

1. When

$$\partial_\alpha M^{\alpha\beta\gamma} = 0 \quad (1)$$

over all space, we must have

$$0 = \int \partial_\alpha M^{\alpha\beta\gamma} d^3x = \partial_0 \int M^{0\beta\gamma} d^3x + \int \partial_i M^{i\beta\gamma} d^3x \quad (2)$$

where by Gauss theorem, the second term turns into a surface integral at infinity, which vanishes if the field is localized. This turns (2) into a conservation law

$$\frac{d}{dt} \int M^{0\beta\gamma} d^3x = 0 \quad (3)$$

When $\beta = i, \gamma = j$,

$$M^{0ij} = \Theta^{0i} x^j - \Theta^{0j} x^i = \frac{1}{4\pi} \epsilon^{ijk} [(\mathbf{E} \times \mathbf{B}) \times \mathbf{x}]_k \quad (4)$$

Thus (3) is recognized as the conservation of total angular momentum of the field.

2. When $\beta = 0$, we have

$$M^{00\gamma} = \Theta^{00} x^\gamma - \Theta^{0\gamma} x^0 = u x^\gamma - c g^i \cdot c t \quad (5)$$

(3) becomes

$$\frac{d}{dt} \int u \mathbf{x} d^3x = c^2 \frac{d}{dt} \int \mathbf{g} t d^3x \quad \Rightarrow \quad \frac{d\mathbf{X}}{dt} = \frac{c^2 \int \mathbf{g} d^3x}{\int u d^3x} = \frac{c^2 \mathbf{P}_{\text{em}}}{E_{\text{em}}} \quad (6)$$

which describes the center of mass motion of the electromagnetic field.