1. In cylindrical coordinate representation (ρ,ϕ,z), the charge density and current density are

$$\eta(\mathbf{x},t)=q\frac{\delta(\rho-R)}{R}\delta(z)\delta(\phi-\omega_0 t) \tag{1}$$
$$\mathbf{J}(\mathbf{x},t)=\eta(\mathbf{x},t)\mathbf{v}(\mathbf{x})=\eta(\mathbf{x},t)R\omega_0\hat{\boldsymbol{\phi}}=q\omega_0\delta(\rho-R)\delta(z)\delta(\phi-\omega_0 t)\hat{\boldsymbol{\phi}} \tag{2}$$

The n-th Fourier component of the current density is

$$\begin{aligned}\mathbf{J}_n(\mathbf{x})&=\frac{1}{T}\int_0^T\mathbf{J}(\mathbf{x},t)e^{in\omega_0 t}dt\\&=\hat{\boldsymbol{\phi}}q\omega_0\delta(\rho-R)\delta(z)\frac{1}{T}\int_0^T\delta(\phi-\omega_0 t)e^{in\omega_0 t}dt\\&=\hat{\boldsymbol{\phi}}q\omega_0\delta(\rho-R)\delta(z)\cdot\frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0}\delta\left[\omega_0\left(\frac{\phi}{\omega_0}-t\right)\right]e^{in\omega_0 t}dt\\&=\hat{\boldsymbol{\phi}}q\omega_0\delta(\rho-R)\delta(z)\cdot\frac{\omega_0}{2\pi}\frac{1}{\omega_0}e^{in\phi}\\&=\hat{\boldsymbol{\phi}}\frac{q\omega_0}{2\pi}\delta(\rho-R)\delta(z)e^{in\phi}\end{aligned} \tag{3}$$

Thus

$$\begin{aligned}\mathcal{M}_n(\mathbf{x})&=\frac{\mathbf{x}\times\mathbf{J}_n(\mathbf{x})}{2}=\frac{q\omega_0}{4\pi}\delta(\rho-R)\delta(z)e^{in\phi}(\rho\hat{\boldsymbol{\rho}}+z\hat{\mathbf{z}})\times\hat{\boldsymbol{\phi}}\\&=\frac{q\omega_0}{4\pi}\delta(\rho-R)\delta(z)e^{in\phi}(\rho\hat{\mathbf{z}}-z\hat{\boldsymbol{\rho}})\end{aligned} \tag{4}$$

which gives an effective magnetic charge density

$$\eta_n^M(\mathbf{x})=-\nabla\cdot\mathcal{M}_n(\mathbf{x})=-\frac{q\omega_0}{4\pi}e^{in\phi}\left\{\delta(\rho-R)\rho\delta'(z)-\delta(z)z\cdot\frac{1}{\rho}\frac{d[\delta(\rho-R)\rho]}{d\rho}\right\} \tag{5}$$

This may look complicated but the second term's $\delta(z)z$ factor will have no effect if $\eta_n^M(\mathbf{x})$ is to be integrated over the whole space. For the same reason, the magnetization density $\mathcal{M}_n$ can be considered to have only the $\hat{\mathbf{z}}$ component.

2. The magnetic multipole moment due to the n-th harmonic $\mathcal{M}_n$ is given by (9.172)

$$M_{lm}^{(n)}=-\int r^l Y_{lm}^*(\theta,\phi)\nabla\cdot\mathcal{M}_n d^3x \tag{6}$$

To calculate this, it is most convenient if we express the divergence in spherical coordinates, so let's rewrite (1) and (2) in spherical coordinates (r,θ,ϕ):

$$\eta(\mathbf{x},t)=q\frac{\delta(r-R)}{R^2}\delta(\cos\theta)\delta(\phi-\omega_0 t) \tag{7}$$
$$\mathbf{J}(\mathbf{x},t)=\eta(\mathbf{x},t)\mathbf{v}(\mathbf{x})=\eta(\mathbf{x},t)R\sin\theta\,\omega_0\hat{\boldsymbol{\phi}}=q\omega_0\frac{\delta(r-R)}{R}\delta\left(\theta-\frac{\pi}{2}\right)\delta(\phi-\omega_0 t)\hat{\boldsymbol{\phi}} \tag{8}$$

Same Fourier transform as in (3) gives

$$\mathbf{J}_n(\mathbf{x})=\hat{\boldsymbol{\phi}}\frac{q\omega_0}{2\pi}\frac{\delta(r-R)}{R}\delta\left(\theta-\frac{\pi}{2}\right)e^{in\phi} \tag{9}$$

from which we have

$$\mathcal{M}_n(\mathbf{x})=\frac{\mathbf{x}\times\mathbf{J}_n(\mathbf{x})}{2}=\frac{q\omega_0}{4\pi}\frac{\delta(r-R)}{R}\delta\left(\theta-\frac{\pi}{2}\right)e^{in\phi}r\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}}=-\frac{q\omega_0}{4\pi}\frac{r\delta(r-R)}{R}\delta\left(\theta-\frac{\pi}{2}\right)e^{in\phi}\hat{\boldsymbol{\theta}} \tag{10}$$

and

$$\begin{aligned}-\nabla\cdot\mathcal{M}_n(\mathbf{x})&=\frac{q\omega_0}{4\pi}\frac{r\delta(r-R)}{R}e^{in\phi}\cdot\frac{1}{r\sin\theta}\frac{d}{d\theta}\left[\delta\left(\theta-\frac{\pi}{2}\right)\sin\theta\right]\\&=\frac{q\omega_0}{4\pi}\frac{\delta(r-R)}{R}e^{in\phi}\left[\delta'\left(\theta-\frac{\pi}{2}\right)+\delta\left(\theta-\frac{\pi}{2}\right)\cot\theta\right]\end{aligned} \tag{11}$$

Again, the second term $\delta(\theta-\pi/2)\cot\theta$ will have no effect when integrated over the the range $\theta\in[0,\pi]$. Inserting (11) into (6) yields

$$M_{lm}^{(n)}=\frac{q\omega_0}{4\pi}\int r^l Y_{lm}^*(\theta,\phi)\frac{\delta(r-R)}{R}e^{in\phi}\delta'\left(\theta-\frac{\pi}{2}\right)d^3x$$
$$=\frac{q\omega_0 R^{l+1}}{4\pi}\delta_{mn}\cdot 2\pi\sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}\underbrace{\int_0^\pi P_l^m(\cos\theta)\delta'\left(\theta-\frac{\pi}{2}\right)\sin\theta d\theta}_{I} \tag{12}$$

where

$$I=-\frac{d}{d\theta}\left[P_l^m(\cos\theta)\sin\theta\right]\bigg|_{\theta=\pi/2}=\frac{dP_l^m(x)}{dx}\bigg|_0 \tag{13}$$

With the recurrence relation (See 14.10.E5 on DLMF)

$$\left(1-x^2\right)\frac{dP_l^m(x)}{dx}=(l+m)P_{l-1}^m(x)-lxP_l^m(x) \tag{14}$$

we have

$$\frac{dP_l^m(x)}{dx}\bigg|_0=(l+m)P_{l-1}^m(0) \tag{15}$$

which, due to the parity of $P_l^m(x)$, will vanish when $l-m$ is even. Thus for the n-th harmonic, the only non-vanishing magnetic multipole moments are those with $l\geq n$ and $l-n$ odd, the lowest of which is $l=n+1$.

3. For the four charges with alternate signs, we can modify in (10) the sign of q and add the corresponding initial offset to ϕ, which gives

$$\mathcal{M}_n(\mathbf{x})=-\hat{\boldsymbol{\theta}}\frac{q\omega_0}{4\pi}\frac{r\delta(r-R)}{R}\delta\left(\theta-\frac{\pi}{2}\right)e^{in\phi}\left[\overbrace{1-e^{-in\pi/2}+e^{-in\pi}-e^{-in3\pi/2}}^{1+(-1)^n-i^n-(-i)^n}\right] \tag{16}$$

This vanishes unless $n=4k+2$, e.g., only the 2nd, 6th harmonic etc. will exist for magnetic multipole moments. For $n=2$, the lowest order is $l=3$. For the E2 radiation, the contribution from electric multipole moment is from $l=2$, but the contribution from magnetic multipole moment is from $l=3$. Thus the magnetic contribution is much weaker than the electric contribution.