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Author(s): Della Dumbaugh and Joachim Schwermer
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The collaboration of Emil Artin and George Whaples: Artin's mathematical circle extends to America

Della Dumbaugh · Joachim Schwermer

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Abstract In his biography of Emil Artin, Richard Brauer describes the years from 1931–1941 as a time when "Artin spoke through his students and through the members of his mathematical circle" rather than through written publications. This paper explores these seemingly quiet years when Artin immigrated to America and disseminated ideas about algebraic number theory during this time in his collaboration with George Whaples, a young American mathematician who had just completed his Ph.D. at the University of Wisconsin. The main result of their work is the use of the product formula for valuations to come up with an axiomatic characterization of both algebraic number fields and algebraic function fields with a finite field of constants. These two families of fields are exactly the fields for which class field theory is known to hold. We situate their mathematical work in the broader context of algebraic number theory and their lives within the broader historical context.

1 Introduction

Solomon Lefschetz had two points in mind when he wrote to Father John F. O'Hara, President of Notre Dame University, in early January, 1937. After congratulating

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D. Dumbaugh (✉)
Department of Mathematics, University of Richmond, Richmond, VA, USA
e-mail: ddumbaugh@richmond.edu

J. Schwermer
Faculty of Mathematics, University of Vienna, Nordbergstrasse 15, 1090 Vienna, Austria
e-mail: Joachim.Schwermer@univie.ac.at

J. Schwermer
Erwin Schrödinger International, Institute for Mathematical Physics, Boltzmanngasse 9, 1090 Vienna, Austria

O'Hara on Notre Dame's recent appointment of Karl Menger to their faculty, Lefschetz offered a "constructive suggestion" about another European mathematician. "I permit myself," Lefschetz wrote to O'Hara

> to name for your strong consideration another absolutely first rate man, the algebraist E. Artin, at the present time Professor at the University of Hamburg. He is an Austrian Aryan, but his wife is one-half Jewish. They have a couple of small children and you know the rest. Like Menger, Artin is in the middle thirties, famous not only as a first rate scientist but also as a teacher, and inspirer of youth, and is a most attractive personality. Although still very young he was in 1930, runner-up for the post of successor to Professor David Hilbert at Göttingen, himself an outstanding mathematical genius of all times (Lefschetz to O'Hara, 12 January, 1937).

Apparently, then, from his position within the AMS, Lefschetz learned of Artin's situation and took up his cause.[1] Fortunately, Father O'Hara "made a place for him on the [Notre Dame] faculty....in order to relieve his mind of the strain under which he labored in Germany (O'Hara to H. B. Wells, 11 June, 1938)." Thus it was a personal letter and a commitment from an institution, and not one of the organized committees, that initially brought Artin to America.[2]

For all of Lefschetz's (accurate) accolades, he did not mention that Artin had not published since 1932. In fact, this request and subsequent appointment occurred during a time that a "casual observer" might describe as "ten years of silence" (roughly 1931–1941) in terms of written publications where, instead, "Artin spoke through his students and through the members of his mathematical circle (Brauer 1967, p. 36)."[3] This paper begins to explore these "ten years of silence" and how Artin disseminated ideas about class field theory during this time and, in particular, how Artin began to work with George Whaples, a young American mathematician who had just completed his Ph.D. at the University of Wisconsin.

[1] Discouraged by the "efficacy" (or, lack thereof) of the organized committees to aid refugees, Harlow Shapley, Oswald Veblen and Hermann Weyl adopted this strategy of appealing directly to an academic institution on behalf of a particular scholar in the late 1930s (Rider 1984, pp. 151–153). Weyl, for example, arranged for a position at Trinity University for Bernard Baule, his former colleague at Graz. Writing candidly, Weyl described the position as "nothing to be particularly enthusiastic about, but certainly a lot better than a concentration camp (Weyl to American Friends Service, as quoted in Rider 1984, p. 119)."

[2] The work of these committees has been examined elsewhere in more general works such as Rider (1984), Reingold (1985), and Siegmund-Schultze (2009), to name only three.

[3] In fact, this work will serve as something of a case study for Brauer's more detailed assessment of Artin. As Brauer puts it, "[t]he ten year period 1921–1931 of Artin's life had seen an activity not often equalled in the life of a mathematician. They were followed by what may appear to a casual observer as a period of ten years of silence. This would be a false impression. It is true that Artin developed a strong aversion against the writing of papers for publication. But the essential change was that from now on, Artin spoke through his students and through the members of his mathematical circle. He gave his own ideas generously to his students. Some of the dissertations written under Artin were probably mostly his own work. Sometimes, he had had an idea before and led his student to find it for himself. On all of his students, Artin exerted a profound influence. It would be impossible to separate what was Artin's work and what was that of others and we shall not attempt this. Artin detested discussion of questions of priority. It did not matter in the least to him, whether some work was done by him or by somebody else. What mattered was that it was done the way he felt it should be done (Brauer 1967, pp. 36–37)."

Artin and Whaples met at the University of Indiana in 1939 and eventually collaborated on three papers. Their main result is the use of the product formula for valuations to come up with an axiomatic characterization of both algebraic number fields and algebraic function fields with a finite field of constants. These two families of fields are exactly the fields for which class field theory is known to hold. Their work was very much inspired by the notion of the idele group, a new approach to class field theory introduced by Claude Chevalley. Artin and Whaples defined valuation vectors as the additive counterpart of the ring of ideles attached to an algebraic number field. This association enabled them to derive the fundamental results of number theory from simple axioms.

The current paper puts the work of Artin and Whaples into a larger context by tracing relevant personal and professional points in their individual and collective lives. For Artin, this discussion begins with his important lectures on class field theory at the University of Hamburg in 1931/1932, which Chevalley among others attended, passes over to his forced emigration to the United States in 1937, and his moves to the University of Indiana at Bloomington in 1938 and Princeton University in 1946. For Whaples, we focus on his work as a Ph.D. student at Wisconsin, his collaboration with Artin at Indiana, and his subsequent short-term position at the Institute for Advanced Study in 1941–1942. We also include a discussion of the influence of the work of Artin and Whaples on the development of algebraic number theory in the following years. In the final section of the paper, we offer a fuller analysis of the salient underlying point of this time in Artin's career, that of his life as an immigrant.

2 Courses in class field theory in 1931/1932

In the summer of 1931, Artin informed Helmut Hasse that "Ich habe die Klassenkörperbeweise jetzt endlich aufgeschrieben und werde sie Ihnen hoffentlich bald zuschicken können. Es hat doch länger gedauert als ich annehmen konnte (Artin to Hasse, 24 August, 1931, also in: Frei and Roquette 2008, p. 396)."[4] Unfortunately, these theorems never appeared in manuscript form and it seems doubtful that Artin sent them to Hasse since they are not located in the Hasse Nachlass (Cod. Ms. H. Hasse, NSUB). Shortly afterwards, however, Artin gave a course in Hamburg in 1931/1932 on class field theory that had a lasting and, in some sense, measureable influence on mathematics. Shokichi Iyanaga and Rockefeller Fellow Claude Chevalley were among the members of Artin's audience for these lectures and discussion (Iyanaga 2006, p. 52). Chevalley had already worked in class field theory and communicated his results, one jointly with Jacques Herbrand, to the Acadmie des Sciences de Paris (Chevalley 1930, 1931, 1932, 1933; Chevalley and Herbrand 1931). Thus, he came to Artin's course not only with some background but also with some contributions to class field theory. Within this course, Artin provided new and simpler proofs for certain statements in class field theory (Chevalley 1933, p. 370). As Iyanaga (2006, p. 52) reports, Chevalley suggested some of these simplifications. "Chevalley had the idea for improving the presentation in Artin's course. He communicated these ideas to me and then to Artin who,

[4] "Finally I have written up the class field proofs and, hopefully, I will soon be able to send them to you (our translation)."

in turn, passed them on to the audience at the beginning of the following hour (Iyanaga 2006, p. 52)." Chevalley (1933) describes his thesis "Sur la théorie du corps de classes dans les corps finis et les corps locaux" as a "new exposition in class field theory" and thanks Artin, among others, for simplifications of theorems. Thus, although Chevalley was not an "official" Ph.D. student of Artin, his thesis represents a prominent example of Artin's success in "speaking through the members of his mathematical circle." We note the somewhat obvious contrast between the "silence" of Artin's promised theorems of August, 1931 that never appeared and the lasting influence of a course begun shortly thereafter. Perhaps this historical record offers contemporary academicians a more expanded view of how to measure the success of a mathematician?

Continuing with what was an apparently good idea, in 1932, Artin also offered a course in class field theory at Göttingen. Olga Taussky wrote up the notes for this course and sold them for 1 Mark to cover printing costs. A translation of these notes is now preserved in (Cohn 1978, pp. 277–304). Taussky comments that in these notes, which "were very much used despite their limited circulation" Artin "made use of the ideas of Herbrand and was more modern than probably any other publication on some of the basic facts in class field theory for some time (Cohn 1978, p. 276)." In the summer of 1933, Artin gave a course on algebraic number theory, designated in the list of courses as "Algebra," at the University of Hamburg. We know the content of this course from the notes taken by E.A. Eichelbrunner (Artin 1933). In particular, in this course, Artin presented the basic material regarding the theory of valuations. It is worth noting that, at the time, Artin had only the original papers of Kurt Hensel, Josef Kürschak, and Alexander Ostrowski, among others, at his disposal, accompanied by the course material of Hensel published in 1908 and 1913 (Hensel 1913). Ostrowski' s seminal paper "Untersuchungen zur arithmetischen Theorie der Körper (Die Theorie der Teilbarkeit in allgemeinen Körpern)" was already submitted at that time but only appeared in 1935 (Ostrowski 1935).

After the three manuscripts and one book Artin published in 1931 and 1932 (Artin 1931a,b, 1932a,b), he would not publish again until 1940. During the time before he left Hamburg, however, he not only hosted international visitors such as the young Herbrand, Chevalley and Iyanaga, but he also advised several students including Max Zorn (1931) and Hans Zassenhaus (1934). (In 1927 he advised his first student, Käte Hey (1927), in her thesis work that extended the theory of the Dedekind Zeta function to the semi-simple hypercomplex number systems). As Brauer notes, although this may have seemed like a period of silence, indeed Artin was distributing mathematical ideas through the meaningful avenues of courses and dissertations. He would continue this trend when he arrived in America as well as resume publishing mathematics. George Whaples would be among the first young Americans to join—and benefit from—Artin's mathematical circle.

3 Artin and Whaples

Artin and Whaples met in 1939 when their paths overlapped at Indiana University. Artin had immigrated to America only 2 years earlier and taken a short-term position at Notre Dame University in South Bend, Indiana (Fenster 2007). News of Artin's arrival in America did not go unnoticed at Indiana University, some 174 miles south of Notre

Dame, particularly by the Chairman of the Mathematics Department, K. P. Williams. "It seems to me that departments should be strengthened from time to time as occasion offers," K. P. Williams wrote to his dean. "There is the opportunity to strengthen this one... There is Professor Artin at Notre Dame, almost on our doorstep, perhaps the leading man in algebra in the world, and one of the outstanding mathematicians of all fields (Williams to Payne, 6 April 1938)." Williams apparently made a convincing case to the dean since Indiana University offered Artin a permanent position beginning in the 1938–1939 academic year.

In 1939, Whaples earned his Ph.D. from the University of Wisconsin under the direction of Mark H. Ingraham. In his dissertation, "On the structure of modules with a commutative algebra as operator domain," as Whaples described it, he "worked on the problem of finding necessary and sufficient conditions for the similarity of two matrix representations of a commutative algebra (Whaples, Application to IAS, 10 February, 1941)." Ingraham would later call Whaples "one of the strongest students we have ever had at Wisconsin. His mind is rather the original type, and more like E. H. Moore in always wishing to pass from the general to the concrete rather than in the reverse order than any student I have had (Ingraham to Veblen, 11 February, 1941)." To be compared to E. H. Moore in American circles was the highest of praise. In this case, this comment had particular meaning since Ingraham and Veblen were both students of Moore at Chicago (Archibald 1938, p. 146).

Whaples spent 1939–1941 at Indiana in a post-doctoral position where he learned class field theory and worked with Emil Artin (Whaples, Application to IAS, February, 1941). This collaboration with Artin ultimately resulted in the publication of three papers: "The Theory of Simple Rings," "Axiomatic Characterization of Fields By the Product Formula for Valuations", and "A Note on Axiomatic Characterization of Fields." In the first paper, Artin and Whaples dealt with the structure of simple rings and extended existing theorems for simple algebras to simple rings. In their second and third publications, Artin and Whaples provided an axiomatic characterization of what is nowadays called a global field by means of the product formula. The commitment to clarify the role played by the basic result in a theory was fundamental to Artin's approach to mathematics. Later, Whaples would capture the essence of Artin's style as he had experienced it in his collaboration when he described

> the tradition of the lectures of Emil Artin, who enjoyed developing a subject from first principles and devoted much research to finding the simplest proofs at every stage. From him we learned how important it is to do this: it is a matter of honor that one should have in his memory the complete proofs of all the theorems he uses; the proof of a new theorem often is found by adapting and extending a method used to prove an old one; and sometimes a known proof using hard computations with the results of lemmas can be replaced by an elegant conceptual one using arguments and sub-results from the proofs of those lemmas (Whaples 1965).

While working with Artin, Whaples also published his own results on the "ramification of class fields and of the theorem of Grunwald on [the] existence of cyclic fields with given local properties (Whaples, Application to IAS, February, 1941; Whaples 1942)." Artin would later cite his non-analytic approach in this work in his letter of recommendation for Whaples to the Institute for Advanced Study without, apparently, noticing the mistake that his own student Shianghaw Wang would find in 1948 (Artin to Veblen, 13 February, 1941).

4 Joint papers of Artin and Whaples

4.1 Artin/Whaples "The Theory of Simple Rings" (1943)

Given the focus of Whaples' dissertation, it seems not so surprising that the first joint work Artin and Whaples pursued was on simple associative rings. In this paper, Artin and Whaples used a new approach to prove structure theorems that generalized known theorems of simple algebras. Specifically, Artin and Whaples presented the foundation for a structure theory of simple associative rings, that is, associative rings R containing no two sided ideal other than itself and 0, such that $R^2 \neq 0$. They proved theorems that generalized results in the structure theory of simple associative algebras as initiated by J. H. M. Wedderburn in his theory of hypercomplex number systems and pursued by A. Adrian Albert and Artin among others. "The reader will be able, if he wishes" Artin and Whaples point out in their introduction, "to extract from our discussion proofs of the structure theorems for algebras which are shorter than those usually given (Artin and Whaples 1943, p. 87)." This applies, for example, to their "Theorem 4" where the authors indicate in a footnote that this result was first proved (for simple algebras) by Wedderburn (Wedderburn 1907). It concerns Wedderburn's groundbreaking result that, given a finite-dimensional simple algebra A over a field F, there exists a finite-dimensional division algebra D (unique up to isomorphism) and a unique integer r such that A is isomorphic to the matrix algebra $M_r(D)$. As Artin would later reflect,

> [t]he essential point in the definition of an algebra is that it is a vector space of finite dimension over a field. This fact allows us to conclude that ascending and descending chains of subalgebras will terminate. After the great success that Emmy Noether had in her ideal theory in rings with ascending chain condition, it seemed reasonable to expect that in rings where the ascending and descending chain condition holds for left ideals one should obtain results similar to those of Wedderburn (Artin 1950, p. 67).

The new point of view introduced by Emmy Noether was the notion of a representation space, that is, a vector space where the elements of the algebra act as linear transformations. The work of Artin and Whaples showed that this treatment of simple algebras could be generalized to a wider class of rings. This new point of view would also allow Chevalley and Nathan Jacobson, independently of one another, to generalize Wedderburn's fundamental theorem.

4.2 Artin/Whaples "Axiomatic Characterization of Fields by the Product Formula for Valuations" (1945) and "A Note on Axiomatic Characterization of Fields" (1946)

Artin and Whaples' "Axiomatic Characterization of Fields by the Product Formula for Valuations" appeared in the *Bulletin of the American Mathematical Society* in 1945 and represented a conceptual breakthrough in algebraic number theory in so far as it clarified the role played by the product formula in characterizing the basic objects of algebraic number theory from a valuation theoretic point of view. At the same time, the paper emphasized the close analogy between the theory of algebraic number fields and the theory of algebraic function fields with a Galois field as the field of constants. The axiomatic method used in this paper unifies these two cases, nowadays bound together by the general notion of a global field as the basic object of study. These two families of fields are exactly the fields for which class field theory is known to hold.

The ideal theoretic approach as developed by Dedekind and his successors had dominated the development in algebraic number theory for a long time. However, Hensel (1913) introduced a new methodological approach to questions in number theory alongside the ideal theoretic approach at the turn of the century. Hensel's new concept of p-adic numbers marked the beginning of the theory of valuations.[5]

In 1921–1924, Hasse had demonstrated the importance of Hensel's ideas for classical questions in number theory, in particular, in the arithmetic theory of quadratic forms over number fields (Hasse 1923a,b, 1924a,b). At that time, Artin was still quite "in favor" of the ideal theoretic approach to algebraic number theory. "I remember numerous conversations," Hasse later recalled,

> which I had with Artin in the years 1922–1925 about the p-adics. Artin was very much in favor (with fire and flame) of Dedekind's theory of ideals in which the greater elegance and simplicity deeply impressed him, and he had the same

[5] The valuation theoretic approach to number theory grew out of a new view of prime ideals in the ring of integers in an algebraic number field k. In analogy to the usual absolute values on k which correspond to the real, respectively complex embeddings of k, one introduced absolute values associated with prime ideals.
More precisely, a valuation $|\ \ |$ on k is a function defined on k with values in the non-negative real numbers satisfying the following axioms:

(1) $|a| = 0$ if and only if $a = 0$
(2) $|ab| = |a||b|$ for all $a, b \in k$
(3) $|a + b| \leq |a| + |b|$ for all $a, b \in k$.

One calls a valuation *non-archimedean* if in addition to (3) it satisfies $|a + b| \leq \max(|a|, |b|)$. If it is not non-archimedean, then it is called *archimedean*. The valuation $|a| = 1$ for all $a \neq 0$ is called the trivial valuation. Two valuations $|\ \ |_1$, $|\ \ |_2$ on the same field are equivalent if there is a $c > 0$ such that $|a|_1^c = |a|_2$ for all $a \in k$. Equivalence is clearly an equivalence relation.
A set of equivalent and non-trivial valuations of the field k is called a prime (divisor) or a place of that field, and denoted by letters like p, $\mathfrak{p}$ or v. We denote by k_v the completion of k with respect to the topology induced on k by the place v. There are two types of valuations possible. First, there are a finite number of completions $k_v = \mathbb{R}$ or $\mathbb{C}$ corresponding to the places given by embeddings of k into $\mathbb{R}$ or $\mathbb{C}$ (up to complex conjugation in the latter case). These valuations are archimedean. In other words, all real embeddings of k and all pairs of conjugate imaginary embeddings give rise to archimedean valuations. These are all the archimedean places an algebraic number field admits. Second, there are an infinite number of non-archimedean places, one for each prime ideal in the ring of algebraic integers of k. For example, on the field $\mathbb{Q}$ of rational numbers there is one non-archimedean place for each prime $p > 0$, the p-adic valuation defined by

$$|p^\alpha \frac{x}{y}|_p = p^{-\alpha}$$

for $\alpha, x, y \in \mathbb{Z}$, x, y not divisible by p. It is a fundamental result of Ostrowski that the only non-trivial valuations on $\mathbb{Q}$ are those equivalent to the $|\ \ |_p$ or the ordinary absolute value $|\ \ |_\infty$.

> regretful smile for the p - adics as my Göttingen teachers at the time. ...Only the later great success of the p -adics in the theory of algebras and class fields made it possible to call his attention to Hensel's method, to allow him to fully acknowledge its value and lead him to contribute papers on the foundation of the p -adic (valuations of algebraic number fields) (Hasse 1950, p. 10).[6]

At the same time Hasse developed the idea of the p-adic transfer from the "small" to the "large," later to be called his local–global principle (Schwermer 2009). It served as a far reaching focus in his research in the following years and came to full fruition in his joint work with Brauer and Emmy Noether when they established the structure theory for central simple algebras defined over an algebraic number field in late 1931 (Brauer et al. 1932).

At end of the 1920s and in the early 1930s, Artin shifted his own thinking from the ideal theoretic approach to the valuation theoretic approach. It might very well have been the success of the local–global principle in the theory of central simple algebras that ultimately convinced Artin to give the valuation theoretic method or point of view a more central role in his own work. His first contribution to the theory of valuations was his short note " Über die Bewertungen algebraischer Zahlkörper" which he submitted on August 24, 1931, to Crelle's Journal for publication in the special volume in honor of Kurt Hensel's 70th birthday. In this short note (Artin 1932b), Artin determined all possible valuations that an algebraic number field admits.

Although Artin had initially embraced the ideal theoretic approach to class field theory—with no less than what Hasse called "fire and flame"—he ultimately integrated the valuation theoretic approach in his work, not only in his research but also in his teaching. In particular, he imparted that approach to Whaples and it formed the essence of their second and third papers.

The second paper is based on an invited address delivered by Artin on April 23, 1943 to the Chicago meeting of the American Mathematical Society. In Section 1 of the written version, Artin and Whaples briefly recall the background material in the theory of valuations on fields. However, the first section culminates with a result not previously found in the literature in this form, namely the so-called Weak Approximation Theorem. This result reads as follows (where k denotes a field):

> THEOREM. If we are given any n nontrivial inequivalent valuations $|\ \ |_v$ of k, an element α_v of k for each valuation, and an $\epsilon > 0$, then we can find an element α of k such that
>
> $$|\alpha - \alpha_v|_v \leq \epsilon \quad \text{for each } \ v = 1, 2, \ldots n$$
>
> (Artin and Whaples 1945, p. 472).

[6] Ich entsinne mich an zahlreiche Gespräche, die ich in den Jahren 1922–1925 mit Artin über den Wert der p-adik führte. Artin war Feuer und Flamme für die Dedekindsche Idealtheorie, in der ihn die grössere Eleganz und Einfachheit bestach, und hatte für die p-adik dasselbe mitleidige Lächeln wie seinerseits meine Göttinger Lehrer... Erst die späteren grossen Erfolge der p-adik in der Theorie der Algebren und Klassenkörper vermochten es, ihn für die Henselsche Methode zu gewinnen, ihn zur vollen Anerkennung ihres Wertes zu bringen und gar eigene Arbeiten von ihm zur Begründung der p-adik (Bewertungen algebraischer Zahlkörper) herbeizuführen.

We briefly discuss the relation of the statement of this theorem to another approximation result, namely, the "Chinese Remainder Theorem," the generalization of a classical result for the ring of integers $\mathbb{Z}$ to all Dedekind rings. Let $\mathcal{O}_k$ be the ring of algebraic integers in the given algebraic number field k, and let $\mathfrak{p}_1, \ldots, \mathfrak{p}_n$ denote n distinct prime ideals in $\mathcal{O}_k$. For each of these prime ideals there is a corresponding valuation $|\ \ |_{\mathfrak{p}}$. Let $x_1, \ldots, x_n$ be elements of $\mathcal{O}_k$, and let ν be a positive integer. Then, by the Chinese Remainder Theorem, one can find an element $y \in \mathcal{O}_k$, rather than just in k, such that $y \equiv x_i \mod \mathfrak{p}_i^{\nu}$ for $i = 1, \ldots, n$. In terms of the corresponding valuation this reads as $|y - x_i|_{\mathfrak{p}_i} \leq t^{\nu}$ for some real number t with $0 < t < 1$.

The "Chinese Remainder Theorem" only covers valuations corresponding to prime ideals, i.e., non-archimedean valuations whereas the Weak Approximation Theorem includes archimedean valuations as well. However, it is worth noting that the "Chinese Remainder Theorem" places integrality conditions at all places.

Artin and Whaples' Weak Approximation Theorem has important consequences. First, given an algebraic number field k, let $\sigma_1, \ldots, \sigma_r$ denote r distinct embeddings $k \to \mathbb{R}$ of k into the field of real numbers, and let ϵ_i, $i = 1, \ldots, r$, be an element in the "set of signs" $\{+1, -1\}$. Then the Approximation Theorem implies that there exists an element $a \in k^*$ with $\mathrm{sign}(\sigma_i(a)) = \epsilon_i$. This result is very useful in the arithmetic theory of algebraic groups, in particular, orthogonal groups attached to quadratic forms over number fields. It permits, for example, the construction of quadratic forms f over k which are indefinite at one real place, say σ, of k and whose conjugates f^{σ_i}, $\sigma_i \neq \sigma$, are positive definite.

Second, as pointed out by Artin and Whaples, one has the independence of valuations as a corollary:

> COROLLARY. If $|\ \ |_1, |\ \ |_2, \ldots, |\ \ |_n$ are nontrivial and inequivalent then a relation
>
> $$|x|_1^{\nu_1}\ |x|_2^{\nu_2} \cdots |x|_n^{\nu_n} = 1$$
>
> is true for all $x \in k$, $x \neq 0$, if and only if all $\nu_i = 0$ (Artin and Whaples 1945, p. 473).

However, for the circle of questions of interest to Artin and Whaples, the approximation theorem implies that no "finite number of valuations can be interrelated in a field. Such an interrelation may nevertheless happen for an infinite number of valuations. In the case of the ordinary function fields and number fields that is not only the case but this fact may even be used to derive all the properties of these fields on a common basis (Artin and Whaples 1945, p. 473)." As Artin and Whaples put it, a prime divisor of a given field k is a set of equivalent and non-trivial valuations of that field. Then they introduce the following axiom:

> AXIOM 1. There is a set $\mathcal{M}$ of prime divisors $\mathfrak{p}$ of k and a fixed set of valuations $|\ \ |_{\mathfrak{p}}$, one for each $\mathfrak{p} \in \mathcal{M}$, such that, for every $\alpha \neq 0$ of k, $|\alpha|_{\mathfrak{p}} = 1$ for all but a finite number of $\mathfrak{p} \in \mathcal{M}$ and

$$\prod_{\mathfrak{p}} |\alpha|_{\mathfrak{p}} = 1$$

where this product is extended over all $\mathfrak{p} \in \mathcal{M}$ (Artin and Whaples 1945, p. 473).

One should point out that the set $\mathcal{M}$ may or may not be the set of all prime divisors of k. Suppose that the field k satisfies this axiom then the set $\mathcal{M}$ can contain only a finite number of archimedean divisors, and if $\mathcal{M}$ contains some of these archimedean divisors then there is no field of constants, that is, we are not in the case of a function field.

Associated to the set $\mathcal{M}$ is a certain space of vectors $\mathfrak{a}$ with one component $\alpha_{\mathfrak{p}}$ for each divisor $\mathfrak{p}$. The component $\alpha_{\mathfrak{p}}$ may range freely over the $\mathfrak{p}$-completion $k_{\mathfrak{p}}$ of k. If the set $\mathcal{M}$ is the set of all prime divisors of k then the space is what is known as the space of valuation vectors. Artin and Whaples did not introduce this notion yet but they point out that "the idèles of Chevalley are special cases of these vectors; for an idèle we must have $\alpha_{\mathfrak{p}} \neq 0$ for all $\mathfrak{p}$ and $|\alpha|_{\mathfrak{p}} = 1$ for all but a finite number of $\mathfrak{p}$ (Artin and Whaples 1945, p. 473)." Artin and Whaples continue, "Our field k may be considered a subset of this space inasmuch as $\alpha \in k$ may also be considered as the vector whose $\mathfrak{p}$-coordinate is the element α of $k_{\mathfrak{p}}$ (Artin and Whaples 1945, p. 474)." This first axiom is supplemented by the second axiom

AXIOM 2. The set $\mathcal{M}$ of Axiom 1 contains at least one prime $\mathfrak{q}$, which is one of the following two types:
1. Discrete, with a residue class field of finite order $N\mathfrak{q}$.
2. Archimedean, with a completed field $k_{\mathfrak{q}}$ which is either the real or the complex field (Artin and Whaples 1945, p. 475).

The main result of the paper is stated in Artin and Whaples' Theorem 3 (Artin and Whaples 1945, p. 484):

If k is a field that satisfies the Axioms 1 and 2 it is an extension of a finite degree n either of the rational field R or of the field $R = k_0(z)$ of rational functions over its field of constants k_0. All valuations satisfy Axiom 2. $\mathcal{M}$ consists of all extensions of the well known valuations of R.

Artin and Whaples concluded this paper by deriving from the two axioms some basic results in algebraic number theory that were fundamental in the previous non-axiomatic approach. These included, among others, Dirichlet's unit theorem and the finiteness of the class number.

Ten months after submitting this paper, Artin and Whaples added a short note where they were able to weaken one of the assumptions in Axiom 1, namely they replaced "that for every $\alpha \neq 0$ of k, $|\alpha|_{\mathfrak{p}} = 1$ for all but a finite number of $\mathfrak{p} \in \mathcal{M}$" by the assumption that the product of all valuations converges absolutely to the limit 1 for all α. They easily adapted the proof of the previously given axiomatic characterization of fields to the new axiom with a few modifications.

This work of Artin and Whaples played a critical role in the development of algebraic number theory. Their work is important because it is strongly related with the emergence of the notions of idèles, due to Chevalley, and valuation vectors. The modern term for these valuation vectors is adèles.

This view was previously expressed by Andre Weil in his review of the *The Collected Papers of Emil Artin* where he notes

> [t]he joint work with Whaples in 1945 played an important role in the introduction of the adele concept in algebraic number theory (Weil 1967a).

Weil was even more explicit in the foreword to his book *Basic Number Theory* where he explained in some detail that "[t]o improve upon Hecke, in a treatment along the classical lines of the theory of algebraic numbers, would be a futile and impossible task." Instead, the point of view Weil adopted in this book was the adelic one. He pointed out

> [i]n the days of Dirichlet and Hermite, and even of Minkowski, the appeal to "continuous variables" in arithmetical questions may well have seemed to come out of some magician's bag of tricks. In retrospect, we see now that the real numbers appear there as one of the infinitely many completions of the prime field, one which is neither more nor less interesting to the arithmetician than its p-adic companions, and that there is at least one language and one technique, that of the adeles, for bringing them all together under one roof and making them cooperate for a common purpose (Weil 1967b).

As mathematics advanced, the concept of adeles and the accompanying *Local–Global Principle* became the language of choice in the arithmetic theory of algebraic groups, their representation theory and the theory of automorphic forms as well in large parts of arithmetic algebraic geometry.

These collaborative papers underscore Artin's willingness to share his ideas with his students and to include them in the publication of that work. Whaples had arrived at Indiana in 1939 without any apparent deep knowledge of algebraic number theory, in particular, class field theory. By April, 1943, he and Whaples had begun to think about the structural and conceptual underpinnings of the main results in class field theory. As they point out, these results hold for algebraic number fields and for function fields with a Galois field as the field of constants. Indeed, number fields and function fields are characterized by their possession of a product formula. In turn, as Artin and Whaples assert, "this shows that the theorems of class field theory are consequences of two simple axioms concerning the valuations (Artin and Whaples 1945, p. 469)." Their work represents a significant contribution to algebraic number theory and, in particular, to the very foundations of class field theory, a field that Weyl would describe as one of the "most difficult and elaborate theor[ies] in all of mathematics (Secretary of IAS School of Mathematics to Professor Tracy Thomas, 20 June, 1947)." While Artin may have served as the leader on these collaborations, Weyl noted that he knew "that Whaples's contributions to these joint papers were very considerable (Secretary of IAS School of Mathematics to Professor Tracy Thomas, 20 June, 1947)." Later, Artin and Tate's lectures on class field theory would develop the subject further using the methods of cohomology theory (Artin and Tate 1968). It was, however, the work of Artin and Whaples that began this transition in the approach to class field theory.

5 Whaples on his "own feet"

Whaples' work and association with Artin opened the door for him to visit the Institute for Advanced Study in Princeton (IAS). In early 1941, apparently while working on these papers with Whaples, Artin contacted Hermann Weyl at the IAS about the possibility of Whaples serving as an assistant to Weyl for the 1941–1942 academic year.[7] Weyl indicated that Artin's proposition was indeed a possibility and sent the requisite instructions and application materials for Whaples. Whaples' application and associated letters of recommendation provide insight into Whaples as a mathematician, teacher, and man.

In his outline of his plans for his mathematical work during the year at the IAS, Whaples indicated that while he had focused on finding matrix representations of a commutative algebra in his dissertation, he had spent most of the past year at Indiana "learning class field theory (Whaples 1941)." That would be learning class field theory from Artin. Whaples showed a great deal of vision when he wrote "I am trying to find a non-analytic proof of the theorem on primes in arithmetic progressions, and am also seeking existence theorems for non-abelian fields with given local properties. I plan to continue work on problems of this sort next year, with the general goal of simplifying class field theory, finding a generalization of it to non-abelian fields, and applying it to the question of what groups can be Galois groups of extension fields (Whaples, Application to the IAS, February, 1941, p. 2)."[8] In many ways, then, Whaples' research plan represented a continuation of research in class field theory, only now in America and by an American. The items of his planned research were very much in line with the current research topics in algebraic number theory, namely, to acquire some insight into non-abelian class field theory. Initiated by Artin, Hasse and Emmy Noether among others, one line of thought was to view the commutative case, i.e., algebraic number theory, as a special case of a general theory of hypercomplex number systems. This focus gave rise to a full development of that theory but was not successful in reaching the sort of conclusion to class field theory one had desired. Today, it is still only in the abelian case that class field theory leads to the laws of decomposition for prime ideals within a field extension and to the law of reciprocity.

All three letters of recommendation described Whaples as "shy" and with a dream-like quality about him. Whaples' dream-like, absent minded qualities made it difficult for him in the classroom where he was apparently not inspiring (MacDuffee to Veblen, 17 February, 1941). However, C. C. MacDuffee also noted that Whaples' mannerisms were "deceptive" since "his mind is very alert. I think he has unusual powers of concentration, and will make a typical specimen of the absent-minded professor (MacDuffee to Veblen, 17 February, 1941)". It may have been this combination of qualities that prompted Artin to cite Whaples' affable personality that helped him prove "very inspiring" to other graduate students at Princeton. Or, perhaps, indeed

[7] Here, again, we aim to give further shape to Brauer's insight into Artin's desire "to help [his students] to develop their own mathematical personalities, to assist them to stand on their own feet, to kindle in them a deep love of mathematics (Brauer 1967, p. 39).

[8] Whaples applied for a stipend but was ultimately funded as Weyl's assistant with a salary of $1,500 for the year (Weyl to Whaples, 13 March, 1941).

Whaples had "out grown" some of these tendencies as Ingraham surmised. In any case, his teaching ability really did not matter at the IAS since the position focused on research. "Of course," Weyl described the position to Whaples,

> this means that you would not be entirely your own master, that you would have to help me a little with my lectures and with my mathematical endeavors, and there would be some routine work to be attended to. But it wouldn't be very much and it would not interfere with your research work. I am quite anxious that you continue your work in close contact with Professor Chevalley (Weyl to Whaples, 18 March, 1941).

In some sense, Weyl picked up where Artin left off, particularly with creating opportunities for Whaples to interact with Chevalley who at that time was a professor at Princeton University.

While at the IAS in 1941–1942, Whaples gave a course on class field theory and a series of lectures titled "Remarks on class field theory." Thus, what began at Indiana in a post-doctoral position now continued at the celebrated IAS. Weyl's other assistant, Alfred T. Brauer, gave at least one lecture on "On the Solvability of the Linear Diophantine Equation in Positive Integers." Brauer, the brother of Richard Brauer, had arrived in the U.S. in 1939 and worked at the Emergency Aid for Displaced German Scholars. Weyl hoped to help him by appointing him as his assistant in 1941. In addition to Weyl, during the year Whaples spent at the IAS, then as now, a veritable who's-who of celebrated mathematicians came into his sphere including James Alexander, Marston Morse, John von Neumann, Wolfgang Pauli, and Carl L. Siegel among the more permanent members and A. Adrian Albert, Richard Courant, Paul Erdös, Jacques Hadamard, Saunders Mac Lane, and Oscar Zariski for shorter visits.

In March, 1942, Weyl wrote a letter of recommendation for Whaples for a Benjamin Pierce Instructorship at Harvard. Weyl mentioned that he "should not describe him as brilliant, and yet someday he might achieve the unexpected; there is something about him of the still waters that run deep (Weyl to Stone, 12 March, 1942)." Apparently Whaples did not secure this position at Harvard; instead, he took a position as an instructor of mathematics at Johns Hopkins University. Weyl commented that Whaples "is not irreplaceable at this time in his present position, but this is much less the case in his new position as instructor of mathematics at Johns Hopkins (Weyl to no name given, 21 May, 1942)." Whaples assumed a position at the University of Pennsylvania in January, 1944 and then moved to Indiana in 1947, perhaps not coincidentally the year after Artin left Indiana for Princeton.

6 Beyond the work of Artin and Whaples

In 1950, nearly two decades after Chevalley took Artin's course on class field theory at Hamburg and almost a decade after Artin and Whaples began their work on valuation vectors at Indiana University, John Tate completed his Princeton doctoral thesis under Artin's supervision on "Fourier analysis in number fields and Hecke's zeta-functions." In his thesis, Tate provided the most elegant and unified treatment of the analytic properties, the analytic continuation and the functional equation of the generalized zeta-functions. Hecke had introduced these zeta-functions in 1920 in his two seminal papers " Über eine neue Art von Zetafunktionen und ihre Beziehungen zur Verteilung der Primzahlen" (Hecke 1918, 1920). Tate followed a new methodological approach that relied on abelian harmonic analysis in the conceptual framework of the notions of idéles and adeles as introduced by Chevalley and Artin. Margaret Matchett had already taken a first step in this direction in her 1946 thesis, "On the Zeta Function for Ideles," written under the direction of Artin at Indiana. She redefined classical Zeta-Functions in terms of integrals over the idéle group. She interpreted the characters of Hecke as exactly those characters of the ideal group that could be derived from idéle characters. She did not, however, succeed in proving the functional equation for the zeta-functions in terms of the idélic approach. Tate established this last result in his thesis. Tate summarized the importance and position of the contributions of Chevalley and Artin and Whaples to algebraic number theory in the introduction to his thesis. "In a work," Tate began,

> the main purpose of which was to take analysis *out* of class field theory, Chevalley introduced the excellent notion of the idéle group, as a refinement of the ideal group. In idéles Chevalley had not only found the best approach to class field theory, but to algebraic number theory generally. This is shown by Artin and Whaples (1945). They defined valuation vectors as the additive counterpart of idéles, and used these notions to derive from simple axioms all of the basic statements of algebraic number theory (Tate 1950, p. 306).

A year later, Artin published his *Algebraic Numbers and Algebraic Functions* (Artin 1951), a text Artin viewed as a collection of lecture notes. "These lecture notes," as Artin described them in 1967, "represent the content of a course given at Princeton University during the academic year 1950/1951. This course was a revised and extended version of a series of lectures given at New York University during the preceding summer. They cover the theory of valuations, local class field theory, the elements of algebraic number theory and the theory of algebraic function fields of one variable."

Artin credited Mr. I. Adamson for preparing the notes and Tate for helpful discussions that led to simplifications of some proofs. Artin arranged the lecture notes into three parts:

- General Valuation Theory (chapters 1–5)
- Local class field theory (chapters 6–10)
- Product formula and function fields in one variable (chapters 11–17).

In chapter 12, familiarly titled "Characterization of Fields by the Product Formula," Artin covers the material of the Artin–Whaples paper of similar title. He introduces Axiom 1 and Axiom 2 from the original paper and refers to fields that satisfy these axioms as Product Formula Fields, or PF-fields. The main result here is the same as the Artin–Whaples paper: A PF-field is either an algebraic number field, or a finite extension of a field of rational functions.

It took little time for what Artin called his "lecture notes" to influence young and advanced scholars. In December, 1951, for example, Kenkichi Iwasawa submitted his paper "On the rings of valuation vectors" to the *Annals of Mathematics*. In this work, Iwasawa refers to specific results in Artin's *Algebraic Numbers and Algebraic Functions*. In his 1958 publication, "On the zeta-function of the simple algebra over the field of rational numbers," G. Fujisaki extended Tate's methods for dealing with zeta-functions over number fields to one of a simple algebra over a number field [Fujisaki 1958]. In his introduction, Fujusaki outlines that "[i[n section 5 we define, *after* Artin [*Algebraic Numbers and Algebraic Functions*] the concepts of valuation vectors and idéles in A [our emphasis]." Fujusaki's reference suggests that no other text on valuation vectors was available, or, at least, that Artin had created an accessible, helpful text.

Thus, Artin's professional opportunities from the course in Hamburg in 1931/1932 to the courses in New York and Princeton in 1950/1951 highlight the critical importance of well-conceived lectures that hinged, in part, on an integration of ideas rather than a more traditional delivery. In between, Artin's direct work with students calls attention to his natural inclination to exchange ideas freely in a different setting. These exchanges were mutually beneficial. In the introduction to Artin's *Collected Works*, Lang and Tate commented that Artin's "lectures and seminars ... inspired his students, towards whom his generosity and affection were unsurpassed [Artin, Collected Works, p. x]." In one of his last conversations with Richard Brauer, reflecting on his students John Tate and Serge Lang, Artin remarked that "[t]his happens only once to a man. Not many mathematicians have been that lucky (Brauer 1967, pp. 28–29)." These observations give further insight into Hans Zassenhaus's characterization of Artin. Specifically, in his obituary of Artin, immediately after his overview of Artin's life, Zassenhaus chose to focus first on Artin's teaching and then on his mathematical contributions. "In my memory," Zassenhaus reflected, "Artin stands out as a great teacher....A teacher in our field can work through many channels of communication: by formal lectures, by research papers, by textbooks, by private conversation, and by generating an infectious spirit of doing research in a large group of students, working through a few (Zassenhaus 1964, pp. 1–2]." Research papers and textbooks represent one form of teaching. Lectures, private conversation and an infectious spirit form another. So Brauer was right. There were no years of "silence" in Artin's professional career. Lefschetz was also right: Artin was "famous ... as a teacher, and inspirer of youth."

The mathematics discussed in this paper occurred during a time of tremendous upheaval in Artin's life. Artin immigrated to America in 1937 with his (then) two children and wife. They were a young family displaced by the Nazis. He initially held a (temporary) position at Notre Dame, then moved to Indiana University and finally to Princeton. Artin seemed to embody the quintessential attributes of scholarship and adaptability which later scholars would designate as critical to an emigrant's success (Ash 2003; Siegmund-Schultze 2009). Artin was remarkably fortunate to have a position at an institution waiting for him when he arrived in America and equally as fortunate to secure a position at Princeton less than a decade after his arrival in America. He willingly distributed his mathematical ideas to American students, women included, and ultimately wielded a mighty influence in the country he called home for two decades.

As Mitchell Ash points out in his "Forced Migration and Scientific Change," "[e]migré scientists and scholars were not only victims, but also agents (Ash 2003,

p. 255)." This investigation of Artin's work suggests that Artin worked as something of an "agent" in terms of distributing ideas about class field theory and algebraic number theory to America. In terms of Brauer's characterization of his "mathematical circles," the work of Artin and Whaples highlights Artin's ever-expanding mathematical circle. "Adaptability" was an issue for immigrants to America (and elsewhere for that matter). Carl Ludwig Siegel, for example, after a return visit to Europe admitted that "I no longer have the hope, which led me to America four years ago, of finding a tolerable position abroad. My character is too clearly developed, and I can no longer suppress my asocial instincts and individualistic tendencies. I can no longer adapt, I am too much of a Prussian (Siegel to Courant, March 22, 1939, as quoted in Siegmund-Schultze 2009, p. 238)."

Artin's ease at folding students into his mathematical circle suggests he was adept at adapting, at least professionally. Ash rightly urges caution when it comes to establishing "causal relationships between forced migration and scientific change (Ash 2003, p. 255)." In the case of Artin, however, even with this cautionary approach in mind, it seems reasonable to suggest that his forced migration played a pivotal role in bringing class field theory to America. It was a confluence of critical events that shaped this moment in the history of mathematics. Artin had given a course on class field theory in Hamburg in the early 1930s that Chevalley, among others, had attended. The political situation in Germany forced Artin's departure for America not long after. The "for America" cannot be overstated. Had a similar political situation occurred even 40 years earlier, a mathematical research community in America would not have existed for Artin to emigrate *to*. Whaples, a second-generation, American-trained mathematician, with an advisor who worked under the influential E. H. Moore at the University of Chicago, had the opportunity to work closely with Artin, a distinguished European mathematician. The published papers of Artin and Whaples show their advancements in algebraic number theory, advancements that helped simplify fundamental proofs of class field theory. Whaples' year at the IAS highlights other benefits of this association, including the chance to work with Chevalley, another distinguished European mathematician. Matchett and Tate's thesis topics point to other lasting implications of this work for students at Indiana and Princeton.

This investigation of one sliver of Artin's work calls attention to an ever-expanding, international mathematical circle that persisted, even flourished, in the most adverse of circumstances. It highlights the significance of Artin's migration "both for its consequences and for what it discloses about historical processes of human and intellectual transfer (Reingold 1985, p. 176)." Without intentionally setting out to explore the effects of "forced migration" on Artin and his work, this broader study suggests a remarkably seamless "human and intellectual transfer" in this particular case.

For Artin, that seamless transfer seemed to gain momentum as his time in America progressed. In 7 years at Indiana, Artin authored at least six papers and advised two students. At Princeton he advised 18 students and wrote 16 papers and at least 2 books or lecture notes. He not only adapted to the new culture in America, but he also integrated his culture into the mathematical milieu, particularly at Princeton (Rota and Palombi 2008). His black leather jacket with a belt at his waist was as much a part of Artin in Princeton as it had been in Europe nearly a decade earlier. For Artin, then, "acquiring membership in the culture of a given country by adapting to the local academic habitus and/or the social standards of the educated elites was" not, apparently, "an important precondition" for his acceptance as a mathematician or scholar (Ash 2003, p. 251). Artin was comfortable as Artin, in America or Germany.

Artin seems to have possessed a certain serenity that helped him adjust to his new surroundings. If we look more broadly, particularly to larger studies on immigration such as Reingold, Ash, and Siegmund-Schultze, then we see that Artin's life suggests that a sort of internal strength may have served as a prerequisite for an immigrant to survive,.even thrive, in a host country. It could also be that this inner serenity allowed Artin to remain comfortable with his pursuit of mathematics for the sake of mathematics rather than for the publication of mathematics. His *Collected Papers* span fewer than 560 pages. By comparison, the *Collected Papers* of Leonard Dickson, an American algebraist and number theorist slightly older than Artin, for example, include six volumes totaling more than 3,600 pages. Even here, in this comparison offered 50 years after his death, Artin makes himself heard. Mathematics was about more than publications.

Thus, this work calls to mind what Peter Galison refers to as "microhistory." As he describes it, "[m]icrohistory is supposed to be exemplification, a display through particular detail of something general, something more than itself. It is supposed to elicit the subtle interconnections of procedures, values and symbols that mark science in a place and time, not as a method but more as a kind of scientific culture (Galison 2008, p. 120)." What is particularly interesting here is that Artin seems not to be marked in a place and time, at least not as we expect him to be marked for this place or time, but, rather, surprisingly *unmarked* by the time and place. In particular, in contrast to so much of what Ash describes, the culture of American mathematics had room for Artin or, perhaps, Artin made room for his own style of mathematics, which included his own version of *Viennese Kultur* (Rota and Palombi 2008).

Galison compares these microhistories, these case studies as it were, to stepping stones and raises the important question "where does the path lead?" If this study serves as one of these stepping stones, we suggest that it leads to a deeper understanding of Artin and how he plied his trade as a mathematician, particularly relative to students; greater insights into what Brauer calls a "mathematical circle," what they are and how they grow; an introductory look at George Whaples and this early phase in his mathematical career; and, at least one almost tangible example of the intangible, inextricable link between the human and intellectual transfer that came with forced migration.

Quite fittingly, this last comment brings us full circle, back to another observation about Artin by Brauer. After his discussion of Artin's mathematical trajectory beginning with his 1921 Ph.D. from Leipzig, to his Privatdozent position at the University of Hamburg in 1923, and his full professorship there three years later, Brauer noted that "[t]he intellectual atmosphere of German universities of that period is remembered with nostalgia by all who knew it." Relative to Artin, he added, that

> Artin, with his wide interests in all fields of human endeavor became the stimulating center of a circle of friends. His strange nickname "Ma" which he always preferred to his given name Emil goes back to those days. It is short for

"Mathematics"; he simply appeared to these young men as an embodiment of mathematics (Brauer 1967, p. 27).

It seems that Artin's "embodiment of mathematics" created a special sense around him of the importance of mathematics, a style that worked well in both Germany and America. In particular, Artin fostered an intellectual atmosphere that drew in other young men (and women), including Chevalley, Iyanaga, Whaples, Matchett, and Tate, among others, at formative stages in their careers. Robin Rider rightly points out that behind the numerical count of mathematicians dismissed from their positions in Germany were "individuals, each with a story, often a poignant one (Rider 1984, p. 110)." Artin's story, particularly his work with Whaples and other young mathematicians, shows he worked best *with* a circle of mathematical friends surrounding him.

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