

$$j_l(x) = A_l(x) \frac{\sin x}{x} + B_l(x) \cos x \quad (1)$$

$$A_0 = 1 \quad B_0 = 0 \quad (2)$$

$$A_1 = \frac{1}{x} \quad B_1 = -\frac{1}{x} \quad (3)$$

$$Z_{l+1}(x) = \frac{2l+1}{x} Z_l(x) - Z_{l-1}(x) \quad \text{for } Z = A, B \quad (4)$$

$$\frac{d[xj_l(x)]}{dx} = j_l(x) + xj'_l(x) = xj_{l-1}(x) - lj_l(x) \quad \text{use } j'_l(x) = j_{l-1}(x) - \frac{l+1}{x}j_l(x) \quad (5)$$

$$\mathbf{E}_{lm}^{\text{TE}} = -ij_l(kr)\Phi_{lm}e^{-i\omega t} = j_l(kr)\Phi_{lm}e^{-i(\omega t + \pi/2)} \quad (6)$$

$$\mathbf{H}_{lm}^{\text{TE}} = \left\{ \frac{l(l+1)}{kr} j_l(kr) \mathbf{Y}_{lm} + \frac{1}{kr} \frac{d[krj_l(kr)]}{d(kr)} \Psi_{lm} \right\} e^{-i\omega t} \quad (7)$$

$$\mathbf{H}_{lm}^{\text{TM}} = -ij_l(kr)\Phi_{lm}e^{-i\omega t} = j_l(kr)\Phi_{lm}e^{-i(\omega t + \pi/2)} \quad (8)$$

$$\mathbf{E}_{lm}^{\text{TM}} = -\left\{ \frac{l(l+1)}{kr} j_l(kr) \mathbf{Y}_{lm} + \frac{1}{kr} \frac{d[krj_l(kr)]}{d(kr)} \Psi_{lm} \right\} e^{-i\omega t} \quad (9)$$

$$\text{Re}(ze^{-i\xi}) = \text{Re}(z) \cos \xi + \text{Im}(z) \sin \xi \quad (10)$$