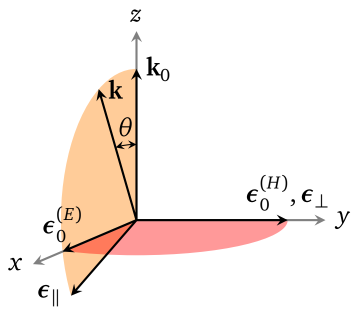

1. **General setup**

   The relevant vectors are shown in the diagram above, where the initial polarization $\boldsymbol{\epsilon}_0$ is either $\boldsymbol{\epsilon}_0^{(E)} = \hat{\mathbf{x}}$ for E plane or $\boldsymbol{\epsilon}_0^{(H)} = \hat{\mathbf{y}}$ for H plane. The outgoing polarization $\boldsymbol{\epsilon}$ is spanned by

$$\boldsymbol{\epsilon}_\parallel = \cos\theta\,\hat{\mathbf{x}} - \sin\theta\,\hat{\mathbf{z}} \qquad\qquad \boldsymbol{\epsilon}_\perp = \hat{\mathbf{y}} \tag{1}$$

   The total differential scattering cross section is obtained by

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_\parallel}{d\Omega} + \frac{d\sigma_\perp}{\Omega} = \frac{r^2\dfrac{1}{2Z_0}\left(\left|\boldsymbol{\epsilon}_\parallel^* \cdot \mathbf{E}_{\text{sc}}\right|^2 + \left|\boldsymbol{\epsilon}_\perp^* \cdot \mathbf{E}_{\text{sc}}\right|^2\right)}{\dfrac{1}{2Z_0}\left|\boldsymbol{\epsilon}_0^* \cdot \mathbf{E}_{\text{inc}}\right|} = \frac{\left|\boldsymbol{\epsilon}_\parallel^* \cdot \mathbf{F}_{\text{sc}}\right|^2 + \left|\boldsymbol{\epsilon}_\perp^* \cdot \mathbf{F}_{\text{sc}}\right|^2}{\left|\boldsymbol{\epsilon}_0^* \cdot \mathbf{E}_{\text{inc}}\right|^2} \tag{2}$$

   For plane incident wave $\mathbf{E}_{\text{inc}} = \boldsymbol{\epsilon}_0 E_0 e^{i\mathbf{k}_0\cdot\mathbf{x}}$, the above becomes

$$\frac{d\sigma}{d\Omega} = \frac{1}{E_0^2}\left(\left|\boldsymbol{\epsilon}_\parallel^* \cdot \mathbf{F}_{\text{sc}}\right|^2 + \left|\boldsymbol{\epsilon}_\perp^* \cdot \mathbf{F}_{\text{sc}}\right|^2\right) \tag{3}$$

   With $ka \gg 1$, we can use (10.127) and (10.132) to find $\boldsymbol{\epsilon}^* \cdot \mathbf{F}_{\text{sc}}$ (where $\boldsymbol{\epsilon}$ is either $\boldsymbol{\epsilon}_\parallel$ or $\boldsymbol{\epsilon}_\perp$)

$$\begin{aligned}\boldsymbol{\epsilon}^* \cdot \mathbf{F}_{\text{sc}} &= \boldsymbol{\epsilon}^* \cdot \mathbf{F}_{\text{sh}} + \boldsymbol{\epsilon}^* \cdot \mathbf{F}_{\text{ill}} \\ &= ika^2 E_0 \left(\boldsymbol{\epsilon}^* \cdot \boldsymbol{\epsilon}_0\right) \frac{J_1(ka\sin\theta)}{ka\sin\theta} + E_0 \frac{a}{2} e^{-2ika\sin(\theta/2)} \left(\boldsymbol{\epsilon}^* \cdot \boldsymbol{\epsilon}_r\right)\end{aligned} \tag{4}$$

   where

$$\boldsymbol{\epsilon}_r = -\boldsymbol{\epsilon}_0 + 2\left(\mathbf{n}_r \cdot \boldsymbol{\epsilon}_0\right)\mathbf{n}_r \tag{5}$$

$$\mathbf{n}_r = \frac{\mathbf{k} - \mathbf{k}_0}{|\mathbf{k} - \mathbf{k}_0|} = \frac{\sin\theta\,\hat{\mathbf{x}} - (1 - \cos\theta)\,\hat{\mathbf{z}}}{2\sin(\theta/2)} = \cos\frac{\theta}{2}\hat{\mathbf{x}} - \sin\frac{\theta}{2}\hat{\mathbf{z}} \tag{6}$$

2. ***E* plane**

   For this part, $\boldsymbol{\epsilon}_0 = \boldsymbol{\epsilon}_0^{(E)} = \hat{\mathbf{x}}$, so

$$\boldsymbol{\epsilon}_r = -\hat{\mathbf{x}} + 2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}\hat{\mathbf{x}} - \sin\frac{\theta}{2}\hat{\mathbf{z}}\right) = \cos\theta\,\hat{\mathbf{x}} - \sin\theta\,\hat{\mathbf{z}} \tag{7}$$

   we see from (4) that $\boldsymbol{\epsilon}_\perp^* \cdot \mathbf{F}_{\text{sc}} = 0$, so the total scattering cross section is

$$\begin{aligned}\frac{d\sigma}{d\Omega} = \frac{d\sigma_\parallel}{d\Omega} &= \left| ika^2\cos\theta \frac{J_1(ka\sin\theta)}{ka\sin\theta} + \frac{a}{2} e^{-2ika\sin(\theta/2)} \right|^2 \\ &= \left| \frac{a}{2}\cos\left(2ka\sin\frac{\theta}{2}\right) + i\left[a\cot\theta J_1(ka\sin\theta) - \frac{a}{2}\sin\left(2ka\sin\frac{\theta}{2}\right)\right]\right|^2 \\ &= \frac{a^2}{4}\left[4\cot^2\theta J_1^2(ka\sin\theta) + 1 - 4\cot\theta J_1(ka\sin\theta)\sin\left(2ka\sin\frac{\theta}{2}\right)\right]\end{aligned} \tag{8}$$

3. H **plane**

In this case $\boldsymbol{\epsilon}_0 = \boldsymbol{\epsilon}_0^{(H)} = \hat{\mathbf{y}}$, giving $\boldsymbol{\epsilon}_r = -\hat{\mathbf{y}}$. Thus $\boldsymbol{\epsilon}_\parallel$ has no contribution in the total differential cross section (4), and

$$\begin{aligned}
\frac{d\sigma}{d\Omega} = \frac{d\sigma_\perp}{d\Omega} &= \left| ika^2 \frac{J_1(ka\sin\theta)}{ka\sin\theta} - \frac{a}{2} e^{-2ika\sin(\theta/2)} \right|^2 \\
&= \left| -\frac{a}{2}\cos\left(2ka\sin\frac{\theta}{2}\right) + i\left[a\csc\theta J_1(ka\sin\theta) + \frac{a}{2}\sin\left(2ka\sin\frac{\theta}{2}\right)\right]\right|^2 \\
&= \frac{a^2}{4}\left[4\csc^2\theta J_1^2(ka\sin\theta) + 1 + 4\csc\theta J_1(ka\sin\theta)\sin\left(2ka\sin\frac{\theta}{2}\right)\right] \qquad (9)
\end{aligned}$$