1. Besides the current loop, the space inside the spherical cavity is current free, so it can be described by a scalar potential. By linearity, this scalar potential inside the cavity is the superposition of the scalar potential by the current loop Φ_{loop} and the scalar potential due to the presence of the iron material Φ_{iron}, where Φ_{iron} satisfies the Laplace equation in a cylindrically symmetric setup, therefore can be written as

$$\Phi_{\text{iron}} = \sum_{l=0}^{\infty} c_l r^l P_l(\cos\theta) \tag{1}$$

We won't need to come up with an expression for Φ_{loop}, since we have readily obtained the magnetic induction flux generated by the loop in equation (5.48) and (5.49), i.e.,

$$B_{r,\text{loop}}(r,\theta) = \frac{\mu_0 I a}{2r}\sum_{n=0}^{\infty}\frac{(-1)^n(2n+1)!!}{2^n n!}\frac{r_<^{2n+1}}{r_>^{2n+2}}P_{2n+1}(\cos\theta) \tag{2}$$

$$B_{\theta,\text{loop}}(r,\theta) = -\frac{\mu_0 I a^2}{4}\sum_{n=0}^{\infty}\frac{(-1)^n(2n+1)!!}{2^n(n+1)!}P^1_{2n+1}(\cos\theta)\begin{cases}-\left(\dfrac{2n+2}{2n+1}\right)\dfrac{1}{a^3}\left(\dfrac{r}{a}\right)^{2n} & \text{for } r<a\\ \dfrac{1}{r^3}\left(\dfrac{a}{r}\right)^{2n} & \text{for } r\geq a\end{cases} \tag{3}$$

We are going to determine the coefficients c_l by imposing the boundary condition. Recall the comment below equation (5.89), the magnetic field (hence magnetic induction flux) at $r\to b^-$ must have zero tangential component, i.e.,

$$B_\theta(b,\theta) = B_{\theta,\text{loop}}(b,\theta) + B_{\theta,\text{iron}}(b,\theta) = 0 \tag{4}$$

Setting $r=b$ in (3) yields

$$B_{\theta,\text{loop}}(b,\theta) = -\frac{\mu_0 I a^2}{4}\sum_{n=0}^{\infty}\frac{(-1)^n(2n+1)!!}{2^n(n+1)!}\frac{1}{b^3}\left(\frac{a}{b}\right)^{2n}P^1_{2n+1}(\cos\theta) \tag{5}$$

Yet by (1)

$$B_{\theta,\text{iron}}(b,\theta) = -\frac{1}{r}\frac{\partial\Phi_{\text{iron}}}{\partial\theta}\bigg|_{r=b} = -\sum_{l=1}^{\infty}c_l b^{l-1}\overbrace{P_l'(\cos\theta)(-\sin\theta)}^{P_l^1(\cos\theta)} \tag{6}$$

Due to the orthogonality of $P_l^m(x)$, for (4) to hold, we must have matching coefficients, i.e.,

$$c_{2n}=0 \qquad \text{and} \qquad c_{2n+1} = -\frac{\mu_0 I a^2}{4}\frac{(-1)^n(2n+1)!!}{2^n(n+1)!}\frac{1}{b^3}\left(\frac{a}{b}\right)^{2n}\frac{1}{b^{2n}} \tag{7}$$

With this, we can calculate $\mathbf{B}_{\text{iron}}$ everywhere inside the cavity:

$$\begin{aligned}B_{r,\text{iron}}(r,\theta) &= -\frac{\partial\Phi_{\text{iron}}}{\partial r} = -\sum_{n=0}^{\infty}(2n+1)c_{2n+1}r^{2n}P_{2n+1}(\cos\theta)\\ &= \frac{\mu_0 I a^2}{4}\sum_{n=0}^{\infty}\frac{(-1)^n(2n+1)(2n+1)!!}{2^n(n+1)!}\frac{1}{b^3}\left(\frac{a}{b}\right)^{2n}\left(\frac{r}{b}\right)^{2n}P_{2n+1}(\cos\theta)\end{aligned} \tag{8}$$

$$\begin{aligned}B_{\theta,\text{iron}}(r,\theta) &= -\frac{1}{r}\frac{\partial\Phi_{\text{iron}}}{\partial\theta} = -\sum_{n=0}^{\infty}c_{2n+1}r^{2n}P^1_{2n+1}(\cos\theta)\\ &= \frac{\mu_0 I a^2}{4}\sum_{n=0}^{\infty}\frac{(-1)^n(2n+1)!!}{2^n(n+1)!}\frac{1}{b^3}\left(\frac{a}{b}\right)^{2n}\left(\frac{r}{b}\right)^{2n}P^1_{2n+1}(\cos\theta)\end{aligned} \tag{9}$$

At $r\to 0$, the 0th order of (2), (3), (8), (9) will dominate, which are

$$B^{(0)}_{r,\text{loop}} = \frac{\mu_0 I}{2a}\cos\theta \qquad B^{(0)}_{r,\text{iron}} = \frac{\mu_0 I a^2}{4}\cdot\frac{1}{b^3}\cos\theta = \frac{a^3}{2b^3}B^{(0)}_{r,\text{loop}} \tag{10}$$

$$B^{(0)}_{\theta,\text{loop}} = \frac{\mu_0 I}{2a}(-\sin\theta) \qquad B^{(0)}_{\theta,\text{iron}} = \frac{\mu_0 I a^2}{4}\frac{1}{b^3}(-\sin\theta) = \frac{a^3}{2b^3}B^{(0)}_{\theta,\text{loop}} \tag{11}$$

After superposition, this gives an overall factor of $1+a^3/2b^3$ compared to situations without the iron.

2. Clearly from (8) and (9), if we consider only the 0th order, for all points inside the cavity ($r<b$), the iron's effect will be equivalent to a loop with radius $2b^3/a^2$ carrying the same current I. But for higher orders, we can not attribute the effect of iron to one single image current, since they have different coefficients according to (8) and (9).