1. Plugging the expansion of $\mathbf{E}(\mathbf{x}, t-\tau)$ around $(\mathbf{x}, t)$,

$$\mathbf{E}(\mathbf{x}, t-\tau) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{\partial^k \mathbf{E}}{\partial t^k}(\mathbf{x}, t)\, \tau^k \tag{1}$$

into the expression for the nonlocal connection gives

$$\begin{aligned} C &= \epsilon_0 \int_{-\infty}^{\infty} d\tau\, G(\tau)\, \mathbf{E}(\mathbf{x}, t-\tau) \\ &= \epsilon_0 \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{\partial^k \mathbf{E}}{\partial t^k}(\mathbf{x}, t) \int_{-\infty}^{\infty} G(\tau)\, \tau^k d\tau && \text{use (7.110)} \\ &= \epsilon_0 \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{\partial^k \mathbf{E}}{\partial t^k}(\mathbf{x}, t) \cdot \left(\frac{\omega_p^2}{\nu_0}\right) \underbrace{\int_0^{\infty} e^{-\gamma\tau/2} \sin \nu_0 \tau \cdot \tau^k d\tau}_{I_k} \end{aligned} \tag{2}$$

Following (7.109), define

$$\omega_{1,2} = -\frac{i\gamma}{2} \pm \nu_0 \qquad \text{where } \nu_0^2 = \omega_0^2 - \frac{\gamma^2}{4} \tag{3}$$

Then I_k can be written

$$\begin{aligned} I_k &= \int_0^{\infty} e^{-\gamma\tau/2} \left(\frac{e^{i\nu_0\tau} - e^{-i\nu_0\tau}}{2i}\right) \tau^k d\tau = \int_0^{\infty} \left(\frac{e^{-i\omega_2\tau} - e^{-i\omega_1\tau}}{2i}\right) \tau^k d\tau \\ &= \frac{1}{2i}\left[\frac{k!}{(i\omega_2)^{k+1}} - \frac{k!}{(i\omega_1)^{k+1}}\right] = -\left(\frac{k!}{2i^k}\right)\left(\frac{1}{\omega_2^{k+1}} - \frac{1}{\omega_1^{k+1}}\right) \\ &= -\left(\frac{k!}{2i^k}\right)\left(\frac{1}{\omega_2} - \frac{1}{\omega_1}\right)\left(\sum_{l=0}^{k} \frac{1}{\omega_1^l \omega_2^{k-l}}\right) = \left(\frac{k!\,\nu_0}{\omega_0^2 i^k}\right)\left(\sum_{l=0}^{k} \frac{1}{\omega_1^l \omega_2^{k-l}}\right) \end{aligned} \tag{4}$$

Hence (2) becomes

$$C = \epsilon_0 \left(\frac{\omega_p^2}{\omega_0^2}\right) \sum_{k=0}^{\infty} i^k \frac{\partial^k \mathbf{E}}{\partial t^k}(\mathbf{x}, t) \left(\sum_{l=0}^{k} \frac{1}{\omega_1^l \omega_2^{k-l}}\right) \tag{5}$$

2. On the other hand,

$$\begin{aligned} \frac{\epsilon(\omega)}{\epsilon_0} - 1 &= \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\gamma\omega} = -\frac{\omega_p^2}{(\omega_1 - \omega)(\omega_2 - \omega)} \\ &= -\frac{\omega_p^2}{\omega_1\omega_2}\left(1 - \frac{\omega}{\omega_1}\right)^{-1}\left(1 - \frac{\omega}{\omega_2}\right)^{-1} \\ &= \left(\frac{\omega_p^2}{\omega_0^2}\right)\left[\sum_{p=0}^{\infty}\left(\frac{\omega}{\omega_1}\right)^p\right]\left[\sum_{q=0}^{\infty}\left(\frac{\omega}{\omega_2}\right)^q\right] = \left(\frac{\omega_p^2}{\omega_0^2}\right)\left[\sum_{k=0}^{\infty} \omega^k \left(\sum_{l=0}^{k} \frac{1}{\omega_1^l \omega_2^{k-l}}\right)\right] \end{aligned} \tag{6}$$

Thus with the formal substitution $\omega \to i\partial/\partial t$, the nonlocal connection can be written

$$C = \epsilon_0 \left[\frac{\epsilon\left(i\dfrac{\partial}{\partial t}\right)}{\epsilon_0} - 1\right] \mathbf{E}(\mathbf{x}, t) = \epsilon_0 \left(\frac{\omega_p^2}{\omega_0^2}\right) \sum_{k=0}^{\infty} i^k \frac{\partial^k \mathbf{E}}{\partial t^k}(\mathbf{x}, t) \left(\sum_{l=0}^{k} \frac{1}{\omega_1^l \omega_2^{k-l}}\right) \tag{7}$$

agreeing with (5).