

We shall give a detailed derivation of (9.18).
From (9.16)

$$\mathbf{A}(\mathbf{x}) = -\frac{i\mu_0\omega}{4\pi} \mathbf{p} \frac{e^{ikr}}{r} \quad (1)$$

Then by (9.4)

$$\begin{aligned} \mathbf{H} &= \frac{1}{\mu_0} \nabla \times \mathbf{A} = -\frac{i\omega}{4\pi} \nabla \times \left(\mathbf{p} \frac{e^{ikr}}{r} \right) = -\frac{i\omega}{4\pi} \hat{\mathbf{e}}_k \epsilon_{ijk} \frac{\partial}{\partial x_i} \left(p_j \frac{e^{ikr}}{r} \right) \\ &= -\frac{i\omega}{4\pi} \hat{\mathbf{e}}_k \epsilon_{ijk} p_j \left(\frac{r \cdot ike^{ikr} \frac{x_i}{r} - e^{ikr} \frac{x_i}{r}}{r^2} \right) = -\frac{i\omega}{4\pi} \hat{\mathbf{e}}_k \epsilon_{ijk} n_i p_j \left(\frac{ik}{r} - \frac{1}{r^2} \right) e^{ikr} \\ &= -\frac{i\omega ik}{4\pi} (\mathbf{n} \times \mathbf{p}) \left(1 - \frac{1}{ikr} \right) \frac{e^{ikr}}{r} = \frac{ck^2}{4\pi} (\mathbf{n} \times \mathbf{p}) \frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr} \right) \end{aligned} \quad (2)$$

And by (9.5)

$$\begin{aligned} \mathbf{E} &= \frac{i}{k} \sqrt{\frac{\mu_0}{\epsilon_0}} \nabla \times \mathbf{H} = \frac{i}{k} \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \frac{k^2}{4\pi\sqrt{\mu_0\epsilon_0}} \nabla \times \left[(\mathbf{n} \times \mathbf{p}) \frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr} \right) \right] \\ &= \frac{ik}{4\pi\epsilon_0} \nabla \times \left[(\mathbf{n} \times \mathbf{p}) \frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr} \right) \right] = \frac{ik}{4\pi\epsilon_0} \left\{ \overbrace{\nabla \left[\frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr} \right) \right]}^{\mathbf{a}} \times (\mathbf{n} \times \mathbf{p}) + \frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr} \right) \overbrace{\nabla \times (\mathbf{n} \times \mathbf{p})}^{\mathbf{b}} \right\} \end{aligned} \quad (3)$$

Let's calculate \mathbf{a} first:

$$\left\{ \begin{aligned} \nabla \left(\frac{e^{ikr}}{r} \right) &= \frac{r \cdot ike^{ikr} \mathbf{n} - e^{ikr} \mathbf{n}}{r^2} = \mathbf{n} \frac{e^{ikr}}{r} \left(ik - \frac{1}{r} \right) \\ \nabla \left(\frac{e^{ikr}}{r} \frac{1}{ikr} \right) &= \frac{1}{ik} \left(\frac{r^2 \cdot ike^{ikr} \mathbf{n} - e^{ikr} \cdot 2r\mathbf{n}}{r^4} \right) = \frac{1}{ik} \mathbf{n} e^{ikr} \left(\frac{ik}{r^2} - \frac{2}{r^3} \right) \end{aligned} \right\} \Rightarrow \mathbf{a} = \mathbf{n} e^{ikr} \left(\frac{ik}{r} - \frac{2}{r^2} + \frac{2}{ikr^3} \right) \quad (4)$$

For \mathbf{b} , with the identity,

$$\nabla \times (\mathbf{n} \times \mathbf{p}) = -\mathbf{p} (\nabla \cdot \mathbf{n}) + (\mathbf{p} \cdot \nabla) \mathbf{n} \quad (5)$$

we have

$$\left\{ \begin{aligned} \nabla \cdot \mathbf{n} &= \frac{\partial}{\partial x_i} \left(\frac{x_i}{r} \right) = \frac{3}{r} - \frac{x_i^2}{r^3} = \frac{2}{r} \\ (\mathbf{p} \cdot \nabla) \mathbf{n} &= p_i \frac{\partial}{\partial x_i} \left(\frac{\hat{\mathbf{e}}_j x_j}{r} \right) = \hat{\mathbf{e}}_j p_i \left(\frac{r \delta_{ij} - x_j n_i}{r^2} \right) = \frac{\mathbf{p} - \mathbf{n} (\mathbf{n} \cdot \mathbf{p})}{r} \end{aligned} \right\} \Rightarrow \mathbf{b} = -\left[\frac{\mathbf{p} + \mathbf{n} (\mathbf{n} \cdot \mathbf{p})}{r} \right] \quad (6)$$

The curly brace of (3) is now

$$\begin{aligned} \{ \} &= \mathbf{n} \times (\mathbf{n} \times \mathbf{p}) e^{ikr} \left(\frac{ik}{r} - \frac{2}{r^2} + \frac{2}{ikr^3} \right) - \frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr} \right) \left[\frac{\mathbf{p} + \mathbf{n} (\mathbf{n} \cdot \mathbf{p})}{r} \right] \\ &= ik \frac{e^{ikr}}{r} \mathbf{n} \times (\mathbf{n} \times \mathbf{p}) + e^{ikr} \left(-\frac{2}{r^2} + \frac{2}{ikr^3} \right) \mathbf{n} \times (\mathbf{n} \times \mathbf{p}) - \frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr} \right) \left[\frac{\mathbf{p} + \mathbf{n} (\mathbf{n} \cdot \mathbf{p})}{r} \right] \\ &= ik \frac{e^{ikr}}{r} \mathbf{n} \times (\mathbf{n} \times \mathbf{p}) + e^{ikr} \left(-\frac{2}{r^2} + \frac{2}{ikr^3} \right) [\mathbf{n} (\mathbf{n} \cdot \mathbf{p}) - \mathbf{p}] - \frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr} \right) \left[\frac{\mathbf{p} + \mathbf{n} (\mathbf{n} \cdot \mathbf{p})}{r} \right] \\ &= ik \frac{e^{ikr}}{r} \mathbf{n} \times (\mathbf{n} \times \mathbf{p}) + \mathbf{p} \left[\frac{e^{ikr}}{r^2} \left(2 - \frac{2}{ikr} \right) - \frac{e^{ikr}}{r^2} \left(1 - \frac{1}{ikr} \right) \right] + \mathbf{n} (\mathbf{n} \cdot \mathbf{p}) \left[\frac{e^{ikr}}{r^2} \left(-2 + \frac{2}{ikr} \right) - \frac{e^{ikr}}{r^2} \left(1 - \frac{1}{ikr} \right) \right] \\ &= ik \frac{e^{ikr}}{r} \mathbf{n} \times (\mathbf{n} \times \mathbf{p}) - \frac{e^{ikr}}{r^2} \left(1 - \frac{1}{ikr} \right) [3\mathbf{n} (\mathbf{n} \cdot \mathbf{p}) - \mathbf{p}] \end{aligned} \quad (7)$$

which gives

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \left[k^2 \frac{e^{ikr}}{r} (\mathbf{n} \times \mathbf{p}) \times \mathbf{n} \right] + [3\mathbf{n} (\mathbf{n} \cdot \mathbf{p}) - \mathbf{p}] e^{ikr} \left(\frac{1}{r^3} - \frac{ik}{r^2} \right) \quad (8)$$