1. In cylindrical coordinate representation (r, ϕ, z) (we use r here since ρ is already used for charge density), the charge density and current density is

$$\rho(\mathbf{x}, t) = q\frac{\delta(r-R)}{r}\delta(z)\delta(\phi-\omega_0 t) \tag{1}$$

$$\mathbf{J}(\mathbf{x}, t) = \rho(\mathbf{x}, t)\mathbf{v}(\mathbf{x}) = \rho(\mathbf{x}, t)R\omega_0(-\sin\phi\hat{\mathbf{x}}+\cos\phi\hat{\mathbf{y}}) = q\omega_0\delta(r-R)\delta(z)\delta(\phi-\omega_0 t)(-\sin\phi\hat{\mathbf{x}}+\cos\phi\hat{\mathbf{y}}) \tag{2}$$

Its n-th Fourier component is

$$\begin{aligned}
\mathbf{J}_n(x) &= \frac{1}{T}\int_0^T \mathbf{J}(\mathbf{x}, t)e^{in\omega_0 t}dt \\
&= q\omega_0\delta(r-R)\delta(z)(-\sin\phi\hat{\mathbf{x}}+\cos\phi\hat{\mathbf{y}})\frac{1}{T}\int_0^T \delta(\phi-\omega_0 t)e^{in\omega_0 t}dt \\
&= q\omega_0\delta(r-R)\delta(z)(-\sin\phi\hat{\mathbf{x}}+\cos\phi\hat{\mathbf{y}})\cdot\frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0}\delta\left[\omega_0\left(\frac{\phi}{\omega_0}-t\right)\right]e^{in\omega_0 t}dt \\
&= q\omega_0\delta(r-R)\delta(z)(-\sin\phi\hat{\mathbf{x}}+\cos\phi\hat{\mathbf{y}})\cdot\frac{\omega_0}{2\pi}\frac{1}{\omega_0}e^{in\phi} \\
&= \frac{q\omega_0}{2\pi}\delta(r-R)\delta(z)e^{in\phi}(-\sin\phi\hat{\mathbf{x}}+\cos\phi\hat{\mathbf{y}})
\end{aligned} \tag{3}$$

Thus

$$\begin{aligned}
\mathcal{M}_n(\mathbf{x}) = \frac{\mathbf{x}\times\mathbf{J}_n(\mathbf{x})}{2} &= \frac{q\omega_0}{4\pi}\delta(r-R)\delta(z)e^{in\phi}(R\cos\phi\hat{\mathbf{x}}+R\sin\phi\hat{\mathbf{y}})\times(-\sin\phi\hat{\mathbf{x}}+\cos\phi\hat{\mathbf{y}}) \\
&= \frac{qR\omega_0}{4\pi}\delta(r-R)\delta(z)e^{in\phi}\hat{\mathbf{z}}
\end{aligned} \tag{4}$$

which gives an effective magnetic charge density

$$\rho_n^M(\mathbf{x}) = -\nabla\cdot\mathcal{M}_n(\mathbf{x}) = -\frac{qR\omega_0}{4\pi}\delta(r-R)\delta'(z)e^{in\phi} \tag{5}$$

2. The magnetic multipole moment due to the n-th harmonic $\mathcal{M}_n$is given by (9.172)

$$\begin{aligned}
M_{lm}^{(n)} &\propto \int r^l Y_{lm}^*\nabla\cdot\mathcal{M}_n d^3x \propto \int r^l Y_{lm}^*\delta(r-R)\delta'(z)e^{in\phi}r^2 dr d\Omega \\
&\propto \int_{-1}^1 P_l^m(\cos\theta)\delta'(R\cos\theta)d(\cos\theta)\int_0^{2\pi}e^{i(n-m)\phi}d\phi \propto \left.\frac{dP_l^m(x)}{dx}\right|_0\cdot\delta_{mn}
\end{aligned} \tag{6}$$

With the recurrence relation (See 14.10.E5 on DLMF)

$$\left(1-x^2\right)\frac{dP_l^m(x)}{dx} = (l+m)P_{l-1}^m(x) - lxP_l^m(x) \tag{7}$$

we have

$$\left.\frac{dP_l^m(x)}{dx}\right|_0 = (l+m)P_{l-1}^m(0) \tag{8}$$

which, due to the parity of $P_l^m(x)$, will vanish when $l-m$ is even. Thus for the n-th harmonic, the only non-vanishing magnetic multipole moments are those with $l \geq n$ and $l-n$ odd, the lowest of which is $l = n+1$.

3. For the four charges with alternate sign, we can modify in (4) the sign of q and add the corresponding initial offset to ϕ, which gives

$$\mathcal{M}_n(\mathbf{x}) = \hat{\mathbf{z}}\frac{qR\omega_0}{4\pi}\delta(r-R)\delta(z)\left[1+(-1)^n - i^n - (-i)^n\right] \tag{9}$$

which vanishes unless $n = 4k+2$, e.g., only the 2nd, 6th harmonic etc. will exist for magnetic multipole moments. For $n = 2$, the lowest order is $l = 3$. For the E2 radiation, the contribution from electric multipole moment is from $l = 2$, but the contribution from magnetic multipole moment is from $l = 3$. Thus the magnetic contribution is much weaker than the electric contribution.