1. At nonrelativistic limit $\beta \to 0$, we have

$$\lambda \to \frac{\omega}{v} \tag{1}$$

With $\beta \to 0$ as well as the small argument approximation

$$K_0(x) \to -\ln\left[\left(\frac{x}{2}\right)+\gamma_E\right] = \ln\left(\frac{2e^{-\gamma_E}}{x}\right) \approx \ln\left(\frac{1.123}{x}\right) \qquad K_1(x) \to \frac{1}{x} \tag{2}$$

(13.36) becomes

$$\left(\frac{dE}{dx}\right)_{b>1/k_D} = \frac{2}{\pi}\frac{(ze)^2}{v^2}\int_0^\infty \mathrm{Re}\left[\frac{i\omega}{\epsilon(\omega)}\right]\ln\left(\frac{1.123k_D v}{\omega}\right)d\omega \tag{3}$$

2. When

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma} \tag{4}$$

the integral in (3) becomes

$$\mathrm{Re}\int_0^\infty \left[\frac{i\omega\left(\omega^2+i\omega\Gamma\right)}{\omega^2-\omega_p^2+i\omega\Gamma}\right]\ln\left(\frac{1.123k_D v}{\omega}\right)d\omega = -\mathrm{Im}\int_0^\infty \frac{F(\omega)}{\omega^2-\omega_p^2+i\omega\Gamma}d\omega \tag{5}$$

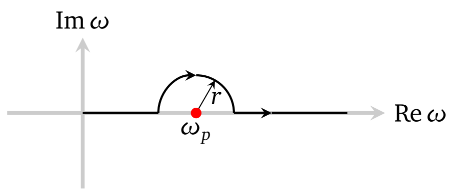
where

$$F(\omega) = \omega\left(\omega^2+i\omega\Gamma\right)\ln\left(\frac{1.123k_D v}{\omega}\right) \tag{6}$$

The integrand of (5) has poles at

$$\omega_\pm = \frac{-i\Gamma \pm \sqrt{-\Gamma^2+4\omega_p^2}}{2} \approx \pm\omega_p - \frac{i\Gamma}{2} \tag{7}$$

which approaches $\pm\omega_p$ from below as $\Gamma \to 0^+$.



The integral on the RHS of (5) becomes

$$\int_0^\infty = \lim_{r\to 0}\left(\int_0^{\omega_j - r} + \int_{\omega_j + r}^\infty + \int_{\text{semi-circle}}\right) \tag{8}$$

The sum of the first two terms is the principal value integral which is real and has no contribution to the imaginary part in (5). The integral over the semi-circle is

$$\lim_{r\to 0}\int_{\text{semi-circle}} = \int_\pi^0 \frac{F\left(\omega_p\right) ire^{i\phi}d\phi}{2\omega_p re^{i\phi}} = -\frac{i\pi}{2}\frac{F\left(\omega_p\right)}{\omega_p} \tag{9}$$

With $\Gamma \to 0$, substituting $F(\omega)$ using (6) turns (5) to

$$-\mathrm{Im}\int_0^\infty \frac{F(\omega)}{\omega^2-\omega_p^2+i\omega\Gamma}d\omega = \frac{\pi}{2}\omega_p^2\ln\left(\frac{1.123k_D v}{\omega_p}\right) \tag{10}$$

giving the energy loss

$$\left(\frac{dE}{dx}\right)_{b>1/k_D} = \frac{z^2e^2}{v^2}\omega_p^2\ln\left(\frac{1.123k_D v}{\omega_p}\right) \tag{11}$$