1. Let the incident electric field be of the form

$$\mathbf{E}(\mathbf{x},t) = \boldsymbol{\epsilon}_1 E(x)\, e^{ik_0 z - i\omega t} \tag{1}$$

where we chose $\hat{\mathbf{z}}$ to align with the beam's propagation direction $\mathbf{k}_0$, and $\hat{\mathbf{x}}$ to be the transverse direction that is in the plane of incidence, but the polarization direction $\boldsymbol{\epsilon}_1$ can be any direction in the plane perpendicular to $\hat{\mathbf{z}}$.

If we write $E(x)$ in its Fourier decomposition

$$E(x) = \int d\kappa A(\kappa)\, e^{i\kappa x} \tag{2}$$

the incident wave can be written as

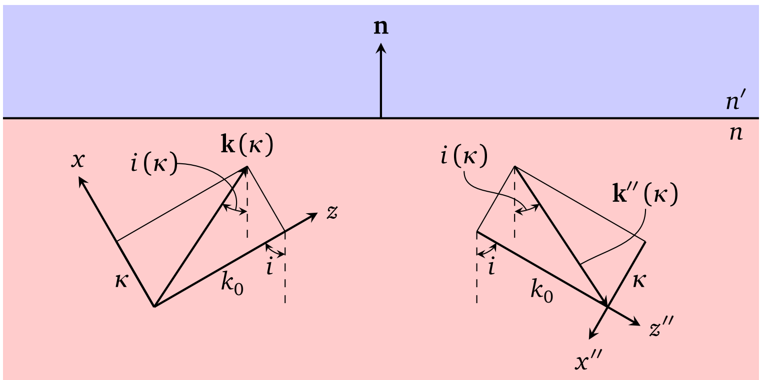
$$\mathbf{E}(\mathbf{x},t) = \boldsymbol{\epsilon}_1 \int d\kappa A(\kappa)\, e^{i(\kappa x + k_0 z) - i\omega t} \tag{3}$$

This form has a clear interpretation which is the superposition of infinite plane waves, each having amplitude $A(\kappa)$ and wave vector $\mathbf{k}(\kappa) = \kappa\hat{\mathbf{x}} + k_0\hat{\mathbf{z}}$ and polarized in the $\boldsymbol{\epsilon}_1$ direction.

2. When a plane wave's incident angle is greater than the critical angle i_0, the text has shown that the ratio of the reflected wave's amplitude to the incident wave's amplitude has unit modulus and a phase change $e^{i\phi(i,i_0)}$. Applying this to each of the constituent wave in the Fourier decomposition (3), we have

$$\mathbf{E}''(\mathbf{x},t) = \boldsymbol{\epsilon}_1 \int d\kappa A(\kappa)\, e^{i\phi[i(\kappa),i_0]} e^{i\mathbf{k}''(\kappa)\cdot\mathbf{x} - i\omega t} \tag{4}$$

where $i(\kappa)$ is the incident angle of the wave $A(\kappa)/\mathbf{k}(\kappa)$, $\mathbf{k}''(\kappa)$ is the reflected wave vector of $\mathbf{k}(\kappa)$.



With the help of the diagram above, we see that

$$\mathbf{k}''(\kappa) = \kappa\hat{\mathbf{x}}'' + k_0\hat{\mathbf{z}}'' \tag{5}$$

This turns (4) into

$$\mathbf{E}''(\mathbf{x},t) = \boldsymbol{\epsilon}_1 \int d\kappa A(\kappa)\, e^{i\phi[i(\kappa),i_0]} e^{i\kappa x'' + ik_0 z'' - i\omega t} \tag{6}$$

From elementary geometry, we know

$$i(\kappa) = i - \tan^{-1}\left(\frac{\kappa}{k_0}\right) \tag{7}$$

If the Fourier coefficient $A(\kappa)$ has support only in the neighborhood of $\kappa = 0$, we can make the approximation

$$\phi[i(\kappa),i_0] \approx \phi(i,i_0) - \left.\frac{d\phi}{di}\right|_i \cdot \tan^{-1}\left(\frac{\kappa}{k_0}\right) \approx \phi(i,i_0) - \left.\frac{d\phi}{di}\right|_i \cdot \frac{\kappa}{k_0} \equiv \phi(i,i_0) - \kappa\cdot\delta \tag{8}$$

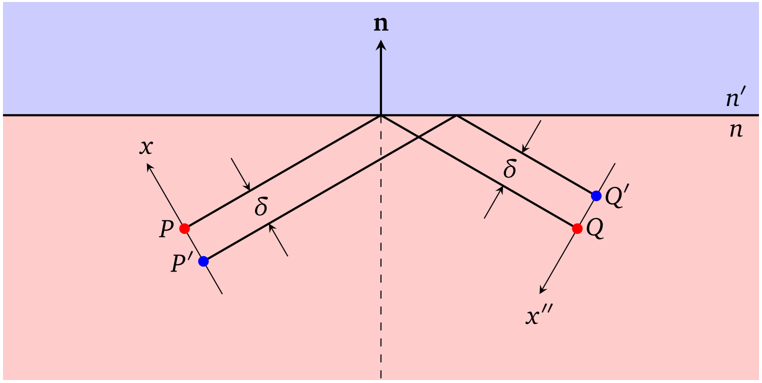
with

$$\delta \equiv \left.\frac{d\phi}{di}\right|_i \cdot \frac{1}{k_0} \tag{9}$$

(Even though there is an obvious abuse of the symbol i, I hope it is clear what it means when it shows up at different positions in an equation.)

Now (6) becomes

$$\mathbf{E}''(\mathbf{x},t) \approx \boldsymbol{\epsilon}_1 \left[\int d\kappa A(\kappa) e^{i\kappa\left(x''-\delta\right)}\right] e^{i\phi(i,i_0)} e^{ik_0 z''-i\omega t} = \boldsymbol{\epsilon}_1 E\left(x''-\delta\right) \cdot e^{i\phi(i,i_0)} e^{ik_0 z''-i\omega t} \tag{10}$$



We can use the diagram above to understand this result. Let P be a point at the center of the incident beam, i.e., $x=0$. Let Q be a point at the center of the totally reflected beam, i.e., $x''=0$. (10) claims that the electric field at Q is the result of subjecting the electric field at P' to the phase change $e^{i\phi(i,i_0)}$ as required by the total internal reflection. This means the actual reflected field is shifted from the geometrically mirrored beam in the direction $Q' \to Q$ with amount δ.

This appears to be in the opposite direction of what figure 7.7 depicted. But if we carry out the derivative in (9) for the perpendicular case and parallel case, we will see that δ is actually a negative quantity.

3. For perpendicular polarization, equation (7.39) indicates that

$$\phi_\perp(i,i_0) = -2\tan^{-1}\left(\frac{\sqrt{\sin^2 i - \sin^2 i_0}}{\cos i}\right) \qquad \Longrightarrow \qquad \frac{d\phi_\perp}{di} = -2\frac{\sin i}{\sqrt{\sin^2 i - \sin^2 i_0}} \tag{11}$$

Thus

$$\delta_\perp = -2\frac{\sin i}{\sqrt{\sin^2 i - \sin^2 i_0}}\frac{1}{k_0} = -\frac{\lambda}{\pi}\frac{\sin i}{\sqrt{\sin^2 i - \sin^2 i_0}} \tag{12}$$

For parallel polarization, equation (7.41) gives

$$\phi_\parallel(i,i_0) = -2\tan^{-1}\left(\frac{\sqrt{\sin^2 i - \sin^2 i_0}}{\sin^2 i_0 \cos i}\right) \qquad \Longrightarrow \qquad \frac{d\phi_\parallel}{di} = -2\frac{\sin i}{\sqrt{\sin^2 i - \sin^2 i_0}} \cdot \frac{\sin^2 i_0}{\sin^2 i - \cos^2 i \sin^2 i_0} \tag{13}$$

Thus

$$\delta_\parallel = \delta_\perp \cdot \frac{\sin^2 i_0}{\sin^2 i - \cos^2 i \sin^2 i_0} \tag{14}$$