1. Our goal is to show equation (6.96), i.e., the microscopic current density

$$\begin{aligned}\langle j_\alpha(\mathbf{x},t)\rangle =& J_\alpha(\mathbf{x},t)+\frac{\partial}{\partial t}\left[D_\alpha(\mathbf{x},t)-\epsilon_0 E_\alpha(\mathbf{x},t)\right]+\sum_{\beta\gamma}\epsilon_{\alpha\beta\gamma}\frac{\partial}{\partial x_\beta}M_\gamma(\mathbf{x},t)\\ &+\sum_\beta\frac{\partial}{\partial x_\beta}\left\langle\sum_{n(\text{mol})}\left[(\mathbf{p}_n)_\alpha(\mathbf{v}_n)_\beta-(\mathbf{p}_n)_\beta(\mathbf{v}_n)_\alpha\right]\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle\\ &-\frac{1}{6}\sum_{\beta\gamma}\frac{\partial^2}{\partial x_\beta\partial x_\gamma}\left\langle\sum_{n(\text{mol})}\left[\left(Q'_n\right)_{\alpha\beta}(\mathbf{v}_n)_\gamma-\left(Q'_n\right)_{\gamma\beta}(\mathbf{v}_n)_\alpha\right]\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle+\cdots \end{aligned}\tag{1}$$

where $\mathbf{J}(\mathbf{x},t)$ is the macroscopic current density:

$$\mathbf{J}(\mathbf{x},t)=\left\langle\sum_{j(\text{free})}q_j\mathbf{v}_j\delta\left(\mathbf{x}-\mathbf{x}_j\right)+\sum_{n(\text{mol})}q_n\mathbf{v}_n\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle\tag{2}$$

and $\mathbf{M}(\mathbf{x},t)$ is the macroscopic magnetization:

$$\mathbf{M}(\mathbf{x},t)=\left\langle\sum_{n(\text{mol})}\mathbf{m}_n\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle\tag{3}$$

and $\mathbf{m}_n$ is the the molecular magnetic moment:

$$\mathbf{m}_n=\sum_{j(n)}\frac{q_j}{2}\left(\mathbf{x}_{jn}\times\mathbf{v}_{jn}\right)\tag{4}$$

Microscopically, the charge density is composed of

$$\eta(\mathbf{x},t)=\eta_{\text{free}}(\mathbf{x},t)+\eta_{\text{bound}}(\mathbf{x},t)\tag{5}$$

where

$$\eta_{\text{free}}(\mathbf{x},t)=\sum_{j(\text{free})}q_j\delta\left(\mathbf{x}-\mathbf{x}_j\right)\tag{6}$$

$$\eta_{\text{bound}}(\mathbf{x},t)=\sum_{n(\text{mol})}\eta_n(\mathbf{x},t)\qquad\qquad \eta_n(\mathbf{x},t)=\sum_{j(n)}q_j\delta\left(\mathbf{x}-\mathbf{x}_j\right)\tag{7}$$

We can see that the first part of $\mathbf{J}(\mathbf{x},t)$ in (2) is accounted for by the movement of $\eta_{\text{free}}(\mathbf{x},t)$, i.e.,

$$\mathbf{J}_{\text{free}}(\mathbf{x},t)=\left\langle\sum_{j(\text{free})}q_j\mathbf{v}_j\delta\left(\mathbf{x}-\mathbf{x}_j\right)\right\rangle\tag{8}$$

hence in the following, we are going to focus solely on the bound charges.
Using (6.89) - (6.92), we know

$$D_\alpha(\mathbf{x},t)-\epsilon_0E_\alpha(\mathbf{x},t)=\left\langle\sum_{n(\text{mol})}(\mathbf{p}_n)_\alpha\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle-\frac{1}{6}\sum_\beta\frac{\partial}{\partial x_\beta}\left\langle\sum_{n(\text{mol})}\left(Q'_n\right)_{\alpha\beta}\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle+\cdots\tag{9}$$

Then it is clear that without having to consider the free charges, (1) is proved if for each molecule n

$$\begin{aligned}\left\langle(j_n)_\alpha(\mathbf{x},t)\right\rangle=&\left\langle q_n(\mathbf{v}_n)_\alpha\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle+\frac{\partial}{\partial t}\left\langle(\mathbf{p}_n)_\alpha\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle+\frac{\partial}{\partial t}\left[-\frac{1}{6}\sum_\beta\frac{\partial}{\partial x_\beta}\left\langle\left(Q'_n\right)_{\alpha\beta}\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle\right]\\ &+\sum_{\beta\gamma}\epsilon_{\alpha\beta\gamma}\frac{\partial}{\partial x_\beta}\left\langle(\mathbf{m}_n)_\gamma\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle+\sum_\beta\frac{\partial}{\partial x_\beta}\left\langle\left[(\mathbf{p}_n)_\alpha(\mathbf{v}_n)_\beta-(\mathbf{p}_n)_\beta(\mathbf{v}_n)_\alpha\right]\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle\\ &-\frac{1}{6}\sum_{\beta\gamma}\frac{\partial^2}{\partial x_\beta\partial x_\gamma}\left\langle\left[\left(Q'_n\right)_{\alpha\beta}(\mathbf{v}_n)_\gamma-\left(Q'_n\right)_{\gamma\beta}(\mathbf{v}_n)_\alpha\right]\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle+\cdots\end{aligned}\tag{10}$$

By definition of the "average" operation:

$$\langle F(\mathbf{x},t)\rangle = \int d^3x' f\left(\mathbf{x}'\right) F\left(\mathbf{x}-\mathbf{x}',t\right) \tag{11}$$

we can write the RHS of (10) as

$$\begin{aligned}
\mathrm{RHS}_{(10)} = & \overbrace{q_n(\mathbf{v}_n)_\alpha f(\mathbf{x}-\mathbf{x}_n)}^{R_1} + \overbrace{\frac{\partial}{\partial t}\left[(\mathbf{p}_n)_\alpha f(\mathbf{x}-\mathbf{x}_n)\right]}^{R_2} \overbrace{-\frac{1}{6}\frac{\partial}{\partial t}\sum_\beta \frac{\partial}{\partial x_\beta}\left[\left(Q'_n\right)_{\alpha\beta} f(\mathbf{x}-\mathbf{x}_n)\right]}^{R_3} \\
& + \overbrace{\sum_{\beta\gamma}\epsilon_{\alpha\beta\gamma}\frac{\partial}{\partial x_\beta}\left[(\mathbf{m}_n)_\gamma f(\mathbf{x}-\mathbf{x}_n)\right]}^{R_4} + \overbrace{\sum_\beta \frac{\partial}{\partial x_\beta}\left\{\left[(\mathbf{p}_n)_\alpha (\mathbf{v}_n)_\beta - (\mathbf{p}_n)_\beta (\mathbf{v}_n)_\alpha\right] f(\mathbf{x}-\mathbf{x}_n)\right\}}^{R_5} \\
& \underbrace{-\frac{1}{6}\sum_{\beta\gamma}\frac{\partial^2}{\partial x_\beta \partial x_\gamma}\left\{\left[\left(Q'_n\right)_{\alpha\beta}(\mathbf{v}_n)_\gamma - \left(Q'_n\right)_{\gamma\beta}(\mathbf{v}_n)_\alpha\right] f(\mathbf{x}-\mathbf{x}_n)\right\}}_{R_6} + \cdots
\end{aligned} \tag{12}$$

By (7), the LHS of (10) can be written as

$$\mathrm{LHS}_{(10)} = \sum_{j(n)} q_j\left[(\mathbf{v}_n)_\alpha + \left(\mathbf{v}_{jn}\right)_\alpha\right] f\left(\mathbf{x}-\mathbf{x}_n-\mathbf{x}_{jn}\right) \tag{13}$$

If we expand $f\left(\mathbf{x}-\mathbf{x}_n-\mathbf{x}_{jn}\right)$ into Taylor series around $\mathbf{x}-\mathbf{x}_n$:

$$f\left(\mathbf{x}-\mathbf{x}_n-\mathbf{x}_{jn}\right) = f(\mathbf{x}-\mathbf{x}_n) - \mathbf{x}_{jn}\cdot(\nabla f)(\mathbf{x}-\mathbf{x}_n) + \frac{1}{2}\sum_{\beta\gamma}\left(\mathbf{x}_{jn}\right)_\beta\left(\mathbf{x}_{jn}\right)_\gamma\left(\frac{\partial^2 f}{\partial x_\beta \partial x_\gamma}\right)(\mathbf{x}-\mathbf{x}_n) + \cdots \tag{14}$$

we will end up with five terms for $\mathrm{LHS}_{(10)}$ up to the second order derivative:

$$\begin{aligned}
L_1: &\quad \sum_{j(n)} q_j(\mathbf{v}_n)_\alpha f(\mathbf{x}-\mathbf{x}_n) \\
L_2: &\quad \sum_{j(n)} q_j\left(\mathbf{v}_{jn}\right)_\alpha f(\mathbf{x}-\mathbf{x}_n) \\
L_3: &\quad -\sum_{j(n)} q_j(\mathbf{v}_n)_\alpha\left[\mathbf{x}_{jn}\cdot(\nabla f)(\mathbf{x}-\mathbf{x}_n)\right] \\
L_4: &\quad -\sum_{j(n)} q_j\left(\mathbf{v}_{jn}\right)_\alpha\left[\mathbf{x}_{jn}\cdot(\nabla f)(\mathbf{x}-\mathbf{x}_n)\right] \\
L_5: &\quad \frac{1}{2}\sum_{j(n)} q_j(\mathbf{v}_n)_\alpha\left[\sum_{\beta\gamma}\left(\mathbf{x}_{jn}\right)_\beta\left(\mathbf{x}_{jn}\right)_\gamma\left(\frac{\partial^2 f}{\partial x_\beta \partial x_\gamma}\right)(\mathbf{x}-\mathbf{x}_n)\right]
\end{aligned}$$

where in this proof, we will leave

$$L_6: \quad \frac{1}{2}\sum_{j(n)} q_j\left(\mathbf{v}_{jn}\right)_\alpha\left[\sum_{\beta\gamma}\left(\mathbf{x}_{jn}\right)_\beta\left(\mathbf{x}_{jn}\right)_\gamma\left(\frac{\partial^2 f}{\partial x_\beta \partial x_\gamma}\right)(\mathbf{x}-\mathbf{x}_n)\right]$$

unmatched with R_{1-6} since it is of order $O\left(\left|\mathbf{x}_{jn}\right|^3\right)$.
We will show below that $L_1+L_2+L_3+L_4+L_5 = R_1+R_2+R_3+R_4+R_5+R_6$.
It is clear that $R_1 = L_1$.
Also

$$\begin{aligned}
R_2 &= \frac{\partial}{\partial t}\left[\sum_{j(n)} q_j\left(\mathbf{x}_{jn}\right)_\alpha f(\mathbf{x}-\mathbf{x}_n)\right] \\
&= \sum_{j(n)} q_j\left(\mathbf{v}_{jn}\right)_\alpha f(\mathbf{x}-\mathbf{x}_n) - \sum_{j(n)} q_j\left(\mathbf{x}_{jn}\right)_\alpha \mathbf{v}_n\cdot(\nabla f)(\mathbf{x}-\mathbf{x}_n) \\
&= L_2 - (\mathbf{p}_n)_\alpha \mathbf{v}_n\cdot(\nabla f)(\mathbf{x}-\mathbf{x}_n)
\end{aligned} \tag{15}$$

and the sum in R_5 can be written more concisely in the form of dot products,

$$R_5 = (\mathbf{p}_n)_\alpha \mathbf{v}_n \cdot (\nabla f)(\mathbf{x}-\mathbf{x}_n) \underbrace{-(\mathbf{v}_n)_\alpha \mathbf{p}_n \cdot (\nabla f)(\mathbf{x}-\mathbf{x}_n)}_{=L_3} \tag{16}$$

Summing (15) and (16) gives us $R_2 + R_5 = L_2 + L_3$, so it remains to prove $R_3 + R_4 + R_6 = L_4 + L_5$.
In fact

$$\begin{aligned}
R_3 =& -\frac{1}{6}\frac{\partial}{\partial t}\sum_\beta \frac{\partial}{\partial x_\beta}\left[3\sum_{j(n)} q_j \left(\mathbf{x}_{jn}\right)_\alpha \left(\mathbf{x}_{jn}\right)_\beta f\left(\mathbf{x}-\mathbf{x}_n\right)\right] \\
=& -\frac{1}{2}\frac{\partial}{\partial t}\sum_{j(n)} q_j \left(\mathbf{x}_{jn}\right)_\alpha \mathbf{x}_{jn} \cdot (\nabla f)(\mathbf{x}-\mathbf{x}_n) \\
=& \underbrace{-\frac{1}{2}\sum_{j(n)} q_j \left(\mathbf{v}_{jn}\right)_\alpha \mathbf{x}_{jn} \cdot (\nabla f)(\mathbf{x}-\mathbf{x}_n)}_{R_{31}} \underbrace{-\frac{1}{2}\sum_{j(n)} q_j \left(\mathbf{x}_{jn}\right)_\alpha \mathbf{v}_{jn} \cdot (\nabla f)(\mathbf{x}-\mathbf{x}_n)}_{R_{32}} \\
& \underbrace{-\frac{1}{2}\sum_{j(n)} q_j \left(\mathbf{x}_{jn}\right)_\alpha \mathbf{x}_{jn} \cdot \frac{\partial}{\partial t}\left[(\nabla f)(\mathbf{x}-\mathbf{x}_n)\right]}_{R_{33}}
\end{aligned} \tag{17}$$

$$\begin{aligned}
R_4 =& \sum_{\beta\gamma} \epsilon_{\alpha\beta\gamma} (\mathbf{m}_n)_\gamma \left[(\nabla f)(\mathbf{x}-\mathbf{x}_n)\right]_\beta \\
=& \left\{\left[(\nabla f)(\mathbf{x}-\mathbf{x}_n)\right] \times \mathbf{m}_n\right\}_\alpha \\
=& \frac{1}{2}\sum_{j(n)} q_j \left\{\left[(\nabla f)(\mathbf{x}-\mathbf{x}_n)\right] \times \left(\mathbf{x}_{jn} \times \mathbf{v}_{jn}\right)\right\}_\alpha \\
=& \underbrace{\frac{1}{2}\sum_{j(n)} q_j \left(\mathbf{x}_{jn}\right)_\alpha \mathbf{v}_{jn} \cdot (\nabla f)(\mathbf{x}-\mathbf{x}_n)}_{=-R_{32}} \underbrace{-\frac{1}{2}\sum_{j(n)} q_j \left(\mathbf{v}_{jn}\right)_\alpha \mathbf{x}_{jn} \cdot (\nabla f)(\mathbf{x}-\mathbf{x}_n)}_{=R_{31}}
\end{aligned} \tag{18}$$

where we have used the vector identity

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\,\mathbf{b} - (\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c} \tag{19}$$

Thus we see $R_{31} + R_{32} + R_4 = 2R_{31} = L_4$, so it remains to show $R_{33} + R_6 = L_5$.
Indeed, we can break R_6 into

$$R_6 = \underbrace{-\frac{1}{6}\sum_{\beta\gamma} \left(Q_n'\right)_{\alpha\beta} (\mathbf{v}_n)_\gamma \left(\frac{\partial^2 f}{\partial x_\beta \partial x_\gamma}\right)(\mathbf{x}-\mathbf{x}_n)}_{R_{61}} + \underbrace{\frac{1}{6}\sum_{\beta\gamma} \left(Q_n'\right)_{\gamma\beta} (\mathbf{v}_n)_\alpha \left(\frac{\partial^2 f}{\partial x_\beta \partial x_\gamma}\right)(\mathbf{x}-\mathbf{x}_n)}_{=L_5} \tag{20}$$

Evaluation of R_{33} needs some care. To see it clearly, we write

$$\mathbf{g}(\mathbf{x}-\mathbf{x}_n) \equiv (\nabla f)(\mathbf{x}-\mathbf{x}_n) \tag{21}$$

where the dependency of $\mathbf{g}$ on t is through $\mathbf{x}_n(t)$. By the chain rule

$$\frac{\partial}{\partial t}\mathbf{g}(\mathbf{x}-\mathbf{x}_n) = \sum_\beta \frac{\partial}{\partial t}\left[g_\beta(\mathbf{x}-\mathbf{x}_n)\right]\hat{\mathbf{e}}_\beta = \sum_\beta \left[-\mathbf{v}_n \cdot \left(\nabla g_\beta\right)(\mathbf{x}-\mathbf{x}_n)\right]\hat{\mathbf{e}}_\beta = -\sum_{\beta\gamma} (\mathbf{v}_n)_\gamma \left(\frac{\partial^2 f}{\partial x_\beta \partial x_\gamma}\right)(\mathbf{x}-\mathbf{x}_n)\,\hat{\mathbf{e}}_\beta \tag{22}$$

which completes the proof since

$$\begin{aligned}
R_{33} =& \frac{1}{2}\sum_{j(n)}\sum_{\beta\gamma} q_j \left(\mathbf{x}_{jn}\right)_\alpha \left(\mathbf{x}_{jn}\right)_\beta (\mathbf{v}_n)_\gamma \left(\frac{\partial^2 f}{\partial x_\beta \partial x_\gamma}\right)(\mathbf{x}-\mathbf{x}_n) \\
=& \frac{1}{6}\sum_{\beta\gamma} \left(Q_n'\right)_{\alpha\beta} (\mathbf{v}_n)_\gamma \left(\frac{\partial^2 f}{\partial x_\beta \partial x_\gamma}\right)(\mathbf{x}-\mathbf{x}_n) = -R_{61}
\end{aligned} \tag{23}$$

2. For this part, first let's say a few words about Jackson (6.99):

$$\left(\frac{\mathbf{B}}{\mu_0}-\mathbf{H}\right)_\alpha = M_\alpha + \overbrace{\left\langle \sum_{n(\mathrm{mol})} (\mathbf{p}_n \times \mathbf{v}_n)_\alpha \, \delta(\mathbf{x}-\mathbf{x}_n) \right\rangle}^{U_\alpha} \overbrace{-\frac{1}{6}\sum_{\beta\gamma\delta} \epsilon_{\alpha\beta\gamma} \frac{\partial}{\partial x_\delta} \left\langle \sum_{n(\mathrm{mol})} \left(Q_n'\right)_{\delta\beta} (\mathbf{v}_n)_\gamma \, \delta(\mathbf{x}-\mathbf{x}_n) \right\rangle}^{W_\alpha} + \cdots \tag{24}$$

The interpretation of this is that both sides represent the α-component of a vector, or in vector identity

$$\frac{\mathbf{B}}{\mu_0} - \mathbf{H} = \mathbf{M} + \mathbf{U} + \mathbf{W} \tag{25}$$

It is supposed to be consistent with the result of inserting (1) to the averaged microscopic Maxwell equation (6.70), which says

$$\frac{1}{\mu_0}\nabla \times \mathbf{B} - \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \langle \mathbf{j} \rangle \tag{26}$$

The result of such insertion followed by the macroscopic interpretation that

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} \tag{27}$$

yields

$$\nabla \times \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{H}\right) = \langle \mathbf{j} \rangle - \mathbf{J} - \frac{\partial}{\partial t}(\mathbf{D} - \epsilon_0 \mathbf{E}) \tag{28}$$

Thus for (24) to be consistent with (28), we need to show that

$$\begin{aligned}(\nabla \times \mathbf{M})_\alpha + (\nabla \times \mathbf{U})_\alpha + (\nabla \times \mathbf{W})_\alpha = &\sum_{\beta\gamma} \epsilon_{\alpha\beta\gamma} \frac{\partial}{\partial x_\beta} M_\gamma(\mathbf{x}, t) + \\ &\sum_\beta \frac{\partial}{\partial x_\beta} \left\langle \sum_{n(\mathrm{mol})} \left[(\mathbf{p}_n)_\alpha (\mathbf{v}_n)_\beta - (\mathbf{p}_n)_\beta (\mathbf{v}_n)_\alpha\right] \delta(\mathbf{x}-\mathbf{x}_n) \right\rangle - \\ &\frac{1}{6}\sum_{\beta\gamma} \frac{\partial^2}{\partial x_\beta \partial x_\gamma} \left\langle \sum_{n(\mathrm{mol})} \left[\left(Q_n'\right)_{\alpha\beta} (\mathbf{v}_n)_\gamma - \left(Q_n'\right)_{\gamma\beta} (\mathbf{v}_n)_\alpha\right] \delta(\mathbf{x}-\mathbf{x}_n) \right\rangle\end{aligned} \tag{29}$$

where the RHS is the α component of $\langle \mathbf{j} \rangle - \mathbf{J} - \partial(\mathbf{D} - \epsilon_0 \mathbf{E})/\partial t$ obtained using (1).

Indeed, there is a term-for-term equality here.

Firstly

$$(\nabla \times \mathbf{M})_\alpha = \sum_{\beta\gamma} \epsilon_{\alpha\beta\gamma} \frac{\partial}{\partial x_\beta} M_\gamma(\mathbf{x}, t) \tag{30}$$

is the definition of cross product.

Secondly,

$$\begin{aligned}(\nabla \times \mathbf{U})_\alpha &= \sum_{\beta\gamma} \epsilon_{\alpha\beta\gamma} \frac{\partial}{\partial x_\beta} U_\gamma \\ &= \sum_{\beta\gamma} \epsilon_{\alpha\beta\gamma} \frac{\partial}{\partial x_\beta} \left\langle \sum_{n(\mathrm{mol})} (\mathbf{p}_n \times \mathbf{v}_n)_\gamma \, \delta(\mathbf{x}-\mathbf{x}_n) \right\rangle \\ &= \sum_{\beta\gamma} \epsilon_{\alpha\beta\gamma} \frac{\partial}{\partial x_\beta} \left\langle \sum_{n(\mathrm{mol})} \sum_{\mu\nu} \epsilon_{\mu\nu\gamma} (\mathbf{p}_n)_\mu (\mathbf{v}_n)_\nu \, \delta(\mathbf{x}-\mathbf{x}_n) \right\rangle \\ &= \sum_\beta \frac{\partial}{\partial x_\beta} \left\langle \sum_{n(\mathrm{mol})} \sum_{\mu\nu} \left(\sum_\gamma \epsilon_{\alpha\beta\gamma} \epsilon_{\mu\nu\gamma}\right) (\mathbf{p}_n)_\mu (\mathbf{v}_n)_\nu \, \delta(\mathbf{x}-\mathbf{x}_n) \right\rangle \\ &= \sum_\beta \frac{\partial}{\partial x_\beta} \left\langle \sum_{n(\mathrm{mol})} \sum_{\mu\nu} \left(\delta_{\alpha\mu}\delta_{\beta\nu} - \delta_{\alpha\nu}\delta_{\beta\mu}\right) (\mathbf{p}_n)_\mu (\mathbf{v}_n)_\nu \, \delta(\mathbf{x}-\mathbf{x}_n) \right\rangle \\ &= \sum_\beta \frac{\partial}{\partial x_\beta} \left\langle \sum_{n(\mathrm{mol})} \left[(\mathbf{p}_n)_\alpha (\mathbf{v}_n)_\beta - (\mathbf{p}_n)_\beta (\mathbf{v}_n)_\alpha\right] \delta(\mathbf{x}-\mathbf{x}_n) \right\rangle\end{aligned} \tag{31}$$

And lastly,

$$
\begin{aligned}
(\boldsymbol{\nabla}\times\mathbf{W})_\alpha &= \sum_{\mu\nu}\epsilon_{\alpha\mu\nu}\frac{\partial}{\partial x_\mu}W_\nu \\
&= \sum_{\mu\nu}\epsilon_{\alpha\mu\nu}\frac{\partial}{\partial x_\mu}\left[-\frac{1}{6}\sum_{\beta\gamma\delta}\epsilon_{\nu\beta\gamma}\frac{\partial}{\partial x_\delta}\left\langle\sum_{n(\mathrm{mol})}\left(Q'_n\right)_{\delta\beta}(\mathbf{v}_n)_\gamma\,\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle\right] \\
&= -\frac{1}{6}\sum_{\mu\delta}\frac{\partial^2}{\partial x_\mu\partial x_\delta}\left\langle\sum_{n(\mathrm{mol})}\sum_{\beta\gamma}\left(\sum_\nu\epsilon_{\alpha\mu\nu}\epsilon_{\nu\beta\gamma}\right)\left(Q'_n\right)_{\delta\beta}(\mathbf{v}_n)_\gamma\,\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle \\
&= -\frac{1}{6}\sum_{\mu\delta}\frac{\partial^2}{\partial x_\mu\partial x_\delta}\left\langle\sum_{n(\mathrm{mol})}\sum_{\beta\gamma}\left(\delta_{\alpha\beta}\delta_{\mu\gamma}-\delta_{\alpha\gamma}\delta_{\mu\beta}\right)\left(Q'_n\right)_{\delta\beta}(\mathbf{v}_n)_\gamma\,\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle \\
&= -\frac{1}{6}\sum_{\mu\delta}\frac{\partial^2}{\partial x_\mu\partial x_\delta}\left\langle\sum_{n(\mathrm{mol})}\left[\left(Q'_n\right)_{\delta\alpha}(\mathbf{v}_n)_\mu-\left(Q'_n\right)_{\delta\mu}(\mathbf{v}_n)_\alpha\right]\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle
\end{aligned}
\tag{32}
$$

which is exactly the third term of RHS of (29) with the dummy index relabel $\delta\leftrightarrow\beta,\mu\leftrightarrow\gamma$.
In the bulk motion where all $\mathbf{v}_n=\mathbf{v}$, to see (6.100):

$$
\frac{1}{\mu_0}\mathbf{B}-\mathbf{H}=\mathbf{M}+(\mathbf{D}-\epsilon_0\mathbf{E})\times\mathbf{v}
\tag{33}
$$

recall (6.92), (6.89) and (6.90), which gives

$$
D_\alpha-\epsilon_0E_\alpha=\underbrace{\left\langle\sum_{n(\mathrm{mol})}(\mathbf{p}_n)_\alpha\,\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle}_{U'_\alpha}\underbrace{-\frac{1}{6}\sum_\beta\frac{\partial}{\partial x_\beta}\left\langle\sum_{n(\mathrm{mol})}\left(Q'_n\right)_{\alpha\beta}\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle}_{W'_\alpha}
\tag{34}
$$

where we have defined vector $\mathbf{U}',\mathbf{W}'$ via its α component.
Thus (33) is proved from (24) by recognizing (refer to $\mathbf{U},\mathbf{W}$ definition in (24))

$$
\mathbf{U}'\times\mathbf{v}=\left\langle\sum_{n(\mathrm{mol})}(\mathbf{p}_n\times\mathbf{v})\,\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle=\mathbf{U}
\tag{35}
$$

$$
\begin{aligned}
\left(\mathbf{W}'\times\mathbf{v}\right)_\gamma &= \sum_{\alpha\beta}\epsilon_{\alpha\beta\gamma}W'_\alpha v_\beta \\
&= \sum_{\alpha\beta}\epsilon_{\alpha\beta\gamma}\left[-\frac{1}{6}\sum_\delta\frac{\partial}{\partial x_\delta}\left\langle\sum_{n(\mathrm{mol})}\left(Q'_n\right)_{\alpha\delta}v_\beta\,\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle\right] \\
&= -\frac{1}{6}\sum_{\alpha\beta\delta}\epsilon_{\alpha\beta\gamma}\frac{\partial}{\partial x_\delta}\left\langle\sum_{n(\mathrm{mol})}\left(Q'_n\right)_{\alpha\delta}v_\beta\,\delta(\mathbf{x}-\mathbf{x}_n)\right\rangle=W_\gamma \qquad\Longrightarrow\qquad \mathbf{W}'\times\mathbf{v}=\mathbf{W}
\end{aligned}
\tag{36}
$$