We have to be extremely careful when dealing with completeness relations in an arbitrary range. Usually it's the safest to go back to the definition of the δ-function.

For this, let

$$U_m(\phi) = A_m \sin\left(\frac{m\pi\phi}{\beta}\right) \qquad V_m(\phi) = B_m \cos\left(\frac{m\pi\phi}{\beta}\right) \tag{1}$$

be the complete set of orthonormal functions over the range $[0, \beta]$.

Normality requires

$$\begin{aligned} \int_0^\beta A_m^2 \sin^2\left(\frac{m\pi\phi}{\beta}\right) d\phi = 1 & \qquad \Longrightarrow \\ A_m^2 \frac{\beta}{m\pi} \int_0^{m\pi} \frac{1-\cos 2x}{2} dx = 1 & \qquad \Longrightarrow \\ A_m^2 = \frac{2}{\beta} & \end{aligned} \tag{2}$$

By completeness, arbitrary $f(\phi)$ over $[0, \beta]$ must have the expansion

$$f(\phi) = \sum_m a_m U_m(\phi) + b_m V_m(\phi) \tag{3}$$

If our boundary condition requires $f(0) = f(\beta) = 0$, then all the b_m's will vanish.

The coefficients for U_m's are

$$a_m = \int_0^\beta f(\phi')U_m(\phi')d\phi' \tag{4}$$

by which we have

$$\begin{aligned} f(\phi) &= \sum_m \left[\int_0^\beta f(\phi')U_m(\phi')d\phi'\right] U_m(\phi) \\ &= \int_0^\beta f(\phi')d\phi' \left[\sum_m A_m^2 \sin\left(\frac{m\pi\phi}{\beta}\right)\sin\left(\frac{m\pi\phi'}{\beta}\right)\right] \\ &= \int_0^\beta f(\phi')d\phi' \left[\sum_m \frac{2}{\beta} \sin\left(\frac{m\pi\phi}{\beta}\right)\sin\left(\frac{m\pi\phi'}{\beta}\right)\right] \end{aligned} \tag{5}$$

Therefore we must identify the content inside the square bracket as $\delta(\phi - \phi')$:

$$\frac{2}{\beta}\sum_m \sin\left(\frac{m\pi\phi}{\beta}\right)\sin\left(\frac{m\pi\phi'}{\beta}\right) = \delta(\phi - \phi') \tag{6}$$