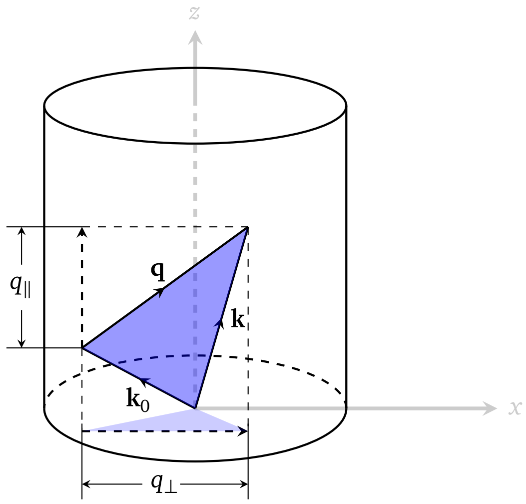


1. Let the axis of the cylinder be along the z direction, and choose x axis so that it is parallel with $\mathbf{q}_\perp$ (see diagram above), where $\mathbf{q}=\mathbf{k}-\mathbf{k}_0$.

   To the first order of $\delta\epsilon/\epsilon$, we can use Born approximation (10.31) for the outgoing polarization $\boldsymbol{\epsilon}$ and initial polarization $\boldsymbol{\epsilon}_0$,

$$\begin{aligned}\frac{\boldsymbol{\epsilon}^*\cdot\mathbf{A}_{\mathrm{sc}}^{(1)}}{D^{(0)}} &= \frac{k^2}{4\pi}\int d^3x e^{i\mathbf{q}\cdot\mathbf{x}}\left(\boldsymbol{\epsilon}^*\cdot\boldsymbol{\epsilon}_0\right)\left(\frac{\delta\epsilon}{\epsilon}\right)\\ &= \frac{k^2}{4\pi}\left(\boldsymbol{\epsilon}^*\cdot\boldsymbol{\epsilon}_0\right)\left(\frac{\delta\epsilon}{\epsilon}\right)\int_0^L dz e^{iq_\parallel z}\int_0^a \rho d\rho\int_0^{2\pi} d\phi e^{iq_\perp\rho\cos\phi} \qquad \text{see steps from (10.112) to (10.113)}\\ &= \frac{k^2}{2}\left(\boldsymbol{\epsilon}^*\cdot\boldsymbol{\epsilon}_0\right)\left(\frac{\delta\epsilon}{\epsilon}\right)\cdot\left(\frac{e^{iq_\parallel L}-1}{iq_\parallel}\right)\cdot a^2\frac{J_1(q_\perp a)}{q_\perp a} \end{aligned} \tag{1}$$

   This gives

$$\begin{aligned}\frac{d\sigma}{d\Omega}(\boldsymbol{\epsilon}_0,\boldsymbol{\epsilon}) = \left|\frac{\boldsymbol{\epsilon}^*\cdot\mathbf{A}_{\mathrm{sc}}^{(1)}}{D^{(0)}}\right|^2 &= \frac{k^4a^4}{4}\left|\boldsymbol{\epsilon}^*\cdot\boldsymbol{\epsilon}_0\right|^2\left(\frac{\delta\epsilon}{\epsilon}\right)^2\left|\frac{2\sin\left(q_\parallel L/2\right)}{q_\parallel}\cdot\frac{J_1(q_\perp a)}{q_\perp a}\right|^2\\ &= \frac{k^4a^4L^2}{16}\left|\boldsymbol{\epsilon}^*\cdot\boldsymbol{\epsilon}_0\right|^2\left(\frac{\delta\epsilon}{\epsilon}\right)^2\left[\frac{\sin\left(q_\parallel L/2\right)}{q_\parallel L/2}\cdot\frac{2J_1(q_\perp a)}{q_\perp a}\right]^2 \end{aligned} \tag{2}$$

   Summing over all outgoing polarizations and averaging over all initial polarizations will turn $\left|\boldsymbol{\epsilon}^*\cdot\boldsymbol{\epsilon}_0\right|^2$ into $\left(1+\cos^2\theta\right)/2$ (see (10.10)), where θ is the angle between $\mathbf{k}$ and $\mathbf{k}_0$. Thus the total differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{k^4a^4L^2}{32}\left(1+\cos^2\theta\right)\left(\frac{\delta\epsilon}{\epsilon}\right)^2\left[\frac{\sin\left(q_\parallel L/2\right)}{q_\parallel L/2}\cdot\frac{2J_1(q_\perp a)}{q_\perp a}\right]^2 \tag{3}$$

2. With varying orientations of the cylinder, we keep $\mathbf{q}$ as the z direction in this part, and let the cylinder's axis be described by the polar angle β and azimuthal angle γ, then (3) can be rewritten as

$$\frac{d\sigma}{d\Omega}(\beta,\gamma) = \frac{k^4a^4L^2}{32}\left(1+\cos^2\theta\right)\left(\frac{\delta\epsilon}{\epsilon}\right)^2\left[\frac{\sin(qL\cos\beta/2)}{qL\cos\beta/2}\cdot\frac{2J_1(qa\sin\beta)}{qa\sin\beta}\right]^2 \tag{4}$$

   When $ka\ll 1$, we have $qa=2ka\sin(\theta/2)\ll 1$. For small argument $J_1(x)\to x/2$, so

$$\frac{d\sigma}{d\Omega}(\beta,\gamma) \approx \frac{k^4a^4L^2}{32}\left(1+\cos^2\theta\right)\left(\frac{\delta\epsilon}{\epsilon}\right)^2\left[\frac{\sin(qL\cos\beta/2)}{qL\cos\beta/2}\right]^2 \tag{5}$$

   Averaging over $\beta\in[0,\pi],\gamma\in[0,2\pi]$, we have

$$\left\langle\frac{d\sigma}{d\Omega}\right\rangle \approx \frac{k^4a^4L^2}{32}\left(1+\cos^2\theta\right)\left(\frac{\delta\epsilon}{\epsilon}\right)^2\frac{\overbrace{\int_0^\pi\left[\frac{\sin(qL\cos\beta/2)}{qL\cos\beta/2}\right]^2\sin\beta d\beta}^{I}\int_0^{2\pi}d\gamma}{4\pi} \tag{6}$$

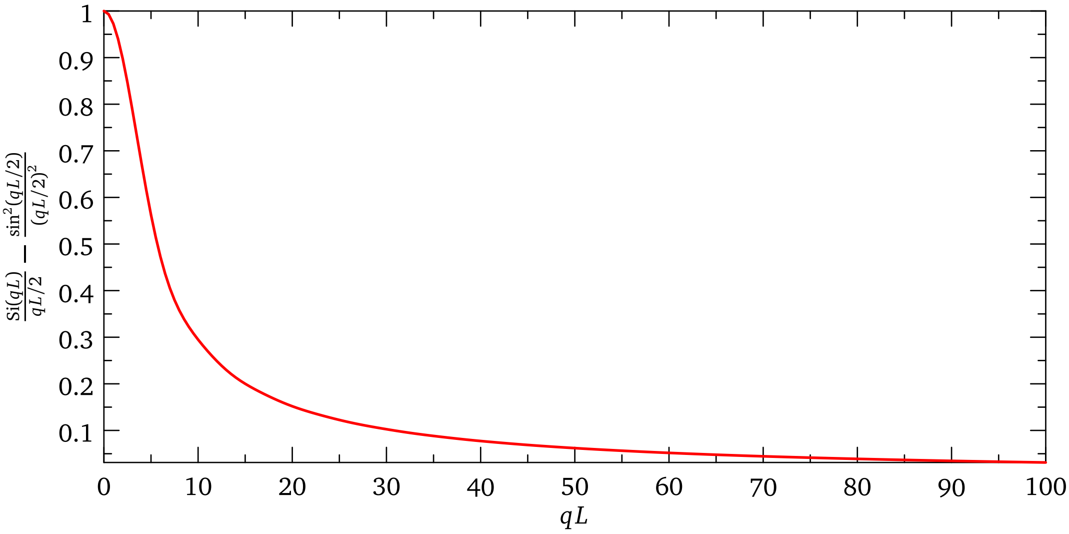
The first integral on the numerator is evaluated explicitly as follows:

$$
\begin{aligned}
I &= \int_0^\pi \frac{\sin^2(qL\cos\beta/2)}{(qL\cos\beta/2)^2}\sin\beta\, d\beta & \text{let } t \equiv qL\cos\beta/2 \\
&= \frac{2}{qL}\int_{-qL/2}^{qL/2}\frac{\sin^2 t}{t^2}dt = \frac{2}{qL}\cdot 2\int_0^{qL/2}\sin^2 t\frac{d(-1/t)}{dt}dt \\
&= \frac{2}{qL}\cdot 2\left(-\frac{\sin^2 t}{t}\bigg|_0^{qL/2} + \int_0^{qL/2}\frac{\sin 2t}{t}dt\right) & \text{note } \lim_{t\to 0}\frac{\sin^2 t}{t} = 0 \\
&= \frac{2}{qL}\cdot 2\left[-\frac{\sin^2(qL/2)}{qL/2} + \mathrm{Si}(qL)\right] = 2\left[\frac{\mathrm{Si}(qL)}{qL/2} - \frac{\sin^2(qL/2)}{(qL/2)^2}\right] & (7)
\end{aligned}
$$

Plugging this back to (6) gives the desired result

$$\left\langle\frac{d\sigma}{d\Omega}\right\rangle = \frac{k^4a^4L^2}{32}\left(1+\cos^2\theta\right)\left(\frac{\delta\epsilon}{\epsilon}\right)^2\left[\frac{\mathrm{Si}(qL)}{qL/2} - \frac{\sin^2(qL/2)}{(qL/2)^2}\right] \tag{8}$$

3. The plot of the square-bracketed term with varying qL is shown below (not sure why the problem asks to plot as function of q^2L^2).



When $kL \ll 1$, the square-bracketed term is very close to unity, giving

$$\left\langle\frac{d\sigma}{d\Omega}\right\rangle \approx \frac{k^4a^4L^2}{32}\left(1+\cos^2\theta\right)\left(\frac{\delta\epsilon}{\epsilon}\right)^2 \tag{9}$$

Comparing this with the dielectric sphere result (10.10)

$$\frac{d\sigma}{d\Omega} = \frac{k^4a^6}{2}\left(\frac{\epsilon_r-1}{\epsilon_r+2}\right)^2\left(1+\cos^2\theta\right) \approx \frac{k^4a^6}{18}\left(1+\cos^2\theta\right)\left(\frac{\delta\epsilon}{\epsilon}\right)^2 \tag{10}$$

we see that there is a factor of $(9/16)(L/a)^2$ which can be attributed to the shape difference (i.e., cylinder v.s. sphere). When $kL \gg 1$ while keeping $ka \ll 1$, note that $\mathrm{Si}(x) \approx \pi/2 - \cos x/x$ for $x \gg 1$, we can estimate the total cross section by integrating (8)

$$
\begin{aligned}
\sigma = \int d\Omega\left\langle\frac{d\sigma}{d\Omega}\right\rangle &\approx \frac{k^4a^4L^2}{32}\left(\frac{\delta\epsilon}{\epsilon}\right)^2\int_0^{2\pi}d\phi\int_0^\pi \sin\theta\, d\theta\left(1+\cos^2\theta\right)\left[\frac{\pi}{2kL\sin(\theta/2)} + O\left(\frac{1}{k^2L^2}\right)\right] \\
&= \frac{k^3a^4L}{32}\left(\frac{\delta\epsilon}{\epsilon}\right)^2\cdot 2\pi^2\left[\underbrace{\int_0^\pi \cos\frac{\theta}{2}\left(1+\cos^2\theta\right)d\theta}_{44/15} + O\left(\frac{1}{kL}\right)\right] \\
&= \frac{11\pi^2k^3a^4L}{60}\left(\frac{\delta\epsilon}{\epsilon}\right)^2\left[1+O\left(\frac{1}{kL}\right)\right] & (11)
\end{aligned}
$$

Unlike the $kL \ll 1$ case, the cross section is proportional to ω^3.