1. By (6.47)

$$\begin{aligned}
\Psi(\mathbf{x},t) &= \int \frac{[f(\mathbf{x}',t')]_{\text{ret}}}{|\mathbf{x}-\mathbf{x}'|} d^3x' \\
&= \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dz' \delta(x')\delta(y')\delta(t') \frac{\delta\left[t' - \left(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)\right]}{|\mathbf{x}-\mathbf{x}'|} \\
&= \int_{-\infty}^{\infty} dz' \frac{\delta\left[t - \frac{\sqrt{\rho^2+(z-z')^2}}{c}\right]}{\sqrt{\rho^2+(z-z')^2}}
\end{aligned} \tag{1}$$

Now let

$$z'' \equiv z' - z, \qquad u \equiv \frac{\sqrt{\rho^2+z''^2}}{c} \qquad \Longrightarrow \qquad z'' = \pm\sqrt{c^2u^2-\rho^2} \qquad \Longrightarrow \qquad dz'' = \pm\frac{c^2u}{\sqrt{c^2u^2-\rho^2}} \tag{2}$$

The integrand in (1) is even in $z''$, so

$$\begin{aligned}
\Psi(\mathbf{x},t) &= 2\int_0^{\infty} dz'' \frac{\delta\left(t - \frac{\sqrt{\rho^2+z''^2}}{c}\right)}{\sqrt{\rho^2+z''^2}} \\
&= 2\int_{\rho/c}^{\infty} \frac{\delta(t-u)}{cu} \cdot \frac{c^2u}{\sqrt{c^2u^2-\rho^2}} du \\
&= 2c\int_{\rho/c}^{\infty} \frac{\delta(t-u)}{\sqrt{c^2u^2-\rho^2}}
\end{aligned} \tag{3}$$

We see that the $\delta$ function has support in the integral range only if $t > \rho/c$, which yields $2c/\sqrt{c^2t^2-\rho^2}$, and for smaller $t$, the integral yields zero. This result can be summarized using the step function

$$\Psi(\mathbf{x},t) = \frac{2c\Theta(ct-\rho)}{\sqrt{c^2t^2-\rho^2}} \tag{4}$$

2. The sheet source function is

$$f(\mathbf{x}',t') = \delta(x')\delta(t') \tag{5}$$

Thus by (6.47)

$$\begin{aligned}
\Psi(\mathbf{x},t) &= \int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dz' \int_{-\infty}^{\infty} dx' \delta(x')\delta(t') \frac{\delta\left[t' - \left(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)\right]}{|\mathbf{x}-\mathbf{x}'|} \\
&= \int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dz' \frac{\delta\left[t - \frac{\sqrt{x^2+(y-y')^2+(z-z')^2}}{c}\right]}{\sqrt{x^2+(y-y')^2+(z-z')^2}} && \text{let } (y',z') = (y,z)+(\rho,\phi) \\
&= \int_0^{2\pi} d\phi \int_0^{\infty} \rho d\rho \frac{\delta\left(t - \frac{\sqrt{x^2+\rho^2}}{c}\right)}{\sqrt{x^2+\rho^2}} && \text{let } u \equiv \frac{\sqrt{x^2+\rho^2}}{c} \\
&= 2\pi\int_{|x|/c}^{\infty} \sqrt{c^2u^2-x^2}\frac{\delta(t-u)}{cu} \cdot \frac{c^2u}{\sqrt{c^2u^2-x^2}} du \\
&= 2\pi c\Theta(ct-|x|)
\end{aligned} \tag{6}$$