

1. Let \mathbf{x}_a represent the center of the atom, and let q_j represent the j th charge of the atom, whose relative position to the center is \mathbf{x}_{ja} . This gives the charge density function

$$\rho(\mathbf{x}) = \sum_j q_j \delta(\mathbf{x} - \mathbf{x}_a - \mathbf{x}_{ja}) \quad (1)$$

Thus the electric force

$$\begin{aligned} \mathbf{F}_e &= \int \rho(\mathbf{x}) \mathbf{E}(\mathbf{x}) d^3x \\ &= \int \sum_j q_j \delta(\mathbf{x} - \mathbf{x}_a - \mathbf{x}_{ja}) \mathbf{E}(\mathbf{x}) d^3x \\ &= \sum_j q_j \mathbf{E}(\mathbf{x}_a + \mathbf{x}_{ja}) \\ &\approx \sum_j q_j \left[\mathbf{E}(\mathbf{x}_a) + \sum_\alpha \hat{\mathbf{e}}_\alpha \sum_\beta \frac{\partial E_\alpha}{\partial x_\beta} \Big|_{\mathbf{x}_a} (x_{ja})_\beta \right] \quad \text{recall atom is neutral} \\ &= \sum_\alpha \hat{\mathbf{e}}_\alpha \sum_\beta \frac{\partial E_\alpha}{\partial x_\beta} \Big|_{\mathbf{x}_a} \underbrace{\sum_j q_j (x_{ja})_\beta}_{d_\beta} = [(\mathbf{d} \cdot \nabla) \mathbf{E}](\mathbf{x}_a) \end{aligned} \quad (2)$$

And the magnetic force

$$\begin{aligned} \mathbf{F}_m &= \int \mathbf{J}(\mathbf{x}) \times \mathbf{B}(\mathbf{x}) d^3x \\ &= \int \sum_j q_j \delta(\mathbf{x} - \mathbf{x}_a - \mathbf{x}_{ja}) (\mathbf{v}_a + \mathbf{v}_{ja}) \times \mathbf{B}(\mathbf{x}) d^3x \\ &= \sum_j q_j (\mathbf{v}_a + \mathbf{v}_{ja}) \times \mathbf{B}(\mathbf{x}_a + \mathbf{x}_{ja}) \\ &\approx \sum_j q_j (\mathbf{v}_a + \mathbf{v}_{ja}) \times \left[\mathbf{B}(\mathbf{x}_a) + \sum_\alpha \hat{\mathbf{e}}_\alpha \sum_\beta \frac{\partial B_\alpha}{\partial x_\beta} \Big|_{\mathbf{x}_a} (x_{ja})_\beta \right] \\ &= \sum_j q_j \mathbf{v}_{ja} \times \mathbf{B}(\mathbf{x}_a) + O(v_{ja} x_{ja}) \\ &\approx \dot{\mathbf{d}} \times \mathbf{B}(\mathbf{x}_a) \end{aligned} \quad (3)$$

where we ignored the contribution from higher order multipoles.

2. Assume the medium is linear dielectric, i.e.,

$$\mathbf{d} = (\epsilon - \epsilon_0) \mathbf{E} \quad (4)$$

If our field is plane wave, let \mathbf{E} be in the \mathbf{x} direction, and \mathbf{B} be in the \mathbf{y} direction, so the wave travels to the \mathbf{z} direction. Then the electric force

$$\mathbf{F}_e = (\mathbf{d} \cdot \nabla) \mathbf{E} = (\epsilon - \epsilon_0) (\mathbf{E} \cdot \nabla) \mathbf{E} = 0 \quad (5)$$

because the field \mathbf{E} has no variance in \mathbf{x} direction.

On the other hand

$$\mathbf{F}_m = (\epsilon - \epsilon_0) \frac{\partial \mathbf{E}}{\partial t} \times \mathbf{B} = (\epsilon - \epsilon_0) \mu_0 \frac{1}{2} \frac{\partial}{\partial t} (\mathbf{E} \times \mathbf{H}) = \frac{(\epsilon \mu_0 - \epsilon_0 \mu_0) c^2}{2} \frac{\partial}{\partial t} \left(\frac{\mathbf{E} \times \mathbf{H}}{c^2} \right) = \left(\frac{n^2 - 1}{2} \right) \frac{\partial \mathbf{g}_{\text{em}}}{\partial t} \quad (6)$$