From problem 8.2, we have determined the form of the fields in TEM mode,

$$\mathbf{E}(\mathbf{x}) = \frac{C\hat{\boldsymbol{\rho}}}{\rho} \qquad \mathbf{B} = \frac{\hat{\mathbf{z}}\times\mathbf{E}}{c} \tag{1}$$

For the voltage between the inner and outer conductors to be $V$, we must have

$$\int_a^b \frac{C}{\rho}d\rho = V \qquad \Longrightarrow \qquad C = \frac{V}{\ln\left(\frac{b}{a}\right)} \tag{2}$$

giving

$$\mathbf{E} = \frac{V}{\ln\left(\frac{b}{a}\right)}\frac{\hat{\boldsymbol{\rho}}}{\rho} \qquad c\mathbf{B} = \frac{V}{\ln\left(\frac{b}{a}\right)}\frac{\hat{\boldsymbol{\phi}}}{\rho} \tag{3}$$

We use Smythe-Kirchhoff approximation (10.109) to estimate the radiation-zone field on the other side of the infinite screen,

$$\mathbf{E}(\mathbf{x}) = \frac{ie^{ikr}}{2\pi r}\mathbf{k}\times\int_{\text{ring}}\hat{\mathbf{z}}\times\mathbf{E}\left(\mathbf{x}'\right)e^{-i\mathbf{k}\cdot\mathbf{x}'}da' \tag{4}$$

We can parameterize $\mathbf{k},\mathbf{x}'$ by the following

$$\mathbf{k} = k\left(\sin\beta\cos\gamma\hat{\mathbf{x}} + \sin\beta\sin\gamma\hat{\mathbf{y}} + \cos\beta\hat{\mathbf{z}}\right) \qquad \mathbf{x}' = \rho\cos\phi\hat{\mathbf{x}} + \rho\sin\phi\hat{\mathbf{y}} \tag{5}$$

then the integrand becomes

$$\begin{aligned}
\hat{\mathbf{z}}\times\mathbf{E}\left(\mathbf{x}'\right)e^{-i\mathbf{k}\cdot\mathbf{x}'} &= \frac{\hat{\boldsymbol{\phi}}}{\rho}\frac{V}{\ln\left(\frac{b}{a}\right)}e^{-ik\rho\sin\beta\cos(\phi-\gamma)} \\
&= \frac{V}{\ln\left(\frac{b}{a}\right)}\left(\frac{-\sin\phi\hat{\mathbf{x}} + \cos\phi\hat{\mathbf{y}}}{\rho}\right)e^{-ik\rho\sin\beta\cos(\phi-\gamma)}
\end{aligned} \tag{6}$$

We can use the integral representation of the Bessel function (see problem 3.16 (d))

$$J_m(x) = \frac{1}{2\pi i^m}\int_0^{2\pi} e^{ix\cos\alpha - im\alpha}d\alpha \tag{7}$$

and its parity property

$$J_m(-x) = J_{-m}(x) = (-1)^m J_m(x) \tag{8}$$

to get

$$\begin{aligned}
\int_0^{2\pi} e^{-ik\rho\sin\beta\cos(\phi-\gamma)}\cos\phi\, d\phi &= \int_0^{2\pi} e^{-ik\rho\sin\beta\cos(\phi-\gamma)}\left(\frac{e^{i\phi}+e^{-i\phi}}{2}\right)d\phi \qquad \text{let } \alpha = \phi-\gamma \\
&= \int_0^{2\pi} e^{-ik\rho\sin\beta\cos\alpha}\left[\frac{e^{i(\alpha+\gamma)}+e^{-i(\alpha+\gamma)}}{2}\right]d\alpha \\
&= \frac{1}{2}\left[-2\pi i e^{i\gamma}J_{-1}(-k\rho\sin\beta) + 2\pi i e^{-i\gamma}J_1(-k\rho\sin\beta)\right] \\
&= -2\pi i\cos\gamma J_1(k\rho\sin\beta)
\end{aligned} \tag{9}$$

$$\begin{aligned}
\int_0^{2\pi} e^{-ik\rho\sin\beta\cos(\phi-\gamma)}\sin\phi\, d\phi &= \int_0^{2\pi} e^{-ik\rho\sin\beta\cos(\phi-\gamma)}\left(\frac{e^{i\phi}-e^{-i\phi}}{2i}\right)d\phi \\
&= \int_0^{2\pi} e^{-ik\rho\sin\beta\cos\alpha}\left[\frac{e^{i(\alpha+\gamma)}-e^{-i(\alpha+\gamma)}}{2i}\right]d\alpha \\
&= \frac{1}{2i}\left[-2\pi i e^{i\gamma}J_{-1}(-k\rho\sin\beta) - 2\pi i e^{-i\gamma}J_1(-k\rho\sin\beta)\right] \\
&= -2\pi i\sin\gamma J_1(k\rho\sin\beta)
\end{aligned} \tag{10}$$

Thus (4) becomes

$$
\begin{aligned}
\mathbf{E}(\mathbf{x}) &= \frac{ie^{ikr}}{2\pi r} \cdot \frac{V}{\ln\left(\dfrac{b}{a}\right)} (-2\pi i) \overbrace{\mathbf{k} \times (-\sin\gamma \hat{\mathbf{x}} + \cos\gamma \hat{\mathbf{y}})}^{k\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}}=-k\hat{\boldsymbol{\theta}}} \int_a^b J_1(k\rho \sin\beta)\, d\rho \qquad \text{use } J_1(x) = -J_0'(x) \\
&= \frac{e^{ikr}}{r} \frac{V}{\ln\left(\dfrac{b}{a}\right)} \left[ \frac{J_0(kb\sin\beta) - J_0(ka\sin\beta)}{\sin\beta} \right] \hat{\boldsymbol{\theta}}
\end{aligned} \tag{11}
$$

The angular distribution of power is obtained by the usual routine

$$
\frac{dP}{d\Omega} = \frac{1}{2Z_0} |\mathbf{E}|^2 r^2 = \frac{V^2}{2Z_0 \ln^2\left(\dfrac{b}{a}\right)} \left[ \frac{J_0(kb\sin\beta) - J_0(ka\sin\beta)}{\sin\beta} \right]^2 \tag{12}
$$

and the total radiated power is obtained by integrating (12) over $\beta \in [0, \pi/2]$ and $\gamma \in [0, 2\pi]$, which does not seem to have closed forms,

$$
P = \int \frac{dP}{d\Omega} d\Omega = \frac{\pi V^2}{Z_0 \ln^2\left(\dfrac{b}{a}\right)} \int_0^{\pi/2} \frac{[J_0(kb\sin\beta) - J_0(ka\sin\beta)]^2}{\sin\beta} d\beta \tag{13}
$$