1. For the given Lagrangian density

$$\mathscr{L} = -\frac{1}{8\pi}\partial_\mu A_\nu \partial^\mu A^\nu - \frac{1}{c}J_\mu A^\mu = -\frac{1}{8\pi}g_{\lambda\mu}g_{\nu\sigma}\partial^\lambda A^\sigma \partial^\mu A^\nu - \frac{1}{c}J_\mu A^\mu \tag{1}$$

we have

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial\left(\partial^\beta A^\alpha\right)} &= -\frac{1}{8\pi}g_{\lambda\mu}g_{\nu\sigma}\left(\delta_\beta{}^\lambda\delta_\alpha{}^\sigma\partial^\mu A^\nu + \delta_\beta{}^\mu\delta_\alpha{}^\nu\partial^\lambda A^\sigma\right)\\ &= -\frac{1}{8\pi}\left(g_{\beta\mu}g_{\nu\alpha}\partial^\mu A^\nu + g_{\lambda\beta}g_{\alpha\sigma}\partial^\lambda A^\sigma\right)\\ &= -\frac{1}{4\pi}\partial_\beta A_\alpha\end{aligned} \tag{2}$$

so

$$\partial^\beta\left[\frac{\partial \mathscr{L}}{\partial\left(\partial^\beta A^\alpha\right)}\right] = -\frac{1}{4\pi}\partial^\beta\partial_\beta A_\alpha = -\frac{1}{4\pi}\Box A_\alpha \tag{3}$$

On the other hand

$$\frac{\partial \mathscr{L}}{\partial A^\alpha} = -\frac{1}{c}J_\alpha \tag{4}$$

The Euler-Lagrange equation

$$\partial^\beta\left[\frac{\partial \mathscr{L}}{\partial\left(\partial^\beta A^\alpha\right)}\right] = \frac{\partial \mathscr{L}}{\partial A^\alpha} \tag{5}$$

gives the equation of motion

$$\Box A_\alpha = \frac{4\pi}{c}J_\alpha \tag{6}$$

which is the Maxwell equation (11.133) under Lorenz guage condition

$$\partial_\alpha A^\alpha = 0 \tag{7}$$

2. The Lagrangian density from (12.86) is

$$\begin{aligned}\mathscr{L} &= -\frac{1}{16\pi}g_{\lambda\mu}g_{\nu\sigma}\left(\partial^\mu A^\sigma - \partial^\sigma A^\mu\right)\left(\partial^\lambda A^\nu - \partial^\nu A^\lambda\right) - \frac{1}{c}J_\mu A^\mu\\ &= -\frac{1}{16\pi}g_{\lambda\mu}g_{\nu\sigma}\left(\partial^\mu A^\sigma\partial^\lambda A^\nu - \underbrace{\partial^\sigma A^\mu\partial^\lambda A^\nu}_{X} - \underbrace{\partial^\mu A^\sigma\partial^\nu A^\lambda}_{Y} + \partial^\sigma A^\mu\partial^\nu A^\lambda\right) - \frac{1}{c}J_\mu A^\mu\end{aligned} \tag{8}$$

We see that X and Y are identical since g, being symmetric, allows the exchange $\lambda \longleftrightarrow \mu$, $\nu \longleftrightarrow \sigma$. Also we see that the other two terms after the expansion are equal

$$\overbrace{g_{\lambda\mu}g_{\nu\sigma}\partial^\mu A^\sigma\partial^\lambda A^\nu}^{\partial_\lambda A_\nu\partial^\lambda A^\nu} = \overbrace{g_{\lambda\mu}g_{\nu\sigma}\partial^\sigma A^\mu\partial^\nu A^\lambda}^{\partial_\nu A_\lambda\partial^\nu A^\lambda} \tag{9}$$

which is also equal to $\partial_\mu A_\nu\partial^\mu A^\nu$ in (1). So the difference between the two Lagrangian densities defined in (1) and (8) is

$$\begin{aligned}\Delta\mathscr{L} = -\frac{1}{8\pi}g_{\lambda\mu}g_{\nu\sigma}\left(\partial^\sigma A^\mu\partial^\lambda A^\nu\right) &= -\frac{1}{8\pi}\partial_\nu A_\lambda\partial^\lambda A^\nu && \text{derivative of products}\\ &= -\frac{1}{8\pi}\left[\partial_\nu\left(A_\lambda\partial^\lambda A^\nu\right) - A_\lambda\left(\partial_\nu\partial^\lambda A^\nu\right)\right] && \partial_\nu\partial^\lambda = \partial^\lambda\partial_\nu\\ &= -\frac{1}{8\pi}\left[\partial_\nu\left(A_\lambda\partial^\lambda A^\nu\right) - A_\lambda\partial^\lambda\left(\partial_\nu A^\nu\right)\right]\end{aligned} \tag{10}$$

Under Lorenz gauge condition (7), the second term vanishes. The quantity $A_\lambda\partial^\lambda A^\nu$ is a contravariant vector B^ν, which renders $\Delta\mathscr{L}$ a 4-divergence.

The difference in action is given by the integral $\int \Delta\mathscr{L}\, d^4x$, which will vanish for $\Delta\mathscr{L}$ being a 4-divergence (assuming B^ν vanishes at infinity).