1. **Prob 11.2**

The relevant transforms between K'/K'', K/K' and K/K'' are

$$x'' = f\left(v_2^2\right)x' - v_2 f\left(v_2^2\right)t' \qquad t'' = g\left(v_2^2\right)t' - v_2 h\left(v_2^2\right)x' \tag{1}$$
$$x' = f\left(v_1^2\right)x - v_1 f\left(v_1^2\right)t \qquad t' = g\left(v_1^2\right)t - v_1 h\left(v_1^2\right)x \tag{2}$$
$$x'' = f\left(v_3^2\right)x - v_3 f\left(v_3^2\right)t \qquad t'' = g\left(v_3^2\right)t - v_3 h\left(v_3^2\right)x \tag{3}$$

Plugging (2) into (1) gives

$$\begin{aligned} x'' &= \left[f\left(v_2^2\right)f\left(v_1^2\right) + v_2 v_1 f\left(v_2^2\right)h\left(v_1^2\right)\right]x - \left[v_1 f\left(v_1^2\right)f\left(v_2^2\right) + v_2 f\left(v_2^2\right)g\left(v_1^2\right)\right]t \\ t'' &= \left[g\left(v_2^2\right)g\left(v_1^2\right) + v_2 v_1 h\left(v_2^2\right)f\left(v_1^2\right)\right]t - \left[v_1 g\left(v_2^2\right)h\left(v_1^2\right) + v_2 h\left(v_2^2\right)f\left(v_1^2\right)\right]x \end{aligned} \tag{4}$$

Matching (3) and (4) requires

$$f\left(v_3^2\right) = f\left(v_2^2\right)f\left(v_1^2\right) + v_2 v_1 f\left(v_2^2\right)h\left(v_1^2\right) \tag{5}$$
$$v_3 f\left(v_3^2\right) = v_1 f\left(v_1^2\right)f\left(v_2^2\right) + v_2 f\left(v_2^2\right)g\left(v_1^2\right) \tag{6}$$
$$g\left(v_3^2\right) = g\left(v_2^2\right)g\left(v_1^2\right) + v_2 v_1 h\left(v_2^2\right)f\left(v_1^2\right) \tag{7}$$
$$v_3 h\left(v_3^2\right) = v_1 g\left(v_2^2\right)h\left(v_1^2\right) + v_2 h\left(v_2^2\right)f\left(v_1^2\right) \tag{8}$$

From problem 11.1, we know $f = g$, so equating the RHS of (5) and (7) gives

$$f\left(v_2^2\right)h\left(v_1^2\right) = h\left(v_2^2\right)f\left(v_1^2\right) \qquad \Longrightarrow \qquad h/f = \text{constant} \tag{9}$$

which is also consistent with (6) and (8).

If we denote the constant $1/C^2$ (subject to the experimental verification that $h/f > 0$), then we can use the relation (developed in problem 11.1) $f^2 - v^2 f h = 1$, to recover the Lorentz transformation

$$f = \frac{1}{\sqrt{1 - v^2/C^2}} \tag{10}$$

2. **Prob 11.3**

Let K' be moving with velocity v_1 in the x-direction relative to K. Let K'' be moving with velocity v_2 in the x-direction relative to K'. Then the K/K', K'/K'' Lorentz transformations are

$$t' = \gamma_{v_1}\left(t - \frac{v_1 x}{c^2}\right) \qquad x' = \gamma_{v_1}\left(x - v_1 t\right) \tag{11}$$
$$t'' = \gamma_{v_2}\left(t' - \frac{v_2 x'}{c^2}\right) \qquad x'' = \gamma_{v_2}\left(x' - v_2 t'\right) \tag{12}$$

With (11) plugged into (12), we get

$$\begin{aligned} t'' &= \gamma_{v_2}\left[\gamma_{v_1}\left(t - \frac{v_1 x}{c^2}\right) - \frac{v_2 \gamma_{v_1}\left(x - v_1 t\right)}{c^2}\right] & x'' &= \gamma_{v_2}\left[\gamma_{v_1}\left(x - v_1 t\right) - v_2 \gamma_{v_1}\left(t - \frac{v_1 x}{c^2}\right)\right] \\ &= \gamma_{v_1}\gamma_{v_2}\left[\left(1 + \frac{v_1 v_2}{c^2}\right)t - \left(\frac{v_1 + v_2}{c^2}\right)x\right] & &= \gamma_{v_1}\gamma_{v_2}\left[\left(1 + \frac{v_1 v_2}{c^2}\right)x - \left(v_1 + v_2\right)t\right] \\ &= \gamma_{v_1}\gamma_{v_2}\left(1 + \frac{v_1 v_2}{c^2}\right)\left[t - \left(\frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}\right)\frac{x}{c^2}\right] & &= \gamma_{v_1}\gamma_{v_2}\left(1 + \frac{v_1 v_2}{c^2}\right)\left[x - \left(\frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}\right)t\right] \end{aligned} \tag{13}$$

If we define

$$v \equiv \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} \tag{14}$$

then elementary algebra shows

$$\gamma_v = \gamma_{v_1}\gamma_{v_2}\left(1 + \frac{v_1 v_2}{c^2}\right) \tag{15}$$

which enables us to deem v as the velocity of K'' relative to K, and consistently read (13) as the Lorentz transformations between K/K''

$$t'' = \gamma_v\left(t - \frac{v x}{c^2}\right) \qquad x'' = \gamma_v\left(x - v t\right) \tag{16}$$