ECSE 310 Thermodynamics of Computing, Fall 2025

Homework 3: SMI and information theory (Problem Analysis)

Assigned: October 27th 2025

Due: November 16th 2025, 11:59 pm

Late policy: 1% deduction per hour

Instructions: Please use the MyCourses tool to submit your answers. If you need to attach a scanned or photographed handwritten page please check to make sure that it is legible. Illegible pages will be rejected and zero credit will be given.

Academic integrity reminder: In submitting this assignment on MyCourses you are attesting that it is the result of your own work.

Weights for each question are shown in square brackets. Homework has 50 points total

1. A coin has a probability of *x* of landing heads and *1-x* of landing tails, where *x* has a value between 0 and 1. Prove that the SMI of the coin toss is maximized when *x*=1/2. [5 points]

2. The attached spreadsheet ("dataset homework 3 fall 2025.csv") provides data on age (A), number of years of post-primary education (Y), gender (G), hours worked per week (W) and income (I) for 48,882 individuals taken from a census. **Important**: For each question below please provide evidence of the calculation technique that you used with your answer (i.e. upload the spreadsheet, Python code, Matlab, Mathematica etc that you used) or no points will be given.
    a. Using this data calculate the marginal SMI associated with each column, i.e. $H(A), H(Y), H(G), H(W), H(I)$. [5 points]
    b. What is the joint entropy for the entire data set? [2 points]
    c. Which variable provides the greatest information about income: number of years of education, gender or number of hours worked? Please quantify your answer. [4 points]
    d. If you learn that someone on that list is female, how much information do you gain or lose about the number of hours that they work? [4 points]?

[15 points total]

3. During an ice hockey game there are two teams with 6 players each on the ice at any time. The dimensions of the rink are 60 m x 30 m (taken as rectangular for the purposes of this question). We will assume that players skate at an average speed of 20 km/hour. We can assume that their speed follows a normal distribution with a standard deviation of 5 km/hour. Based on this information calculate:
    a. The differential entropy (in bits) associated with the position of all of the players assuming that we care about the identity of each player and that positions are specified to the nearest meter. [4 points]
    b. The differential entropy (in bits) associated with the position of all of the players assuming that we only care about the team that the player is on (but not their individual identity) [3 points]
    c. The differential entropy (in bits) associated with the speed of all of the players, assuming that speed is measured to the nearest 0.1 km/hour. [4 points]
    d. Use this to estimate the minimum bandwidth (in bits/second) that would be needed to transmit the instantaneous position and speed of all members of both teams assuming that we would need to send an update each time the average player position changes by more than 0.5 meter and assuming that we only care about the team that the player is on (but not their individual identity). Please note that we assume that there are no line changes during that time! [4 points]

    [15 points]

4. A monatomic gas at low pressure (10 kPa) is contained at room temperature (293 K) in a cubic box of dimensions 1 mm on all sides. If we consider the instantaneous pressure difference Δp between two halves of the chamber (due to random molecular movements), then 50% of the time we would expect the value of Δp to be less than what value? Please explain your calculations.

    [15 points]