

1. TEM mode is a 2D static field problem. By cylindrical symmetry, the magnetic field goes along the $\hat{\phi}$ direction. In the space between the two conductors, $\nabla \times \mathbf{H} = 0$ implies

$$\mathbf{H}(\rho) = \hat{\phi} H_0 \frac{a}{\rho} \quad (1)$$

and by (8.28)

$$\mathbf{E}(\rho) = \pm \hat{\rho} \sqrt{\frac{\mu}{\epsilon}} H_0 \frac{a}{\rho} \quad (2)$$

So the time-averaged power is

$$P = 2\pi \int_a^b \left| \frac{1}{2} \mathbf{E}(\rho) \times \mathbf{H}^*(\rho) \right| \rho d\rho = \pi a^2 |H_0|^2 \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right) \quad (3)$$

2. When we consider energy loss to the conductor, by (8.12), we have

$$\left. \frac{dP_{\text{loss}}(z)}{dz} \right|_{\rho=a} = \frac{\mu\omega\delta}{4} |H_0(z)|^2 \quad \left. \frac{dP_{\text{loss}}(z)}{dz} \right|_{\rho=b} = \frac{\mu\omega\delta}{4} |H_0(z)|^2 \left(\frac{a}{b}\right)^2 \quad (4)$$

Thus from $z \rightarrow z + dz$, the total energy loss is

$$dP_{\text{loss}}(z) = \frac{\mu\omega\delta}{4} |H_0(z)|^2 \cdot 2\pi \left(a + \frac{a^2}{b} \right) dz \quad (5)$$

From (3), we know

$$|H_0(z)|^2 = \frac{\frac{P(z)}{\pi a^2} \sqrt{\frac{\epsilon}{\mu}}}{\ln\left(\frac{b}{a}\right)} \quad (6)$$

hence (5) becomes

$$\frac{dP_{\text{loss}}(z)}{dz} = \frac{\mu\omega\delta}{4} \cdot 2P(z) \sqrt{\frac{\epsilon}{\mu}} \left[\frac{\frac{1}{a} + \frac{1}{b}}{\ln\left(\frac{b}{a}\right)} \right] = \frac{1}{2\sigma\delta} \cdot 2P(z) \sqrt{\frac{\epsilon}{\mu}} \left[\frac{\frac{1}{a} + \frac{1}{b}}{\ln\left(\frac{b}{a}\right)} \right] \quad (7)$$

Clearly, this describes a differential equation

$$\frac{dP(z)}{dz} = -2\gamma P(z) \quad \text{where} \quad \gamma = \frac{1}{2\sigma\delta} \sqrt{\frac{\epsilon}{\mu}} \left[\frac{\frac{1}{a} + \frac{1}{b}}{\ln\left(\frac{b}{a}\right)} \right] \quad (8)$$

3. For TEM mode, the voltage difference V and the current I can be calculated as the following

$$V = \int_a^b E(\rho) d\rho = \sqrt{\frac{\mu}{\epsilon}} H_0 a \ln\left(\frac{b}{a}\right) \quad I = 2\pi a H_0 \quad (9)$$

giving the impedance

$$Z_0 = \frac{V}{I} = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln\left(\frac{b}{a}\right) \quad (10)$$

4. We treat the waveguide as a series circuit with per unit length resistance R , which can be deduced by the the energy loss per unit length

$$\frac{dP_{\text{loss}}}{dz} = \frac{1}{2} I_{\text{peak}}^2 R \quad \Rightarrow \quad R = \frac{2dP_{\text{loss}}/dz}{I_{\text{peak}}^2} = \frac{2 \cdot \frac{1}{2\sigma\delta} |H_0(z)|^2 \cdot 2\pi \left(a + \frac{a^2}{b} \right)}{4\pi^2 a^2 |H_0(z)|^2} = \frac{1}{2\pi\sigma\delta} \left(\frac{1}{a} + \frac{1}{b} \right) \quad (11)$$

The equivalent inductance per unit length, L , can be computed from the total magnetic energy stored per length

$$\frac{dW_{\text{mag}}}{dz} = \frac{1}{4} I_{\text{peak}}^2 L \quad \Rightarrow \quad L = \frac{4dW_{\text{mag}}/dz}{I_{\text{peak}}^2} \quad (12)$$

Magnetic field exists in all of the three regions,

$$\frac{dW_{\text{mag}}}{dz} = 2\pi \left[\overbrace{\int_0^a \frac{\mu_c}{4} H_{\rho < a}^2(\rho) \rho d\rho}^{W_1} + \overbrace{\int_a^b \frac{\mu_c}{4} H_{a < \rho < b}^2(\rho) \rho d\rho}^{W_2} + \overbrace{\int_b^\infty \frac{\mu_c}{4} H_{\rho > b}^2(\rho) \rho d\rho}^{W_3} \right] \quad (13)$$

Assuming $\delta \ll a < b$, we can ignore the curvature and use (8.9) to approximate $H_{\rho < a}, H_{\rho > b}$, which gives

$$\begin{aligned} W_1 &= \frac{\mu_c}{4} \int_0^a H_0^2 e^{-2(a-\rho)/\delta} \rho d\rho \\ &= \frac{\mu_c H_0^2}{4} e^{-2a/\delta} \int_0^a e^{2\rho/\delta} \rho d\rho \\ &= \frac{\mu_c H_0^2}{4} \left[\frac{\delta a}{2} - \frac{\delta^2}{4} (1 - e^{-2a/\delta}) \right] \end{aligned} \quad (14)$$

$$\begin{aligned} W_3 &= \frac{\mu_c}{4} \int_b^\infty H_0^2 \left(\frac{a}{b} \right)^2 e^{-2(\rho-b)/\delta} \rho d\rho \\ &= \frac{\mu_c H_0^2}{4} \left(\frac{a}{b} \right)^2 e^{2b/\delta} \int_b^\infty e^{-2\rho/\delta} \rho d\rho \\ &= \frac{\mu_c H_0^2}{4} \left(\frac{a}{b} \right)^2 \left(\frac{\delta b}{2} + \frac{\delta^2}{4} \right) \end{aligned} \quad (15)$$

Also,

$$W_2 = \frac{\mu}{4} \int_a^b H_0^2 \left(\frac{a}{\rho} \right)^2 \rho d\rho = \frac{\mu H_0^2 a^2}{4} \ln \left(\frac{b}{a} \right) \quad (16)$$

Ignoring $O(\delta^2)$ in (14) and (15), and putting everything back to (12), we eventually get

$$\begin{aligned} L &\approx \frac{2\pi \left[\mu_c H_0^2 \left(\frac{\delta a}{2} \right) + \mu_c H_0^2 \left(\frac{\delta a^2}{2b} \right) + \mu H_0^2 a^2 \ln \left(\frac{b}{a} \right) \right]}{4\pi^2 a^2 H_0^2} \\ &= \frac{\mu}{2\pi} \ln \left(\frac{b}{a} \right) + \frac{\mu_c \delta}{4\pi} \left(\frac{1}{a} + \frac{1}{b} \right) \end{aligned} \quad (17)$$