1. By (10.101), the electric field satisfies the integral relation

$$\mathbf{E}(\mathbf{x}) = \frac{1}{2\pi}\boldsymbol{\nabla}\times\int_{\Omega}(\mathbf{n}\times\mathbf{E})\frac{e^{ikR}}{R}da' = \frac{1}{2\pi}\int_{\Omega}\boldsymbol{\nabla}\left(\frac{e^{ikR}}{R}\right)\times(\mathbf{n}\times\mathbf{E})\,da' \tag{1}$$

where Ω represents the surface of the aperture.
From (9.98), assuming $r_> = r, r_< = r'$, we have

$$\frac{e^{ikR}}{R} = 4\pi\cdot ik\sum_{l=0}^{\infty} j_l\left(kr'\right)h_l^{(1)}(kr)\sum_{m=-l}^{l}Y_{lm}^{*}\left(\theta',\phi'\right)Y_{lm}(\theta,\phi) = ik\sum_{l=0}^{\infty} j_l\left(kr'\right)h_l^{(1)}(kr)(2l+1)P_l(\cos\gamma) \tag{2}$$

where we have used the addition theorem. Note this is an *exact* expansion for e^{ikR}/R.
When $kr' \ll 1$, the small-argument expansion of the spherical Bessel function

$$j_l(x) = \frac{x^l}{(2l+1)!!}\left[1-\frac{x^2}{2(2l+3)}+\cdots\right] \tag{3}$$

ensures an accuracy up to the first order of kr' even if we only keep the $l=0, l=1$ terms in (2), i.e.,

$$\frac{e^{ikR}}{R} \approx ik\left[h_0^{(1)}(kr)+h_l^{(1)}(kr)\cdot kr'\cos\gamma\right] \qquad \text{accurate up to } O\left(kr'\right) \tag{4}$$

With the explicit forms of the spherical Hankel functions

$$h_0^{(1)}(x) = -i\frac{e^{ix}}{x} \qquad h_1^{(1)}(x) = -\frac{e^{ix}}{x}\left(1+\frac{i}{x}\right) \tag{5}$$

(4) can be written as

$$\frac{e^{ikR}}{R} \approx ik\frac{e^{ikr}}{kr}\left[-i-\left(1+\frac{i}{kr}\right)kr'\cos\gamma\right] = \frac{e^{ikr}}{r}\left[1-ik\left(\hat{\mathbf{r}}\cdot\mathbf{x}'\right)+\frac{\hat{\mathbf{r}}\cdot\mathbf{x}'}{r}\right] \tag{6}$$

of which the gradient is

$$\boldsymbol{\nabla}\left(\frac{e^{ikR}}{R}\right) \approx \boldsymbol{\nabla}\left(\frac{e^{ikr}}{r}\right)\left[1-ik\left(\hat{\mathbf{r}}\cdot\mathbf{x}'\right)+\frac{\hat{\mathbf{r}}\cdot\mathbf{x}'}{r}\right]+\frac{e^{ikr}}{r}\left[-ik\boldsymbol{\nabla}\left(\hat{\mathbf{r}}\cdot\mathbf{x}'\right)+\boldsymbol{\nabla}\left(\frac{\hat{\mathbf{r}}\cdot\mathbf{x}'}{r}\right)\right] \tag{7}$$

It is straightforward to obtain

$$\boldsymbol{\nabla}\left(\frac{e^{ikr}}{r}\right) = \frac{e^{ikr}}{r}\left(ik-\frac{1}{r}\right)\hat{\mathbf{r}} \qquad \boldsymbol{\nabla}\left(\hat{\mathbf{r}}\cdot\mathbf{x}'\right) = \frac{\mathbf{x}'-(\hat{\mathbf{r}}\cdot\mathbf{x}')\hat{\mathbf{r}}}{r} \tag{8}$$

which turns (7) into

$$\boldsymbol{\nabla}\left(\frac{e^{ikR}}{R}\right) \approx \frac{e^{ikr}}{r}\left\{\overbrace{\left(ik-\frac{1}{r}\right)\hat{\mathbf{r}}}^{A}-\underbrace{\left(ik-\frac{1}{r}\right)\frac{\mathbf{x}'}{r}}_{B}+\overbrace{\left[k^2+\frac{3}{r}\left(ik-\frac{1}{r}\right)\right]\left(\hat{\mathbf{r}}\cdot\mathbf{x}'\right)\hat{\mathbf{r}}}^{C}\right\} \tag{9}$$

It is worth emphasizing that the only approximation we have made so far is the long-wavelength approximation $kr' \ll 1$ and the "$\approx$" in (9) is accurate to the first order of kr'. In particular, in this derivation we don't need r to be in the radiation zone (except that $r_> = r, r_< = r'$ when applying (9.98)).
The contribution of the A term of (9) to the integral (1) is

$$\mathbf{E}_A = \frac{1}{2\pi}\frac{e^{ikr}}{r}\left(ik-\frac{1}{r}\right)\hat{\mathbf{r}}\times\int_{\Omega}\mathbf{n}\times\mathbf{E}da' \tag{10}$$

Comparing this to the exact magnetic dipole field (9.36)

$$\mathbf{E}_{\mathbf{m}} = -\frac{Z_0}{4\pi}k^2\frac{e^{ikr}}{r}\left(1-\frac{1}{ikr}\right)(\hat{\mathbf{r}}\times\mathbf{m}) \tag{11}$$

we can immediately identify the effective magnetic dipole moment as

$$\mathbf{m} = \frac{2}{i\omega\mu}\int_\Omega \mathbf{n}\times\mathbf{E}da' \tag{12}$$

Following (9.31), we see that

$$\left(\hat{\mathbf{r}}\cdot\mathbf{x}'\right)(\mathbf{n}\times\mathbf{E}) = \overbrace{\left\{\frac{(\hat{\mathbf{r}}\cdot\mathbf{x}')(\mathbf{n}\times\mathbf{E})+[\hat{\mathbf{r}}\cdot(\mathbf{n}\times\mathbf{E})]\mathbf{x}'}{2}\right\}}^{\text{symmetric part}} + \underbrace{\left\{\frac{[\mathbf{x}'\times(\mathbf{n}\times\mathbf{E})]\times\hat{\mathbf{r}}}{2}\right\}}_{\text{asymmetric part}} \tag{13}$$

If we further define

$$\mathbf{p} \equiv \epsilon\int_\Omega \mathbf{x}'\times(\mathbf{n}\times\mathbf{E})\,da' = \epsilon\mathbf{n}\int_\Omega \mathbf{x}'\cdot\mathbf{E}da' \tag{14}$$

we see that the contribution of the B term and the asymmetric part of the C term of (9) to the integral (1) is

$$\begin{aligned}\mathbf{E}_{B+C_{\text{asym}}} &= \frac{1}{4\pi\epsilon}\frac{e^{ikr}}{r}\left\{-2\left(\frac{ik}{r}-\frac{1}{r^2}\right)\mathbf{p}+\left[k^2+3\left(\frac{ik}{r}-\frac{1}{r^2}\right)\right]\hat{\mathbf{r}}\times(\mathbf{p}\times\hat{\mathbf{r}})\right\}\\ &= \frac{1}{4\pi\epsilon}\frac{e^{ikr}}{r}\left\{k^2\hat{\mathbf{r}}\times(\mathbf{p}\times\hat{\mathbf{r}})+\left(\frac{ik}{r}-\frac{1}{r^2}\right)[\mathbf{p}-3(\hat{\mathbf{r}}\cdot\mathbf{p})\hat{\mathbf{r}}]\right\}\end{aligned} \tag{15}$$

which is equivalent to the exact electric dipole field (9.18)

$$\mathbf{E}_{\mathbf{p}} = \frac{1}{4\pi\epsilon}\left\{k^2(\hat{\mathbf{r}}\times\mathbf{p})\times\hat{\mathbf{r}}\frac{e^{ikr}}{r}+[3\hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\mathbf{p})-\hat{\mathbf{p}}]\left(\frac{1}{r^3}-\frac{ik}{r^3}\right)e^{ikr}\right\} \tag{16}$$

justifying (14) as the effective electric dipole moment of the aperture.

To see the effect of the symmetric part of C term, let's identify $\mathbf{n}\times\mathbf{E}$ as the surface *magnetic current* $\mathbf{J}_m$. We can do this because (1) is the E-M dual of the usual *electric current* radiation relation (see (9.3))

$$\mathbf{B} = \nabla\times\mathbf{A} = \frac{\mu}{4\pi}\nabla\times\int_\Omega \mathbf{J}_e\left(\mathbf{x}'\right)\frac{e^{ikR}}{R}da' \tag{17}$$

Thus the aperture problem at hand is the E-M dual of the radiation problem – due to electric current – discussed in section 9.2 and 9.3. Just as the symmetric part of (9.31) is attributed to the electric quadrupole moment (see (9.37), (9.42)), term C's symmetric part will be attributed to the aperture's magnetic quadrupole moment.

2. From the Maxwell equation $\nabla\times\mathbf{E} = i\omega\mathbf{B}$, we have

$$\begin{aligned}i\omega\int_\Omega \mathbf{x}'(\mathbf{n}\cdot\mathbf{B})\,da' &= \hat{\mathbf{e}}_l\int_\Omega x_l'\left[\mathbf{n}\cdot\left(\nabla'\times\mathbf{E}\right)\right]da' = \hat{\mathbf{e}}_l\int_\Omega x_l'\nabla'\cdot(\mathbf{E}\times\mathbf{n})da'\\ &= \hat{\mathbf{e}}_l\int_\Omega \nabla'\cdot\left(x_l'\mathbf{E}\times\mathbf{n}\right)da' - \hat{\mathbf{e}}_l\int_\Omega \hat{\mathbf{e}}_l\cdot(\mathbf{E}\times\mathbf{n})\,da'\\ &= \hat{\mathbf{e}}_l\oint_{\partial\Omega} x_l'(\mathbf{E}\times\mathbf{n})\cdot\mathbf{n}_{\partial\Omega}dl' + \int_\Omega \mathbf{n}\times\mathbf{E}da'\end{aligned} \tag{18}$$

where in the last step, we have used the 2D divergence theorem and $\mathbf{n}_{\partial\Omega}$ is the outward normal (pointing from aperture to the conductor) of the boundary edge of the aperture. One may argue that $\mathbf{E}\times\mathbf{n}$ is tangential to the screen plane thus must vanish at the edge, but when the screen is infinitely thin, it may be singular (see (3.186)). Instead, we can see that when the screen had a finite thickness, $\mathbf{E}$ must be normal to the conductor's surface. Thus when we let the thickness go to zero, $\mathbf{E}$ must remain parallel (or antiparallel) to $\mathbf{n}_{\partial\Omega}$, making the entire $(\mathbf{E}\times\mathbf{n})\times\mathbf{n}_{\partial\Omega}$ term vanish. Thus we obtain the alternative form of the effective magnetic dipole moment

$$\mathbf{m} = \frac{2}{\mu}\int_\Omega \mathbf{x}'(\mathbf{n}\cdot\mathbf{B})\,da' \tag{19}$$