Arch. Hist. Exact Sci. (2016) 70:543–552
DOI 10.1007/s00407-016-0178-z

An analysis of the *Tabulae magistrales* by Giovanni Bianchini

José Chabás[1]

Received: 15 January 2016 / Published online: 29 February 2016

Abstract Bianchini called *Tabulae magistrales* a set of eight tables he compiled to solve problems in spherical astronomy. This set, which is the object of this paper, consists of auxiliary and trigonometric functions, including the sine and the tangent functions, for radii 10,000 and 60,000, and seems to be the first set of tables in Latin specifically devoted to mathematical tools for computational astronomy. Bianchini presented some of his tables in decimal form, which meant that for the first time one of the oldest astronomical tradition, the sexagesimal base ($R = 60$), was abandoned.

Giovanni Bianchini (b. ca. 1400, d. after 1469) spent most of his life in Ferrara, where he served as administrator of the estate of the d'Este family. He had previously been a merchant in Venice until 1427, where it is likely that he attended some abacus school. In that year Marquis Nicolò d'Este (1383–1441) invited him to join his court in Ferrara. He found there the proper milieu to become a celebrated mathematician and astronomer. Bianchini is mostly known for an extensive and original set of astronomical tables for the motion of the Sun, the Moon, and the planets that he completed in 1442.[1] The tables were addressed to his patron Leonello d'Este (d. 1450), Nicolò's son and

[1] See Chabás and Goldstein (2009). For additional biographical information on Bianchini, see Federici Vescovini (1968).

Communicated by: Noel Swerdlow.

A preliminary version of this paper was presented in a workshop held in Paris (SYRTE-Observatoire de Paris) in June 2015, "Analysing and editing numerical tables from ancient astral sciences." I thank all participants for their comments and suggestions. I am also grateful to Bernard R. Goldstein (Pittsburgh) for his helpful remarks on a draft of this paper.

✉ José Chabás
jose.chabas@upf.edu

1 Universitat Pompeu Fabra, Barcelona, Spain

successor, and in 1452 they were presented and dedicated to the Emperor Frederick III, during a visit to the local court in Ferrara, then ruled by Borso d'Este (d. 1471). Bianchini's highly innovative tables were compiled in the framework of the Alfonsine Tables, as recast and developed in Paris a century earlier, and were printed for the first time under the title *Tabulae astronomiae* in 1495 in Venice. There were two later editions, one also produced in Venice in 1526 and another in Basel in 1553. Bianchini's planetary tables were appreciated and used by the most outstanding astronomers of his time, including Georg Peurbach (1423–1461) and Regiomontanus (1436–1476).[2]

Bianchini is also the author of others texts and shorter collections of tables concerning mathematical astronomy.[3] In a letter dated November 21, 1463, addressed to Regiomontanus, Bianchini listed some of his other works: *Flores Almagesti*, written around 1440 and consisting of several treatises on arithmetic and algebra to serve as a mathematical introduction to astronomy; a short text dated 1442 on an instrument to measure the altitude of the stars; a set of tables with canons on the *primum mobile*;[4] and a set called *Tabulae magistrales* to facilitate the computational tasks of practitioners of astronomy.[5] This is the same letter where Bianchini presents an algebraic problem to Regiomontanus "Find two numbers in the ratio 5 to 8 whose sum equals its product."[6] The analysis of the "magistral" tables mentioned in this letter is the object of the present paper.

The *Tabulae magistrales* are a set of auxiliary tables consisting of six or eight trigonometric functions, depending on the manuscript, to solve a variety of problems in spherical astronomy. As far as I can determine, this is the first set of tables in Latin exclusively devoted to mathematical tools for computational astronomy. The need for such a set of tools to solve specific astronomical problems was already felt by early Arabic astronomers, who compiled for that purpose sets of auxiliary functions, consisting of basic trigonometric functions, such as the sine and the tangent functions, or combinations of them. This was the case, among others, of Ḥabash al-Ḥāsib (ca. 850), who included in his zijes the tangent function with a norm of 60, and Abū Naṣr Manṣūr Ibn ᶜIrāq (ca. 1000), who provided compound auxiliary functions as well as instructions on how to apply them to a number of astronomical problems.[7] Bianchini's auxiliary tables differ from those compiled by his Arabic predecessors.

As is most often the case in medieval astronomy, the principle guiding table-makers was to produce labor-saving and user-friendly tables, as Bianchini explicitly explains:

[2] In a letter dated 1456 to Johann Nihil, court astrologer to Emperor Frederick III, Peurbach described the calculation of ephemerides he had computed together with Regiomontanus, for which they used Bianchini's tables. While in Vienna in 1460 Regiomontanus made a copy of these tables for his own use and wrote abridged canons to them entitled, *Canones breviati in tabulas Ioannis de Blanchinis*. On Peurbach and Regiomontanus, see Swerdlow (forthcoming).

[3] On the trigonometrical tables of Bianchini, see Rosińska (1981). For a list of Bianchini's various sets of tables extant in Cracow, see Rosińska (1984).

[4] This term refers to the outermost celestial sphere, responsible for diurnal rotation, and can be translated literally as "first movable." The term was also used by John of Lignères and Regiomontanus, and eventually became standard, to characterize tables for spherical astronomy, most of which concern the diurnal rotation.

[5] See Magrini (1917), especially Letter II, pp. 24 and XIV. See also Gerl (1989).

[6] The two numbers sought are 13/5 and 13/8.

[7] See Jensen (1972), King (2004).

"composui tabulas magistrales iocundas et breves et utiles ad concluendum multos calculos" (BJ 556, 13r). And so he did in his *Tabulae magistrales*.

This set of tables is extant in several manuscripts, including Bologna, Biblioteca Comunale, MS 1601, 66v, 85v–92v (8 tables); Cracow, Biblioteka Jagiellońska, MS 556, 48r–54v, 93r (8 tables), and MS 606, 62r–69r (Tables 1–6); Paris, Bibliothèque nationale de France, MS 7270, 183v–186r, 188r, 228r–230v (Tables 1–7), MS 7271, 181v–187v, 245v (Tables 1–7), MS 7286, 132v, 149r–154v (Tables 1–7), and MS 10265, 223v–232r, 237r–239r (Tables 1–6 and 8). For the presentation of the tables in this paper we take as base manuscript Cracow, MS 556 (henceforth BJ 556), copied in Rome about 1464 for Gregorius de Cracovia, astrologer of Pope Paul II. The sequence of the tables in this manuscript does not follow the expected order: Table 8 does not follow Table 7 but Table 1.

The tables

Table 1 (BJ 556, f. 48r–v): *Divisio tabule magistralis primae sinus secundi maxime declinacionis per sinum primum cuiuslibet gradus*

The title refers to two basic trigonometric functions: sine (*sinus primus*) and cosine (*sinus secundus*). The entries are given at intervals of 0;10° of the argument, θ, from 0;10° to 90;0°. The last entry, $T_1(90°)$, is 9167, which is a rounding of 10,000 $\cdot \cos 23;33,30° = 10{,}000 \cdot 0.916654$.

The function underlying *Tabula magistralis 1,* expressed in modern notation, is

$$T_1\,(\theta) = R\frac{\cos \varepsilon}{\sin \theta},$$

with R = 10,000, that is, the cosecant function with a coefficient R cos ε = 9167. That 23;33,30° is the value used for the obliquity of the ecliptic, ε, is clear from Bianchini's text (BJ 556, f. 7ra) where this value is explicitly mentioned, and from the *Tabula novissima declinacionis per arcum secundum Iohannem Blanchinum* (BJ 556, f. 59r), where the maximum entry is 23;33,30°. Note that Bianchini's value for the obliquity of the ecliptic is the same as that used in the Toledan Tables and in other previous tables for the same purpose. However, all other entries in Bianchini's declination table differ,[8] thus providing a good example of how inappropriate it is in medieval astronomy to identify two tables for the same purpose based only on a shared parameter. It happens that some astronomers or table-makers did not limit themselves to copying a pre-existing table; rather, they computed its entries independently.[9] For the recomputation of selected entries in this and other tables, see Table A, below.

As can be seen in Fig. 1, at the right of the column for $T_1\,(\theta)$ there is another column, headed "Equatio," intended for interpolation. To illustrate this, consider the row for argument 0;10°, where we are given the number 523610. This is indeed the number to be subtracted from $T_1(0;10°) = 3142857$ to obtain the following entry,

8 See Chabás and Goldstein (2012).

9 For another example, see Levi ben Gerson's table of declination which is based on the same parameter used by Abraham Ibn Ezra, but the entries in their respective tables are all different: Goldstein (1974, p. 96).

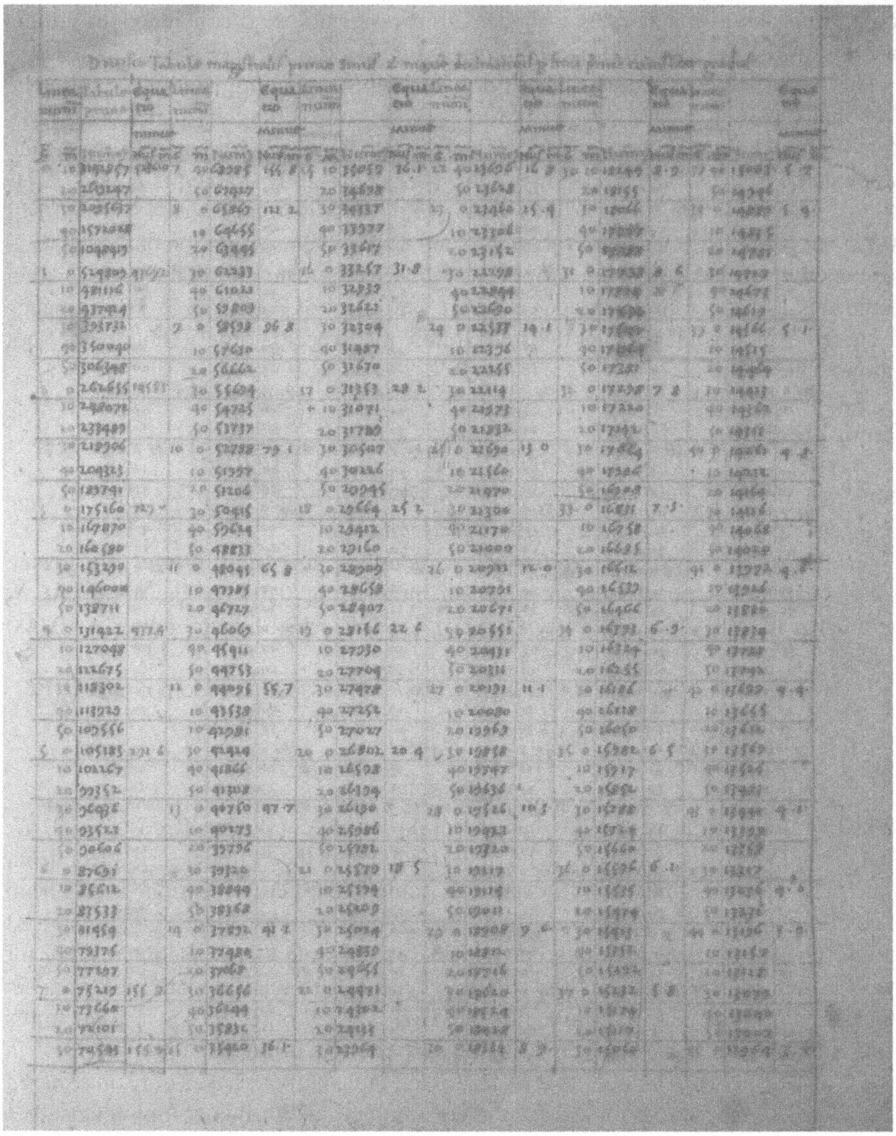

Fig. 1 *Tabula magistralis 1* in Cracow, BJ 556, 48r (excerpt)

$T_1(0;20°) = 2619247$. This method is valid for large numbers, but for the rest of the table in this column there are two numbers, one headed "numerus" and another headed "minuti," normally separated by a blank space or a dot. The purpose is the same, interpolation. Consider, for example, the entry for argument 17°, $T_1(17;0°) = 31353$. The associated numbers in the second column are 28 and 2. And indeed, $T_1(17;10°) = 31071$ results from subtraction of 282 from 31353. Furthermore, to find the entries between 17;0° and 17;10° we are supposed to use linear interpolation and subtract the integer 28 (here called "numerus") and 2 parts (here called "minuti"), as many times as needed. Here "minuti" are to be understood as decimal fractions (2/10 in this case). Thus, $T_1(17;1°) = 31353 - 28.2$. Columns for the "Equatio" are found in all other tables, except for *Tabula magistralis 7*.

Table 8 (BJ 556, f. 49r–v): *Tabula magistralis octava. Numerus divisionis sinus primi cuiuslibet gradus per sinum secundum maxime declinacionis*

The entries in this table are also given at intervals of 0;10° of the argument, θ, from 0;10° to 90;0°. The last entry, $T_8(90°)$, is 10905, which is a close approximation of 10,000/cos 23;33,30° = 10,000/0.916654 = 10,909.25. The function underlying *Tabula magistralis 8* can be represented in modern notation by

$$T_8(\theta) = R\frac{\sin\theta}{\cos\varepsilon},$$

with R = 10,000, that is, the secant function with a coefficient R/cos ε = 10,905. Note that the functions $T_1(\theta)$ and $T_8(\theta)$ are reciprocal, and $T_1(\theta) \cdot T_8(\theta) = 10^8$.

Table 2 (BJ 556, f. 50r–v): *Tabula magistralis secunda. Sinus primi differencie ascensionis cuiuslibet gradus eclyptice in circulum directum et circulum obliquum in regione graduum 45*

The entries in this table represent $\sin\gamma$, where γ is the ascensional difference, that is, the difference between the right ascension and the oblique ascension of a point on the ecliptic rising on the horizon.[10] The entries in this table are displayed at intervals of 0;10° of the argument, from 0;10° to 90;0°. The independent variable here is the longitude, λ. The ascensional difference depends on the geographical latitude, φ, and can be computed by means of the modern formula

$$\sin\gamma = \tan\delta\tan\varphi,$$

where the declination, δ, is found in a declination table with the longitude of the point, λ, as argument or can be computed with the expression

$$\delta(\lambda) = \arcsin(\sin\lambda \cdot \sin\varepsilon).$$

Bianchini compiled this table for Ferrara and considered its latitude to be $\varphi = 45°$; hence $\tan\varphi = 1$. This assumption considerably facilitates computation, because in this case

$$\sin\gamma = \tan\delta.$$

When $\lambda = 90°$, $\sin\lambda = 1$, and $\delta = \varepsilon$; it follows that $\sin\gamma = \tan\varepsilon$. Now, the last entry, $T_2(90°)$, is 26161, which is indeed tan 23;33,30° times a coefficient, 60,000, not 10,000 as was the case for the two previous tables. Then it is easy to verify that the function underlying *Tabula magistralis 2* can be represented by

$$T_2(\lambda) = R\tan\delta(\lambda),$$

[10] See, e.g., Chabás and Goldstein (2012) (ref. 7), p. 30.

where $R = 60{,}000$.

Table 3 (BJ 556, f. 51r–v): *Tabula magistralis tercia. Productum sinus primi maxime declinacionis per sinum eius secundum et numerus quociens multiplicatus per sinum secundum ascensionis stelle secundum Iohannem Blanchinum que posite sunt in tabula radicum ascensionum*

In this table the entries are given at intervals of 0;10° of the argument, but from 0;0°, rather than 0;10° as in the previous cases, to 90;0°. The entries agree in most cases with those in *Tabula magistralis 2*, but in the reverse order. The heading of the table suggests an expression of the type

$$T_3(\theta) = R\frac{\sin\varepsilon}{\cos\varepsilon}\cdot\cos\alpha,$$

where α is the right ascension. The *Tabula radicum ascensionum* mentioned in the heading is indeed found in Cracow, MS BJ 556, f. 59v. It is a table for right ascension given at intervals of 1° beginning at Aries 0°, with the longitude, λ, as independent variable. The large interval makes interpolation necessary, with uncertain accuracy. To check the entries, the right ascension can also be computed with the expression

$$\alpha = \arcsin(\tan\delta/\tan\varepsilon),$$

where $\tan\delta\,(\lambda)$ has already been tabulated in *Tabula magistralis 2*. The first entry here, for $\lambda = 0°$, is $T_3(0°) = 26160$, and results from multiplying 60,000 by tan 23;33,30°. We note that in *Tabula magistralis 2*, this product was taken to be 26161. Again, for $\lambda = 90°$, $\sin\lambda = 1$ and $\delta = \varepsilon$; thus $\alpha = 90°$, and $\cos\alpha = 0$. Hence $T_3(90°) = 0$. We have recomputed selected entries (see Table A), by means of

$$T_3(\lambda) = R\tan\ \varepsilon\cdot\cos\alpha,$$

with R = 60,000, and the expression for α given above. The agreement of text and recomputation is very good.

Table 4 (BJ 556, f. 52r–v): *Tabula magistralis quarta. Numerus divisionis sinus primi cuiuslibet gradus quarti circuli per sinus eius secundum*[11]

The entries in this Table (see Fig. 2) are given at intervals of 0;10° of the argument, θ, from 0;10° to 89;50°. The entry for 45° is 10,000, wrongly copied in MS BJ 556 as 1000. The function underlying the table is

$$T_4(\theta) = R\frac{\sin\theta}{\cos\theta} = R\tan\theta,$$

that is, the tangent function normed for a radius R = 10,000.

Table 5 (BJ 556, f. 53r–v): *Tabula magistralis quinta*

The entries in this table are also given at intervals of 0;10° of the argument, θ, from 0;10° to 90°. In this case $T_5(30°) = 20{,}000$ and $T_5(90°) = 10{,}000$. The underlying function is thus

[11] Excerpts of Tables 4 and 5 were published by Rosińska (1981) (ref. 3), pp. 49–50.

Table A Selected entries from the *Tabulae magistrales* in Cracow, BJ 556, and recomputed values (in italics)

Argument	TM 1	TM 8	TM 2	TM 3	TM 4	TM 5	TM 6
0;0	–	–	–	26160	–	–	–
				26161			
0;10	3142857	31	70	26158	29	3428571	12
	3151227	*32*	*70*	*26161*	*29*	*3437752*	*13*
…							
30;0	18334	5453	12130[b]	22163[c]	5773	20000	2517
	18333	*5455*	*12237*	*22136*	*5774*	*20000*	*2517*
…							
45;0	12964	7711	17655	17683	1000[d]	14143	4360
	12963	*7714*	*17678*	*17678*	*10000*	*14142*	*4360*
…							
60;0	10595[a]	9444	22136	12234	17321	11548	7551
	10585	*9448*	*22136*	*12237*	*17321*	*11547*	*7552*
…							
89;50	9167	10905	26159	70	3420851	10000	1491491
	9167	*10909*	*26161*	*70*	*3437737*	*10000*	*1498934*
90;0	9167	10905	26161	0	–	10000	–
	9167	*10909*	*26161*	*0*		*10000*	

[a] Instead of 10585, as in Bibliothèque nationale de France (henceforth BnF), MS 10265, 224r
[b] Instead of 12230, as in BnF 7271, 182v; BnF 7286, 150r; BnF 10265, 225r; but 12130 in BnF 7270, 183v
[c] Instead of 22136, as in BnF 7270, 185v; BnF 7271, 183v; BnF 7286, 151r; BnF 10265, 226v
[d] Instead of 10000, as in BnF 7270, 228r; BnF 7271, 185r; BnF 7286, 152r; BnF 10265, 228v

$$T_5\,(\theta) = \frac{R}{\sin\theta} = R\,\mathrm{cosec}\,\theta,$$

that is, the cosecant table multiplied by a coefficient R = 10,000. We note that $T_5\,(\theta)$ results from $T_1\,(\theta)$ by division by the constant cos 23;33,30°.

Table 6 (BJ 556, f. 54r–v): *Tabula magistralis sexta*

The argument in this table ranges from 0;10° to 89;50°, at intervals of 0;10°. The entry for 45°, $T_6(45°)$, is 4360; which is a rounding of 10,000 · tan 23;33,30° = 10,000 · 0.4360236. The function underlying the table is

$$T_6\,(\theta) = R\tan\theta\,\tan\,\varepsilon,$$

that is, the tangent table multiplied by a coefficient 4360 $= R\tan\varepsilon$, with R = 10,000. This function results from multiplying $T_4\,(\theta)$ by the constant tan 23;33,30°.

Table 7 (BJ 556, f. 93r): *Tabula ad inveniendum gradum ascensionum cuiuscumque stelle cum medietate arcus ipsius diurni in sexto climate cuius latitudo est gradum 45, in qua intratur cum gradibus ascensionum ipsius in medio celi, que tabula dilatatur in tabulis sequentibus. Tabula magistralis septima*

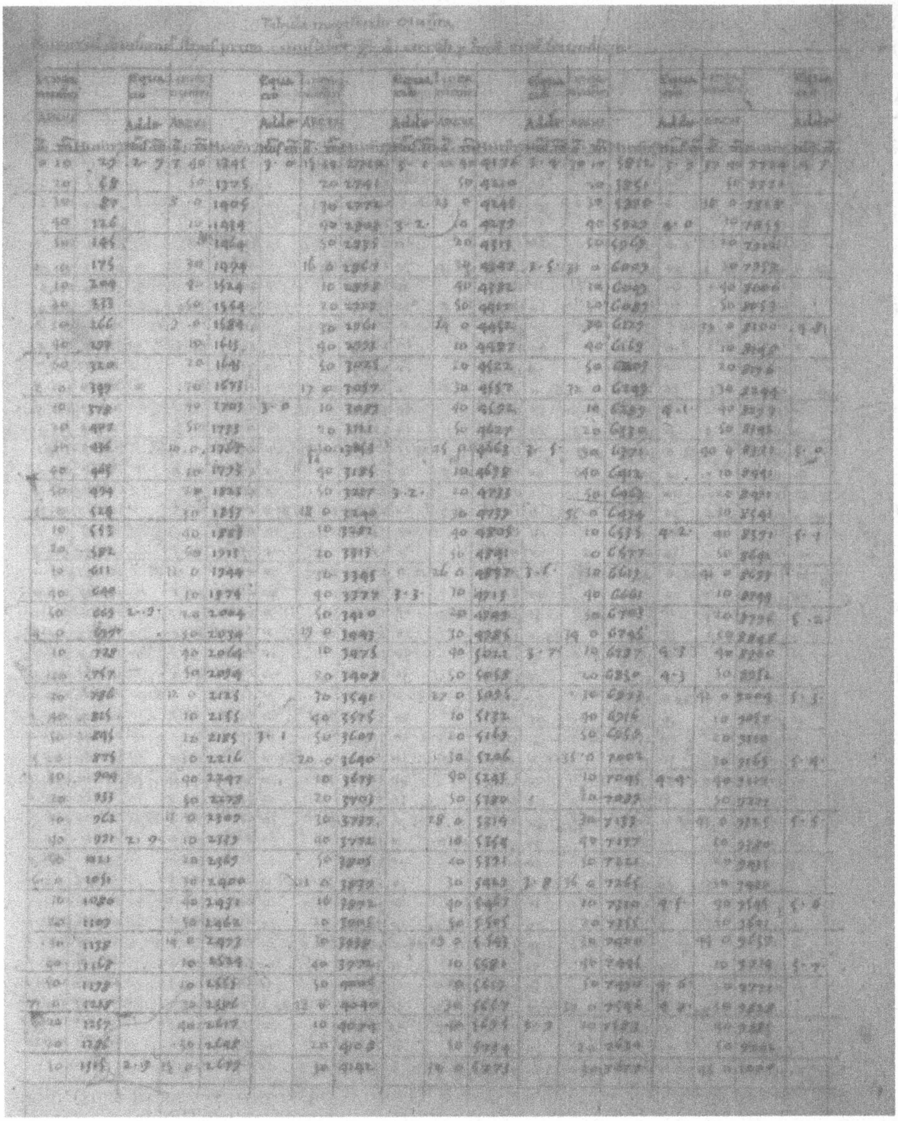

Fig. 2 *Tabula magistralis 4* in Cracow, BJ 556, f. 52r (excerpt)

This table is different from those preceding it, for it involves quantities such as oblique ascension, ascensional difference, and semidiurnal arc that need to be computed previously with other auxiliary tables: see Table B.

The argument of this table is right ascension, α, given for each integer degree from 1° to 360°. Column 2 displays the oblique ascension for geographical latitude 45° (Ferrara), where $\tan \varphi = 1$. The oblique ascension, ρ, is the difference between the right ascension and the ascensional difference, γ. To recompute this column, we note that *Tabula magistralis 2*, only valid when $\tan \varphi = 1$, gives $\sin \gamma$, as a function of the

Table B *Tabula magistralis 7* (excerpt)

Right ascension (°)	Oblique ascension (°)	Semidiurnal arc (°)	Right ascension (°)	Oblique ascension (°)	Semidiurnal arc (°)
1	0;33	90;27	...		
...					
30	17;24	102;35	210	222;34	77;25
...			...		
60	37;51	112;10	240	262;10	67;50
...			...		
90	64;9	115;51	270	295;51	64;9
...			...		
120	97;50	112;10	300	322;10	67;50
...			...		
150	137;25	102;35	330	342;35	77;25
...			...		
180	180;0	90;0	360	360;0	90;0

longitude, λ, not of α. Therefore, for each value of the argument, one first computes λ and then finds $\sin\gamma$ in *Tabula magistralis 2* to determine the ascensional difference. Then, an entry in column 2 is found by adding the ascensional difference just computed to the argument. To illustrate the procedure, consider $\alpha = 60°$. The expression $\tan\alpha = \cos\varepsilon \cdot \tan\lambda$ gives $\lambda = 62;6{,}39°$, which corresponds to $\sin\gamma = 22631$ in *Tabula magistralis* 2, and thus $\gamma = 22;9{,}34° \approx 22;10°$. Therefore, the ascensional difference is $60° - 22;10° = 37;50°$, almost in agreement with the entry (37;51°). Column 3 is the semidiurnal arc, and it can be computed by adding 90° to the ascensional difference. In the example considered, the arc sought is $90° + 22;10° = 112;10°$, which is the entry in the table.

Other tables

In Cracow, BJ 556 there are also two other trigonometric tables by Bianchini that do not belong to the *Tabule magistrales*, although they are closely related to them: one displays the sine and the cosine functions, at intervals of 0;10° of the argument (ff. 57v–58v), normed R = 60,000, and the other is for the *umbra* (shadow), that is, the cotangent function (f. 47r–v), with the same interval, but normed R = 10,000. Note that Bianchini abandoned in this case the coefficient 12, or multiples of it, traditionally and immovably used in tables of shadows until then.

The *Tabule magistrales* contain tables for radii 10,000 and 60,000. The sexagesimal base (R = 60) goes back at least to Ptolemy in his table of chords in *Almagest* I.11, and was also used in the tables for the sine function. The entries were usually displayed at intervals of half a degree of the argument. To enhance precision Bianchini compiled new tables enlarging the radius to 60,000 and shortening the intervals to 0;10°. However, decimalization was the real innovation in Bianchini's *Tabule magistrales*, for five of these tables were compiled for the first time for R = 10,000, thus abandoning in this particular case one of the oldest astronomical traditions, the sexagesimal base.

This new approach was soon followed by many other astronomers, starting with a young contemporary of Bianchini with whom he corresponded, Regiomontanus, who in 1468 compiled a table for the sine function normed $R = 10^7$.[12] Of special interest is also Bianchini's table for tangents (*Tabula magistralis 4*), with $R = 10^4$ and intervals of 0;10°, which Regiomontanus modified to a table that he called "fecunda," with a norm 10 times bigger ($R = 10^5$), but with a drastically reduced number of entries (intervals of 1°). Regiomontanus's *tabula fecunda* was part of his *Tabulae directionum profectionumque* (1467), published for the first time in 1490 and many times after (eight times in the sixteenth century). On the other hand, Bianchini's *Tabule magistrales* were not printed. This might explain why Regiomontanus, and not Bianchini, has often been considered by modern scholars responsible for "providing the model for our modern tables."

References

Chabás, J., and B.R. Goldstein. 2009. *The astronomical tables of Giovanni Bianchini*. Boston: Leiden.

Chabás, J., and B.R. Goldstein. 2012. *A Survey of European Astronomical Tables in the Late Middle Ages*. Boston: Leiden.

Federici Vescovini, G. 1968. Bianchini, Giovanni. *Dizionario biografico degli Italiani* 10: 194–196.

Gerl, A. 1989. Trigonometrisch-astronomisches Rechnen kurz vor Copernicus: Der Briefwechsel Regiomontanus–Bianchini. *Boethius: Texte und Abhandlungen zur Geschichte der exakten Wissenschaften*, 21, Stuttgart.

Goldstein, B.R. 1974. *The Astronomical Tables of Levi ben Gerson*, 96. Transactions of the Connecticut Academy of Arts and Sciences, 45. Hamden, CT.

Jensen, C. 1972. Abū Naṣr Manṣūr's Approach to Spherical Astronomy as Developed in His Treatise "The Table of Minutes". *Centaurus* 16: 1–19.

King, A. 2004. *In Synchrony with the Heavens*. Vol. 1: *The Call of the Muezzin*, 114–183. Boston: Leiden.

Magrini, S. 1917. Joannes de Blanchinis Ferrariensis e il suo carteggio scientifico col Regiomontano (1463–1464). *Atti e memorie della deputazione ferrarese di storia patria* 22(3): 1–37.

Rosen, E. 1975. Regiomontanus, Johannes, *Dictionary of Scientific Biography*. New York, VI: 348–352.

Rosińska, G. 1980. L'audience de Regiomontanus à Cracovie au XVe et au début du XVIe siècle. In *Regiomontanus studies*. ed. G. Hamann, 315–333. Vienna.

Rosińska, G. 1981. Tables trigonométriques de Giovanni Bianchini. *Historia Mathematica* 8: 46–55.

Rosińska, G. 1984. *Scientific Writings and Astronomical Tables in Cracow: A Census of Manuscript Sources (XIVth–XVIth Centuries)*. Wrocław, 476–487.

Swerdlow, N. M., *The renaissance of astronomy: Regiomontanus, Copernicus, Tycho, Kepler, Galileo* (forthcoming).

[12] See, for instance, Rosińska (1980) and Rosen (1975), especially p. 350.