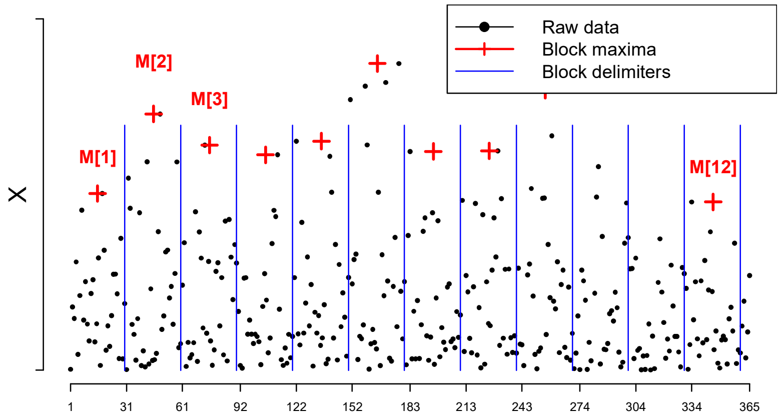

Figure 1.9: Work-flow for the block-maxima method.

Exercise 1.3 (Domains of attraction: illustration)**:**
Hint: Results from Exercise 1.2 will be useful.

1. Illustrate the phenomenon of weak convergence stated in Fisher and Tipett's theorem through the convergence of histograms (built from a random sample) of maxima towards histograms of the limit, for a negative shape parameter:
   - Choose a textbook distribution F in the Weibull domain of attraction and find appropriate norming sequences a_n, b_n such that (MDA) holds.
   - Write a short code allowing to:
     - generate M blocks of size n of independent random variables distributed according to F and normalize the block maxima ;
     - plot a histogram of the M normalized maxima and superimpose the histogram for the limit distribution in a visually illustrative manner.
   - Let M and n vary so as to illustrate weak convergence of maxima as $M \to \infty$. Explain the role of M and n in what you observe. Summarize the results in a figure including ≈ 6 such histograms with different values of M and a single (appropriate) value of n.
2. show (graphically and numerically) uniform convergence of c.d.f.'s. Explain why (i.e. prove that) weak convergence of normalized maxima indeed implies uniform convergence of c.d.f's
3. Change the input distribution and work with a translated Pareto distribution, $\mathbb{P}(X > x) = ((x - \beta)/u)^{\alpha}$, on some for some $\alpha > 0, \beta, u \in \mathbb{R}$, $x \geq u + \beta$. Draw similar outputs as in the previous questions and compare the rate of convergence.
4. With the distribution from question 1 or 3, generate a dataset of an appropriate size and estimate the GEV parameters with a maximum-likelihood method. discuss the convergence towards the true parameters.

Hand out a notebook (R or Python) with maths, code and results.