1. From (10.90) (a.k.a, the Stratton-Chu integral, or the vector Kirchhoff integral)

$$\mathbf{E}(\mathbf{x}) = \oint_S \left[i\omega \left(\mathbf{n}' \times \mathbf{B}\right) G + \left(\mathbf{n}' \times \mathbf{E}\right) \times \nabla' G + \left(\mathbf{n}' \cdot \mathbf{E}\right) \nabla' G \right] da' \qquad G = \frac{1}{4\pi} \frac{e^{ikR}}{R} \tag{1}$$

we can use Kirchhoff approximation by integrating over the aperture and take the integrand to be the unperturbed incident wave

$$\mathbf{E}_{\text{inc}} = E_0 \left(\cos\alpha \boldsymbol{\epsilon}_1 - \sin\alpha \boldsymbol{\epsilon}_3\right) e^{ik(z\cos\alpha + x\sin\alpha)} \tag{2}$$

$$\mathbf{B}_{\text{inc}} = \frac{\hat{\mathbf{k}}_0 \times \mathbf{E}_{\text{inc}}}{c} = \frac{E_0}{c} \left(\sin\alpha \boldsymbol{\epsilon}_1 + \cos\alpha \boldsymbol{\epsilon}_3\right) \times \left(\cos\alpha \boldsymbol{\epsilon}_1 - \sin\alpha \boldsymbol{\epsilon}_3\right) e^{ik(z\cos\alpha + x\sin\alpha)} = \frac{E_0}{c} \boldsymbol{\epsilon}_2 e^{ik(z\cos\alpha + x\sin\alpha)} \tag{3}$$

Also with the radiation-zone approximation

$$G = \frac{1}{4\pi} \frac{e^{ikR}}{R} \approx \frac{1}{4\pi} \frac{e^{ikr}}{r} e^{-i\mathbf{k}\cdot\mathbf{x}'} \qquad \Longrightarrow \qquad \nabla' G = -i\mathbf{k} G \tag{4}$$

we see (1) turns into

$$\mathbf{E}(\mathbf{x}) \approx \frac{ie^{ikr}}{4\pi r} E_0 \int_{\text{aperture}} \left[-k\boldsymbol{\epsilon}_1 + \cos\alpha \left(\mathbf{k} \times \boldsymbol{\epsilon}_2\right) + \sin\alpha \mathbf{k} \right] e^{ikx'\sin\alpha} e^{-i\mathbf{k}\cdot\mathbf{x}'} da' \tag{5}$$

We now follow the same argument in the paragraph below equation (10.91): since $\mathbf{E}(\mathbf{x})$ is transverse to $\mathbf{k}$, the integral from the component parallel to $\mathbf{k}$ in the first term must cancel that from the third term. Using the vector identity

$$\boldsymbol{\epsilon}_1 = \left(\boldsymbol{\epsilon}_1 \cdot \hat{\mathbf{k}}\right) \hat{\mathbf{k}} - \hat{\mathbf{k}} \times \left(\hat{\mathbf{k}} \times \boldsymbol{\epsilon}_1\right) \tag{6}$$

we turn (5) into

$$\mathbf{E}(\mathbf{x}) \approx \frac{ie^{ikr}}{4\pi r} E_0 \int_{\text{aperture}} \left[\frac{\mathbf{k} \times \left(\mathbf{k} \times \boldsymbol{\epsilon}_1\right)}{k} + \cos\alpha \left(\mathbf{k} \times \boldsymbol{\epsilon}_2\right) \right] e^{ikx'\sin\alpha} e^{-i\mathbf{k}\cdot\mathbf{x}'} da' \tag{7}$$

Expanding $\mathbf{k}$ as $\mathbf{k} = \left(\mathbf{k} \cdot \boldsymbol{\epsilon}_1\right) \boldsymbol{\epsilon}_1 + \left(\mathbf{k} \cdot \boldsymbol{\epsilon}_2\right) \boldsymbol{\epsilon}_2 + \left(\mathbf{k} \cdot \boldsymbol{\epsilon}_3\right) \boldsymbol{\epsilon}_3$, we have

$$\frac{\mathbf{k} \times \left(\mathbf{k} \times \boldsymbol{\epsilon}_1\right)}{k} = \frac{\mathbf{k} \times \left[-\left(\mathbf{k} \cdot \boldsymbol{\epsilon}_2\right) \boldsymbol{\epsilon}_3 + \left(\mathbf{k} \cdot \boldsymbol{\epsilon}_3\right) \boldsymbol{\epsilon}_2 \right]}{k} = \cos\theta \left(\mathbf{k} \times \boldsymbol{\epsilon}_2\right) - \sin\theta \sin\phi \left(\mathbf{k} \times \boldsymbol{\epsilon}_3\right) \tag{8}$$

which gives the approximated diffracted field

$$\mathbf{E}(\mathbf{x}) \approx \frac{ie^{ikr}}{2\pi r} E_0 \int_{\text{aperture}} \left[\left(\frac{\cos\theta + \cos\alpha}{2} \right) \left(\mathbf{k} \times \boldsymbol{\epsilon}_2\right) - \frac{\sin\theta \sin\phi}{2} \left(\mathbf{k} \times \boldsymbol{\epsilon}_3\right) \right] e^{ikx'\sin\alpha} e^{-i\mathbf{k}\cdot\mathbf{x}'} da' \tag{9}$$

as claimed.

2. From (9) and the procedure following (10.112), we have

$$\mathbf{E}(\mathbf{x}) = \frac{ie^{ikr}}{r} a^2 E_0 \left[\left(\frac{\cos\theta + \cos\alpha}{2} \right) \left(\mathbf{k} \times \boldsymbol{\epsilon}_2\right) - \frac{\sin\theta \sin\phi}{2} \left(\mathbf{k} \times \boldsymbol{\epsilon}_3\right) \right] \frac{J_1(ka\xi)}{ka\xi} \tag{10}$$

giving the angular distribution of power

$$\frac{dP}{d\Omega} = P_i \frac{(ka)^2}{4\pi} \frac{1}{\cos\alpha} \overbrace{\left| \left(\frac{\cos\theta + \cos\alpha}{2} \right) \left(\hat{\mathbf{k}} \times \boldsymbol{\epsilon}_2\right) - \frac{\sin\theta \sin\phi}{2} \left(\hat{\mathbf{k}} \times \boldsymbol{\epsilon}_3\right) \right|^2}^{\equiv f(\alpha,\theta,\phi)} \left| \frac{2J_1(ka\xi)}{ka\xi} \right|^2 \tag{11}$$

where

$$\begin{aligned} f(\alpha,\theta,\phi) &= \left| \left(\frac{\cos\theta + \cos\alpha}{2} \right) \left(\hat{\mathbf{k}} \times \boldsymbol{\epsilon}_2\right) - \frac{\sin\theta \sin\phi}{2} \left(\hat{\mathbf{k}} \times \boldsymbol{\epsilon}_3\right) \right|^2 \\ &= \left[\left(\frac{\cos\theta + \cos\alpha}{2} \right) \cos\theta + \frac{\sin^2\theta \sin^2\phi}{2} \right]^2 + \\ &\quad \left[\left(\frac{\cos\theta + \cos\alpha}{2} \right) \sin\theta \cos\phi \right]^2 + \left(\frac{\sin^2\theta \sin\phi \cos\phi}{2} \right)^2 \end{aligned} \tag{12}$$

Below are the plots of the angular distribution of power for the scalar approximation (10.119), Smythe-Kirchhoff approximation (10.114) and the approximation based on the Stratton-Chu equation (11). The incident angle is $\alpha = \pi/4$. The upper diagram is viewed from $y-$ direction, and the lower diagram (enlarged 3 times) is viewed from $x+$ direction. For the upper diagram ($\phi = 0, \pi$), the scalar approximation (10.119) and the distribution based on Stratton-Chu (11) are identical function of θ. For the lower diagram ($\phi = \pi/2, 3\pi/2$), they are very close to each other (but not identical).