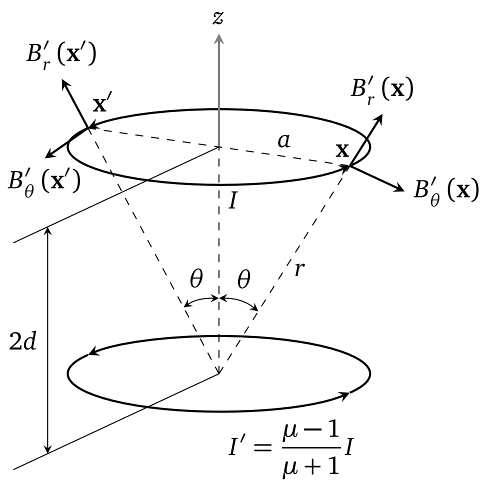
1. From problem 5.17, for the vacuum region, the magnetic field will be the same as replacing the medium with a circular loop of current

$$I' = \frac{\mu - 1}{\mu + 1} I \tag{1}$$

symmetrically located on the other side of the slab.



Now take the center of the image current loop as the origin, by equation (5.48), (5.49), the magnetic induction flux generated by the image current I' has components

$$B_r' = \frac{\mu_0 I' a}{2r} \sum_{n=0}^{\infty} \frac{(-1)^n (2n+1)!!}{2^n n!} \frac{a^{2n+1}}{r^{2n+2}} P_{2n+1}(\cos\theta) \tag{2}$$

$$B_\theta' = -\frac{\mu_0 I' a^2}{4} \sum_{n=0}^{\infty} \frac{(-1)^n (2n+1)!!}{2^n (n+1)!} \frac{1}{r^3} \frac{a^{2n}}{r^{2n}} P_{2n+1}^1(\cos\theta) \tag{3}$$

For purpose of calculating the net force on the original loop, it's sufficient to consider the force excerted from the image current I' on I (since the net self force from I will be zero).

Now consider a pair of antipodal points $\mathbf{x}$ and $\mathbf{x}'$ on the original circular loop. The differential current $Id\mathbf{l}$ at $\mathbf{x}$ and $Id\mathbf{l}'$ at $\mathbf{x}'$ are equal and opposite, but by symmetry $B_z'(\mathbf{x}) = B_z'(\mathbf{x}')$, so the contribution to the net force from $B_z'(\mathbf{x})$ and $B_z'(\mathbf{x}')$ cancel each other. However, the x-y projections $B_\parallel'(\mathbf{x})$ and $B_\parallel'(\mathbf{x}')$ are equal but opposite, so they produce the same contribution along the z-direction at $\mathbf{x}$ and $\mathbf{x}'$.

It's not hard to see that

$$\begin{aligned}
& dF_z = -Idl\left(B_r'\sin\theta + B_\theta'\cos\theta\right) \\
\Longrightarrow \quad & \mathbf{F} = -2\pi I a \left(B_r'\sin\theta + B_\theta'\cos\theta\right)\hat{\mathbf{z}} \\
& = -2\pi I a \left(\frac{\mu_0 I' a}{2}\right) \sum_{n=0}^{\infty} \frac{(-1)^n (2n+1)!!}{2^n n!} \frac{a^{2n+1}}{r^{2n+3}} \left[P_{2n+1}(\cos\theta)\sin\theta - \frac{1}{2(n+1)} P_{2n+1}^1(\cos\theta)\cos\theta\right]\hat{\mathbf{z}} \\
& = -\left(\frac{\mu-1}{\mu+1}\right)\pi\mu_0 I^2 a^2 \sum_{n=0}^{\infty} \frac{(-1)^n (2n+1)!!}{2^n n!} \frac{a^{2n+1}}{r^{2n+3}} \sin\theta \left[P_{2n+1}(\cos\theta) + \frac{P_{2n+1}'(\cos\theta)\cos\theta}{2n+2}\right]\hat{\mathbf{z}}
\end{aligned} \tag{4}$$

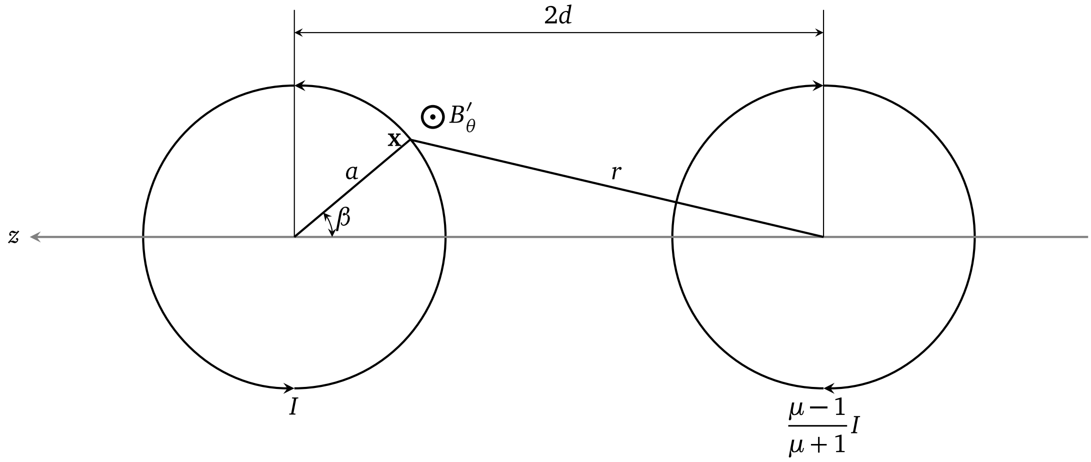
Recall the recurrence relation of Legendre polynomials, see wikipedia

$$P_{l+1}'(x) = (l+1)P_l(x) + xP_l'(x) \tag{5}$$

(3) can be simplified further as

$$
\begin{aligned}
\mathbf{F} &= -\left(\frac{\mu-1}{\mu+1}\right)\pi\mu_0 I^2 a^2 \sum_{n=0}^{\infty} \frac{(-1)^n\,(2n+1)!!}{2^n n!}\,\frac{a^{2n+1}}{r^{2n+3}}\sin\theta\,\frac{P'_{2n+2}(\cos\theta)}{2\,(n+1)}\,\hat{\mathbf{z}} \\
&= \left(\frac{\mu-1}{\mu+1}\right)\pi\mu_0 I^2 \sum_{n=0}^{\infty} \frac{(-1)^n\,(2n+1)!!}{2^{n+1}\,(n+1)!}\left(\frac{a}{r}\right)^{2n+3} P^1_{2n+2}(\cos\theta)\,\hat{\mathbf{z}} \qquad \left(\text{recall } a/r = \sin\theta,\ r = \sqrt{4d^2+a^2}\right) \\
&= \left(\frac{\mu-1}{\mu+1}\right)\pi\mu_0 I^2 \sum_{n=0}^{\infty} \frac{(-1)^n\,(2n+1)!!}{2^{n+1}\,(n+1)!}\sin^{2n+3}\theta\, P^1_{2n+2}(\cos\theta)\,\hat{\mathbf{z}}
\end{aligned}
\tag{6}
$$

2. For this setup, the image current is as depicted below.



At point $\mathbf{x}$ on the original loop parameterized with angle β, the induction flux generated by the image current I' has B'_θ coming out of the paper, with magnitude

$$
B'_\theta = -\frac{\mu_0 I' a^2}{4}\sum_{n=0}^{\infty}\frac{(-1)^n\,(2n+1)!!}{2^n\,(n+1)!}\,\frac{1}{r^3}\,\frac{a^{2n}}{r^{2n}}P^1_{2n+1}(0) \qquad\qquad B'_r = 0 \tag{7}
$$

By symmetry, only the force along the z-direction will accummulate, which can be obtained by

$$
\begin{aligned}
\mathbf{F} &= -\hat{\mathbf{z}}\int_0^{2\pi} I a B'_\theta \cos\beta\, d\beta \\
&= \hat{\mathbf{z}}\left(\frac{\mu-1}{\mu+1}\right)\frac{\mu_0 I^2}{4}\sum_{n=0}^{\infty}\frac{(-1)^n\,(2n+1)!!}{2^n\,(n+1)!}P^1_{2n+1}(0)\int_0^{2\pi}\frac{a^{2n+3}}{r^{2n+3}}\cos\beta\, d\beta
\end{aligned}
\tag{8}
$$

By (5.45)

$$
P^1_{2n+1}(0) = \frac{(-1)^{n+1}\,\Gamma(n+3/2)}{\Gamma(n+1)\,\Gamma(3/2)} = \frac{(-1)^{n+1}\,\dfrac{(2n+1)!!}{2^{n+1}}\sqrt{\pi}}{n!\,\dfrac{\sqrt{\pi}}{2}} = \frac{(-1)^{n+1}\,(2n+1)!!}{2^n n!} \tag{9}
$$

This turns (8) into

$$
\mathbf{F} = -\hat{\mathbf{z}}\left(\frac{\mu-1}{\mu+1}\right)\frac{\mu_0 I^2}{4}\sum_{n=0}^{\infty}\frac{[(2n+1)!!]^2}{2^{2n} n!\,(n+1)!}\int_0^{2\pi}\frac{a^{2n+3}}{r^{2n+3}}\cos\beta\, d\beta \qquad \text{where } r = \sqrt{a^2+4d^2-4ad\cos\beta} \tag{10}
$$

This integral does not have a closed form.

3. When $d \gg a$, for case (a), the 0th order will dominate in (6), where

$$
\sin^3\theta \approx \left(\frac{a}{2d}\right)^3 \qquad\qquad P^1_2(\cos\theta) = -3\cos\theta\sin\theta \approx -\frac{3a}{2d} \tag{11}
$$

which gives

$$\begin{aligned}
\mathbf{F} &\approx \left(\frac{\mu-1}{\mu+1}\right)\pi\mu_0 I^2\cdot\frac{1}{2}\left(\frac{a}{2d}\right)^3\left(-\frac{3a}{2d}\right)\hat{\mathbf{z}} \\
&= -\frac{3\pi\mu_0 I^2}{32}\left(\frac{\mu-1}{\mu+1}\right)\left(\frac{a}{d}\right)^4\hat{\mathbf{z}} \qquad (12)
\end{aligned}$$

This can be understood as the force between two point magnetic dipoles, where the image dipole

$$\mathbf{m}' = \left(\frac{\mu-1}{\mu+1}\right)\pi I a^2\hat{\mathbf{z}} \qquad (13)$$

generates a induction flux at $\mathbf{x}$:

$$\mathbf{B}' = \frac{\mu_0}{4\pi}\left[\frac{3\mathbf{n}(\mathbf{n}\cdot\mathbf{m}')-\mathbf{m}'}{|\mathbf{x}|^3}\right] \qquad (14)$$

When the original dipole

$$\mathbf{m} = \pi I a^2\hat{\mathbf{z}} \qquad (15)$$

is placed at $\mathbf{x} = x\hat{\mathbf{z}}$, the force on $\mathbf{m}$ is given by (5.69)

$$\begin{aligned}
\mathbf{F} = \boldsymbol{\nabla}\left(\mathbf{m}\cdot\mathbf{B}'\right) &= \frac{\mu_0}{4\pi}\boldsymbol{\nabla}\left\{\mathbf{m}\cdot\left[\frac{3\hat{\mathbf{z}}(\hat{\mathbf{z}}\cdot\mathbf{m}')-\mathbf{m}'}{x^3}\right]\right\} \\
&= \frac{\mu_0}{4\pi}\boldsymbol{\nabla}\left(\frac{2|\mathbf{m}||\mathbf{m}'|}{x^3}\right) \\
&= \frac{\mu_0}{4\pi}\cdot 2\left(\frac{\mu-1}{\mu+1}\right)\pi^2 I^2 a^4\boldsymbol{\nabla}\left(\frac{1}{x^3}\right) \\
&= -\frac{3\mu_0\pi I^2 a^4}{2x^4}\left(\frac{\mu-1}{\mu+1}\right)\hat{\mathbf{z}} \qquad (16)
\end{aligned}$$

Plugging in $x = 2d$ gives exactly (12).

For case (b), the 0th order of (10) is

$$\begin{aligned}
\mathbf{F} &\approx -\hat{\mathbf{z}}\left(\frac{\mu-1}{\mu+1}\right)\frac{\mu_0 I^2}{4}\int_0^{2\pi}\frac{a^3}{\sqrt{a^2+4d^2-4ad\cos\beta}^3}\cos\beta\, d\beta \\
&= -\hat{\mathbf{z}}\left(\frac{\mu-1}{\mu+1}\right)\frac{\mu_0 I^2}{4}\int_0^{2\pi}\left(\frac{a}{2d}\right)^3\left[1+\left(\frac{a}{2d}\right)^2-\left(\frac{a}{d}\right)\cos\beta\right]^{-3/2}\cos\beta\, d\beta \\
&\approx -\hat{\mathbf{z}}\left(\frac{\mu-1}{\mu+1}\right)\frac{\mu_0 I^2}{32}\left(\frac{a}{d}\right)^3\int_0^{2\pi}\left(1+\frac{3a}{2d}\cos\beta\right)\cos\beta\, d\beta \\
&= -\hat{\mathbf{z}}\left(\frac{\mu-1}{\mu+1}\right)\frac{\mu_0 I^2}{32}\left(\frac{a}{d}\right)^3\left(\frac{3a}{2d}\right)\pi \\
&= -\frac{3\pi\mu_0 I^2}{64}\left(\frac{\mu-1}{\mu+1}\right)\left(\frac{a}{d}\right)^4\hat{\mathbf{z}} \qquad (17)
\end{aligned}$$

This can also be understood as force between the two dipoles. This time $\mathbf{n}\cdot\mathbf{m}' = 0$ in (14), hence

$$\begin{aligned}
\mathbf{F} = \boldsymbol{\nabla}\left(\mathbf{m}\cdot\mathbf{B}'\right) &= \frac{\mu_0}{4\pi}\boldsymbol{\nabla}\left(\frac{-\mathbf{m}\cdot\mathbf{m}'}{x^3}\right) \qquad \text{(note that } \mathbf{m} \text{ and } \mathbf{m}' \text{ are antiparallel)} \\
&= \frac{\mu_0}{4\pi}\left(\frac{\mu-1}{\mu+1}\right)\pi^2 I^2 a^4\boldsymbol{\nabla}\left(\frac{1}{x^3}\right) \\
&= -\frac{3\mu_0\pi I^2 a^4}{4x^4}\left(\frac{\mu-1}{\mu+1}\right)\hat{\mathbf{z}} \qquad (18)
\end{aligned}$$

Pluggin in $x = 2d$ gives exactly (16).