

We want to express the electric field so its transverse component is circularly polarized, i.e.

$$\mathbf{E}_{\text{trans}}(\mathbf{x}, t) = E_0(x, y)(\hat{\mathbf{e}}_1 \pm i\hat{\mathbf{e}}_2)e^{ikz-i\omega t} \quad (1)$$

However if this is the whole thing, to satisfy $\nabla \cdot \mathbf{E} = 0$, we must have

$$\frac{\partial E_0(x, y)}{\partial x} \pm i \frac{\partial E_0(x, y)}{\partial y} = 0 \quad (2)$$

which means the amplitude E_0 is a constant for all x, y , which violates the finite extent assumption.

This contradiction means the electric field must have a longitudinal component, i.e.,

$$\mathbf{E}(\mathbf{x}, t) = [E_0(x, y)(\hat{\mathbf{e}}_1 \pm i\hat{\mathbf{e}}_2) + F(x, y)\hat{\mathbf{e}}_3]e^{ikz-i\omega t} \quad (3)$$

Then $\nabla \cdot \mathbf{E} = 0$ requires

$$\frac{\partial E_0(x, y)}{\partial x} \pm i \frac{\partial E_0(x, y)}{\partial y} + ikF(x, y) = 0 \quad \implies \quad F(x, y) = \frac{i}{k} \left(\frac{\partial E_0}{\partial x} \pm i \frac{\partial E_0}{\partial y} \right) \quad (4)$$

Moreover, $-\partial \mathbf{B} / \partial t = \nabla \times \mathbf{E}$ implies

$$\begin{aligned} i\omega \mathbf{B} &= \nabla \times \mathbf{E} = e^{ikz-i\omega t} \left\{ \hat{\mathbf{e}}_1 \left[\frac{\partial F}{\partial y} - ik(\pm iE_0) \right] + \hat{\mathbf{e}}_2 \left(ikE_0 - \frac{\partial F}{\partial x} \right) + \hat{\mathbf{e}}_3 \left[\frac{\partial (\pm iE_0)}{\partial x} - \frac{\partial E_0}{\partial y} \right] \right\} \\ &\approx e^{ikz-i\omega t} \left[\pm kE_0\hat{\mathbf{e}}_1 + ikE_0\hat{\mathbf{e}}_2 \pm i \left(\frac{\partial E_0}{\partial x} \pm i \frac{\partial E_0}{\partial y} \right) \hat{\mathbf{e}}_3 \right] \\ &= \pm k [E_0(\hat{\mathbf{e}}_1 \pm i\hat{\mathbf{e}}_2) + F\hat{\mathbf{e}}_3] e^{ikz-i\omega t} = \pm k \mathbf{E} \end{aligned} \quad (5)$$

where the approximation ignores the second order space derivatives $\partial F / \partial x, \partial F / \partial y$.

This gives

$$\mathbf{B} \approx \mp \frac{ik}{\omega} \mathbf{E} = \mp i \sqrt{\mu\epsilon} \mathbf{E} \quad (6)$$