1. From (11.152)

$$E_{\parallel}(t) = -\frac{ze\gamma vt}{(b^2+\gamma^2v^2t^2)^{3/2}} \qquad E_{\perp}(t) = \frac{ze\gamma b}{(b^2+\gamma^2v^2t^2)^{3/2}} \tag{1}$$

Thus the corresponding Fourier components are

$$\begin{aligned} E_{\parallel}(\omega) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{-ze\gamma vte^{i\omega t}dt}{(b^2+\gamma^2v^2t^2)^{3/2}} \qquad \text{let } s \equiv \frac{b}{\gamma v} \\ &= \frac{-ze}{\sqrt{2\pi}}\frac{1}{(\gamma v)^2}\left[\overbrace{\int_{-\infty}^{\infty}\frac{t\cos\omega t dt}{(s^2+t^2)^{3/2}}}^{0} + i\int_{-\infty}^{\infty}\frac{t\sin\omega t dt}{(s^2+t^2)^{3/2}}\right] \\ &= \frac{ize}{\sqrt{2\pi}}\frac{1}{(\gamma v)^2}\int_{-\infty}^{\infty}\frac{d}{dt}\left(\frac{1}{\sqrt{s^2+t^2}}\right)\sin\omega t dt \\ &= -\frac{ize}{\sqrt{2\pi}}\frac{\omega}{(\gamma v)^2}\int_{-\infty}^{\infty}\frac{\cos\omega t}{\sqrt{s^2+t^2}}dt \end{aligned} \tag{2}$$

$$E_{\perp}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{ze\gamma be^{i\omega t}dt}{(b^2+\gamma^2v^2t^2)^{3/2}} = \frac{ze}{\sqrt{2\pi}}\frac{b}{\gamma^2v^3}\int_{-\infty}^{\infty}\frac{\cos\omega t dt}{(s^2+t^2)^{3/2}} \tag{3}$$

From DLMF 10.32.E11 we have the integral representation of the Modified Bessel function

$$K_{\nu}(\omega s) = \frac{\Gamma\left(\nu+\frac{1}{2}\right)(2s)^{\nu}}{\sqrt{\pi}\omega^{\nu}}\int_0^{\infty}\frac{\cos\omega t dt}{(s^2+t^2)^{\nu+\frac{1}{2}}} \tag{4}$$

turning (2) and (3) into

$$E_{\parallel}(\omega) = -\frac{ize}{\sqrt{2\pi}}\frac{\omega}{(\gamma v)^2}\cdot 2K_0(\omega s)\frac{\sqrt{\pi}}{\Gamma\left(\frac{1}{2}\right)} = -\frac{ize}{\gamma vb}\sqrt{\frac{2}{\pi}}\xi K_0(\xi) \qquad \text{where } \xi \equiv \omega s = \frac{\omega b}{\gamma v} \tag{5}$$

$$E_{\perp}(\omega) = \frac{ze}{\sqrt{2\pi}}\frac{b}{\gamma^2v^3}\cdot 2K_1(\xi)\frac{\sqrt{\pi}\omega}{\Gamma\left(\frac{3}{2}\right)\cdot 2s} = \frac{ze}{bv}\sqrt{\frac{2}{\pi}}\xi K_1(\xi) \tag{6}$$

2. From the result of problem 13.2, the energy transfer is

$$\Delta E = \frac{\pi e^2|E(\omega_0)|^2}{m} = \frac{2z^2e^4}{mb^2v^2}\xi^2\left[\frac{1}{\gamma^2}K_0^2(\xi)+K_1^2(\xi)\right] \tag{7}$$

The adiabatic condition (see paragraph around (13.8)) is represented by large $b/\gamma v$, or large $\xi = b/b_{\max}$. The large argument asymptotic form of $K_{\nu}(\xi)$ is given in Jackson (3.104)

$$K_{\nu}(\xi) \to \sqrt{\frac{\pi}{2\xi}}e^{-\xi} \tag{8}$$

for which the energy transfer

$$\Delta E_{\text{adiabatic}} \approx \frac{\pi z^2e^4}{mv^2}\left(1+\frac{1}{\gamma^2}\right)\frac{\xi}{b^2}e^{-2\xi} \tag{9}$$

is exponentially small for large ξ.
When $\xi \ll 1$ or $b \ll b_{\max}$, we can use the small argument asymptotic form given in Jackson (3.103)

$$K_0(\xi) \to -\ln\frac{\xi}{2}+\gamma_{\text{Euler}} \qquad K_1(\xi) \to \frac{1}{\xi} \tag{10}$$

giving an energy transfer

$$\Delta E \approx \frac{2z^2e^4}{mb^2v^2}\xi^2\left[\frac{1}{\gamma^2}\left(-\ln\frac{\xi}{2}+\gamma_{\text{Euler}}\right)^2+\frac{1}{\xi^2}\right] \approx \frac{2z^2e^4}{mb^2v^2} \tag{11}$$

This agrees with the result of problem 13.1 where $b \gg b_{\min}$ (but for ξ to be small, we must in the mean time have $b \ll b_{\max}$.