

## SL08. VC Dimensions

### Introduction:

- Haussler Theorem states that number of examples  $m$ :

$$m \geq \frac{1}{\epsilon} (\ln|H| + \ln \frac{1}{\delta})$$

- The problem is, Haussler Theorem breaks with infinite hypotheses spaces.
- If you have an infinite set of hypotheses, the above equation will be infinite.
- Linear separators, artificial neural networks and decision trees with continuous inputs all have infinite hypotheses spaces.
- Note that there is a notion of syntactic hypothesis space, which is anything that can be written as the hypothesis. However, there is also semantic hypothesis or only the meaningfully different hypothesis space.



### Power of a Hypothesis Space:

- VC Dimension is the largest set of inputs that the hypothesis class can shatter (label in all possible ways).
- Interval training:
  - To prove that a specific VC dimension is possible, you need to prove that there's some set of points you can shatter.
  - To prove that a specific VC dimension is not possible, you need to prove that there's no example that can be shattered.
- For any  $d$ -dimensional hyperplane hypothesis class, the VC dimension will be  $d + 1$ .
- If our hypothesis is points inside convex polygon, VC dimension is infinite.

### Sample Complexity & VC Dimension:

- Adding the VC Dimension to Haussler Theorem:

$$m \geq \frac{1}{\epsilon} (8 \cdot Vc(H) \cdot \log_2 \frac{13}{\epsilon} + 4 \cdot \log_2 \frac{2}{\delta})$$

### VC Dimension of Finite $H$ :

- $H$  is PAC-learnable if, and only if, VC dimension is finite.