho(\mathbf{x}')d^3x' + \oint_S \sigma(\mathbf{x}')da'\right] + \oint_S \sigma'(\mathbf{x})da\left[\int_V \rho(\mathbf{x}')d^3x' + \oint_S \sigma(\mathbf{x}')da'\right] \tag{5}$$

  The equality of LHS and RHS is obvious by exchanging the order of the double integrals, as well as the dummy label of integration variables.

- **1.13**

  In order to apply the Green's reciprocation theorem, we need two setups (unprimed v.s. primed), as shown below.



  In the unprimed setup, the volume charge distribution ρ is the δ-function at q's location, and the surface charge distribution is σ_L, σ_R (not necessarily uniform) for the left and right plate. Both plates are grounded, so $\Phi_L = \Phi_R = 0$.

  In the primed setup, only the right plate is grounded, and left plate is uniformly charged to produce an electric field E between the plates, therefore $\Phi'_L = Ed$. There is no charge between the plates, so $\rho' = 0$.

  With these, we can see the RHS of (3) vanishes since $\rho' = 0$ in V and $\Phi_L = \Phi_R = 0$ on S, but the LHS is

$$\begin{aligned}\text{LHS} &= \int_V \rho\Phi' d^3x + \int_{S_L} \sigma_L\Phi'_L da + \underbrace{\int_{S_R} \sigma_R\Phi'_R da}_{=0} \\ &= q(Eh) + (Ed)\int_{S_L} \sigma_L da \end{aligned} \tag{6}$$

  Equating LHS with RHS gives

$$Q_L = -q\cdot\frac{h}{d} \tag{7}$$