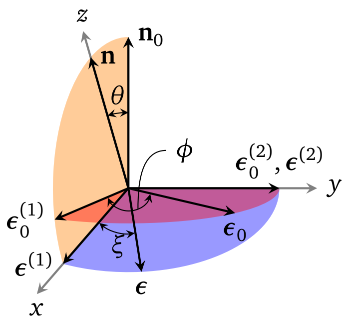

1. For convenience, our setup changes from figure 10.1 by aligning the z axis with $\mathbf{n}$ and x axis with $\boldsymbol{\epsilon}^{(1)}$. In this frame, we have

$$\mathbf{n} = \hat{\mathbf{z}} \qquad \boldsymbol{\epsilon}^{(1)} = \hat{\mathbf{x}} \qquad \boldsymbol{\epsilon}^{(2)} = \hat{\mathbf{y}} \tag{1}$$

$$\mathbf{n}_0 = -\sin\theta\,\hat{\mathbf{x}} + \cos\theta\,\hat{\mathbf{z}} \qquad \boldsymbol{\epsilon}_0^{(1)} = \cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{z}} \qquad \boldsymbol{\epsilon}_0^{(2)} = \hat{\mathbf{y}} \tag{2}$$

Per the diagram above, let the incident polarization vector $\boldsymbol{\epsilon}_0$ be at an angle ϕ to $\boldsymbol{\epsilon}_0^{(1)}$, i.e.,

$$\boldsymbol{\epsilon}_0 = \cos\phi\,\boldsymbol{\epsilon}_0^{(1)} + \sin\phi\,\boldsymbol{\epsilon}_0^{(2)} = \cos\phi\cos\theta\,\hat{\mathbf{x}} + \cos\phi\sin\theta\,\hat{\mathbf{z}} + \sin\phi\,\hat{\mathbf{y}} \tag{3}$$

and let the scattered field's polarization vector $\boldsymbol{\epsilon}$ be at an angle ξ to $\boldsymbol{\epsilon}^{(1)}$, i.e.,

$$\boldsymbol{\epsilon} = \cos\xi\,\hat{\mathbf{x}} + \sin\xi\,\hat{\mathbf{y}} \tag{4}$$

then for the perfectly conducting sphere, the differential cross section in the long-wavelength limit is given by (10.14)

$$\frac{d\sigma}{d\Omega}(\mathbf{n}, \boldsymbol{\epsilon}; \mathbf{n}_0, \boldsymbol{\epsilon}_0) = k^4 a^6 \left| \boldsymbol{\epsilon}^* \cdot \boldsymbol{\epsilon}_0 - \frac{1}{2}(\mathbf{n} \times \boldsymbol{\epsilon}^*) \cdot (\mathbf{n}_0 \times \boldsymbol{\epsilon}_0) \right|^2 \tag{5}$$

Note that

$$\boldsymbol{\epsilon}^* \cdot \boldsymbol{\epsilon}_0 = \cos\phi\cos\theta\cos\xi + \sin\phi\sin\xi \tag{6}$$

$$\mathbf{n} \times \boldsymbol{\epsilon}^* = \cos\xi\,\hat{\mathbf{y}} - \sin\xi\,\hat{\mathbf{x}} \tag{7}$$

$$\mathbf{n}_0 \times \boldsymbol{\epsilon}_0 = \cos\phi\,\hat{\mathbf{y}} - \sin\phi\sin\theta\,\hat{\mathbf{z}} - \sin\phi\cos\theta\,\hat{\mathbf{x}} \tag{8}$$

we can turn (5) into

$$\frac{d\sigma}{d\Omega}(\mathbf{n}, \boldsymbol{\epsilon}; \mathbf{n}_0, \boldsymbol{\epsilon}_0) = k^4 a^6 \left| \cos\phi \left(\cos\theta - \frac{1}{2}\right)\cos\xi + \sin\phi \left(1 - \frac{1}{2}\cos\theta\right)\sin\xi \right|^2 \tag{9}$$

The problem statement asked for the cross section, "*summed over outgoing polarizations*", which could be understood as the integral of (9) with $d\xi$ over $[0, 2\pi]$. But if we interpret the sum as $d\sigma_\parallel/d\Omega + d\sigma_\perp/d\Omega$, as is done in (10.8), then we will end up with

$$\begin{aligned}
\frac{d\sigma_\parallel}{d\Omega} + \frac{d\sigma_\perp}{d\Omega} &= k^4 a^6 \left[\cos^2\phi \left(\cos\theta - \frac{1}{2}\right)^2 + \sin^2\phi \left(1 - \frac{1}{2}\cos\theta\right)^2 \right] \\
&= k^4 a^6 \left[\frac{5}{4} - \overbrace{\cos^2\phi\sin^2\theta}^{|\boldsymbol{\epsilon}_0 \cdot \mathbf{n}|^2} - \frac{1}{4}\overbrace{\sin^2\phi\sin^2\theta}^{|\mathbf{n}\cdot(\mathbf{n}_0 \times \boldsymbol{\epsilon}_0)|^2} - \overbrace{\cos\theta}^{\mathbf{n}_0 \cdot \mathbf{n}} \right] && (10) \\
&= k^4 a^6 \left[\frac{5}{4} - |\boldsymbol{\epsilon}_0 \cdot \mathbf{n}|^2 - \frac{1}{4}|\mathbf{n} \cdot (\mathbf{n}_0 \times \boldsymbol{\epsilon}_0)|^2 - \mathbf{n}_0 \cdot \mathbf{n} \right] && (11)
\end{aligned}$$

If we interpret the sum as integration of (9) with $d\xi$ over $[0, 2\pi]$, the cross term will vanish, but the sum of the square terms will end up with an additional factor of π compared to (10).

2. The result is easily obtained from the alternate form (10).

3. With (10), the ratio is

$$\frac{\frac{d\sigma_{\theta=\pi/2,\phi=0}}{d\Omega}}{\frac{d\sigma_{\theta=\pi/2,\phi=\pi/2}}{d\Omega}} = \frac{1}{4} \tag{12}$$

Note when $\theta = \pi/2, \phi = 0$, the incident electric field's polarization is along the z axis, so is the induced electric dipole. By (9.23), for an observation point along the dipole's axis, there is no electric radiation, thus the only measurement is due to the magnetic dipole. When $\theta = \pi/2, \phi = \pi/2$, the incident electric field polarization is along the y axis, thus the induced electric dipole is along the y axis. By (9.23), for an observation point along the z axis, the electric dipole radiation is maximum. The ratio $1/4$ is justified by noting that the magnetic dipole radiation is a lot weaker than the electric dipole radiation.