1. Inside the conductor where current density is $\mathbf{J}$ and magnetic field is $\mathbf{H}$, the time-averaged force density is $\mathrm{Re}\left(\mu_c \mathbf{J}\times\mathbf{H}^*\right)/2$. Thus the time-averaged force per unit area is the integral

$$\begin{aligned}
\mathbf{f} &= \frac{1}{2}\mathrm{Re}\int_0^\infty d\xi \mu_c \mathbf{J}\times\mathbf{H}^* && \text{by (8.13)}\\
&= \frac{\mu_c}{2\delta}\int_0^\infty d\xi\left[(1-i)\left(\mathbf{n}\times\mathbf{H}_{\|}\right)e^{-\xi/\delta}e^{i\xi/\delta}\right]\times\left(\mathbf{H}_{\|}e^{-\xi/\delta}e^{-i\xi/\delta}\right)\\
&= -\mathbf{n}\left|\mathbf{H}_{\|}\right|^2\cdot\frac{\mu_c}{2\delta}\int_0^\infty e^{-2\xi/\delta}\\
&= -\frac{\mu_c}{4}\mathbf{n}\left|\mathbf{H}_{\|}\right|^2 && (1)
\end{aligned}$$

2. When outside μ is different from μ_c, there can be reflected wave which carries momentum. Additional force is needed to account for this momentum change. However, due to the charge coservation,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\mathbf{J}=0 \qquad\Longrightarrow\qquad \rho=-\frac{1}{i\omega}\nabla\cdot\mathbf{J} \tag{2}$$

Inside the conductor, $\mathbf{J}$ is given by (8.13), which has zero divergence. Therefore the charge density is zero everywhere inside the conductor, hence there is no electric force.

3. Let the tangential magnetic field just outside of the conductor surface be $\mathbf{H}_{\|}(t)$. Its Fourier decomposition is given by

$$\mathbf{H}_{\|}(t)=\int\mathbf{H}_{\|}(\omega)e^{-i\omega t}d\omega \qquad \mathbf{H}_{\|}(\omega)=\frac{1}{2\pi}\int\mathbf{H}_{\|}(t)e^{i\omega t}dt \tag{3}$$

The reality of $\mathbf{H}_{\|}(t)$ implies

$$\mathbf{H}_{\|}(-\omega)=\mathbf{H}^*_{\|}(\omega) \tag{4}$$

From (8.8),

$$\delta(\omega)=\sqrt{\frac{2}{\mu_c\omega\sigma}} \tag{5}$$

for negative frequency $-\omega$, we have the option to choose $\delta(-\omega)=\pm i\delta(\omega)$, but the choice of + sign will correspond to the unphysical solution (see (8.9)), so we must choose

$$\delta(-\omega)=-i\delta(\omega) \tag{6}$$

Applying (8.9) and (8.13) to each frequency, the instantaneous, complex magnetic field and current density at depth ξ can be written

$$\mathbf{H}(\xi,t)=\int\mathbf{H}_{\|}(\omega)e^{-\xi/\delta(\omega)}e^{i\xi/\delta(\omega)}e^{-i\omega t}d\omega \tag{7}$$

$$\mathbf{J}(\xi,t)=\int\left[\frac{1-i}{\delta(\omega)}\right]\left[\mathbf{n}\times\mathbf{H}_{\|}(\omega)\right]e^{-\xi/\delta(\omega)}e^{i\xi/\delta(\omega)}e^{-i\omega t}d\omega \tag{8}$$

whose real parts are the corresponding physical quantity.

It then follows that at depth ξ, the force density is

$$\mathbf{f}(\xi,t)=\mu_c\,\mathrm{Re}\left[\mathbf{J}(\xi,t)\right]\times\mathrm{Re}\left[\mathbf{H}(\xi,t)\right] \tag{9}$$

In general, for two complex harmonic quantities

$$a(t)=A(\omega)e^{-i\omega t} \qquad b(t)=B\left(\omega'\right)e^{-i\omega' t} \tag{10}$$

we have

$$\begin{aligned}
\mathrm{Re}\,[a\,(t)]\,\mathrm{Re}\,[b\,(t)] &= (\mathrm{Re}A\cos\omega t + \mathrm{Im}A\sin\omega t)\left(\mathrm{Re}\,B\cos\omega' t + \mathrm{Im}\,B\sin\omega' t\right)\\
&= \mathrm{Re}A\,\mathrm{Re}\,B\cos\omega t\cos\omega' t + \mathrm{Re}A\,\mathrm{Im}\,B\cos\omega t\sin\omega' t +\\
&\quad \mathrm{Im}A\,\mathrm{Re}\,B\sin\omega t\cos\omega' t + \mathrm{Im}A\,\mathrm{Im}\,B\sin\omega t\sin\omega' t\\
&= \mathrm{Re}A\,\mathrm{Re}\,B\left[\frac{\cos(\omega+\omega')\,t + \cos(\omega-\omega')\,t}{2}\right] + \mathrm{Re}A\,\mathrm{Im}\,B\left[\frac{\sin(\omega+\omega')\,t - \sin(\omega-\omega')\,t}{2}\right] +\\
&\quad \mathrm{Im}A\,\mathrm{Re}\,B\left[\frac{\sin(\omega+\omega')\,t + \sin(\omega-\omega')\,t}{2}\right] + \mathrm{Im}A\,\mathrm{Im}\,B\left[\frac{\cos(\omega-\omega')\,t - \cos(\omega+\omega')\,t}{2}\right] \qquad (11)
\end{aligned}$$

Taking the time average will yield

$$\langle \mathrm{Re}\,[a\,(t)]\,\mathrm{Re}\,[b\,(t)]\rangle = \begin{cases} \dfrac{\mathrm{Re}A\,\mathrm{Re}\,B + \mathrm{Im}A\,\mathrm{Im}\,B}{2} = \dfrac{1}{2}\,\mathrm{Re}\,[A(\omega)\,B^*(\omega)] & \text{for } \omega = \omega' \\ \dfrac{\mathrm{Re}A\,\mathrm{Re}\,B - \mathrm{Im}A\,\mathrm{Im}\,B}{2} = \dfrac{1}{2}\,\mathrm{Re}\,[A(\omega)\,B(-\omega)] & \text{for } \omega = -\omega' \end{cases} \qquad (12)$$

Taking the time average of (9) and applying (12), we see the double integral collapses into a single integral (selecting $\omega' = \pm\omega$). Furthermore, with (4) and (6), we have

$$\langle \mathbf{f}(\xi, t)\rangle = \mu_c\,\mathrm{Re}\int \left[\frac{1-i}{\delta\,(\omega)}\right]\left[\mathbf{n}\times\mathbf{H}_{\|}\,(\omega)\right]\times\mathbf{H}^*_{\|}\,(\omega)\,e^{-2\xi/\delta(\omega)}d\omega \qquad (13)$$

Integrating ξ from $0 \to \infty$ gives the per unit area force

$$\langle \mathbf{f}(t)\rangle = -\frac{\mu_c}{2}\mathbf{n}\int \mathbf{H}_{\|}\,(\omega)\cdot\mathbf{H}^*_{\|}\,(\omega)\,d\omega = -\frac{\mu_c}{2}\mathbf{n}\left\langle \mathbf{H}_{\|}\,(t)\cdot\mathbf{H}_{\|}\,(t)\right\rangle \qquad (14)$$