1. Since inside the cylinder, $\nabla' \cdot \mathbf{M}(\mathbf{x}') = 0$, equation (5.100) can be simplified into

$$\Phi_M = \frac{1}{4\pi} \oint_S \frac{\mathbf{n}' \cdot \mathbf{M}(\mathbf{x}')\, da'}{|\mathbf{x} - \mathbf{x}'|} \tag{1}$$

If we take the center of cylinder as origin and let $z$-axis point to the direction of $\mathbf{M}$, the scalar potential of the point on the axis can be written as

$$\begin{aligned}\Phi_M(0,0,z) &= \frac{1}{4\pi} \oint_S \frac{\mathbf{n}' \cdot \mathbf{M} da'}{|\mathbf{x} - \mathbf{x}'|} \\ &= \frac{M_0}{4\pi}\left[\int_0^a \frac{2\pi\rho\, d\rho}{\sqrt{\rho^2 + (L/2 - z)^2}} - \int_0^a \frac{2\pi\rho\, d\rho}{\sqrt{\rho^2 + (L/2 + z)^2}}\right] \\ &= \frac{M_0}{2}\left[\sqrt{a^2 + \left(\frac{L}{2} - z\right)^2} - \left|\frac{L}{2} - z\right| - \sqrt{a^2 + \left(\frac{L}{2} + z\right)} + \left|\frac{L}{2} + z\right|\right]\end{aligned} \tag{2}$$

Or, treating inside point and outside point differently,

$$\Phi_M(0,0,z) = \begin{cases} \dfrac{M_0}{2}\left[\sqrt{a^2 + \left(\dfrac{L}{2} - z\right)^2} - \sqrt{a^2 + \left(\dfrac{L}{2} + z\right)^2} + 2z\right] & \text{for} |z| < \dfrac{L}{2} \\ \dfrac{M_0}{2}\left[\sqrt{a^2 + \left(\dfrac{L}{2} - z\right)^2} - \sqrt{a^2 + \left(\dfrac{L}{2} + z\right)^2} + \operatorname{sgn}(z)\, L\right] & \text{for } |z| > \dfrac{L}{2} \end{cases} \tag{3}$$

Thus

$$\mathbf{H}(0,0,z) = -\frac{\partial \Phi_M}{\partial z}\hat{\mathbf{z}} = \begin{cases} -\dfrac{\mathbf{M}}{2}\left[\dfrac{z - L/2}{\sqrt{a^2 + (L/2 - z)^2}} - \dfrac{z + L/2}{\sqrt{a^2 + (L/2 + z)^2}} + 2\right] & \text{for } |z| < \dfrac{L}{2} \\ -\dfrac{\mathbf{M}}{2}\left[\dfrac{z - L/2}{\sqrt{a^2 + (L/2 - z)^2}} - \dfrac{z + L/2}{\sqrt{a^2 + (L/2 + z)^2}}\right] & \text{for } |z| < \dfrac{L}{2} \end{cases} \tag{4}$$

$$\mathbf{B}(0,0,z) = \mu_0(\mathbf{H} + \mathbf{M}) = -\frac{\mathbf{M}}{2}\left[\frac{z - L/2}{\sqrt{a^2 + (L/2 - z)^2}} - \frac{z + L/2}{\sqrt{a^2 + (L/2 + z)^2}}\right] \tag{5}$$

2. Plot is below.