1. Energy conservation of individual charged particle implies

$$e\Phi + \gamma mc^2 = \text{constant} \tag{1}$$

For the sake of momentum accounting, we can treat this constant to be zero. Substituting the current density

$$\mathbf{J} = en\mathbf{v} \tag{2}$$

into the total momentum of the charged particles gives

$$\mathbf{P}_{\text{mech}} = \int n\gamma m\mathbf{v}d^3x = \int \frac{\gamma m\mathbf{J}}{e}d^3x = -\frac{1}{c^2}\int \Phi\mathbf{J}d^3x \tag{3}$$

where we have used (1) with the zero constant.

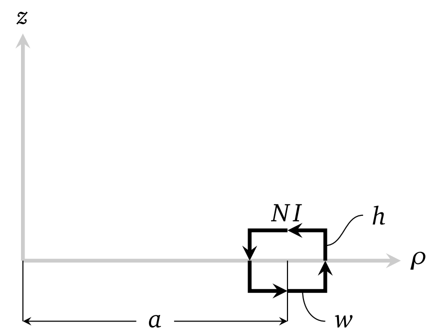
This "hidden" momentum (i.e., mechanical momentum of the charged particles in the electric field) cancels the field momentum calculated in problem 6.5 (a).

2. The electric potential at the inner vertical edge and outer vertical edge are

$$\Phi_{\text{in}} = \frac{Q}{a - w/2} \qquad\qquad \Phi_{\text{out}} = \frac{Q}{a + w/2} \tag{4}$$

The difference in electric energy for the charged particle at the two edges is

$$\Phi_{\text{in}} - \Phi_{\text{out}} = \frac{Q}{(a - w/2)} - \frac{Q}{(a + w/2)} \approx \frac{Qw}{a^2} \tag{5}$$



With energy conservation, the difference in γ factor between the outer-edge and inner-edge particle thus satisfies

$$(\gamma_{\text{out}} - \gamma_{\text{in}})\, mc^2 + e\,(\Phi_{\text{out}} - \Phi_{\text{in}}) = 0 \qquad \Longrightarrow \qquad \gamma_{\text{out}} - \gamma_{\text{in}} = \frac{e\,(\Phi_{\text{in}} - \Phi_{\text{out}})}{mc^2} \approx \frac{eQw}{a^2mc^2} \tag{6}$$

Let n be the number of charges per unit length in the wire, to maintain the current I in the wire, we must have

$$NI = nev \tag{7}$$

which gives a net z direction momentum due to the difference in γ

$$\mathbf{P}_{\text{mech}} = \hat{\mathbf{z}}\,(\gamma_{\text{out}} - \gamma_{\text{in}})\, nmv \cdot h \approx \hat{\mathbf{z}}\frac{eQw}{a^2mc^2} \cdot m \cdot \frac{NI}{e} \cdot h = \hat{\mathbf{z}}\frac{QNIwh}{a^2c^2} \tag{8}$$

which is the opposite of the field momentum calculated in problem 6.6 (a), adjusted for Gaussian units.