Equation (9.8) was presented with an intuitive argument. We will now derive it from the exact equation (9.11) under the radiation-zone approximation $kr \gg 1$.

Recall equation (9.11):

$$\mathbf{A}(\mathbf{x}) = \mu_0 ik \sum_{l,m} h_l^{(1)}(kr) Y_{lm}(\theta,\phi) \int \mathbf{J}(\mathbf{x}') j_l(kr') Y_{lm}^*(\theta',\phi') d^3x' \tag{1}$$

When $kr \gg 1$, by (9.89)

$$h_l^{(1)}(kr) \to (-i)^{l+1} \frac{e^{ikr}}{kr} \tag{2}$$

In the approximation $kr \to \infty$, (1) becomes

$$\begin{aligned}
\lim_{kr\to\infty} \mathbf{A}(\mathbf{x}) &= \mu_0 i \sum_{l,m} (-i)^{l+1} \frac{e^{ikr}}{r} Y_{lm}(\theta,\phi) \int \mathbf{J}(\mathbf{x}') j_l(kr') Y_{lm}^*(\theta',\phi') d^3x' \\
&= \mu_0 \frac{e^{ikr}}{r} \int \mathbf{J}(\mathbf{x}') d^3x' \sum_{l,m} (-i)^l j_l(kr') Y_{lm}(\theta,\phi) Y_{lm}^*(\theta',\phi') && \text{by Addition Theorem (3.62)} \\
&= \mu_0 \frac{e^{ikr}}{r} \int \mathbf{J}(\mathbf{x}') d^3x' \sum_{l} (-i)^l j_l(kr') \left(\frac{2l+1}{4\pi}\right) P_l(\cos\gamma) && (3)
\end{aligned}$$

where γ is the angle between $\mathbf{n}$ and $\mathbf{x}'$.

From DLMF 10.60.E7

$$\sum_n (2n+1) i^n j_n(z) P_n(\cos\alpha) = e^{iz\cos\alpha} \tag{4}$$

and the reflection formula DLMF 10.47.E14

$$j_n(-z) = (-1)^n j_n(z) \tag{5}$$

we have

$$\sum_n (2n+1)(-i)^n j_n(kr) P_n(\cos\alpha) = e^{-ikr\cos\alpha} \tag{6}$$

by letting $z = -kr$ in (4).

Applying (6) to (3) gives the desired approximation (9.8):

$$\lim_{kr\to\infty} \mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int \mathbf{J}(\mathbf{x}') e^{-ikr\cos\gamma} d^3x' = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int \mathbf{J}(\mathbf{x}') e^{-ik\mathbf{n}\cdot\mathbf{x}'} d^3x' \tag{7}$$