1. Here we can use the result developed in the solution of problem 7.4. Recall in the setting of problem 7.4, the metal medium has a complex permittivity

$$\widetilde{\epsilon'} = \epsilon' + \frac{i\sigma}{\omega} \tag{1}$$

and we have calculated the real and imaginary part of the transmitted wave vector for normal incidence

$$k_R'^2 = \frac{k^2}{2}\left(\frac{\epsilon'}{\epsilon_0}\right)\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon'}\right)^2}+1\right] = \frac{k^2}{2}\left(\frac{\left|\widetilde{\epsilon'}\right|}{\epsilon_0}+\frac{\epsilon'}{\epsilon_0}\right) \tag{2}$$

$$k_I'^2 = \frac{k^2}{2}\left(\frac{\left|\widetilde{\epsilon'}\right|}{\epsilon_0}-\frac{\epsilon'}{\epsilon_0}\right) \tag{3}$$

On the other hand, if we separate the complex index of refraction into real and imaginary parts,

$$\widetilde{n'} = n_R' + i n_I' \tag{4}$$

we must have

$$n_R'^2 - n_I'^2 = \mathrm{Re}\,\widetilde{n'}^2 = \frac{\epsilon'}{\epsilon_0} \qquad n_R'^2 + n_I'^2 = \left|\widetilde{n'}\right|^2 = \frac{\left|\widetilde{\epsilon'}\right|}{\epsilon_0} \tag{5}$$

Thus we can write k_R', k_I' in terms of n_R', n_I'

$$k_R' = k n_R' \qquad k_I' = k n_I' \tag{6}$$

By problem 7.4, the reflected wave's amplitude is

$$\frac{E_0''}{E_0} = \frac{1-k_z'/k}{1+k_z'/k} \tag{7}$$

where $k_z' = k_R' + i k_I'$. This gives

$$R = \frac{\left|E_0''\right|^2}{\left|E_0\right|^2} = \left|\frac{1-\widetilde{n'}}{1+\widetilde{n'}}\right|^2 \tag{8}$$

and hence

$$T = 1 - R = \frac{4n_R'}{\left|1+\widetilde{n'}\right|^2} \tag{9}$$

2. Let $\hat{\mathbf{z}}$ be the direction of the incident and transmitted wave vector $\mathbf{k}, \mathbf{k}'$, and let the boundary be the $z = 0$ plane. Without loss of generality, let $\mathbf{E}_0 = \hat{\mathbf{y}} E_0$.
In the half space $z < 0$, we have

$$\mathbf{E}\cdot\mathbf{D}^* = \epsilon_0\left|E_0\right|^2 \qquad \mathbf{B}\cdot\mathbf{H}^* = \left(\frac{\mathbf{k}\times\mathbf{E}_0}{\omega}\right)\cdot\left(\frac{\mathbf{k}\times\mathbf{E}_0}{\omega\mu}\right)^* = \frac{k^2}{\omega^2\mu}\left|E_0\right|^2 = \epsilon_0\left|E_0\right|^2 \tag{10}$$

which means the rate of change per volume is zero for $z < 0$.
On the other hand, for $z > 0$

$$\mathbf{E}\cdot\mathbf{D}^* = \left(\hat{\mathbf{y}}E_0' e^{-k_I' z} e^{i k_R' x}\right)\cdot\left(\widetilde{\epsilon'}\hat{\mathbf{y}}E_0' e^{-k_I' z} e^{i k_R' x}\right)^* = \widetilde{\epsilon'}^*\left|E_0'\right|^2 e^{-2k_I' z} \tag{11}$$

$$\mathbf{B}\cdot\mathbf{H}^* = \left(\frac{\mathbf{k}'\times\mathbf{E}_0'}{\omega}\right)\cdot\left(\frac{\mathbf{k}'\times\mathbf{E}_0'}{\omega\mu}\right)^* = \frac{k_z' k_z'^*}{\omega^2\mu}\left|E_0'\right|^2 e^{-2k_I' z} \tag{12}$$

When we calculate the rate of rate for the energy density

$$\frac{\partial u(z)}{\partial t} = \mathrm{Re}\left[i\omega\left(\frac{\mathbf{E}\cdot\mathbf{D}^* - \mathbf{B}\cdot\mathbf{H}^*}{2}\right)\right] \tag{13}$$

the magnetic term (12) is real hence has no contribution, which leaves

$$\frac{\partial u(z)}{\partial t}=\frac{\omega}{2}\left|E_0'\right|^2 e^{-2k_I'z}\cdot\operatorname{Im}\widetilde{\epsilon'} \tag{14}$$

Integrating (14) from $z\to\infty$ gives the total power above the unit area:

$$P'=\int_0^z\frac{\partial u(z)}{\partial t}dz=\frac{\omega}{4k_I'}\left|E_0'\right|^2\operatorname{Im}\widetilde{\epsilon'}=\frac{\omega}{4kn_I'}\left|E_0'\right|^2\left(2\epsilon_0 n_R'n_I'\right)=\frac{\sqrt{\epsilon_0/\mu_0}n_R'\left|E_0'\right|^2}{2} \tag{15}$$

The incident energy flux is

$$\operatorname{Re}\left(\frac{\mathbf{E}\times\mathbf{H}^*}{2}\right)=\frac{\sqrt{\epsilon_0/\mu_0}\left|E_0\right|^2}{2}\hat{\mathbf{z}} \tag{16}$$

which gives the ratio of transmitted power to the influx power

$$T=n_R'\frac{\left|E_0'\right|^2}{\left|E_0\right|^2} \tag{17}$$

From solution to problem 7.4

$$\frac{\left|E_0'\right|^2}{\left|E_0\right|^2}=\frac{4}{\left|1+\widetilde{n'}\right|^2} \tag{18}$$

which shows that (17) recovers the earlier calculation (9).

3. In the limit of large σ, problem 7.5 has shown that

$$k_R'=k_I'\approx\frac{1}{\delta}\qquad\Longrightarrow\qquad n_R'=n_I'\approx\frac{1}{k\delta} \tag{19}$$

Then by (14) the rate of density change

$$\frac{\partial u(z)}{\partial t}=\frac{\omega}{2}\left|E_0'\right|^2 e^{-2k_I'z}\left(2\epsilon_0 n_R'n_I'\right)=\left(\frac{1}{\omega\mu_0\delta^2}\right)\left|E_0'\right|^2 e^{-2k_I'z} \tag{20}$$

The dependency of T on δ has been calculated in problem 7.4 part (b), i.e., $T\approx 2\omega\delta/c$. Also by (14)

$$\frac{\partial u(z)}{\partial t}=\frac{\omega}{2}\left|E_0'\right|^2 e^{-2k_I'z}\cdot\operatorname{Im}\widetilde{\epsilon'}=\frac{\sigma}{2}\left|E_0'\right|^2 e^{-2k_I'z}=\frac{1}{2}\mathbf{J}(z)\cdot\mathbf{E}'^*(z) \tag{21}$$

This indicates that all the transmitted energy is converted into heat.