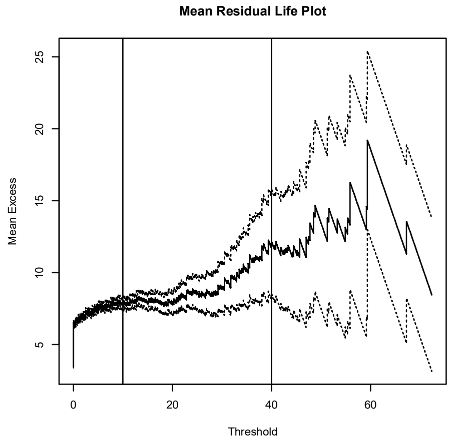

Figure 2.2: mean residual life plot for the `rain` dataset

Exercise 2.1 (Quantile estimation with simulated data)**:**
(Recommended **R** package: `evd`.)

Generate $n = 1e + 5$ independent data $X_i, i \leq n$ from a Fréchet distribution with *c.d.f.*

$$G_{a,b,\alpha}(x) = e^{-[(x-b)/a]^{-alpha}}$$

with $\alpha = 3, b = 1, a = 0$. In the sequel pretend that you don't know the true distribution.

1. Fit a GPD disribution above some threshold t. Choose t with a stability plot (function `tcplot`).

2. Estimate the quantile z_p of the distribution of the X_i at level $p = 1 - 5.10^{-4}$, using a plug-in method based on the GPD model above t and an empirical estimate $\hat{\zeta}$ of $\zeta = \mathbb{P}(X_1 > t)$.

3. Investigate the variability of the quantile estimator based on a Monte-Carlo approximation. Namely repeat the experiment $N_{expe} = 100$ times (which means simulating N_{expe} datasets). For each replication i, fit a GPD model above some fixed threshold and use it to estimated a quantile $\hat{z}_i$. In the end, consider the empirical interquantile range (e.g. at level 0.90) of the $\hat{z}_i$'s. Does the true quantile belong to this interquantile range? Compute the mean squared error (based on the N_{expe} experiments) of the quantile estimate.

4. Repeat the latter question (i.e. compute the mean squared error) with a Fréchet shape $\alpha = 1.1$, and $\alpha = 10$ (you may have to change your threshold). Comment on how the shape parameter affects the mean squared error.

2.3 Regular variation of a real function

We now focus on a most useful case in terms of risk management applications in relation with very large events, that is, the case $\gamma > 0$. We shall see that the maximum domain of attraction of Fréchet type is in fact the same as the family of regularly varying (tail) distribution functions. We start with elementary definitions and first properties.