1. **Prob 7.10**
   If for a single frequency ω, the incident wave can be expressed as

$$\psi_{\text{inc}}(x,\omega,t) = e^{ik(\omega)x - i\omega t} \tag{1}$$

the coherent superposition of many frequencies can be written as

$$\psi_{\text{inc}}(x,t) = \int d\omega A(\omega)\psi_{\text{inc}}(x,\omega,t) = \int d\omega A(\omega) e^{ik(\omega)x-i\omega t} \tag{2}$$

Applying the form of single-frequency transmitted wave

$$\psi_{\text{trans}}(x,\omega,t) = \tau(\omega) e^{i\phi(\omega)} e^{ik(\omega)(x-L)-i\omega t} \tag{3}$$

to each frequency gives the superposition of transmitted wave

$$\psi_{\text{trans}}(x,t) = \int d\omega A(\omega)\tau(\omega) e^{i\phi(\omega)} e^{ik(\omega)(x-L)-i\omega t} \tag{4}$$

If we make the following approximation

$$\tau(\omega) \approx \tau(\omega_0) \tag{5}$$

$$\phi(\omega) \approx \phi(\omega_0) + \underbrace{\left.\frac{d\phi}{d\omega}\right|_{\omega_0}}_{T}(\omega-\omega_0) \tag{6}$$

(4) becomes

$$\begin{aligned}
\psi_{\text{trans}}(x,t) &\approx \tau(\omega_0)\int d\omega A(\omega) e^{i[\phi(\omega_0)-T\omega_0]} e^{ik(\omega)(x-L)-i\omega(t-T)} \qquad \text{let } \beta \equiv \phi(\omega_0) - T\omega_0 \\
&= \tau(\omega_0) e^{i\beta}\int d\omega A(\omega) e^{ik(\omega)(x-L)-i\omega(t-T)} \\
&= \tau(\omega_0) e^{i\beta}\psi_{\text{inc}}(x-L, t-T)
\end{aligned} \tag{7}$$

2. **Prob 7.11**
   (a) By solution from problem 7.2, we know

$$\frac{E_{\text{trans}}}{E_{\text{inc}}} = \left(\frac{2n}{n+1}\right) e^{i[k_2(\omega)-k(\omega)]d} \cdot \left[\frac{2}{(1+r_{23})+(1-r_{23})n}\right] \tag{8}$$

where

$$r_{23} = \left(\frac{n-1}{n+1}\right) e^{i2k_2(\omega)d} \tag{9}$$

and where $k(\omega)$ is the wave number of the incident wave in the air, and $k_2(\omega)$ is the wave number in the medium.
This gives

$$\frac{E_{\text{trans}}}{E_{\text{inc}}} = \left[\frac{4n}{(n+1)^2-(n-1)^2 e^{i2k_2(\omega)d}}\right] e^{i[k_2(\omega)-k(\omega)]d} \tag{10}$$

Identifying $z = n\omega d/c = k_2(\omega)d$ and comparing with the form (3), we have

$$\tau(\omega) = \left|\frac{4n}{(n+1)^2-(n-1)^2 e^{i2z}}\right| = \frac{4n}{\sqrt{\left[(n+1)^2-(n-1)^2\cos 2z\right]^2 + \left[(n-1)^2\sin 2z\right]^2}} \tag{11}$$

$$\phi(\omega) = z + \tan^{-1}\left[\frac{(n-1)^2\sin 2z}{(n+1)^2-(n-1)^2\cos 2z}\right] \tag{12}$$

(b) For $z = 0$ or $z = \pi$, straightforward substitution into (11) and calculation of derivative of (12) will give

$$|\tau| = 1 \qquad\qquad \frac{cT}{d} = \frac{n^2+1}{2} \tag{13}$$

and for $z = \pi/2$

$$|\tau| = \frac{2n}{n^2+1} \qquad\qquad \frac{cT}{d} = \frac{2n^2}{n^2+1} \tag{14}$$

I don't understand what the remaining question means by "average over any integer number of quarter-wavelength optical paths".

(c) The $\tau \sim \omega$ and $\phi \sim \omega$ plots are shown below.