1. The effective transitional magnetization is

$$\begin{aligned}
\mathscr{M}(\mathbf{x},t) &= \frac{\mathbf{x}\times\mathbf{J}(\mathbf{x},t)}{2} = -\frac{iv_0}{4}\cdot\frac{a_0}{z}\mathbf{x}\times\hat{\mathbf{z}}\rho(\mathbf{x},t) \\
&= -\frac{iv_0a_0}{4}\left(\frac{r\sin\theta\sin\phi\hat{\mathbf{x}}-r\sin\theta\cos\phi\hat{\mathbf{y}}}{r\cos\theta}\right)\rho(\mathbf{x},t) \\
&= -\frac{i\alpha ca_0}{4}\tan\theta\left(\hat{\mathbf{x}}\sin\phi-\hat{\mathbf{y}}\cos\phi\right)\rho(\mathbf{x},t) \\
&= \frac{i\alpha ca_0}{4}\tan\theta\rho(\mathbf{x},t)\hat{\boldsymbol{\phi}} \qquad (1)
\end{aligned}$$

Since $\mathscr{M}$ has only $\hat{\boldsymbol{\phi}}$ component and that component is independent of ϕ, we know

$$\nabla\cdot\mathscr{M}(\mathbf{x},t) = \frac{1}{r\sin\theta}\frac{\partial\mathscr{M}_\phi(\mathbf{x},t)}{\partial\phi} = 0 \qquad (2)$$

For long-wavelength, (9.172) applies, so all the magnetic multipole moments vanish.

For electric multipole moments, by definition

$$\begin{aligned}
Q_{lm} &= \int Y^*_{lm}(\theta,\phi)r^l\rho(\mathbf{x})d^3x = \int Y^*_{lm}(\theta,\phi)r^l\frac{2q_e}{\sqrt{6}a_0^4}re^{-3r/2a_0}Y_{00}(\theta,\phi)Y_{10}(\theta,\phi)d^3x \\
&= \delta_{l1}\delta_{m0}\cdot\frac{2q_e}{\sqrt{6}a_0^4}\sqrt{\frac{1}{4\pi}}\int_0^\infty r^4e^{-3r/2a_0}dr \qquad \text{recall } \int_0^\infty x^ne^{-ax} = \frac{n!}{a^{n+1}} \\
&= \delta_{l1}\delta_{m0}\frac{2q_e}{\sqrt{6}a_0^4}\sqrt{\frac{1}{4\pi}}\frac{4!}{(3/2a_0)^5} = \delta_{l1}\delta_{m0}\frac{256}{81\sqrt{6\pi}}q_ea_0 \qquad (3)
\end{aligned}$$

We see the only multipole moment that enters is Q_{10}, which corresponds to the harmonically oscillating dipole along the $\hat{\mathbf{z}}$ direction.

With the presence of $\mathbf{J}$, or equivalently the effective magnetization $\mathscr{M}$, there is also a contribution to the electric multipole from magnetization source (9.170),

$$Q'_{lm} = -\frac{ik}{(l+1)c}\int r^lY^*_{lm}(\theta,\phi)\overbrace{\nabla\cdot(\mathbf{x}\times\mathscr{M})}^{-\mathbf{x}\cdot(\nabla\times\mathscr{M})}d^3x \qquad (4)$$

Note

$$\begin{aligned}
\mathbf{x}\cdot(\nabla\times\mathscr{M}) &= \mathbf{x}\cdot\left[\nabla\times\left(\frac{i\alpha ca_0}{4}\tan\theta\frac{2q_e}{\sqrt{6}a_0^4}re^{-3r/2a_0}\sqrt{\frac{1}{4\pi}}\sqrt{\frac{3}{4\pi}}\cos\theta\hat{\boldsymbol{\phi}}\right)\right] \\
&= \frac{i\alpha ca_0}{4}\cdot\frac{2q_e}{\sqrt{6}a_0^4}\sqrt{\frac{1}{4\pi}}\sqrt{\frac{3}{4\pi}}r^2e^{-3r/2a_0}\frac{1}{r\sin\theta}\frac{d\sin^2\theta}{d\theta} \\
&= \frac{i\alpha ca_0}{4}\cdot2\rho(\mathbf{x}) \qquad (5)
\end{aligned}$$

Thus

$$Q'_{lm} = -\frac{k\alpha a_0}{2(l+1)}Q_{lm} = -\frac{k\alpha a_0}{4}\delta_{l1}\delta_{m0}Q_{10} \qquad (6)$$

At typical wavelengths, $Q'_{10} \ll Q_{10}$.

2. Ignoring Q'_{10}, we can use (4.5) to find $\mathbf{p}$ from Q_{10},

$$\mathbf{p} = \sqrt{\frac{4\pi}{3}}Q_{10}\hat{\mathbf{z}} \qquad (7)$$

The total power is given by (9.24)

$$P = \frac{c^2Z_0k^4}{12\pi}|\mathbf{p}|^2 = \frac{c^2Z_0k^4}{9}Q_{10}^2 = \left(\frac{2}{3}\right)^8\cdot\hbar\omega_0\frac{\alpha^4c}{a_0} \qquad \text{where } \omega_0 = \frac{3q_e^2}{32\pi\epsilon_0\hbar a_0} \qquad (8)$$

3. Numerically, the reciprocal time is

$$\left(\frac{2}{3}\right)^8 \frac{\alpha^4 c}{a_0} \approx 6.27 \times 10^8 \text{ s}^{-1} \tag{9}$$

4. Under this model, the dipole will be

$$\mathbf{p}(t) = 2q_e a_0 (\cos\omega t, \sin\omega t, 0) = 2q_e a_0 \,\mathrm{Re}\left[e^{-i\omega t}(1, i, 0)\right] \tag{10}$$

The power derived from this dipole is

$$P = \frac{c^2 Z_0 k^4}{12\pi} \cdot 8q_e^2 a_0^2 \tag{11}$$

The ratio to the power from part b is

$$\frac{P_{(\mathrm{d})}}{P_{(\mathrm{b})}} = \frac{8}{\left(\frac{256}{81\sqrt{6\pi}}\sqrt{\frac{4\pi}{3}}\right)^2} \approx 3.60 \tag{12}$$