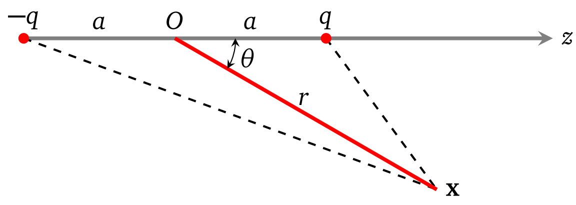
1. Refer to the diagram below,



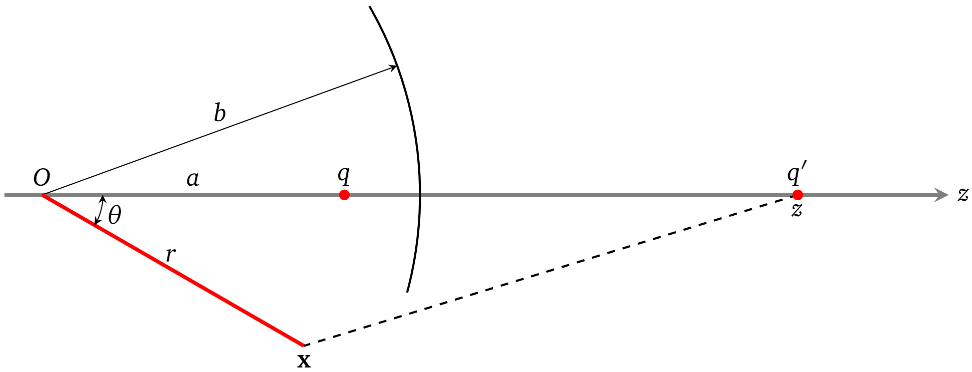
the potential at point $\mathbf{x}$ is easily seen to be

$$\begin{aligned}\Phi(\mathbf{x}) &= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{r^2+a^2-2ar\cos\theta}} - \frac{1}{\sqrt{r^2+a^2+2ar\cos\theta}}\right) \\ &= \frac{q}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\left(\frac{r_<^l}{r_>^{l+1}}\right)[P_l(\cos\theta)-P_l(-\cos\theta)] && \text{(even } l\text{s drop out)} \\ &= \frac{q}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{r_<^{2l+1}}{r_>^{2l+2}}\cdot 2P_{2l+1}(\cos\theta) && (1)\end{aligned}$$

With $a \to 0$, we have $r_> = r, r_< = a$, which gives

$$\begin{aligned}\Phi(\mathbf{x}) &= \frac{q}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{a^{2l+1}}{r^{2l+2}}\cdot 2P_{2l+1}(\cos\theta) \\ &= \frac{2qa}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{a^{2l}}{r^{2l+2}}P_{2l+1}(\cos\theta) && \text{(only } l=0 \text{ survives with } a\to 0) \\ &\longrightarrow \frac{p}{4\pi\epsilon_0}\frac{1}{r^2}\cos\theta && (2)\end{aligned}$$

2. When we enclose $\pm q$ with a grounded sphere with radius b, the interior potential can be obtained using method of images.



The quantity and location of the image charge of q are

$$q' = -\frac{qb}{a} \qquad\qquad z = \frac{b^2}{a} \tag{3}$$

Thus the potential contribution from this image charge at point $\mathbf{x}$ is

$$\Phi_{q'}(\mathbf{x}) = \frac{q'}{4\pi\epsilon_0}\frac{1}{\sqrt{r^2+z^2-2rz\cos\theta}} \tag{4}$$

For image charges of both $\pm q$, the potential will thus be

$$\Phi_{\pm q'}(\mathbf{x}) = \frac{q'}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{r^2+z^2-2rz\cos\theta}} - \frac{1}{\sqrt{r^2+z^2+2rz\cos\theta}}\right) \tag{5}$$

As $a \to 0$, the image charges tend to infinity, hence (5) will expand into

$$
\begin{aligned}
\Phi_{\pm q'}(\mathbf{x}) &= \frac{q'}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{r^l}{z^{l+1}}\left[P_l(\cos\theta) - P_l(-\cos\theta)\right] \\
&= \frac{-qb/a}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{r^{2l+1}}{(b^2/a)^{2l+2}}\cdot 2P_{2l+1}(\cos\theta) \\
&= -\frac{2qa}{4\pi\epsilon_0}\sum_{l=0}^{\infty}\frac{r^{2l+1}a^{2l}}{b^{4l+3}}P_{2l+1}(\cos\theta) \qquad \text{(only } l=0 \text{ survives with } a\to 0\text{)} \\
&\longrightarrow -\frac{p}{4\pi\epsilon_0}\frac{r}{b^3}\cos\theta
\end{aligned}
\tag{6}
$$

In combination with (2), the potential at any interior point is

$$
\Phi(\mathbf{x}) = \frac{p}{4\pi\epsilon_0}\left(\frac{1}{r^2} - \frac{r}{b^3}\right)\cos\theta \tag{7}
$$