1. Note that the scattered field (10.57) is derived with incident amplitude $\boldsymbol{\epsilon}_1 \pm i\boldsymbol{\epsilon}_2$, but in this problem, the incident amplitude is $(\boldsymbol{\epsilon}_1 \pm i\boldsymbol{\epsilon}_2)/\sqrt{2}$, so the scattered field has a factor of $1/\sqrt{2}$ on top of (10.57), i.e.,

$$\mathbf{E}_{\text{sc}} = \frac{1}{2\sqrt{2}} \sum_{l=1}^{\infty} i^l \sqrt{4\pi(2l+1)} \left\{ \alpha_\pm(l) h_l^{(1)}(kr) \mathbf{X}_{l,\pm 1} \pm \frac{\beta_\pm(l)}{k} \boldsymbol{\nabla} \times \left[h_l^{(1)}(kr) \mathbf{X}_{l,\pm 1} \right] \right\} \tag{1}$$

With the asymptotic form

$$h_l^{(1)}(kr) \to (-i)^{l+1} \frac{e^{ikr}}{kr} \quad \text{as} \quad r \to \infty \tag{2}$$

(1) becomes

$$\mathbf{E}_{\text{sc}} \to \frac{1}{i} \sqrt{\frac{\pi}{2}} \sum_{l=1}^{\infty} \sqrt{2l+1} \left\{ \alpha_\pm(l) \frac{e^{ikr}}{kr} \mathbf{X}_{l,\pm 1} \pm \frac{\beta_\pm(l)}{k} \boldsymbol{\nabla} \times \left[\frac{e^{ikr}}{kr} \mathbf{X}_{l,\pm 1} \right] \right\} \tag{3}$$

By (10.60)

$$\boldsymbol{\nabla} \times \left[\frac{e^{ikr}}{kr} \mathbf{X}_{lm} \right] = \frac{i\mathbf{n}\sqrt{l(l+1)}}{r} \frac{e^{ikr}}{kr} Y_{lm} + i \frac{e^{ikr}}{r} \mathbf{n} \times \mathbf{X}_{lm} \tag{4}$$

we see that the radial component is $O(1/r)$ order higher than the $\mathbf{n} \times \mathbf{X}_{lm}$ component, which can be ignored as $r \to \infty$, i.e.,

$$\mathbf{E}_{\text{sc}} \to \frac{e^{ikr}}{r} \frac{1}{ik} \sqrt{\frac{\pi}{2}} \sum_{l=1}^{\infty} \sqrt{2l+1} \left[\alpha_\pm(l) \mathbf{X}_{l,\pm 1} \pm i\beta_\pm(l) \mathbf{n} \times \mathbf{X}_{l,\pm 1} \right] \tag{5}$$

which gives

$$\mathbf{f} = \frac{1}{ik} \sqrt{\frac{\pi}{2}} \sum_{l=1}^{\infty} \sqrt{2l+1} \left[\alpha_\pm(l) \mathbf{X}_{l,\pm 1} \pm i\beta_\pm(l) \mathbf{n} \times \mathbf{X}_{l,\pm 1} \right] \tag{6}$$

2. By the optical theorem (10.139)

$$\sigma_{\text{tot}} = \frac{4\pi}{k} \operatorname{Im}\left[\boldsymbol{\epsilon}_0^* \cdot \mathbf{f}(\mathbf{k} = \mathbf{k}_0) \right] \tag{7}$$

For convenience, let $\mathbf{k}_0$ be along the $\hat{\mathbf{z}}$ direction and let $\boldsymbol{\epsilon}_0 = \boldsymbol{\epsilon}_\pm = (\hat{\mathbf{x}} \pm i\hat{\mathbf{y}})/\sqrt{2}$, thus we take $\mathbf{n} = \hat{\mathbf{z}}$ in (6) for $\mathbf{f}(\mathbf{k} = \mathbf{k}_0)$. With (9.119)

$$\mathbf{X}_{lm} = \frac{1}{\sqrt{l(l+1)}} \mathbf{L} Y_{lm} \tag{8}$$

we have

$$\boldsymbol{\epsilon}_0^* \cdot \mathbf{f}(\mathbf{k} = \mathbf{k}_0) = \frac{1}{ik} \sqrt{\frac{\pi}{2}} \sum_{l=1}^{\infty} \sqrt{2l+1} \left\{ \alpha_\pm(l) \frac{(\boldsymbol{\epsilon}_\mp \cdot \mathbf{L}) Y_{l,\pm 1}}{\sqrt{l(l+1)}} \pm i\beta_\pm(l) \frac{[\boldsymbol{\epsilon}_\mp \cdot (\hat{\mathbf{z}} \times \mathbf{L})] Y_{l,\pm 1}}{\sqrt{l(l+1)}} \right\} \tag{9}$$

Note that

$$\boldsymbol{\epsilon}_\mp \cdot \mathbf{L} = \frac{L_\mp}{\sqrt{2}} \qquad\qquad \boldsymbol{\epsilon}_\mp \cdot (\hat{\mathbf{z}} \times \mathbf{L}) = \left(\frac{\hat{\mathbf{x}} \mp i\hat{\mathbf{y}}}{\sqrt{2}} \right) \cdot \left(L_x \hat{\mathbf{y}} - L_y \hat{\mathbf{x}} \right) = \mp i \left(\frac{L_x \mp iL_y}{\sqrt{2}} \right) = \frac{\mp iL_\mp}{\sqrt{2}} \tag{10}$$

as well as

$$L_\mp Y_{l,\pm 1} = \sqrt{l(l+1)} Y_{l0} \tag{11}$$

where $Y_{l0} = \sqrt{(2l+1)/4\pi}$ for forward scattering ($\theta = 0$).
Now (9) yields

$$\boldsymbol{\epsilon}_0^* \cdot \mathbf{f}(\mathbf{k} = \mathbf{k}_0) = \frac{1}{ik} \sqrt{\frac{\pi}{2}} \sum_{l=1}^{\infty} \sqrt{2l+1} \left[\frac{\alpha_\pm(l) + \beta_\pm(l)}{\sqrt{2}} \right] \sqrt{\frac{2l+1}{4\pi}} = \frac{1}{ik} \frac{1}{4} \sum_{l=1}^{\infty} (2l+1) \left[\alpha_\pm(l) + \beta_\pm(l) \right] \tag{12}$$

and (7) gives

$$\sigma_{\text{tot}} = -\frac{\pi}{k^2} \sum_{l=1}^{\infty} (2l+1) \operatorname{Re}\left[\alpha_\pm(l) + \beta_\pm(l) \right] \tag{13}$$

agreeing with (10.62). (Note that 10.62 is not affected by the fact its incident wave has amplitude $\sqrt{2}\boldsymbol{\epsilon}_\pm$, since the same factor $\sqrt{2}$ also appears in denominator incident flux, thus is canceled out.)