

1. Recall (9.18)

$$\mathbf{H}(\mathbf{x}) = \frac{ck^2}{4\pi} (\mathbf{n} \times \mathbf{p}) \frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr}\right) \quad (1)$$

$$\mathbf{E}(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \left\{ k^2 (\mathbf{n} \times \mathbf{p}) \times \mathbf{n} \frac{e^{ikr}}{r} + [3\mathbf{n}(\mathbf{n} \cdot \mathbf{p}) - \mathbf{p}] \left(\frac{1}{r^3} - \frac{ik}{r^2} \right) e^{ikr} \right\} \quad (2)$$

Let \mathbf{p} be along the $\hat{\mathbf{z}}$ direction, then

$$\mu_0 |\mathbf{H}|^2 = \frac{\mu_0 c^2 k^4 |\mathbf{p}|^2}{(4\pi)^2 r^2} \sin^2 \theta \left(1 + \frac{1}{k^2 r^2}\right) = \frac{|\mathbf{p}|^2}{(4\pi)^2 \epsilon_0} \sin^2 \theta \frac{k^2}{r^4} (1 + k^2 r^2) \quad (3)$$

Hence

$$\int \mu_0 |\mathbf{H}|^2 d\Omega = \frac{|\mathbf{p}|^2}{(4\pi)^2 \epsilon_0} \frac{k^2}{r^4} (1 + k^2 r^2) \int \sin^2 \theta d\Omega = \frac{|\mathbf{p}|^2}{(4\pi)^2 \epsilon_0} \frac{k^2 (1 + k^2 r^2)}{r^4} \cdot \frac{8\pi}{3} \quad (4)$$

For the electric part, we have

$$\epsilon_0 |\mathbf{E}|^2 = \frac{1}{(4\pi)^2 \epsilon_0} \left\{ \frac{k^4}{r^2} \overbrace{|\mathbf{n} \times \mathbf{p}|^2}^{\sin^2 \theta} + \left(\frac{1 + k^2 r^2}{r^6} \right) \overbrace{|3\mathbf{n}(\mathbf{n} \cdot \mathbf{p}) - \mathbf{p}|^2}^{|\mathbf{p}|^2(1+3\cos^2 \theta)} - 2 \frac{k^2}{r^4} \overbrace{[(\mathbf{n} \times \mathbf{p}) \times \mathbf{n}] \cdot \mathbf{p}}^{|\mathbf{p}|^2 \sin^2 \theta} \right\} \quad (5)$$

giving

$$\begin{aligned} \int \epsilon_0 |\mathbf{E}|^2 d\Omega &= \frac{|\mathbf{p}|^2}{(4\pi)^2 \epsilon_0} \left[\frac{k^4}{r^2} \int \sin^2 \theta d\Omega + \left(\frac{1 + k^2 r^2}{r^6} \right) \int \overbrace{(1 + 3\cos^2 \theta)}^{8\pi} d\Omega - 2 \frac{k^2}{r^4} \int \sin^2 \theta d\Omega \right] \\ &= \frac{|\mathbf{p}|^2}{(4\pi)^2 \epsilon_0} \left[\frac{k^4 r^4 + 3(1 + k^2 r^2) - 2k^2 r^2}{r^6} \right] \cdot \frac{8\pi}{3} \end{aligned} \quad (6)$$

and finally

$$\int (\epsilon_0 |\mathbf{E}|^2 - \mu_0 |\mathbf{H}|^2) d\Omega = \frac{|\mathbf{p}|^2}{2\pi\epsilon_0 r^6} \quad (7)$$

2. By (6.140), the reactance outside the sphere of radius a is

$$\begin{aligned} X_a &= \frac{4\omega}{|I_i|^2} \int_a^\infty r^2 dr \int (w_m - w_e) d\Omega \\ &= -\frac{\omega}{|I_i|^2} \int_a^\infty r^2 dr \int (\epsilon_0 |\mathbf{E}|^2 - \mu_0 |\mathbf{H}|^2) d\Omega \\ &= -\frac{\omega |\mathbf{p}|^2}{2\pi\epsilon_0 |I_i|^2} \int_a^\infty \frac{dr}{r^4} \\ &= -\frac{\omega |\mathbf{p}|^2}{6\pi\epsilon_0 |I_i|^2 a^3} \end{aligned} \quad (8)$$

3. The center-fed antenna of 9.2 has a dipole moment of

$$p = \frac{iI_0 d}{2\omega} \quad (9)$$

which gives a reactance of

$$X_a = -\frac{d^2}{24\pi\epsilon_0 \omega a^3} \quad (10)$$