1. By (9.116), the free space solution for TE and TM modes are

$$\mathbf{E}_{lm}^{\mathrm{TE}} = Z_0 g_l(kr)\mathbf{L}Y_{lm}(\theta,\phi) \qquad \mathbf{H}_{lm}^{\mathrm{TE}} = -\frac{i}{kZ_0}\nabla\times\mathbf{E}_{lm}^{\mathrm{TE}} \tag{1}$$

$$\mathbf{H}_{lm}^{\mathrm{TM}} = f_l(kr)\mathbf{L}Y_{lm}(\theta,\phi) \qquad \mathbf{E}_{lm}^{\mathrm{TM}} = \frac{iZ_0}{k}\nabla\times\mathbf{H}_{lm}^{\mathrm{TM}} \tag{2}$$

where

$$(f|g)_l(kr) = A_l^{(1)}h_l^{(1)}(kr) + A_l^{(2)}h_l^{(2)}(kr) \tag{3}$$

Since we are dealing with the radiation of a conductor ball, we expect the $r\to\infty$ behavior to be an outgoing wave (as opposed to an incoming wave), so we choose the Hankel function of the first kind $h_l^{(1)}(kr)$ for the radial functions. As is done in problem 9.22 and 9.23, we now express the solution in terms of vector spherical harmonics

$$\mathbf{E}_{lm}^{\mathrm{TE}} = -iZ_0H_0h_l^{(1)}(kr)\mathbf{\Phi}_{lm} \qquad \mathbf{H}_{lm}^{\mathrm{TE}} = \frac{H_0}{k}\left\{\frac{l(l+1)}{r}h_l^{(1)}(kr)\mathbf{Y}_{lm} + \frac{1}{r}\frac{d\left[rh_l^{(1)}(kr)\right]}{dr}\mathbf{\Psi}_{lm}\right\} \tag{4}$$

$$\mathbf{H}_{lm}^{\mathrm{TM}} = -iH_0h_l^{(1)}(kr)\mathbf{\Phi}_{lm} \qquad \mathbf{E}_{lm}^{\mathrm{TM}} = -\frac{Z_0H_0}{k}\left\{\frac{l(l+1)}{r}h_l^{(1)}(kr)\mathbf{Y}_{lm} + \frac{1}{r}\frac{d\left[rh_l^{(1)}(kr)\right]}{dr}\mathbf{\Psi}_{lm}\right\} \tag{5}$$

The boundary conditions are such that electric field is normal and magnetic field is tangential at the surface of the ball. This is achieved by setting

$$h_l^{(1)}(ka) = 0 \qquad \text{for TE mode} \tag{6}$$

$$\left.\frac{d}{dr}\left[rh_l^{(1)}(kr)\right]\right|_{r=a} = 0 \qquad \text{for TM mode} \tag{7}$$

Thus (6) and (7) are the characteristic equation for the (complex) wave number k since $h_l^{(1)}(z)$ takes complex value and it does not have real zeroes. In fact, all zeroes of $h_l^{(1)}(z)=0$ have negative imaginary parts, so do the zeroes of $d\left[zh_l^{(1)}(z)\right]/dz=0$. *We don't have a proof for these claims except for the explicit calculations for $l=1$ and $l=2$ in the next part.*

If we write the zero of (6) or (7) as real and imaginary parts

$$z_{ln} = k_{ln}a = \gamma_{ln} - i\eta_{ln} \qquad \eta_{ln} > 0 \tag{8}$$

then the time dependency becomes

$$e^{-ick_{ln}t} = e^{-ic\gamma_{ln}t/a}e^{-c\eta_{ln}t/a} \tag{9}$$

where we can clearly see the decaying effect of the negative imaginary part, which is typical for a radiation field extending into infinity.

2. Explicitly from (9.87)

$$h_1^{(1)}(z) = -\frac{e^{iz}}{z}\left(1+\frac{i}{z}\right) \qquad \frac{d\left[zh_1^{(1)}(z)\right]}{dz} = e^{iz}\left(\frac{i}{z^2}+\frac{1}{z}-i\right) \tag{10}$$

$$h_2^{(1)}(z) = \frac{ie^{iz}}{z}\left(1+\frac{3i}{z}-\frac{3}{z^2}\right) \qquad \frac{d\left[zh_2^{(1)}(z)\right]}{dz} = ie^{iz}\left(\frac{6}{z^3}-\frac{6i}{z^2}-\frac{3}{z}+i\right) \tag{11}$$

Using form (8), the wavelength and the energy decay time are

$$|\lambda_{lmn}| = \frac{2\pi}{|\gamma_{ln}|}\cdot a \qquad \tau_{lmn} = \frac{1}{2\eta_{ln}}\cdot\frac{a}{c} \tag{12}$$

It is straightforward to calculate the zeroes of (10) and (11),

$$
\begin{aligned}
&\text{TE mode } l=1: && z_{1m1}=-i && \Longrightarrow && \lambda_{1m1}=\infty && \tau_{1m1}=\frac{1}{2}\frac{a}{c} && (13)\\
&\text{TM mode } l=1: && z'_{1m(1|2)}=\pm\frac{\sqrt{3}}{2}-\frac{i}{2} && \Longrightarrow && \left|\lambda_{1m(1|2)}\right|=\frac{4\pi}{\sqrt{3}}a && \tau_{1m(1|2)}=\frac{a}{c} && (14)\\
&\text{TE mode } l=2: && z_{2m(1|2)}=\pm\frac{\sqrt{3}}{2}-\frac{3i}{2} && \Longrightarrow && \left|\lambda_{2m(1|2)}\right|=\frac{4\pi}{\sqrt{3}}a && \tau_{2m(1|2)}=\frac{1}{3}\frac{a}{c} && (15)\\
&\text{TM mode } l=2: && z'_{2m1}\approx-1.596072i && \Longrightarrow && \lambda_{2m1}=\infty && \tau_{2m1}\approx 0.3133\frac{a}{c} && \\
& && z'_{2m(2|3)}\approx\pm 1.807339-0.701964i && \Longrightarrow && \left|\lambda_{2m(2|3)}\right|\approx 3.4765a && \tau_{2m(2|3)}\approx 0.7123\frac{a}{c} && (16)
\end{aligned}
$$