I think in the problem statement, "angular momentum" is to be understood as the "spin" angular momentum as given in problem 7.27 (otherwise the integral will have coordinate dependency, and is not an intrinsic property of the field).

By problem 7.28, the electric field is given as

$$\mathbf{E}(\mathbf{x},t)=\underbrace{\left[E_0(x,y)(\hat{\mathbf{e}}_1\pm i\hat{\mathbf{e}}_2)+\frac{i}{k}\overbrace{\left(\frac{\partial E_0}{\partial x}\pm i\frac{\partial E_0}{\partial y}\right)}^{F_\pm}\hat{\mathbf{e}}_3\right]}_{\mathscr{E}}e^{ikz-i\omega t} \tag{1}$$

Then from the relation $\mathbf{E}=-\partial\mathbf{A}/\partial t$ implied by the Coulomb gauge, we have

$$\mathbf{A}(\mathbf{x},t)=\frac{1}{i\omega}\mathbf{E}(\mathbf{x},t)=\left(\frac{\mathscr{E}}{i\omega}\right)e^{ikz-i\omega t} \tag{2}$$

The time average of the spin density is thus

$$\begin{aligned}\langle j\rangle=\frac{\epsilon_0}{2}\mathscr{E}\times\mathscr{A}^*&=\frac{i\epsilon_0}{2\omega}\left[E_0(\hat{\mathbf{e}}_1\pm i\hat{\mathbf{e}}_2)+\frac{iF_\pm}{k}\hat{\mathbf{e}}_3\right]\times\left[E_0(\hat{\mathbf{e}}_1\mp i\hat{\mathbf{e}}_2)-\frac{iF_\mp}{k}\hat{\mathbf{e}}_3\right]\\&=\frac{i\epsilon_0}{\omega}\left(\mp iE_0^2\hat{\mathbf{e}}_3+\frac{iE_0}{k}\frac{\partial E_0}{\partial x}\hat{\mathbf{e}}_2-\frac{iE_0}{k}\frac{\partial E_0}{\partial y}\hat{\mathbf{e}}_1\right)\\&=\pm\frac{\epsilon_0E_0^2}{\omega}\hat{\mathbf{e}}_3-\frac{\epsilon_0E_0}{k\omega}\frac{\partial E_0}{\partial x}\hat{\mathbf{e}}_2+\frac{\epsilon_0E_0}{k\omega}\frac{\partial E_0}{\partial y}\hat{\mathbf{e}}_1\end{aligned} \tag{3}$$

We see that the spin density component along the $\hat{\mathbf{e}}_3$ direction is $\langle j_3\rangle=\pm\epsilon_0E_0^2/\omega$.

On the other hand, the time average of the energy density is

$$\langle u\rangle=\frac{\epsilon_0}{4}\mathscr{E}\cdot\mathscr{E}^*+\frac{1}{4\mu_0}\mathscr{B}\cdot\mathscr{B}^* \tag{4}$$

where we have proved in problem 7.28 that

$$\mathscr{B}\approx\mp\frac{i}{c}\mathscr{E} \tag{5}$$

This gives

$$\begin{aligned}\langle u\rangle\approx\frac{\epsilon_0}{2}\mathscr{E}\cdot\mathscr{E}^*&=\frac{\epsilon_0}{2}\left[E_0(\hat{\mathbf{e}}_1\pm i\hat{\mathbf{e}}_2)+\frac{iF_\pm}{k}\hat{\mathbf{e}}_3\right]\cdot\left[E_0(\hat{\mathbf{e}}_1\mp i\hat{\mathbf{e}}_2)-\frac{iF_\mp}{k}\hat{\mathbf{e}}_3\right]\\&=\frac{\epsilon_0}{2}\left\{2E_0^2+\frac{1}{k^2}\left[\left(\frac{\partial E_0}{\partial x}\right)^2+\left(\frac{\partial E_0}{\partial y}\right)^2\right]\right\}\end{aligned} \tag{6}$$

If E_0 changes slowly in the transverse direction, we can further ignore the square bracket and write

$$\langle u\rangle\approx\epsilon_0E_0^2\qquad\Longrightarrow\qquad\langle j_3\rangle=\pm\frac{\langle u\rangle}{\omega} \tag{7}$$

Note this relation is exact when the wave is a true plane wave with positive or negative helicity, i.e., where E_0 is a constant.

Additionally, when E_0 is cylindrically symmetric, i.e.

$$E_0(x,y)=E_0(r) \tag{8}$$

by (3), the time average of the total spin along the $\hat{\mathbf{e}}_1$ direction is

$$\begin{aligned}\langle j_1\rangle&=\int_0^{2\pi}d\phi\int_0^R rdr\cdot\frac{\epsilon_0E_0(r)}{k\omega}\frac{\partial E_0}{\partial y}=\frac{\epsilon_0}{k\omega}\int_0^{2\pi}d\phi\int_0^R rdrE_0(r)\frac{dE_0}{dr}\frac{\partial r}{\partial y}\\&=\frac{\epsilon_0}{k\omega}\int_0^R E_0(r)\frac{dE_0}{dr}rdr\int_0^{2\pi}\sin\phi d\phi=0\end{aligned} \tag{9}$$

Similarly $\langle j_2\rangle=0$ too.