

1. For air, let's count only the dominating gases: Nitrogen and Oxygen (80% and 20% by volume respectively). The effective number of electrons per molecule is $Z_{\text{air}} \approx 0.8 \cdot 14 + 0.2 \cdot 16 = 14.4$. The number of molecules per cubic centimeter at NTP is given by the ideal gas law:

$$N_{\text{mol}}^{\text{air}} = \frac{P}{kT} = \frac{1.01 \times 10^5 \text{ N/m}^2}{1.38 \times 10^{-23} \text{ J/K} \cdot 300 \text{ K}} = 2.44 \times 10^{19} / \text{cm}^3 \quad (1)$$

giving the number of electrons per cubic centimeter

$$N_e^{\text{air}} = 14.4 \times 2.44 \times 10^{19} / \text{cm}^3 = 3.51 \times 10^{20} / \text{cm}^3 \quad (2)$$

For other single-atom metal elements, the number of molecules (atoms) and electrons per cubic centimeter are given by

$$N_{\text{mol}}^{\text{metal}} = \frac{\rho N_A}{M} \quad N_e^{\text{metal}} = \frac{Z \rho N_A}{M} \quad (3)$$

where $N_A \approx 6.022 \times 10^{23}$ atoms/mol is the Avogadro number, M is the molar mass of the element, ρ is the density of the element, and Z is the atomic number.

Element	ρ (g/cm ³)	M (g/mol)	Z	N_e / cm^3
Al	2.70	26.98	13	7.83×10^{23}
Cu	8.96	63.55	29	2.46×10^{24}
Pb	11.34	207.2	82	2.70×10^{24}

With these plugged into (13.14), we have the following values for the energy loss per centimeter (or energy loss per g/cm²) for incident proton and mu meson

(a) **Incident proton**

Kinetic Energy (MeV)	dE/dx (MeV/cm)			
	Air	Al	Cu	Pb
10	5.92×10^{-7}	1.22×10^{-3}	7.15×10^{-3}	1.66×10^{-2}
100	9.85×10^{-8}	2.01×10^{-4}	1.25×10^{-3}	3.25×10^{-3}
1000	3.06×10^{-8}	6.24×10^{-5}	3.99×10^{-4}	1.09×10^{-3}

Energy loss in units of MeV/cm for incident proton

Kinetic Energy (MeV)	$dE/\rho dx$ (MeV·cm ² /g)			
	Air	Al	Cu	Pb
10	34.46	35.19	29.46	22.03
100	5.73	5.82	5.16	4.31
1000	1.78	1.80	1.65	1.44

Energy loss in units of MeV·cm²/g for incident proton

(b) Incident mu meson

Kinetic Energy (MeV)	dE/dx (MeV/cm)			
	Air	Al	Cu	Pb
10	1.08×10^{-7}	2.20×10^{-4}	1.36×10^{-3}	3.53×10^{-3}
100	3.14×10^{-8}	6.40×10^{-5}	4.09×10^{-4}	1.11×10^{-3}
1000	3.22×10^{-8}	6.54×10^{-5}	4.29×10^{-4}	1.21×10^{-3}

Energy loss in units of MeV/cm for incident mu meson

Kinetic Energy (MeV)	$dE/\rho dx$ (MeV·cm ² /g)			
	Air	Al	Cu	Pb
10	6.26	6.35	5.62	4.68
100	1.83	1.85	1.69	1.47
1000	1.87	1.89	1.77	1.61

Energy loss in units of MeV·cm²/g for incident mu meson

2. Since $\rho = NAm_p$, with m_p being the mass of the proton or neutron, we can rewrite (13.14) as

$$\begin{aligned}
 \frac{dE}{\rho dx} &= 4\pi \frac{NZ}{\rho} \frac{z^2 e^4}{mc^2 \beta^2} (\ln B_q - \beta^2) \\
 &= \left(\frac{2Z}{A} \right) z^2 \cdot \left(2\pi \frac{e^4}{mc^2 m_p} \right) \frac{1}{\beta^2} (\ln B_q - \beta^2)
 \end{aligned} \tag{4}$$

With

$$\frac{e^2}{1\text{cm}} \approx 1.44 \times 10^{-13} \text{MeV} \quad mc^2 \approx 0.511 \text{MeV} \quad m_p = 1.67 \times 10^{-24} \text{g} \tag{5}$$

we have

$$2\pi \frac{e^4}{mc^2 m_p} \approx 0.15 \text{MeV} \cdot \text{cm}^2/\text{g} \tag{6}$$

leaving the only dependence of $dE/\rho dx$ on the material in $\ln B_q$ (together with a crude approximation $2Z/A \approx 1$, which is somewhat bad approximation for copper or lead), where

$$B_q = \frac{2\gamma^2 \beta^2 mc^2}{Z\hbar |\omega|} \tag{7}$$

This explains the tiny difference (within a factor of 2, mainly due to $2Z/A$) between $dE/\rho dx$ in different materials.