

1. The tangential field in the opening given by (3.186) is

$$\mathbf{E}_{\text{tan}}(\rho) = \frac{E_0}{\pi} \frac{\rho}{\sqrt{R^2 - \rho^2}} \quad (1)$$

Then by (9.72), the effective electric dipole is

$$\begin{aligned} \mathbf{p}_{\text{eff}} &= \epsilon \mathbf{n} \int (\mathbf{x} \cdot \mathbf{E}_{\text{tan}}) da = \epsilon \mathbf{n} \int \frac{E_0}{\pi} \frac{\rho^2}{\sqrt{R^2 - \rho^2}} da && \text{note } \mathbf{n}E_0 = -\mathbf{E}_0 \text{ by the sign convention} \\ &= -\epsilon \frac{\mathbf{E}_0}{\pi} \cdot 2\pi \int_0^R \frac{\rho^2}{\sqrt{R^2 - \rho^2}} \rho d\rho && \text{let } \rho = R \sin \theta \\ &= -2\epsilon R^3 \mathbf{E}_0 \int_0^{\pi/2} \sin^3 \theta d\theta \\ &= -\frac{4\epsilon R^3 \mathbf{E}_0}{3} && \Rightarrow \\ \gamma^E &= -\frac{4R^3}{3} \end{aligned} \quad (2)$$

2. By (9.72),

$$\begin{aligned} i\mu\omega \mathbf{m}_{\text{eff}} &= 2 \int \mathbf{n} \times \mathbf{E}_{\text{tan}} da = 2 \int \frac{E_0}{\pi} \frac{\rho}{\sqrt{R^2 - \rho^2}} \hat{\phi} da \\ &= 2 \frac{E_0}{\pi} \int_0^{2\pi} \underbrace{(-\sin \phi \hat{\mathbf{x}} + \cos \phi \hat{\mathbf{y}})}_0 d\phi \int_0^R \frac{\rho^2}{\sqrt{R^2 - \rho^2}} d\rho = 0 \end{aligned} \quad (3)$$

3. But if we calculate the effective magnetic dipole using (9.74) and (5.132)

$$\begin{aligned} \mathbf{m}_{\text{eff}} &= 2 \int \mathbf{x} (\mathbf{n} \cdot \mathbf{H}) da = 2 \int \rho \frac{2H_0}{\pi} \frac{\rho}{\sqrt{R^2 - \rho^2}} \sin \phi da \\ &= \frac{4H_0}{\pi} \int_0^{2\pi} \underbrace{(\cos \phi \hat{\mathbf{x}} + \sin \phi \hat{\mathbf{y}})}_{\pi \hat{\mathbf{y}}} \sin \phi d\phi \underbrace{\int_0^R \frac{\rho^3}{\sqrt{R^2 - \rho^2}} d\rho}_{R^3 \cdot 2/3} = \frac{8H_0 R^3}{3} \hat{\mathbf{y}} \end{aligned} \quad (4)$$

Refer back to figure 5.15, we see that the direction of \mathbf{H}_0 is along the $\hat{\mathbf{y}}$ direction, which means the effective magnetic polarizability (without referencing to a particular orientation of frame) is

$$\gamma_{\alpha\beta}^M = \frac{8R^3}{3} \delta_{\alpha\beta} \quad (5)$$

4. The difference between (3) and (4) is explained by the text (end of first paragraph of on page 423): *To lowest order their time dependence can be ignored, provided the effective dipole moment is related to \mathbf{E}_0 and the magnetic moment to \mathbf{H}_0 .*