Ibn al-Kammād's "Muqtabis" zij and the astronomical tradition of Indian origin in the Iberian Peninsula
Author(s): José Chabás and Bernard R. Goldstein
Source: *Archive for History of Exact Sciences*, Vol. 69, No. 6 (November 2015), pp. 577-650
Published by: Springer
Stable URL: https://www.jstor.org/stable/24569656

Arch. Hist. Exact Sci. (2015) 69:577–650
DOI 10.1007/s00407-015-0158-8

# Ibn al-Kammād's *Muqtabis* zij and the astronomical tradition of Indian origin in the Iberian Peninsula

**José Chabás[1] · Bernard R. Goldstein[2]**

Received: 22 May 2015 / Published online: 5 August 2015


**Abstract** In this paper, we analyze the astronomical tables in *al-Zīj al-Muqtabis* by Ibn al-Kammād (early twelfth century, Córdoba), based on the Latin and Hebrew versions of the lost Arabic original, each of which is extant in a unique manuscript. We present excerpts of many tables and pay careful attention to their structure and underlying parameters. The main focus, however, is on the impact *al-Muqtabis* had on the astronomy that developed in the Iberian Peninsula and the Maghrib and, more generally, on the transmission and diffusion of Indian astronomy in the West after the arrival of al-Khwārizmī's astronomical tables in al-Andalus (Muslim Spain) in the tenth century. This tradition of Indian origin competed with the Greek tradition represented by al-Battānī's astronomical tables and was much more alive in the Iberian Peninsula and the Maghrib than previously thought. From Spain, the Indian tradition entered mainstream European astronomy, where we find echoes of it in all versions of the Alfonsine Tables, both in manuscript and in the printed editions (beginning in 1483), as well as in Copernicus's *De Revolutionibus*, published in 1543.

More than 20 years ago, we published in this journal a long paper on a set of astronomical tables in *al-Zīj al-Muqtabis* compiled by Ibn al-Kammād, an astronomer active in Córdoba at the very beginning of the twelfth century (Chabás and Goldstein 1994). In that paper, we focused on the tables in this zij and, by means of analyzing their structure and determining the underlying parameters, we identified some of Ibn al-

Communicated by: George Saliba.

✉ José Chabás
josechabas@gmail.com

1 Barcelona, Spain

2 Pittsburgh, PA, USA

Kammād's sources and situated this zij in a long tradition going back to Indian and Greek astronomy. We are now able to elaborate our previous arguments, based on new material and new insights into old material produced in the Iberian Peninsula and beyond. Specifically, we can better assess the impact of Ibn al-Kammād's work on western astronomers, with an emphasis on his role in transmitting astronomical procedures and parameters of Indian origin.

The case of Ibn al-Kammād, although not exceptional, is paradigmatic of the history of western astronomy in the Middle Ages in that it exemplifies the main lines of its development: elaboration of astronomical theories and procedures based on Ptolemaic and non-Ptolemaic materials in the eastern Islamic world, transmission from the early Islamic milieu to the West, reworking in al-Andalus of the material received, diffusion and use of it in the Iberian Peninsula in different cultural and religious contexts, and translation into others languages (Latin and Hebrew in this instance).

Abū Ja$^{c}$far Aḥmad ben Yūsuf Ibn al-Kammād was probably born in Seville and worked in Córdoba. In the anonymous thirteenth-century version of Ibn Isḥāq's zij preserved in a fourteenth-century copy, Hyderabad, Andra Pradesh State Library, MS 298 (no foliation), we are told in chapter 35 that "Abu l-$^{c}$Abbās [Ibn] al-Kammād said in a horoscope he drew in Córdoba in the year 510 Hijra...". This date corresponds to 1116–17 A.D. (cf. Mestres 1996, p. 404). He is said to have been a disciple of Azarquiel, and indeed, several of the tables analyzed below depend on Azarquiel (d. 1100). Most of the little information known about Ibn al-Kammād was assembled by Comes (2007). He is the author of three zijes (*al-Kawr $^{c}$alā l-dawr*, *al-Amad $^{c}$alā l-abad*, and *al-Muqtabis*) as well as of an astrological work on the duration of pregnancy, called *Kitāb Mafātiḥ al-asrār* (Book of the keys to the secrets), which Vernet identified in 1949 in El Escorial, Biblioteca del Monasterio, MS Árabe 939, but the manuscript only contains chapters 10–15 of that work. According to Comes, Baghdad Museum, MS 296 [782], includes another Arabic text by Ibn al-Kammād, hitherto not examined in detail. The first two zijes are not extant in any language, but *al-Kawr $^{c}$alā l-dawr* (The periodic rotations) is partially preserved in the same El Escorial manuscript, as well as in a long quotation in the zij of Ibn Isḥāq (Mestres 1996, pp. 392, 405–406). On the other hand, *al-Amad $^{c}$alā l-abad* (The eternally valid [tables]) is not extant at all. There is also a short text in Castilian in Segovia, Biblioteca de la Catedral, MS 115, entitled *Libro sobre çircunferencia de moto* and attributed to Yuçaf Benacomed, that may belong to one of these two zijes.

The third zij composed by Ibn al-Kammād, *al-Muqtabis,* is probably a summary of the first two. It is the only extant in Latin and Hebrew versions: Madrid, Biblioteca Nacional, MS 10023 (Millás 1942, pp. 231–247; see a digital reproduction at http://bdh-rd.bne.es/viewer.vm?id=0000014503&page=1), and Vatican, MS Heb. 498 (Langermann 1984, and Richler 2008, p. 428). In this paper, MS M denotes the Latin manuscript in Madrid and MS F denotes the Hebrew manuscript in the Vatican: each version is extant in a unique copy. The Latin version is due to an otherwise unknown John of Dumpno, in 1260 in Palermo, as stated in the explicit (f. 18vb):

*Et perfecta fuit translatio et interpretatio istarum portarum in Panormitana ciuitate per Johanem de Dumpno filium quondam Philipii de Dumpno, die ueneris 27 mensis augusti 4 indicione ab annis Domini et Saluatoris nostri Ihesu Xristi*

*1260. Et concordatus est cum ipsis annis nonus decimus dies mensis ramadan ab annis seductionis 658, sub laude et gloria omnipotentis Dei et Saluatoris nostri Ihesu Christi feliciter, amen. Explicit.*

In the canons, Ibn al-Kammād does not refer to any source of Indian origin. The canons in Latin are divided into 30 chapters (ff. 1ra–18vb), each of them called *porta*; the last chapter mentions the other two zijes by Ibn al-Kammād, *al-Kawr ᶜalā l-dawr* and *al-Amad ᶜalā l-abad*, thus indicating that *al-Muqtabis* was written after the other two. In Arabic, only chapter 28 has survived (Algiers, Bibliothèque Nationale, MS 1454, and El Escorial, MS 939: see Comes 2007). After the canons follows a set of chapters (18vb–23rb), some of which are associated with *al-Kawr ᶜalā l-dawr.* A new explicit repeats the name of the translator, John of Dumpno, and the city of Palermo, but the date is now 1262. Folios 23v–24r contain another similar text followed by notes, and ff. 24v–26v are blank. The tables of *al-Muqtabis* are found on ff. 27r–54v, after which there are some additional tables (55r–66r); see below.

The author of the Hebrew version, Solomon Franco (*fl.* 1375), was active in Córdoba and Toledo (Langermann 1993, pp. 31, 41–42). Little is known about him, but he does say in a supercommentary on Abraham Ibn Ezra's Biblical commentary that he studied with Joseph Ibn Waqār (fourteenth century, Castile) whose zij is cited extensively in this paper (see below). In addition to astronomy, his interests included astrology and magic (see Goldstein 2013). Franco's version of *al-Muqtabis* is uniquely preserved in MS F: canons (ff. 2r–19r) and tables (ff. 29r–68v); the introduction to his canons was edited and translated in Goldstein (2013). He provides rules and tables for converting dates among Jewish, Christian, and Muslim calendars that apparently were not part of Ibn al-Kammād's zij; otherwise, the tables in MS F mostly agree with those in MS M, but for textual variants. Moreover, we believe that the two extant versions are independent witnesses to the original Arabic.

As is the case with most zijes, the sources of *al-Muqtabis* are multiple, but among them stand out those of the *Sindhind* tradition that drew on Indian sources, to the point that this zij by Ibn al-Kammād is a crucial link in the transmission of Indian astronomy to the West. Although the dominant tradition in medieval astronomy derives from Ptolemy's *Almagest*, there are other elements involved, but their history is difficult to trace. The reason is that some of the astronomical texts of late antiquity are not extant, and this is also the case for most of the astronomical texts produced in the Islamic world in the eighth and early ninth centuries. As reconstructed on the basis of later sources, the dominant astronomical traditions in early Islam were of Indian origin, later displaced for the most part by Greek traditions. This was particularly true in the eastern Islamic world, whereas many of the Indian traditions were maintained throughout the Middle Ages in the West (in Arabic in al-Andalus and the Maghrib, and subsequently in Latin in western Europe, and in Hebrew). An added difficulty is that few medieval astronomers identified their sources: this task has been undertaken by modern scholars whose works will be cited in this paper. For the Indian traditions in the Islamic world in the eighth and ninth centuries, we depend on the fruits of this scholarly endeavor. Our principal goal is to describe the continued presence of Indian traditions in medieval Spain, based largely on the zij of Ibn al-Kammād and related texts.

Early Arabic astronomers were influenced by a series of texts produced in India in the fifth to the seventh centuries. The works we will cite are the *Sūrya-Siddhānta*, based on Burgess's translation (1860), and the *Khaṇḍakhādyaka* by Brahmagupta (seventh century), based on Sengupta's translation (1934). The history of the text of the *Sūrya-Siddhānta* is complex, and we only note that the earliest version (not extant) is dated ca. 450 (Pingree 1968a, p. 29; Neugebauer and Pingree (eds. and trans.) 1970, 1:13). Fragments of the eighth-century zijes by al-Fazārī and by Ya$^{c}$qūb Ibn Ṭāriq that tell us about the early Indian traditions in the Islamic world were collected by Pingree (1968b and 1970). Moreover, some aspects of Indian astronomy reached the Islamic world by an indirect route, for they were first transmitted to Iran at the time of the Sasanian dynasty (226–652). These traditions were included in the *Zīj al-Shāh*, composed ca. 450 and revised in the sixth and seventh centuries; they only survive in fragments preserved by later Islamic astronomers (Kennedy 1958, and Pingree 1963, p. 242; cf. Goldstein and Sawyer 1977, p. 168). For a fanciful medieval account of the transmission of Indian astronomy to the court of the first Abbasid caliph, al-Saffāḥ (reigned: 750–754), see Goldstein (1967a), pp. 147–149; for an assessment of the historicity of this passage, see Pingree (1970), pp. 101–102. For an overview of Indian astronomy, see Pingree (1978).

The most important astronomical work in this tradition, called the *Sindhind* zij, was composed by Muḥammad ibn Mūsā al-Khwārizmī (*fl.* 840, Baghdad), but it is not extant in its original form. The tables in this version were based on the era of the last king of the Sasanian dynasty, Yazdegird III (epoch: June 16, 632). Subsequently, they were recast to the era of the Hijra (epoch: July 15, 622) by Maslama al-Majrīṭī (d. 1007, Córdoba); this version is also not extant. However, Maslama's version survives in a Latin translation by Adelard of Bath (ca. 1080–ca. 1150), which was published by Heinrich Suter in 1914 and translated in Neugebauer (1962a). This version is known on the basis of nine manuscripts: two of them are complete copies and reasonably close to Adelard's original, but the others are either incomplete or attributed to Petrus Alfonsi or Robert of Chester (Mercier 1987, p. 89). Other information on al-Khwārizmī's zij comes from a commentary by the tenth-century astronomer, Aḥmad Ibn al-Muthannā, probably working in al-Andalus, which only survives in Hebrew and Latin translations (Goldstein 1967a, and Millás Vendrell 1963). Another extant commentary is by Ibn Masrūr (tenth century, Cairo). Both commentaries were written prior to the composition of Maslama's version, that is, they refer to the original form of al-Khwārizmī's zij (van Dalen 1996, pp. 198–200; Pingree 1996; Haddad et al. 1981, pp. 224–225; Samsó [1992] 2011, pp. 84–91). All versions of this zij used sidereal coordinates for the positions of the celestial bodies in contrast to the tropical coordinates used in the *Almagest*. In this essay, we refer to the extant version by Maslama simply as al-Khwārizmī's zij.

In al-Andalus, in addition to those who depended on Ibn al-Kammād, several authors used al-Khwārizmī's *Sindhind* zij, either directly or by means of Maslama's recension (see King, Samsó and Goldstein 2001, p. 56). Among them is Ibn al-Ṣaffār (d. 1035), who was a disciple of Maslama and active in Córdoba and Denia. He wrote a book on the use of the astrolabe and composed a zij of which only seven chapters are extant (Paris, MS Heb. 1102): see Castells and Samsó (1995). Ibn al-Samḥ (d. 1035), another disciple of Maslama, worked in Granada and wrote several texts on mathematics, as well as treatises on the construction and use of the astrolabe and the equatorium (Rius 2007, p. 568). His zij, also in the tradition of al-Khwārizmī, is not extant (see Samsó [1992] 2011, pp. 85, 87). The third author, Ibn Muᶜādh (d. 1093), from Jaén, compiled a zij known as *Tabulae Jahen* of which the Arabic original is not extant, but the text of the canons has been preserved in a Latin translation by Gerard of Cremona, entitled *Liber tabularum Iahen cum regulis suis*, printed in 1549 with the title *Scriptum antiquum saraceni cuiusdam* (Heller 1549, ff. N i r - Z ii r). The author of the underlying Arabic text was identified by Moritz Steinschneider (1863, pp. 10–11), but his findings were ignored, for the most part, in subsequent literature. As Steinschneider noted, the author's name appears as Abumadh (ch. 23, f. X i r) and as Abumad (ch. 25, f. X iiij v), which are corrupt forms of Ibn Muᶜādh. According to the canons, Ibn Muᶜādh adapted the zij of al-Khwārizmī to the coordinates of his city and used some parameters that come from the Indian tradition that are not found in Ibn al-Kammād's zij. On Ibn Muᶜādh, see Hermelink (1964), Goldstein (1977), Villuendas (1979), Samsó (1980, 1994, 1996), Smith and Goldstein (1993), Casulleras (2010), and Samsó [1992] 2011, pp. 152–166, 484–487.

Of particular interest is a twelfth-century incomplete copy of al-Khwārizmī's zij, extant in Paris, Bibliothèque nationale de France, MS 16208, that contains tables for the mean motions, equations, and latitudes of the planets (for a description of this manuscript, see Pedersen 2002, pp. 165–166). For instance, the maximum equations for the Sun and the Moon in this short set of tables are 2;14° and 4;56°, respectively, which are parameters of Indian origin used by al-Khwārizmī. The radices are computed for dates in the Julian calendar, beginning in 1100 and, according to Mercier (1987, p. 101), it would seem that the computations were intended for Toledo.

The impact of the Indian tradition in al-Andalus was not restricted to the compilation of zijes, for it can also be discerned in the Almanac of 1307. This almanac, of which the original in Arabic is lost, was briefly described by Millás (1949, pp. 388–390), based on a Latin translation made in Tortosa that is extant in Madrid, Biblioteca Nacional, MS 17961. Subsequently, this text has been called the "Almanac of Tortosa" but, since other versions of it in Latin, Castilian, Catalan, Hebrew, and Portuguese have been identified, this designation is no longer appropriate (Chabás 1996b). The Almanac of 1307 uses sidereal coordinates and, in addition to tables specific to an almanac for the positions of the Sun, the Moon, and the planets for varying numbers of years beginning in 1307, it also contains tables commonly found in zijes, such as tables for mean motions and equations, as well as lists of the positions of the planetary apogees and nodes. The Almanac of 1307 had a long life for, in various manuscripts, there is a continuation of it beginning in 1391 (Chabás 1996b).

Another direct user of the *Sindhind* of al-Khwārizmī was Abu l-Ḥasan ᶜAlī al-Qusanṭīnī (or Qusunṭīnī), a fourteenth-century astronomer working in Fez, who composed a zij containing various elements of Indian origin, with an explanatory text in verse [Kennedy and King 1982; cf. King, Samsó and Goldstein (2001, p. 63)].

In al-Andalus and the Maghrib, and later in Christian Spain and Portugal, a significant number of astronomers, especially table makers, depended directly on Ibn al-Kammād.

The zij of Ibn Isḥāq al-Tūnisī (ca. 1193–1222, Tunis and Marrakesh) only survives in a version composed by an anonymous author active in Tunis around 1280: it is extant in Hyderabad, Andra Pradesh State Library, MS 298, a manuscript copied in 1317 in the city of Homs, Syria. According to Mestres (1999, pp. 5–6), the sources of Ibn Isḥāq's zij include the zij of al-Khwārizmī, the Toledan Tables, and Azarquiel's works, and among the astronomers who used it were Ibn al-Bannā' (1256–1321, Marrrakesh), Ibn al-Raqqām (d. 1315, Tunis and Granada), Ibn Waqār (mentioned above), and the astronomers in the service of King Alfonso X. In the preface to this work, the anonymous author indicated that Ibn Isḥāq's zij depended on the works of Ibn al-Kammād, Ibn al-Hā'im (thirteenth century, Seville), Ibn Mū$^c$ādh, and Ibn al-Ṣaffār, among others (Mestres 1999, p. 7). Moreover, we note that many of the canons of Ibn Isḥāq's zij in the Hyderabad manuscript were taken from, or follow closely, those of *al-Muqtabis*. In this essay, we generally refer to the anonymous recension in the Hyderabad manuscript simply as the zij of Ibn Isḥāq.

Vatican, Biblioteca Apostolica, MS Pal. lat. 1414 is a late thirteenth-century manuscript containing the Toledan Tables, the Almanach of Azarquiel, as well as tables for Toulouse, and notes for Paris (for a description of this manuscript, see Pedersen 2002, p. 178–179). It also contains a table of sines, normed 150 (see Sect. 2.1, below), a table for the ascensional difference in the tradition of al-Khwārizmī (see Chabás and Goldstein 2012, p. 30, Table 2.6A; Neugebauer and Schmidt 1952, pp. 225–227), and 8 tables on eclipses already in *al-Muqtabis*, as well as a summary of chapter 7 of the zij (ff. 140v–143r). At the present time, it is the only known example of the use of Ibn al-Kammād's tables in Latin and the only text in Latin dependent on al-*Muqtabis* other than MS M.

The impact of Ibn al-Kammād's works or, more generally, of astronomy of Indian origin, can be traced in various zijes composed in the Iberian Peninsula during the fourteenth century. One of them is the Tables of Barcelona, extant in Hebrew, Latin, and Catalan versions, which consist of a set of tables compiled for the meridian of that city and a text accompanying them. The author, Jacob Corsuno, a Jew from Seville, had been called to Barcelona by King Pere el Cerimoniós (1319–1387) for that purpose. The tables use sidereal coordinates for the positions of the Sun, the Moon, and the planets; their epoch is March 1, 1321, and they are arranged according to the Christian calendar (Millás 1962; Chabás 1996a).

Two astronomers in the Jewish community of the Iberian Peninsula, both of whom lived in the fourteenth century, were followers of Ibn al-Kammād: Juan Gil of Burgos, whose zij in Hebrew is only extant in Madrid, Biblioteca Nacional, MS 23078 (formerly London, Jews College, MS 135) [MS G]; and Joseph Ibn Waqār, whose zij is preserved in Arabic in Hebrew characters in Munich, Bayerische Staatsbibliothek, MS Heb. 230 [MS W]. (Note that throughout this essay, we refer to the old foliation of MS G, which is in Hebrew characters.) Little is known about Juan Gil: in a passage in MS G 14b–15a, introduced by the phrase "the copyist (*ha-ma$^c$tiq*) said," the author is identified as Juan Gil (MS G 15a:1) and Abū Ja$^c$far [Ibn] al-Kammād's *Muqtabis* is mentioned (14b:-3). Juan Gil has been identified with the person with the same name in documents of the Archives of the Crown of Aragon dated 1350–1352 (Rubió y Lluch 1908, pp. 155, 164), and who is cited in two astrological texts in Judeo-Portuguese (González Llubera 1953); see also Beaujouan (1969), p. 11. Moreover, a note taken from an astrological text by Juan Gil is preserved in Paris MS Heb. 1031, f. 207b. The epoch for the tables in this zij is 1310 of the Spanish Era (MS G 62b, 75b, et passim), which is equivalent to 1273 A.D. (The epoch of the Spanish Era is January 1, $-37 =$ JDN 1707544: see Neugebauer 1962a, p. 82.) See also Chabás and Goldstein (1997, 2015, pp. 43–44).

Joseph Ibn Waqār of Toledo was a member of a prominent Jewish family; his zij was composed in Arabic in 761 A.H. = 1359/60 A.D. (MS W 4b:13–14), and the canons were translated by the author himself into Hebrew (MS W 4a:16: both versions are preserved in MS W): see Steinschneider (1893), pp. 598–599; Castells (1996). The author's name is given in MS W 1a:2, 4a:17, and 13b:1, and the epoch for the tables is 720 A.H. = 1320/21 A.D. (MS W 20a). The zij, *al-Kawr*, composed by Ibn al-Kammād, is mentioned in MS W 4a:10–11 and 20a:1. See also Chabás and Goldstein (1997, 2015, pp. 44–45).

The anonymous zij, uniquely extant in Vatican, MS Heb. 384, ff. 263a–278b, has as its epoch 1400 A.D. It includes a number of double-argument tables for planetary corrections, and astrological tables are notably absent. There is no indication of familiarity with the Parisian Astronomical Tables that were dominant in Europe at the time outside the Iberian Peninsula, and the zij that it most closely resembles is that of Judah ben Verga (see below). Some of the tables in this zij are in the tradition of Ibn al-Kammād, notably a table for trepidation and a table for the solar equation. For details, see Goldstein (2003).

In the fifteenth century, Judah ben Verga, active in Lisbon from about 1455–1475, compiled a zij called *Ḥuqqot Shamayim* (Ordinances of the Heavens), in which he described an observation he made of an autumnal equinox in September 1456 and compared it with a similar observation by Ptolemy in order to determine the length of the tropical year. In his solar equation table, he depended on Ibn al-Kammād (Goldstein 2001, pp. 232, 244–245, 267–269). However, generally, it has been difficult to establish the exact relationship of this zij with others in the late Middle Ages.

And then, there are a few elements typically of Indian origin that entered the mainstream tradition of European astronomy in the Middle Ages. One of them is the approach given to lunar latitude as a function of the argument of lunar latitude, with a maximum value of 4;30°, as in al-Khwārizmī's zij. This parameter is found in many tables such as the Tables of Barcelona and the zij of Abu l-Ḥasan ᶜAlī al-Qusanṭīnī, among others. In the fourteenth century, Jacob ben David Bonjorn, the author of a set of tables for syzygies (conjunctions and oppositions of the Sun and the Moon) beginning in 1361 and computed for the city of Perpignan, used this particular parameter in his tables for eclipses (Chabás 1991, 1992). So in the fifteenth century did Abraham Zacut, who compiled in Salamanca a lengthy zij (*ha-Ḥibbur ha-Gadol)*, with epoch 1473 (Chabás and Goldstein 2000). Another element in the Indian tradition which is found in western astronomy is the parameter associated with the lunar anomaly, namely a maximum equation of 4;56°. In the Iberian Peninsula and the Maghrib, this parameter was used in the zijes of Ibn Muᶜādh, Ibn Isḥāq, Abu l-Ḥasan ᶜAlī al-Qusanṭīnī, and the Almanac of 1307, to cite a few, and in the Latin world by the astronomers in the service of King Alfonso, as well as in the tables of John Vimond (Paris, ca. 1320) and the Parisian Alfonsine Tables, the most widely disseminated set of tables in the late Middle Ages throughout Europe (*editio princeps*: Venice, 1483), among others. For us, this is a clear indication that the Parisian Alfonsine Tables of the early fourteenth century were a significant vehicle for the transmission of the Iberian tradition to the rest of Europe. Moreover, through the Alfonsine Tables, this parameter for the lunar equation, 4;56°, was even used by Copernicus (*De Rev.* IV.11) who, in this respect, belongs to this long Indian tradition.

## 1 Chronology and calendars

MS M 27r–v

The purpose of the tables is to convert dates from the Arabic calendar to the "Roman" (i.e., Julian) and Egyptian calendars. As indicated in the canons accompanying the tables, the epoch is the Hijra, given as noon, "the day of Mercury" (Wednesday), corresponding to noon, July 14, 622 A.D.: "Know that the radix used in this canon for each of the motions of the centers and the arguments is noon, Wednesday, that preceded the Thursday on which day is the [first day of] Muḥarram in the first year of the Hijra" [*Scias quia radicales positi in hoc canone in singulis motibus centrorum et argumentorum sunt positi super circulo meridiei diei Mercurii qui praecesserat diem Jovis in quo die Jovis intraverat Almuharram in primo anno annorum seductionis...*] (MS M 6r, chapter 9 of the canons). The date mentioned in the text requires an explanation. According to al-Bīrūnī (Sachau 1879, p. 34), in 17 A.H. the Caliph ᶜUmar declared the epoch of the Hijra era to be Thursday, corresponding to July 15, 622 A.D. The convention for the beginning of the civil day in the Hijra calendar is sunset, and the correspondence with the Julian date is for the day that begins at midnight following that sunset. Hence, the beginning of Thursday, Muḥarram 1, 1 A.H. took place at sunset of Wednesday, July 14, 622 A.D. However, the astronomical convention, beginning with Ptolemy, is that the day begins at noon. For Muslim astronomers, this was taken to mean that the epoch of the Hijra took place on Wednesday, noon, preceding the civil day, Thursday, that began about 6 hours later (as is stated in the canons to Ibn al-Kammād's tables), but see the comments on Table 1, below. In this passage and elsewhere in the canons and the tables, the Arabic expression for "years of the Hijra" has been translated into Latin as *anni seductionis*. The term in Latin for the Hijra is unusual, but it occurs elsewhere in this manuscript (see, e.g., MS M 35r, below). We note that both *seductio* and the Arabic root of *Hijra* can mean "separation." To be sure, *Hijra* in this context refers to Muḥammad's "emigration" from Mecca to Medina. In MS F (e.g., 39r), the corresponding expression in Hebrew is *li-teḥillat peraṭ hagar*, meaning (literally) "for the beginning of the era of Hagar," where Hagar represents the Arabs (as the mother of Ishmael in the Hebrew Bible). This meaning of the term *peraṭ* is unusual: see Goldstein (2013), pp. 176–178.

The main table displays the correspondence among three calendars (see Table 1).

The entry 932 Julian years 9 months 17;0 days [$= 932 \times 365.25 + 9 \times 30 + 17;0 = 340700$ days] corresponds to the time interval from the beginning of the Seleucid era (October 1, −311; JDN = 1607739) to the Hijra, whereas the entry 9 Egyptian years 11 months 9 days [$= 9 \times 365 + 11 \times 30 + 9 = 3624$ days] is the time interval between the Hijra and the beginning of the Yazdegird era (June 16, 632; JDN = 1952063). If we consider the dates given here for the epochs of the Seleucid and the Yazdegird eras to be correct, then the date for the Hijra underlying this table is Thursday, July

**Table 1** Conversion from the Arabic to the Julian and Egyptian calendars in MS M (excerpt)

| Arabic years | Collected Julian y. | | | | Collected Egyp. y. | | |
|---|---|---|---|---|---|---|---|
| | years | m. | d. | frac. | years | m. | d. |
| Radix | 932 | 9 | 17 | 0 | 9 | 11 | 9 |
| 90 | 1020 | 1 | 8 | 0 | 77 | 5 | 18 |
| ... | | | | | | | |
| 900 | 1905 | 11 | 23 | 45 | 963 | 10 | 11 |

15, 622 (JDN = 1948439 = 1607739 + 340700 = 1952063 − 3624), despite what is indicated in the canons to these tables.

These tables in MS M entirely, or partially, reproduce calendrical tables in the zij of al-Khwārizmī (Suter 1914, pp. 110–113), and/or in the tables associated with al-Battānī but now attributed to Maslama (Nallino 1903–1907, 2: 301, 304–305).

MS F 29r–32v

Displayed here are tables for the Jewish, Muslim, and Christian calendars that were added to the text of Ibn al-Kammād by the translator, Solomon Franco. The Jewish calendar is called the years of Adam, the Muslim calendar is called the years of Hagar, and the Christian calendar is called the years of Edom (see Goldstein 2013). A note on f. 29r indicates that the beginning of Muḥarram 780 A.H. corresponds to the beginning of Iyar 5138 A.M. (= April 29, 1378).

It is worth noting that chapter 2 (conversion of dates from the Muslim calendar to the Byzantine calendar and vice versa, through calculation) of the canons in Hyderabad, Andra Pradesh State Library, MS 298, reproduces word for word fragments of the canons in Latin of *al-Muqtabis*. Moreover, other chapters, also on calendrical matters, follow closely these canons (Mestres 1999, pp. 16–23). Furthermore, the first two tables in the Hyderabad manuscript are identical with the first two tables in *al-Muqtabis* (Mercier 1996, pp. 420–421).

## 2 Trigonometry and spherical astronomy

### 2.1 Functions related to the Sine

MS M 46v; MS F 54v

Three trigonometric functions are tabulated (see Table 2), and for each integer degree we are given Sin $\theta$ = 60 sin $\theta$, Cos $\theta$ = 60 cos $\theta$, and Vers $\theta$ = 60 − Cos $\theta$ = 60 (1 − cos $\theta$). All three functions are normed for R = 60 and, following Kennedy (1956a, pp. 139–140), we have capitalized the first letter of their abbreviations. The Almanac of Azarquiel (Millás 1943–50, p. 229) has a table with these three functions, but given at intervals of 3°. Except for copying errors, the entries in both manuscripts of *al-Muqtabis* agree.

**Table 2** Functions related to the Sine in MS M (excerpt)

| Arg. (°) | Sin (°) | Cos (°) | Vers (°) |
|---|---|---|---|
| 1 | 1; 3 | 59;58 [b] | 0; 7 [c] |
| 2 | 2; 6 | 59;58 | 0; 3 |
| ... | | | |
| 30 | 30; 2 [a] | 51;58 | 8; 2 |
| ... | | | |
| 60 | 51;58 [b] | 30; 0 | 30; 0 |
| ... | | | |
| 89 | 59;58 | 1; 3 | 58;57 |
| 90 | 60; 0 | 0; 0 | 60; 0 |

[a] Read 30;0, as in MS F
[b] Read 59;59, as in MS F
[c] Read 0;1, as in MS G

The sine function and its derivatives are characteristic of Indian astronomy; in ancient Greek astronomy, one had to depend on the chord function which, while equivalent to the sine function, is much clumsier to use ["the chord function was (...) replaced by the sine in even the most ancient of extant [Indian] texts," Van Brummelen (2009), p. 95]:

$$\mathrm{Sin}\,\theta = 1/2\,\mathrm{Crd}\,2\theta.$$

For a chord table, see *Almagest* I.11.

The norm used here (60) differs from that in al-Khwārizmī's zij (150), a norm previously used in Brahmagupta's *Khaṇḍakhādyaka* (655 A.D.); see Pingree (1976), p. 160.

Note that in MS F there is no column for the Versine: *meytar* is used for Sine and *takhlit ha-meytar* for Cosine. This table with the three functions is also found in the set of tables of Juan Gil, only extant in MS G 105b–106a, with the title: Table of Sines [*mish$^{c}$anim*], Versines [*ḥiṣṣim*], and Cosine [*mish$^{c}$an ha-hashlama*].[1]

Examples of Western tables with sines normed 150 abound: see a list of manuscripts associated with the Toledan Tables (Pedersen 2002, pp. 946–954), as well as others, e.g., Paris, Bibliothèque nationale de France, MS 7316A, f. 116r, and Vatican, Biblioteca Apostolica, MS Pal. lat. 1414, ff. 90r–91r, containing Ibn al-Kammād's tables for eclipses.

Chapter 19 of the canons of Ibn Isḥāq's zij is taken from canon 17 of *al-Muqtabis* and, according to Mestres, the six tables corresponding to the functions related to the sine in the Hyderabad manuscript probably go back to one of the two lost zijes by Ibn al-Kammād of which *al-Muqtabis* is a summary (Mestres 1999, p. 60).

## 2.2 Solar declination

MS M 35v; MS F 40r

[1] The term *mish$^{c}$an* in the sense of Sine is not otherwise attested.

**Table 3** Solar declination in MS M (excerpt)

| Arg. (°) | Declin. (°) |
|---|---|
| 1 | 0;24 |
| 2 | 0;48 |
| ... | |
| 30 | 11;31 |
| ... | |
| 60 | 20;16 |
| ... | |
| 89 | 23;33 |
| 90 | 23;33 |

This table displays the solar declination for each integer degree of the argument, the solar longitude (see Table 3). The entries reach a maximum of 23;33° at 90°. This value for the obliquity of the ecliptic is associated with the *Mumtaḥan* zij (see Vernet 1956, p. 515, and Kennedy 1956a, p. 145). MS F displays the same table, with minor variants. The modern formula for the declination, $\delta$, is

$$\delta = \arcsin(\sin\lambda \times \sin\varepsilon),$$

where $\lambda$ is the longitude, and $\varepsilon$ is the obliquity. The maximum declination is the obliquity of the ecliptic, that is, the angle between the ecliptic and the celestial equator.

This tradition for tables of declination is also found, with variants, in other sets of tables, such as those by Juan Gil (MS G 89b), Ibn Waqār (MS W 55a) and the Tables of Barcelona (Millás 1962, pp. 194–195, and Chabás 1996a, pp. 499–500), as well as in the calendar of Peter of Saint Omer (Pedersen 1983–84, p. 347). For a list of some historical values of the obliquity, see Chabás and Goldstein (2012), pp. 22–24.

## 2.3 Cotangent function

MS M 48r; MS F 57r

The heading for this table in MS M is *Tabula umbre et altitudinis* (Table of shadows and altitude); the entries represent the length of a shadow, s, projected by a gnomon of 12 units as a function of the altitude of the Sun, h, based on the formula

$$\mathrm{s} = 12\,\mathrm{cotan}\,\mathrm{h},$$

where the cotangent function is the reciprocal of the tangent function (see Table 4).

The corresponding table in MS F has minor variants.

Juan Gil included this table in his zij (MS G 107a), as is also the case for the Tables of Barcelona (Millás 1962, p. 235, and Chabás 1996a, pp. 502–504; see also Chabás and Goldstein 2012, p. 25).

**Table 4** Cotangent function in MS M (excerpt)

| Arg. (°) | Cotangent (°) |
|---|---|
| 1 | 687;27 |
| 2 | 343;39 |
| 3 | 228;47 |
| 4 | 181;36 |
| ... | |
| 30 | 20;47 |
| ... | |
| 60 | 6;56 |
| ... | |
| 89 | 0;13 |
| 90 | 0; 0 |

According to Mestres (1999, p. 65), chapter 22 (*On the solar altitude from the shadows and vice versa*) of the canons of Ibn Isḥāq's zij is based on canon 20 of *al-Muqtabis*.

## 2.4 Normed right ascension

MS M 48v–49r; MS F 55r

This table gives the right ascension, increased by 90°, in degrees and minutes as a function of the longitude, beginning at Capricorn 0°; there is a column for each zodiacal sign (see Table 5). The right ascension, $\alpha$, for a given celestial longitude, $\lambda$, is defined as follows:

$$\sin\alpha = \tan\delta/\tan\varepsilon$$

where $\delta$ is the declination of a given $\lambda$, and $\varepsilon$ is the obliquity of the ecliptic. The normed right ascension, $\alpha'$, is defined as:

$$\alpha' = \alpha + 90^\circ.$$

The same table for normed right ascension is found in al-Battānī's zij (Nallino 1903–1907, 2:61–64), for an obliquity of 23;35°, but differs from that in the *Handy Tables* (Stahlman 1959, pp. 206–209) and that in al-Khwārizmī's zij (Suter 1914, pp. 171–173), both calculated for higher values of the obliquity.

As deduced from this table, the rising times of the individual signs are as follows:

32;13° for Capricorn, Cancer, Sagittarius, and Gemini
27;50° for Aries, Pisces, Virgo, and Libra
29;54° for Taurus, Aquarius, Leo, and Scorpio

**Table 5** Normed right ascension in MS M (excerpt)

| Arg. (°) | Cap (°) | Aqr (°) | Psc (°) | Ari (°) | Tau (°) | Gem (°) |
|---|---|---|---|---|---|---|
| 1 | 1; 6 | 33;15 | 63; 4 | 90;55 | 118;50 | 148;50 |
| 2 | 2;11 | 34;17 | 64; 1 | 91;50 | 119;48 | 149;53 |
| ... | | | | | | |
| 29 | 31;10 | 61;10 | 89; 5 | 116;56 | 146;45 | 178;54 |
| 30 | 32;13 | 62; 7 | 90; 0 | 117;53 | 147;47 | 180; 0 |

| Arg. (°) | Cnc (°) | Leo (°) | Vir (°) | Lib (°) | Sco (°) | Sgr (°) |
|---|---|---|---|---|---|---|
| 1 | 181; 6 | 213;15 | 243; 4 | 270;55 | 298;50 | 328;50 |
| 2 | 182;11 | 214;17 | 244; 1 | 271;50 | 299;48 | 329;53 |
| ... | | | | | | |
| 29 | 211;10 | 241;10 | 269; 5 | 296;56 | 326;45 | 358;54 |
| 30 | 212;13 | 242; 7 | 270; 0 | 297;53 | 327;47 | 360; 0 |

The same table, for the same obliquity of the ecliptic, appears in the tables of Juan Gil (MS G 99b–100a), the Tables of Barcelona (Millás 1962, pp. 220–221, and Chabás 1996a, p. 501), and those of Ibn Waqār (MS W 52b), among many others.

### 2.5 Oblique ascension for Córdoba

MS M 49v–51r; MS F 55v–56r

In this table, we are given the oblique ascension for Córdoba, latitude 38;30°, as explicitly stated in the heading of the Hebrew manuscript. As was the case for right ascension, there is a column for each zodiacal sign; here, the first column corresponds to Aries, rather than to Capricorn. In these tables, there are also columns for the length of the diurnal seasonal hours, defined as one twelfth of the time from sunrise to sunset at a given location on a given day of the year (or a given solar longitude).

The occurrence of a table for Córdoba in this zij is probably a sign that Ibn al-Kammād depended on the version of the zij of al-Khwārizmī made by Maslama for Córdoba (see Table 6).

The rising times of the individual signs deduced from this table are as follows:

18;25° for Aries and 18;45° for Pisces
21;48° for Taurus and 21;29° for Aquarius
29;19° for Gemini and Capricorn

**Table 6** Oblique ascension and diurnal seasonal hours for Córdoba in MS M (excerpt)

| Arg. (°) | Aries Ascen. (°) | Aries S. hour (°) | Taurus Ascen. (°) | Taurus S. hour (°) | Gemini Ascen. (°) | Gemini S. hour (°) |
|---|---|---|---|---|---|---|
| 1 | 0;35 | 15; 3 | 19; 5 | 16;37 | 41; 3 | 17;56 |
| 2 | 1;11 | 15; 6 | 19;45 | 16;40 | 42; 8 | 17;58 |
| ... | | | | | | |
| 10 | 5;58 | 15;31 | 25;13 | 17; 3 | 49;19 | 18;10 |
| ... | | | | | | |
| 20 | 12; 5 | 16; 6 | 32;31 | 17;29 | 59; 0 | 18;21 |
| ... | | | | | | |
| 29 | 17;46 | 16;31 [a] | 39,27 | 17;56 | 68;27 | 18;24 |
| 30 | 18;25 | 16;41 [a] | 40;13 | 17;55 | 69;32 | 18;24 |

| Arg. (°) | Cancer Ascen. (°) | Cancer S. hour (°) | Leo Ascen. (°) | Leo S. hour (°) | Virgo Ascen. (°) | Virgo S. hour (°) |
|---|---|---|---|---|---|---|
| 1 | 70;38 | 18;24 | 106; 3 | 17;52 | 144; 0 | 16;31 |
| 2 | 71;44 | 18;24 | 107;22 | 17;50 | 145;15 | 16;28 |
| ... | | | | | | |
| 10 | 80;49 | 18;20 | 117;39 | 17;29 | 155;14 | 16; 3 |
| ... | | | | | | |
| 20 | 92;43 | 18;10 | 130;10 | 17; 0 | 167;38 | 15;31 |
| ... | | | | | | |
| 29 | 103;33 | 17;56 | 141;25 | 16;38 [b] | 178;46 | 15; 3 |
| 30 | 104;44 | 17;55 | 142;54 | 16;34 [b] | 180; 0 | 15; 0 |

[a] With MS F; blank in MS M
[b] With MS F; blank in MS M

35;12° for Cancer and Sagittarius
38;1° for Leo and Scorpio
37;15° for Virgo and Libra

The surprising results for the rising times of Pisces and Aquarius derive from an erroneous entry for Aqr 30° = Psc 0° (341;15° in MS M). MS F has the correct value, 341;35°.

The entries for diurnal seasonal hours reach a maximum of 18;24° at Cancer 0°, a value which is in good agreement with the entry for the half-length of daylight, 7;21h, displayed in another table for Córdoba in MS M 47v (see below). Indeed, 18;24 × 12/15 = 14;43h, which yields a value for the half-daylight of 7;21,30h.

**Table 7** Oblique ascensions for Burgos, Barcelona, and Toledo (excerpts)

| Burgos | | | Barcelona | | | Toledo | | |
|---|---|---|---|---|---|---|---|---|
| Arg. (°) | Aries Ascen. (°) | ... | Arg. (°) | Aries Ascen. (°) | ... | Arg. (°) | Aries Ascen. (°) | ... |
| 1 | 0;23 | | 1 | 0;35 | | 1 | 0;33 | |
| 2 | 1; 6 | | 2 | 1;10 | | 2 | 1;10 | |
| ... | | | ... | | | ... | | |
| 10 | 5;23 | | 10 | 5;46 | | 10 | 5;51 | |
| ... | | | ... | | | ... | | |
| 20 | 11;16 [a] | | 20 | 11;38 | | 20 | 11;50 | |
| ... | | | ... | | | ... | | |
| 29 | [illegible] | | 29 | 17;11 | | 29 | 17;26 | |
| 30 | [illegible] | | 30 | 17;48 | | 30 | 18; 4 | |

[a] The reading for the minutes is uncertain

When recomputing the entries, close, although not exact, agreement is obtained with $\varphi = 38;30°$ and $\varepsilon = 23;51°$; this value for the obliquity produces better agreement than the values found in our text (23;33° and 23;35°).

## 2.6 Oblique ascension for cities other than Córdoba

Each of the followers of Ibn al-Kammād presented tables for the oblique ascension for their own cities. Juan Gil (MS G 100b–101b) has a table for Burgos (latitude 42;18°: G 148b), which includes columns for the seasonal hours. The Tables of Barcelona (Millás 1962, pp. 222–223, and Chabás 1996a, p. 502) have no such table for Córdoba, but one for Barcelona. The data for diurnal seasonal hours are presented in a separate table. Ibn Waqār (MS W 53a) has a table for the oblique ascension for Toledo. The latitude is not specified there, but in a table for geographical coordinates on f. 59a, it is given as 40;0°, whereas in a table for lunar parallax on f. 54a it is given as 39;55°. There are no columns for the seasonal hours in Ibn Waqār's table. For excerpts from tables of oblique ascension, see Table 7.

MS M 59v–61r displays a similar table, including columns for the seasonal hours, for Salé, a place in North Africa near Rabat, whose latitude is given here as 33°.

MS F has three other tables for the oblique ascensions. On ff. 63v–64r, there is a table for the city of Seville whose latitude is given as 37;15°, and it includes entries for the length of seasonal hours. Its maximum value is 18;18°, corresponding to a longest daylight of 14;38h. On ff. 64v–65r, there is another table for the horizon of a city whose latitude is 41;51° (Zaragoza). The maximum value for the length of a seasonal hour is 18;49°, which corresponds to a longest daylight of 15;3h, precisely the number appearing in the heading. The third table is on ff. 65v–66r. As indicated in the heading,

**Table 8** Length of daylight for Córdoba in MS M (excerpt)

| Arg. (°) | Arg. (°) | Cap (h) | Aqr (h) | Psc (h) | Ari (h) | Tau (h) | Gem (h) |
|---|---|---|---|---|---|---|---|
| 1 | 29 | 4;39 | 4;53 | 5;24 | 6; 1 | 6;38 | 7; 9 |
| 2 | 28 | 4;39 | 4;54 | 5;25 | 6; 2 | 6;39 | 7;10 |
| ... | | | | | | | |
| 29 | 1 | 4;51 | 5;22 | 5;59 | 6;36 | 7; 7 | 7;21 |
| 30 | 0 | 4;52 | 5;23 | 6; 0 | 6;37 | 7; 8 | 7;21 |
| | | Sag | Sco | Lib | Vir | Leo | Cnc |

it is valid for the city of Toledo whose latitude is 40;0°; the maximum value for the length of a seasonal hour is 18;34°, which corresponds to a longest daylight of 14;51h.

## 2.7 Length of daylight for Córdoba

MS M 47v; MS F 56v

This table gives the half-length of daylight in Córdoba, as indicated in the heading in both manuscripts, at intervals of one degree of the argument, as a function of the solar longitude. The table begins at Capricorn (see Table 8).

The maximum entry represents half of the longest daylight (M/2), and it is 7;21h for Cancer 0°. This value follows from the formula:

$$\tan\varphi = -\cos(\mathrm{M}/2) \times \cot\varepsilon,$$

where $\varphi = 38;30°$ and $\varepsilon = 23;33°$.

## 2.8 Tangent function

MS M 48r: "Tabula directionis arcus luminis et transitus"; MS F 68r: Table for the correction of the arc of light.

The argument in this table is given in degrees at 3°-intervals from 3° to 90° (with some copying errors). In MS M, columns 2 and 3 are headed "directio arcus luminis" and "minuta diuersitatis transitus," whereas in MS F they are headed "Correction for the arc of light" and "minutes of the crossing and the difference," respectively (see Table 9). Column 2 has entries in degrees and minutes, and column 3, in minutes and seconds. A very similar table is found, but with entries to seconds in both columns, in the Almanac of Azarquiel (Millás 1943–50, p. 226). However, the headings for the columns in the latter are quite different from those here.

Column 2 represents the tangent function. More specifically, the tabulated function is $y = 5 \times \tan x$, where the coefficient 5 has to be understood as 60/12. Note, however,

**Table 9** Tangent function in MS M (excerpt)

| | *directio arcus luminis* [a] (°) | *minuta diuersitatis transitus* [b] (′) |
|---|---|---|
| 3 | 0;14 | 1;23 |
| 6 | 0;31 | 2;46 |
| 9 | 0;47 | 4; 6 |
| 12 | 1; 4 | 5;34 [f] |
| ... | | |
| 30 | 2;53 | 12;12 |
| ... | | |
| 45 | 4;40 | 17;16 |
| ... | | |
| 60 | 8;39 | 20;56 |
| ... | | |
| 78 | 28;31 [c] | 23;20 |
| 81 | 31;25 | 23;15 |
| 84 | 47;35 | 23;23 |
| 87 | 94; 0 [d] | 23;29 |
| 90 | 1507; 0 [e] | 23;33 |

[a] MS F: "Correction for the arc of light"
[b] MS F: "Minutes of the transit [*ha-ma*ᶜ*abar*] and the difference [*ha-shinnuy*]"
[c] MS F: 23;35
[d] MS F: 96;0
[e] MS F: 360;0
[f] MS F: 5;54

that the entry for 45° is erroneously given as 4;40° instead of 5;0°. The entry for 90°, given as 1507;0°, actually corresponds to an argument of 89;49°. This is one of the very few examples in zijes from al-Andalus of the tangent function, already used by Ḥabash al-Ḥāsib with base R = 1 (Debarnot 1987, p. 62). Besides this table and the analogous one in the Almanac of Azarquiel, we only know of one other in the *Kitāb majhūlāt* of Ibn Muᶜādh al-Jayyānī, from Jaén (eleventh century), where the tabulated function is the quotient of the sine and the cosine of the same angle, and the entries are given to seconds (Villuendas 1979, p. 118; Samsó 1980, pp. 60–68). All three tables date from about the same time. Two examples of the use of this auxiliary table, given at intervals of 1°, are known among Maghribi astronomers: Ibn Isḥāq (Mestres 1999, pp. 275 and 278) and Ibn al-Raqqām in his three zijes, *Shāmil, Qawīm*, and *Mustawfī* (Samsó 2014, p. 323).

The tangent function is used to determine the ascensional difference, $\gamma$, of a celestial body whose longitude is $\lambda$ at a geographical latitude $\varphi$. As explained by Neugebauer, following the instructions given in the *Sūrya-Siddhānta* ii.60–63, one needs first to define a "small circle of the Sun," that is, a circle parallel to the equator on which the Sun moves during one day when its longitude is $\lambda$ (see Neugebauer 1962a, pp. 50–51, and Burgess (trans.) [1860] 1935, pp. 101–102; cf. Goldstein 1967a, pp. 78–80, 204–205, and Pingree 1968b, p. 123). The ascensional difference is called "equation of daylight" by al-Bīrūnī (see Lesley 1957, p. 123), which corresponds to the headings in MSS M and F. It was computed by means of verbal instructions, which are equivalent to the modern formula

$$\mathrm{Sin}\,\gamma = s_0\,(R/g)\tan\delta,$$

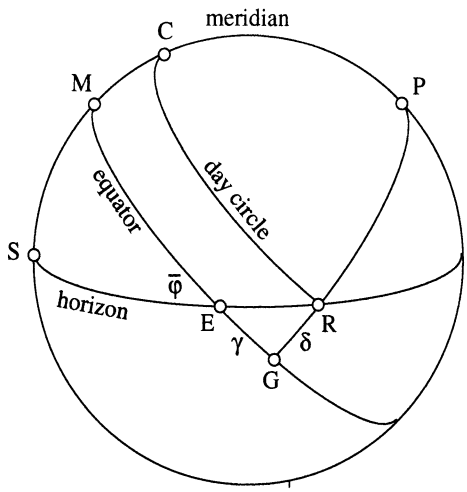


**Fig. 1** The day circle of the Sun for a given $\lambda$, where arc SER is the horizon, arc SMCP is the meridian, arc MEG is the equator whose pole is P, R is the rising point of the Sun for some given $\lambda$, arc RC is half the day circle (from sunrise to noon) on a small circle parallel to the equator, EM is the equator, $\varphi$ is the geographical latitude (and $\bar{\varphi} = 90 - \varphi$), $\gamma$ is the ascensional difference, and $\delta$ is the declination of the Sun at R

where $s_0$ is the length of the equinoctial noon shadow of a gnomon of height g, and $\delta$ is the declination of the Sun. In the expression above, $s_0$ is the only element depending on $\varphi$, whereas (R/g) tan $\delta$ depends on $\lambda$ only and can therefore be used for all climates. When R = 60 and g = 12, (R/g) tan $\delta$ reduces to $5 \times \tan\delta$, which is the function tabulated in column 2. Finally, the length of daylight (or, "arc of light") in equatorial degrees, when the Sun is at a given $\lambda$, is $180° + 2\gamma$ ($\gamma$ is positive for $0° \leq \lambda \leq 180°$, and negative for $180° \leq \lambda \leq 360°$). To justify this formula, see Fig. 1. The standard formula for $\gamma$ in spherical right triangle EGR is:

$$\sin\gamma = \tan\delta \times \tan\varphi.$$

This is equivalent to the previous formula, for Sin $\gamma$ = R sin $\gamma$, and $s_0/g = \tan\varphi$.

The maximum in column 3 is 0;23,33°, which is a sixtieth of the value for the obliquity of the ecliptic used by Ibn al-Kammād. Although the agreement is not very good, we think it likely that the entries in this table are $\delta/60$, where $\delta$ is the declination. This table is also found in the Almanac of Azarquiel (Millás 1943–50, p. 226), and in the two zijes of Ibn al-Raqqām, *Qawīm* and *Mustawfī*, where it is called *nisbat al-mamarrāt* (Samsó 2014, p. 322). We have not succeeded in determining the purpose for which column 3 of this table was computed.

Among the tables in this manuscript that do not belong to *al-Muqtabis*, there is one (MS M 59r) with the title "Tabula partis cordarum supereminentium et partis almudarat solis et planetarum" (see Table 10). It has three columns, the first being for the argument at intervals of 1° from 1° to 90°. The other two columns display entries for the auxiliary functions $y = 5 \times \tan x$ (here called *pars cordarum supereminentium*) and $y = 1/\cos x$ (i.e., the secant function, here called *pars almudarat*); these entries differ from those in Table 9. In contrast to Table 9, the entry for 45° is correct in the case of the tangent. Note that the entry for 60° in the column for the secant should be 2;0°. However, the corresponding entry in the zij of Ibn al-Bannā', where the table is called *nisbat al-madārāt*, is also 2;1° (King 2004, p. 132). The column for the secant

**Table 10** Tangent and secant functions (excerpt)

| | tangent (°) | secant (°) |
|---|---|---|
| 1 | 0; 4 | 1; 0 |
| 2 | 0; 9 | 1; 0 |
| ... | | |
| 30 | 2;53 | 1;10 |
| ... | | |
| 45 | 5; 0 | 1;25 |
| ... | | |
| 60 | 8;39 | 2; 1 |
| ... | | |
| 86 | 59;51 | 12;56 |
| 87 | 96; 1 | 19;19 |
| 88 | [?];27 | 29;15 |
| 89 | [?]; 2 | 56; 0 |
| 90 | [?]; 0 | 60; – [a] |

[a] Blank

function is also found in the Almanac of Azarquiel, at intervals of 3° of the argument, where the entry for sec 60° is given as 2;0,56° (Millás 1943–50, p. 226). The same table, with its three columns, is reproduced by two other Maghribi astronomers, at intervals of 1°: Ibn Isḥāq (Mestres 1999, pp. 275 and 278–281) and Ibn al-Raqqām in his three zijes, *Shāmil*, *Qawīm*, and *Mustawfī* (Samsó 2014, p. 323). As reported by King, both authors give 2;1° for sec 60°.

## 3 Equation of time

MS M 46r; MS F 59r

The entries in this table are given in time degrees, rounded to the nearest integer, and the maximum, 8°, occurs at Sco 2°–12° in the Latin manuscript and at Sco 2°–14° in the Hebrew manuscript (see Table 11).

In MS M, beneath the table, we read: "Mediatus solis in radice posita ad directionem dierum cum noctibus: 10.23.24.50. a puncto capitis arietis," whose meaning is not entirely clear. It probably refers to a position of the mean Sun, which has to be understood as 10s 23;24,50°, corrected for the equation of time. Elsewhere in this manuscript, the term *directio* means equation: see, e.g., MS M 35v. The entries in the table may have been taken from the more precise values given in the zij of al-Battānī (Nallino 1903–1907, 2:61–64; preserved in the Toledan Tables: Toomer 1968, pp. 34–35; Pedersen 2002, pp. 968–983), displayed in time degrees, that differ substantially from those in the zij of al-Khwārizmī (Suter 1914, pp. 181–182), where they are given in hours, minutes, and seconds.

The same table is found in the Tables of Barcelona (Millás 1962, p. 224, and Chabás 1996a, p. 505). Juan Gil gives four tables for the equation of time: two "according to Abū Ja$^{c}$far al-Kammād" (MS G 94b–95a), one with entries in degrees and minutes and

**Table 11** Equation of time in MS M (excerpt)

| Arg. (°) | Ari (°) | Tau (°) | Gem (°) | Cnc (°) | Leo (°) | Vir (°) | Lib (°) | Sco (°) | Sgr (°) | Cap (°) | Aqu (°) | Psc (°) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 4 | 5 | 4 | 3 | 4 | 6 | 7 | 7 | 3 | 0 | 0 |
| 2 | 2 | 4 | 5 | 4 | 3 | 4 | 6 | 8 | 7 | 3 | 0 | 0 |
| ... | | | | | | | | | | | | |
| 10 | 3 | 5 | 5 | 4 | 3 | 4 | 6 | 8 | 6 | 2 | 0 | 0 |
| ... | | | | | | | | | | | | |
| 20 | 4 | 5 | 4 | 3 | 3 | 5 | 7 | 7 | 5 | 1 | 0 | 1 |
| ... | | | | | | | | | | | | |
| 29 | 4 | 5 | 4 | 3 | 3 | 6 | 7 | 7 | 4 | 0 | 0 | 2 |
| 30 | 4 | 5 | 4 | 3 | 3 | 6 | 7 | 7 | 4 | 0 | 0 | 2 |

the other with entries in minutes and seconds of an hour; two other tables "according to the author" (MS G 95b–96a), one with entries in degrees and minutes and the other with entries in minutes and seconds of an hour.

The equation of time is given either in time degrees or in minutes of an hour, where $1° = 0;4$h. In the zij of Ibn Waqār, the equation of time is given in minutes of an hour, as the heading of the table specifies (MS W 36a). The argument is for 1° and multiples of 6°, to 30° in each zodiacal sign. The entries (disregarding the units) are 4 times the corresponding entries in Table 11; it is therefore most likely that Table 11 underlies this table.

## 4 Trepidation and apogees

### 4.1 Trepidation

In *al-Muqtabis* there are two tables for trepidation: one displays a constant motion of 0;5,24,32°/y (where y corresponds to an Arabic year) for the first point of Aries (see Table 12) and the other gives its equation, representing a variable motion that reaches a maximum of 9;59° (see Table 13).

MS M 28v, "Tabula aduenctionis puncti capitis arietis"; MS F 33r, Table for the motion of the head of Aries on the circle of access [*ha-qadima*] and recess [*ha-iḥur*]

Both manuscripts display entries to seconds and give the same radix, 3;51,11°.

Note that the entries are for years and months in the Hijra calendar: the entry for collected years includes the radix and the entry for 90 collected years is obtained by adding the radix (3;51,11°) to the entry for 90 expanded years. The same radix and equivalent entries for the mean motion of the vernal point are found in Azarquiel's *Treatise on the motion of the fixed stars* (Paris, Bibliothèque nationale de France, MS Heb. 1036; see Millás 1943–50, pp. 266, 324), where the entries are given to sexagesimal fourths. The treatise by Azarquiel seems to be a direct antecedent of Ibn al-Kammād's table. The *Liber de motu octave sphere* attributed to Thābit Ibn

**Table 12** Motion of the First Point of Aries in MS M (excerpt)

| Collected years | (s) | (°) | Expanded years | (s) | (°) | Months | (s) | (°) |
|---|---|---|---|---|---|---|---|---|
| Radix | 0 | 3;51,11 | 1 | 0 | 0; 5,24 | Almu. | 0 | 0; 0,27 |
| 90 | 0 | 11;57,59 | 2 | 0 | 0;10,49 | Saph. | 0 | 0; 0,53 |
| 180 | 0 | 20; 4,48 | ... | | | ... | | |
| ... | | | 29 | 0 | 2;36,51 | Dilca. | 0 | 0; 4,53 |
| 720 | 2 | 8;54,37 | 30 | 0 | 2;42,16 | Dilch. | 0 | 0; 5,24 |
| 810 | 2 | 16;52,26 | 60 | 0 | 5;24,32 | | | |
| 900 | 2 | 24;59,14 | 90 | 0 | 8; 6,48 | | | |

**Table 13** Equation of the Motion of the First Point of Aries in MS M (excerpt)

| | | 0/6 | 1/7 | 2/8 |
|---|---|---|---|---|
| 1 | 29 | 0;10 | 5;16 | 8;52 |
| 2 | 28 | 0;20 | 5;25 | 8;56 |
| ... | | | | |
| 10 | 20 | 1;45 | 7;53 [b] | 9;28 |
| ... | | | | |
| 20 | 10 | 3;25 | 7;48 | 9;55 |
| ... | | | | |
| 29 | 1 | 4;38 [a] | 8;41 | 9;59 |
| 30 | 0 | 5; 7 | 8;47 | 9;59 |
| | | 5/11 | 4/10 | 3/9 |

[a] Instead of 4;58; MS F illegible

[b] Judging from the surrounding entries, this entry should probably be 6;50; MS F reads 6;53, and MS G 78b seems to read 6;50, but this number is faint in the copy available to us

Qurra displays a similar table (Millás 1943–50, p. 507; Neugebauer 1962a, p. 296; and Goldstein 1964). On the date of the zero point in theories of trepidation, that is the moment when the value for trepidation is 0°, see Samsó (1997), p. 108; Comes (2001), p. 331; and Goldstein (2011), pp. 78–79.

MS M 35v, "Tabula directionis aduenctionis capitis arietis"; MS F 40r, Table for the equation of access of the head of Aries

Table 13 complements Table 12. For an edition and a reconstruction of it, see Chabás and Goldstein (1994), p. 25, and Chabás and Goldstein (2015), chapter 7. For another analysis of this table, see Mancha (1998); for general discussions of trepidation, see Mercier 1976–1977 and 1996; and Ragep (1996). For the Indian traditions of precession and trepidation, see Pingree (1972).

The entries, e, in this table can be recomputed, although without perfect agreement, by means of the formula

$$\sin e = (r/60) \times \sin i,$$

where $i$ is the argument and $r = 10;24°$ is a parameter appearing in models for trepidation associated with Azarquiel, where arcsin $(10;24/60) = 9;58,54°$ is very close to the maximum value in this table, 9;59° at 90° (Chabás and Goldstein 1994, pp. 24–27; for a similar table see Toomer 1968, p. 118; and Pedersen 2002, pp. 1552–1554). This table is also found in the Tables of Barcelona (Millás 1962, pp. 194–195, table 20(3), and Chabás 1996a, pp. 487–489), in a treatise by al-Marrākushī (J.-J. Sédillot and L.-A. Sédillot 1834, p. 131), and in the tables of Juan Gil (MS G 78b). A table based on the same formula is found in Ibn al-Raqqām's *Mustawfī* zij (Samsó 2014, p. 314).

## 4.2 Solar apogee

MS M 35r has a table headed, "Tabula directionis centri solis et augis eius ad primordium annorum seductionis" (Table for the equation of center for the Sun and its apogee for the beginning of the years of the Hijra). The longitude of the solar apogee for the Hijra is given above the heading, as "Aux" 2s 16;45,21°. Likewise, in MS F 39v, which displays a table for the equation of the Sun, the same value is given for the solar apogee, presumably for the beginning of the Hijra era.

The value of the solar apogee for the Hijra given in *al-Muqtabis* differs from the corresponding value in the zij of al-Khwārizmī, 2s 17;55° (see Neugebauer 1962a, p. 19) which, in turn, came from the *Zīj al-Shāh* and ultimately from Indian sources (Pingree 1965). In contrast, Ibn Mu$^{c}$ādh of Jaén retained al-Khwārizmī's parameter (see Samsó [1992] 2011, p. 154). There are two values for the solar apogee in Latin texts attributed to Thābit Ibn Qurra: 2s 20;45° and 2s 22;55° (Neugebauer 1962b, pp. 266 and 286). Brahmagupta's value for the solar apogee was 2s 20°, later corrected to 2s 17° (Sengupta (trans.) 1934, pp. 13 and 139). For the solar apogee of 2s 20° in an Indian astronomical work composed in the sixth century, see also Neugebauer and Pingree (ed. and trans.) 1970, 1:93 and 2:69.

The Almanac of 1307 gives the solar apogee as Gem 19;6° (Madrid, Biblioteca Nacional, MS 3349, f. 54v) and Gem 10;6° (Madrid, Biblioteca Nacional, MS 17961, ff. 61r and 90v). The first value differs from that of Ibn al-Kammād by 2;21° and from that of al-Khwārizmī in 1;11°, whereas the second value is probably a copyist's error: with the proper motion of 0;0,0,2,2,14,46°/d embedded in Ibn al-Kammād'zij (see Sect. 5.2, below), the solar apogee progresses 2;21,31° in the time interval between the Hijra and the beginning of 1307 (March 1), demonstrating that the position of the solar apogee in the Almanac of 1307 depends on Ibn al-Kammād's parameters.

For Juan Gil, the position of the solar apogee is 2s 18;46° (MS G 79a). In the Tables of Barcelona (Millás 1962, pp. 192–193, and Chabás 1996a, pp. 487–489), the solar apogee for the "first year of King Pere of Aragon" (1321) is given is 2s 19;25,55° in Ripoll, MS 21, and 2s 19;30° in Vatican, MS Heb. 356. In MS W 36b, in the table for the solar equation given by Ibn Waqār, the equation is 0° for longitude 2s 19° exactly, which is therefore the solar apogee (very nearly the same value as in the Tables of Barcelona). It should be noted that the epoch for Ibn Waqār's tables is 720 A.H., corresponding to 1320/21 A.D.

The longitude of the solar apogee at epoch in the zij of Ibn ᶜAzzūz (fourteenth century, Fez) is 2s 16;44,21°, i.e., one minute less than Ibn al-Kammād's value (Samsó 1997, pp. 82, 102).

## 4.3 Planetary apogees

Both in MSS M and F, the values for the apogees of the planets appear in the headings of the tables for the planetary equations (M 37v–44v; F 42r–49r): see Table 14.

We note, first, that three of the values in MS M differ from those in MS F by 0;0,30°; second, that the value given for Venus agrees exactly with that for the Sun; and third, that these values do not correspond to those given in the zij of al-Khwārizmī (Suter 1914, pp. 14–15, Neugebauer 1962a, pp. 41 and 99), where the apogees of the Sun and Venus differ. The planetary apogees at epoch in the zij of Ibn ᶜAzzūz are identical to those in MS F (with the variant that agrees with MS M), which suggests that here MS F is the better witness to the original text of Ibn al-Kammād (see Samsó 1997, p. 102).

The planetary apogees given in the Almanac of 1307 (Madrid, Biblioteca Nacional, MS 3349, f. 54v, and MS 17961, ff. 61r and 90v) are as follows:

| | |
|---|---|
| Saturn | Sgr 20;55° = 260;55° |
| Jupiter | Vir 22;32° = 172;32° |
| Mars | Leo 8;24° = 128;24° |
| Venus | Gem 21;15° = 81;15° |
| Mercury | Sco 14;54° = 224;54° |

In this case, the five values agree with those in the zij of al-Khwārizmī (Suter 1914, pp. 14–15, Neugebauer 1962a, pp. 41 and 99), except for Saturn, where al-Khwārizmī has Sgr 4;55°. Note that the solar apogee (see Sect. 4.2) depends instead on that of Ibn al-Kammād. Therefore, in the Almanac of 1307 the solar apogee was given a proper motion according to Ibn al-Kammād, whereas the apogees of the five planets were assigned their positions for the Hijra according to al-Khwārizmī.

The apogees in the zij of Abu l-Ḥasan ᶜAlī al-Qusanṭīnī (fourteenth century, Fez) are all slightly greater than those of Ibn al-Kammād, and the apogees of the Sun and Venus are identical (Kennedy and King 1982, p. 10):

| | |
|---|---|
| Saturn | 7s 29;43° |
| Jupiter | 5s 9;43° |
| Mars | 4s 2;13° |
| Sun | 2s 17;19° |
| Venus | 2s 17;19° |
| Mercury | 6s 18;27° (emended from 6s 18;24°) |

**Table 14** Planetary apogees in MSS M and F

| | MS M | MS F |
|---|---|---|
| Saturn | 7s 28;38,30° | 7s 28;38,1°. Noted above it: "another version has 7s 28;38,30°". |
| Jupiter | 5s 8;21, 0° | 5s 8;21,30° |
| Mars | 3s 29;41, 0° | 3s 29;41,30° |
| Venus | 2s 16;45;21° | 2s 16;45;21° |
| Mercury | 6s 18;21, 0° | 6s 18;21,30° |

**Table 15** Planetary apogees for 720 A.H. (1321) in the Tables of Barcelona and the zij of Ibn Waqār

| | Tables of Barcelona | MS W |
|---|---|---|
| Saturn | 8s 1;23° | 8s 1° |
| Jupiter | 5s 11; 3° | 5s 11° |
| Mars | 4s 2;23° | 4s 2° |
| Venus | 2s 19;25° | 2s 19° |
| Mercury | 6s 21; 3° [a] | 6s 21° |

[a] With Vatican, MS Heb. 379; Vatican, MS Heb. 356 reads 3s 21;3°, and Ripoll, MS 21, omits the entry

The values for the planetary apogees at epoch in the Tables of Barcelona (1321) are closely related to those in *al-Muqtabis*, for the differences between the corresponding values of the apogees are all about 2;42°(±0;2°): see Chabás (1996a), pp. 496–497. The values in MS W are simply rounded from the values in the Tables of Barcelona. While the significance of the parameter 2;42°(±0;2°) is unclear, it does indicate that for the compilers of these tables the planetary apogees have a proper motion (cf. Samsó and E. Millás 1998, pp. 268–269) (Table 15).

## 5 Mean motions

The headings of the tables indicate that they were intended for the meridian of Córdoba and calculated for Arabic years, months, etc. The tabulated values for the mean motions are given to seconds for the solar center, the longitude of the solar apogee, the longitude of the vernal point, and the lunar center, and to minutes for the rest of the 16 quantities considered (see Table 16).

### 5.1 Sun

MS M 28r; MS F 32v

The entries for the solar center in Table 17 do not agree with either al-Khwārizmī (Suter 1914, p. 115), or al-Battānī (Nallino 1903–1907, 2:20). Note that the entries for collected years incorporate the value of the radix. This is readily seen by checking the entry for, say, 720 years, 7s 16;13,19°, which results from adding to the radix eight times the entry for 90 expanded years, 3s 23;42,14°. This is indeed the case, although the result obtained, 7s 16;13,1°, is close but not in exact agreement (the entry in MS F, 3s 23;42,17°, gives a closer result). With these uncertainties in the table, the underlying daily mean motion of the Sun is 0;59,8,9,21,15,...°/d, and it yields a year-length of 365;15,36,34,...d, which is sidereal. This value for the mean solar motion happens to be the difference between the corresponding value attributed to Azarquiel, which is also embedded in the Toledan Tables (0;59,8,11,28,27,...°/d), and Azarquiel's value for the daily motion of the solar apogee (0;0,0,2,7,10,39,... °/d; see Toomer 1969, p. 319).

For related values for the mean motion of the Sun, see Sects. 15.1 and 16.4.

**Table 16** Mean Motions and Radices in MS M

| | Entry for 720 Arabic years (°) | Recomputed mean motion (°/d) | Radix |
|---|---|---|---|
| Sun (center) | 7s 16;13,19° | 0;59, 8, 9,21,15,... | 1s 6;35, 9° |
| Solar apogee (proper motion) | 0s 2;24,24° | 0; 0, 0, 2, 2,14, 46, ... | 0s 0; 0, 0° |
| Vernal point | 2s 8;45,37° [a] | 0; 0, 0,54,56,57,... | 0s 3;51,11° |
| Moon (center) | 10s 8;37, 1° [b] | 13;10,34,52,46,... | 4s 0;34,42° |
| Moon (anomaly) | 10s 21;12° [c] | 13; 3,53,56,19,... | 3s 18;11° |
| Double elongation | 0s 5;42° | 24;22,53,26,50,... | 0s 14;33° [d] |
| Saturn (center) | 4s 10;30° [e] | 0; 2, 0,24,24,... | 7s 26;52° [f] |
| Saturn (anomaly) * | 9s 23;49° | 0;57, 7,44,57,... | 11s 27;48° |
| Jupiter (center) | 4s 9;37° [g] | 0; 4,59, 5,31,... | 5s 21;58° |
| Jupiter (anomaly) * | 0s 14;46° | 0;54, 9, 3,37,... | 4s 23; 2° [h] |
| Mars (center) | 7s 22;53° | 0;31,26,28,47,... | 3s 1;46° |
| Mars (anomaly) * | 10s 10;46° | 0;27,41,40,34,... | 8s 22;14° |
| Venus (anomaly) | 0s 28; 3° | 0;36,59,29,21,... | 1s 15;21° |
| Lunar node ** | 9s 15;25° | 0; 3,10,46,41,... | 4s 6;30° [i] |
| Mercury (center) | 3s 14;36° | 0;59, 8,11,23,... | 9s 4;58° |
| Mercury (anomaly) | 0s 10; 2° | 3; 6,24, 7,19,... | 2s 14; 1° |

* It is uncommon to tabulate the motion in anomaly of an outer planet since the longitude of the Sun is equal to the sum of the planet's apogee, its mean center, and its mean anomaly

** The location of the lunar node between the tables for the mean motions of Venus and Mercury, in both MSS M and F, is unusual but the reason, most probably, was to maintain the format for the other planets where two quantities are listed on each page (center and anomaly). It is also unusual to have tables for the motion of Mercury's center since, in principle, they should be identical with those for the Sun

[a] MS F: 2s 8;45,38°

[b] MS F: 10s 8;37,11°

[c] MS F: 10s 20;12°

[d] MS F: 0s 34;13°. Note that the digits are the same as those in MS M but in the wrong order, which is very strange in alphanumeric notation

[e] With MS F; MS M reads 4s 9;30° but, a check of successive differences in the table, shows that the reading in MS F is correct

[f] MS F: 7s 26;53°

[g] MSS 4s 8;37°. The emendation is based on an analysis of the surrounding differences in the entries in the table

[h] MS F: 4s 23;0°

[i] MS F: 4s 5;38°

The Almanac of 1307 has a table for the daily mean motion of the Sun, Venus, and Mercury (Madrid, Biblioteca Nacional, MS 3349, f. 54r, and MS 17961, ff. 61r and 90v), where the first entry, 0;59, 8, 9, 21°/d, agrees nicely with that embedded in *al-Muqtabis*. As will be seen below, the same occurs with the mean motion of the Moon and the planets (see Sects. 5.3 and 5.4).

Among his tables, Juan Gil has one (MS G 76b–77a) for the motion of the Sun in collected years of 152(= 19 × 8) years, from 0 to 199 cycles. In the following table (MS G 77a–78b), Juan Gil gives the motion of the Sun in single years from 1 to 152, and for each day of the month in a year beginning on March 1. For cycle 0 the entry is 0s 0;0°, for cycle 1 the entry is 11s 28;30°, for cycle 2 it is 11s 27;0°, and so on. Computation from these entries yields a daily mean motion of the Sun of 0;59,8,9,26,28, ...°/d, which is also a sidereal value, very close to that embedded in *al-Muqtabis* (0;59,8,9,21,15,...°/d).

**Table 17** Mean Motion of the Sun in MS M (excerpt)

| Collected years | (s) | (°) | Expanded years | (s) | (°) | Months | (s) | (°) |
|---|---|---|---|---|---|---|---|---|
| Radix | 1 | 6;35, 9 | 1 | 11 | 18;54, 7 | Almu. | 0 | 29;34, 4 |
| 90 | 5 | 0;17,25 | 2 | 11 | 8;47,22 | Saph. | 1 | 28; 9, 1 |
| 180 | 8 | 23;19,41 | ... | | | ... | | |
| ... | | | 29 | 1 | 18;59,58 | Dilca. | 10 | 20;19,11 |
| 720 | 7 | 16;13,19 | 30 | 1 | 7;54, 5 | Dilch. | 11 | 18;54, 7 |
| 810 [a] | 11 | 9;35,35 [c] | 60 | 2 | 15;48,11 | | | |
| 900 [b] | 3 | 3;37,51 | 90 | 3 | 23;42,14 [d] | | | |

| Days | (s) | (°) | Hours | (s) | (°) | Fractions | (s) | (°) |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0;59, 8 | 1 | 0 | 0; 2,28 | Octavus | 0 | 0; 0,20 |
| 2 | 0 | 1;58,16 | 2 | 0 | 0; 4,55 | Sextus | 0 | 0; 0,25 |
| ... | | | ... | | | ... | | |
| 29 | 0 | 28;34,56 | 23 | 0 | 0;56, 3 | Medius | 0 | 0; 1,14 |
| 30 | 0 | 29;34, 4 | 24 | 0 | 0;59, 8 | Duo terc. | 0 | 0; 1,38 [e] |

[a] 910 in MS M
[b] 1000 in MS M
[c] 9;55,35 in MS F
[d] 23;42,17 in MS F
[e] 0;1,39 in MS F

In the Tables of Barcelona, we are given values for the epoch, beginning in 1321 (11s 5;48°), as well as for each day in a month in a year, for successive years and accumulations of them (Millás 1962, pp. 162–189, and Chabás 1996a, pp. 482–486). The value deduced from the entry for 600 years (11s 9;45°) is 0;59,8,9,10,13, . . . °/d, also in the tradition of *al-Muqtabis*.

In MS W 20a–20[bis]b, Ibn Waqār lists the radices of Ibn al-Kammād's lost *zīj al-Kawr* for the beginning of each year from 720 A.H. to 839 A.H. The entries in this table give the value of the mean longitude of the celestial bodies on the last day of that year. The entry for the Sun for 720 A.H. is 10s 5;29,5° and that for 780 A.H., 0s 21;29,33°. The resulting daily mean motion is 0;59,8,11,26,4,...°/d, a value which is very close to that in *al-Muqtabis*, and agrees with the value, 0;59,8,11,20,56, ...°/d, we deduce from the excess of revolution given by Ibn Waqār in MS W 57b (cf. Castells 1991, pp. 45 and 48).

## 5.2 Solar apogee

MS M 28v; MS F 33r

The solar apogee has a proper motion, and its daily mean motion derived from the tabulated value for 720 A.H. (see Table 16) is 0;0,0,2,2,14,46,...°/d. This is equivalent to a motion of 1° in about 299 Arabic years or in about 290 Julian years, a value which differs from the daily motion of the apogee used by Azarquiel (0;0,0,2,7,10,39,...°/d), corresponding to a motion of 1° in about 279 Julian years.

If we combine the value for the radix of the solar center (1s 6;35,9°) with the radix for the solar apogee (2s 16;45,21°: see above), we obtain the longitude of the Sun at epoch (i.e., the Hijra), 3s 23;20,30°. This value is close to, but not identical with, those given by Azarquiel and al-Khwārizmī (see Toomer 1968, p. 44).

The recomputations show that Ibn al-Kammād used Azarquiel's value for the mean motion of the Sun, but that he incorporated a different parameter for the mean motion of the apogee which does not appear in any extant text prior to this one.

The proper motion of the solar apogee in the Tables of Barcelona is given as a column in a table also containing the mean motion of the first point of Aries and precession (Millás 1962, p. 190, and Chabás 1996a, pp. 486–487). The radix is 0;0,0°, and the entry for 300 years is 1;2,15° in MS Vat. Heb. 356; however, it is erroneously given in MS Ripoll 21 (and in Millás's edition) as 1;0,15°. From the former entry, we compute a value of 0;0,0,2,2,42,38,... °/d, very close to that in *al-Muqtabis*.

Neither Juan Gil nor Ibn Waqār has a table for the motion of the solar apogee.

## 5.3 Moon

MS M 29r–v; MS F 33v–34r

The tables for the lunar mean motions give entries for its center, anomaly, and double elongation. The daily mean motion of the Moon in longitude resulting from the tabulated value for 720 A.H. (see Table 16) is 13;10,34,52,46,. . .°/d. This is exactly al-Khwārizmī's value (cf. Neugebauer 1962a, pp. 42, 92), and very nearly that in the Toledan Tables (cf. Toomer 1968, pp. 44, 48; Pedersen 2002, pp. 1149–1156). For the daily mean motion in anomaly, the result is 13;3,53,56,19,. . .° /d, which is very close to the corresponding value in the zij of al-Battānī (Nallino 1903–1907, 2:20), and in the Toledan Tables (Toomer 1968, pp. 44, 49; Pedersen 2002, pp. 1156–1160). In the case of double elongation, we compute a daily mean motion of 24;22,53,26,50,. . .°/d.

The Almanac of 1307 has tables for the daily mean motion of the Moon in longitude and anomaly for a period of one year, as well as a table for the lunar node at intervals of 3 days for a period of one year (Madrid, Biblioteca Nacional, MS 3349, ff. 20r, 22r, and 24r; and MS 17961, ff. 59v, 61v, and 63r). The mean motions underlying these tables are 13;10,34,52,47°/d, 13;3,53,56,4°/d, and 0;3,10,46,24°/d, respectively. These values agree, to sexagesimal thirds, with those embedded in *al-Muqtabis* (see Table 16).

In his tables for the lunar mean motions (MS G 79b–81a), Juan Gil uses cycles of 152 years for the lunar center, as was the case in his table for the solar mean motion, cycles of 116 years for lunar anomaly, and cycles of 76 years for the double elongation. The daily mean motions we deduce from the tables for these three quantities are 13;10,34,56,55,. . .°/d, 13;3,53,47,53,. . .°/d, and 24;22,53,14,42,. . .°/d, respectively; we note that these values differ somewhat from those in *al-Muqtabis*.

The recomputed parameters for these three quantities from the Tables of Barcelona are, respectively, 13;10,35,1,23,. . .°/d, 13;3,53,54,51, . . .°/d, and 24;22,53,24,26, . . .°/d, (see Chabás 1996a, p. 486), and those in the zij of Ibn Waqār (MS W 20a–

20[bis]b), are 13;10,34,52,44,. . .°/d, 13;3,53,56,56. . .°/d, and 24;22,53,21,45. . .°/d, which are quite close to one another.

## 5.4 Planets

MS M 30r–34v; MS F 34v–39r

For Saturn, Jupiter, Mars, and Mercury, the tables (arranged for the radix, collected years in units of 30 Arabic years, single Arabic years, months, days, and hours) give entries for the mean motion for both the center and the anomaly. For Venus, the entries display the mean motion in anomaly only (Venus's mean motion of center is that of the Sun), and for the lunar node, the entries are the complement in 360° of the mean motion of the ascending node in longitude, to avoid subtractions. All parameters computed from the tabulated entries for the mean motions show close, although not perfect, agreement with those derived from the Toledan Tables (see Table 16); the differences are never greater than 0;0,0,1°. The values for the radices are for the Hijra and, again, they differ from those in the zij of al-Khwārizmī and those in the Toledan Tables.

The sum of the radix for Mercury's center, 9s 4;58°, and the value for Mercury's apogee, 6s 18;21,30° (as in MS F), is 3s 23;19,30°, which is very close to the position of the Sun at epoch according to this zij, 3s 23;20,30°. The entries for the mean motions of center were computed by adding the entry for 720 Arabic years to the appropriate number of complete rotations, subtracting the radix from it and dividing the result by 720y × 354;22d, where an Arabic year is 354;22d. For example, for Saturn

$$0;2,0,24,24,...^\circ/\mathrm{d} = (130;30 + 24 \times 360 - 236;52)/(720 \times 354;22).$$

The sum of the daily mean motions of center and anomaly for each of the outer planets should equal the daily mean solar motion, 0;59,8,9,21,15,. . .°/d. For Saturn, the mean daily motion of center is 0;2,0,24,24,...°/d and the mean daily motion in anomaly is 0;57,7,44,57,...°/d. Hence, the sum for Saturn is 0;59,8,9,21,...°/d which is the value for the Sun.

In the Almanac of 1307, there are tables for the daily mean motion of the planets in longitude (Madrid, Biblioteca Nacional, MS 3349, ff. 54r, and MS 17961, ff. 61r, 90v). The entries for one day are 0;2,0,24,26°/d (Saturn), 0;4,59,5,32°/d (Jupiter), 0;31,26,28,46°/d (Mars), and 0;59,8,9,21°/d (Venus, Mercury, and Sun), all of which agree nicely with those embedded in *al-Muqtabis* (see Table 16).

The daily mean motions of the planets derived from the Tables of Barcelona differ from those in *al-Muqtabis* displayed in Table 16, in the thirds when referring to planetary longitudes, and in the fourths in the case of planetary anomalies (see Chabás 1996a, p. 486). All values for the planets derived from the zij of Ibn Waqār are also close to those used in Ibn al-Kammād's zij. However, in the zij of Ibn Waqār "epoch values according to the [lost] zij *al-Kawr* [by Ibn al- Kammād]" are given for the mean longitudes of each outer planet (as well as for the Sun and the Moon) for the end of each year from 720 A.H. to 839 A.H. together with the mean anomalies for Venus, Mercury, and the Moon at these times, that is, there is no table for radices and motions in collected years in this zij (see MS W 20a–20[bis]b). For some values in this table for the solar mean motion, see Sect. 5.1, above.

# 6 Equations

## 6.1 Sun

MS M 35r; MS F 39v

The entries in this table correspond to the solar equation in degrees, minutes, and seconds as a function of mean argument of center, which is given at 1°-intervals. The maximum value of the solar equation, which occurs at 92°, is 1;52,44°, thus differing from the more common values: 1;59° (Ḥabash al-Ḥāsib, Yaḥyā ibn Abī Manṣūr), 1;59,10° (Toledan Tables, al-Battānī), 2;14° (al-Khwārizmī), 2;23° (Ptolemy). It is therefore likely that Ibn al-Kammād accepted a parameter from Azarquiel. In both manuscripts, the heading gives the position of the solar apogee 2s 16;45,21°, for the Hijra. For an edition and recomputation of this table, see Chabás and Goldstein (1994), pp. 4–10, reprinted in Chabás and Goldstein (2015), pp. 182–190.

The Almanac of 1307 does not display a table for the solar equation, but its maximum value can be estimated from the 4-year table for the daily true solar positions for the period 1307–1310 (Madrid, Biblioteca Nacional, MS 3349, ff 16r–19v, and MS 17691, ff. 55v–59r). We note that this table displays sidereal coordinates, for the entry of the Sun in Aries occurs on March 23. To estimate the maximum solar equation, we use the table for 1308, which has fewer scribal errors than that for 1307 (in the manuscripts cited above, the entries for March–August 1307 were in fact mistakenly copied from those for the same period in 1309). The passage of the Sun through its apogee takes place around mid-June; at that time, the solar equation is zero and the true longitude of the Sun equals the longitude of the solar apogee. Since we are told that the solar apogee at the beginning of 1307 is Gem 19;6° (see Sect. 4.2) and, given its proper motion of 0;0,0,2,2,14,46°/d (see Sect. 5.2), the longitude of the solar apogee in mid-June 1308 should be 79;6,16°. In that year, the Sun reached it apogee between noon June 13 and noon June 14, when the true solar positions were 78;45,19° and 79;42,15°, respectively. Thus, the passage through apogee should have occurred at about 8;50h after noon on June 13. Now, in the two symmetric dates about 93 days before and after that time, the positions of the Sun are such that the solar equation is very close to its extremal values. The nearest dates in the table are March 12 (entry: 348;59,27°) and September 15 (entry: 169;31,22°). If we consider a daily mean solar motion of 0;59,8,9°/d (see Sect. 5.1), in 93d 8;50h (March 12, noon, to June 13, 8;50h), the Sun has progressed 92;0,34° (this is its mean center), whereas its true center is 79;6,16°–348;59,27° = 90;6,49°. The solar equation is the difference between the mean and the true centers of the Sun: 92;0,34° − 90;6,49° = 1;53,45°. Similarly for the other symmetrical point where the solar equation should reach its minimum, the Sun has progressed 92;17,49° between its passage through apogee and September 15, noon. The true center is 169;31,22° − 79;6,16° = 90;25,6°, and the solar equation is thus 90;25,6° − 92;17,49° = −1;52,43°. The values obtained for the solar equation, when close to its maximum and minimum, agree very well, although not perfectly, with that used by Ibn al-Kammād in *al-Muqtabis*, 1;52,44°, indicating that, with respect to the Sun, the Almanac of 1307 depended on Ibn al-Kammād.

In Juan Gil's table for the solar equation (MS G 79a), the argument is given at intervals of 0;30° and the entries are in degrees and minutes; the maximum value is 1;52°, truncated from Ibn al-Kammād's maximum of 1;52,44°.

The solar equation in the Tables of Barcelona is also based on a maximum of 1;52°, where the entries are only given to minutes, but the presentation is quite different from that in MSS M and F and that of Juan Gil, for the argument of the table is the mean solar longitude (rather than the mean argument of center), and thus, the entries correspond to the true solar longitude. The solar equation is obtained by subtracting the entry from the corresponding argument (Millás 1962, pp. 192–193; Chabás 1996a, pp. 489–494). In other words, let $y = e(\bar{\kappa})$ be the function for the solar equation, $e$, where the mean center, $\bar{\kappa}$, serves as the variable listed in the table in *al-Muqtabis*, and $e(\bar{\kappa})$ is negative for $0° \leq \bar{\kappa} \leq 180°$. Then, the corresponding function in the Tables of Barcelona is

$$\lambda = e(\bar{\kappa}) + \bar{\kappa} + \lambda_A = e(\bar{\kappa}) + \bar{\lambda},$$

where $\lambda$ is the true longitude of the Sun, $\bar{\lambda}$ its mean longitude, and $\lambda_A$ the longitude of the solar apogee, here taken as 79°. Note that $\bar{\kappa} = \bar{\lambda} - \lambda_A$. By using mean longitude as the argument, a value for the solar apogee has been assumed: it occurs where the entry is equal to the argument, i.e., where the difference between the mean longitude and the true longitude is zero. This occurs between longitudes 2s 19° (the corresponding entry is 2s 19;1°) and 2s 20° (the corresponding entry is 2s 19;59°), and this is consistent with the solar apogee in the heading in the Tables of Barcelona, 2s 19;25,55°.

A similar table is found in the zij of Ibn Waqār (MS W 36b). No value for the apogee is mentioned in the heading in MS W, but it can easily be determined from the table, where the equation for 2s 19° is exactly 0°; hence, this is Ibn Waqār's solar apogee, and it is very close to the value in the Tables of Barcelona. In Ibn Waqār's table for the solar equation, the maximum value is 1;53° for arguments 5s 18° to 5s 23°; that is, Ibn Waqār rounded Ibn al-Kammād's maximum of 1;52,44° to the nearest minute in contrast to the Tables of Barcelona and the zij of Juan Gil where Ibn al-Kammād's value was truncated.

The table for the solar equation in the zij of Judah ben Verga displays entries to minutes. The maximum is 1;53°, a value rounded from Ibn al-Kammād's, 1;52,44°.

The solar equation in the zij of Abu l-Ḥasan ᶜAlī al-Qusanṭīnī is in the Sindhind tradition rather than the Ptolemaic tradition: the maximum equation is 2;14°, as in the zij of al-Khwārizmī (Kennedy and King 1982, p. 11; cf. Suter 1914, p. 134). The entries in this table were computed by the "method of declinations" that probably came from Sasanian Iran (Kennedy and King 1982, p. 11):

$$e(\bar{\kappa}) = e_{\max} \times \delta(\bar{\kappa})/\varepsilon,$$

where $e(\bar{\kappa})$ is the solar equation of the mean center $\bar{\kappa}$, $e_{\max}$ is the maximum solar equation of 2;14°, $\delta(\bar{\kappa})$ is the declination of $\bar{\kappa}$, and $\varepsilon$ is the obliquity of the ecliptic (cf. Neugebauer 1962a, p. 95).

## 6.2 Moon

MS M 36r–37r; MS F 40v–41v

The five columns in this table display the argument, the equation of center, an interpolation function, the increment (a quantity introduced by Ptolemy corresponding to the difference between the equation of anomaly at quadrature (when the elongation is 90°) and the equation of anomaly at syzygy (when the elongation is either 0° or 180°), and the equation of anomaly. The Latin terms used in MS M for these four quantities, *remotio centri*, *minuta partis remotionis*, *remotio linee*, and *directio centri*, respectively, differ from the standard Latin terminology (see, e.g., the Toledan Tables: Toomer 1968, pp. 58–59; Pedersen 2002, pp. 1250–1258). As shown in Table 18, the equation of center reaches a maximum of 13;9° at 114°, and the equation of anomaly a maximum of 5;1,0° at 95°. These are values found in the zij of al-Battānī and the Toledan Tables, but we note there is no column here for the lunar latitude, in contrast to what is found in these two sets of tables. The entries for the equation of anomaly agree with those in al-Battānī's zij, but differ slightly from those in the Toledan Tables, where the maximum is 5;0,59° at 94°–95°.

The same table is found in the *Almagest* V.8, in the *Handy Tables*, and in many medieval sets of tables, such as the zij of Yaḥyā ibn Abī Manṣūr (Salam and Kennedy 1967, pp. 495–496), the zij of al-Battānī (Nallino 1903–1907, 2:78–83) and the Toledan Tables (Toomer 1968, pp. 58–59; Pedersen 2002, pp. 1250–1258).

The table for the lunar equations by Juan Gil (MS G 86b–89a) has the same entries except for the fact that, as was the case for the solar equation, the entries are given at intervals of 0;30°.

In the Tables of Barcelona (Millás 1962, pp. 191, 196–197, and Chabás 1996a, p. 495), the lunar equations are presented in two tables. One contains the quantities depending on the double elongation (equation of center and minutes of proportion), and the other those depending on the lunar anomaly (increment and equation of anomaly). It is noteworthy that in the table for the equation of center and the minutes of proportion, the two columns were transcribed erroneously (only the entries for even values of the argument were copied), so that the maximum, 13;9°, occurs at 57° (not at 114°: Table 18). Nevertheless, but for copyist's errors, all entries seem to derive from *al-Muqtabis*.

In his zij, Ibn Waqār also presents the lunar equations in two tables, one for quantities depending on the double elongation and another for those depending on the lunar anomaly. The entries in the first table agree with those in *Almagest* V.8. The second table is for the equation of anomaly (with a maximum of 5;1°) and the quantity called "increment" (with a maximum of 2;40°), and the entries also derive from *Almagest* V.8. Instead of presenting these two quantities in two columns (see Table 18), for each zodiacal sign from 0 to 5 we are here given a matrix of 30 rows (one for each integer degree of the lunar anomaly) and 7 columns (for values 0, 10, 20, ..., 60 of the minutes of proportion), and the entries in it, in minutes, are the sum of the equation of anomaly and the corresponding increment depending on the proportion. Thus, the column for 0 min of proportion displays the equation of anomaly as in Table 18, and from the column for 60 min of proportion one can derive, by subtraction, the full "increment" listed in standard tables. This format is unparalleled in any other zij.

**Table 18** Lunar Equations in MS M (excerpt)

| Argument | | Equation of center | Min. prop. | Increment | Equation of anom. |
|---|---|---|---|---|---|
| (°) | (°) | (°) | (′) | (°) | (°) |
| 1 | 359 | 0; 9 | 0 | 0; 3 | 0; 4,50 |
| 2 | 358 | 0;18 | 0 | 0; 5 | 0; 9,40 |
| ... | | | | | |
| 30 | 330 | 4;23 | 3 | 1;10 | 2;19,44 |
| ... | | | | | |
| 60 | 300 | 8;36 | 12 | 2; 3 | 4; 9, 6 |
| ... | | | | | |
| 94 | 266 | 12;20 | 28 | 2;38 | 5; 0,59 |
| 95 | 265 | 12;24 | 29 | 2;38 | 5; 1, 0 |
| 96 | 264 | 12;28 | 30 | 2;38 | 5; 0,59 |
| ... | | | | | |
| 102 | 258 | 12;48 | 33 | 2;39 | 4;58,55 |
| 103 | 257 | 12;51 | 33 | 2;40 | 4;58,11 |
| ... | | | | | |
| 109 | 251 | 13; 4 | 37 | 2;40 | 4;52, 0 |
| 110 | 250 | 13; 5 | 37 | 2;39 | 4;50,33 |
| ... | | | | | |
| 113 | 247 | 13; 8 | 39 | 2;37 | 4;45,47 |
| 114 | 246 | 13; 9 | 39 | 2;36 | 4;44, 6 |
| 115 | 245 | 13; 8 | 40 | 2;36 | 4;42,23 |
| ... | | | | | |
| 150 | 210 | 9;22 | 55 | 1;35 | 2;42,36 |
| ... | | | | | |
| 180 | 180 | 0; 0 | 60 | 0; 0 | 0; 0,30 |

The table for the lunar equation in the Almanac of 1307 differs from that in *al-Muqtabis*, for it only has one equation (for anomaly), with a maximum of 4;56°, and no equation of center. This table is characteristic of the Sindhind tradition, and its entries were computed by the "method of declinations" (described in Sect. 6.1, above). It is found in the zijes of al-Khwārizmī (cf. Suter 1914, p. 134, and Neugebauer 1962a, p. 96; Kennedy and King 1982, p. 11), Ibn Mu<sup>c</sup>ādh (Samsó [1992] 2011, p. 157), Ibn Isḥāq (Mercier 1996, p. 415), Abu l-Ḥasan <sup>c</sup>Alī al-Qusanṭīnī, and in the Latin world in the tables of John Vimond (Chabás and Goldstein 2004, p. 225), and the Parisian Alfonsine Tables, among others (Chabás and Goldstein 2012, p. 72).

## 6.3 Planets

MS M 37v–44v; MS F 42v–49r

The tables for the equations of the five planets are displayed in the same format as that for the lunar equations, except for the fact that instead of a column for the "increment"; we are given two columns, that we call subtractive and additive differ-

**Table 19** Equations of Venus in MS M (excerpt)

| Argument (°) | (°) | Equation of center (°) | Min. prop. (′) | Subtractive diff. (°) | Equation of anom. (°) | Additive diff. (°) |
|---|---|---|---|---|---|---|
| 1 | 359 | 0; 2 | 60 | 0; 0 | 0;26 | 0; 0 |
| 2 | 358 | 0; 4 | 60 | 0; 1 | 0;51 | 0; 1 |
| ... | | | | | | |
| 86 | 274 | 1;58 | 3 | 0;31 | 34;21 | 0;34 |
| 87 | 273 | 1;59 | 2 | 0;32 | 34;42 | 0;34 |
| ... | | | | | | |
| 94 | 266 | 1;59 | 5 | 0;36 | 37; 4 | 0;37 |
| 95 | 265 | 1;58 | 6 | 0;36 | 37;23 | 0;38 |
| ... | | | | | | |
| 134 | 226 | 1;28 | 42 | 1; 9 | 45;57 | 1;13 |
| 135 | 225 | 1;26 | 43 | 1;10 | 45;59 | 1;15 |
| 136 | 224 | 1;25 | 44 | 1;11 | 45;59 | 1;16 |
| 137 | 223 | 1;23 | 44 | 1;12 | 45;58 | 1;17 |
| ... | | | | | | |
| 160 | 200 | 0;42 | 57 | 1;41 | 37,12 | 1;51 |
| 161 | 199 | 0;40 | 57 | 1;42 | 36;12 | 1;52 |
| 162 | 198 | 0;38 | 57 | 1;42 | 35; 7 | 1;52 |
| 163 | 197 | 0;36 | 58 | 1;41 | 33;57 ? | 1;52 |
| 164 | 196 | 0;34 | 58 | 1;41 | 32;44 | 1;51 |
| 165 | 195 | 0;32 | 58 | 1;38 | 31;24 ? | 1;50 |
| ... | | | | | | |
| 179 | 181 | 0; 2 | 60 | 0;11 | 2;36 | 0;12 |
| 180 | 180 | 0; 0 | 60 | 0; 0 | 0; 0 | 0; 0 |

ences, to be applied to the minutes of proportion near apogee or perigee, respectively, when computing the equation of anomaly. In the computation of the longitude of a planet, one has to take into account the position of its apogee, which is the sum of its value at the epoch and the motion of the solar apogee in the time interval since the epoch.

In the tables of Ibn al-Kammād, the maximum solar equation (1;52,44°) differs from the maximum equation of center for Venus (1;59°; see Table 19). This value for Venus is not that of the *Almagest*, but follows al-Battānī and the Toledan Tables.

Except for the equation of center for Venus, the tables for the planetary equations are essentially those found in *Almagest* XI.11 and in the *Handy Tables*. The columns are displayed in the same way as in the zij of al-Battānī (Nallino 1903–1907, 2:8–137) and the Toledan Tables (Toomer 1968, pp. 60–68; Pedersen 2002, pp. 1259–1306), among many others. All we can deduce from these tables is that Ibn al-Kammād accepted the Ptolemaic tradition, as displayed in al-Battānī's zij, followed by most Muslim astronomers, and that Ibn al-Kammād's contribution was restricted to solar theory, without introducing any changes in planetary theory (see Table 20). As mentioned above, the apogees are listed in the heading of the tables for the plan-

**Table 20** Planetary Equations in MS M

| | Eq. of center | Eq. of anomaly |
|---|---|---|
| Saturn | 6;31° (90° – 94°) [a] | 6;13° (94° – 98°) |
| Jupiter | 5;15° (89° – 96°) | 11; 3° (99° – 102°) |
| Mars | 11;24° (91° – 96°) [b] | 41; 9° (130° – 132°) |
| Venus | 1;59° (88° – 94°) [c] | 45;59° (135° – 136°) |
| Mercury | 3; 2° (93° – 97°) | 22; 2° (111° – 112°) |

[a] MS F: 89° – 94°
[b] MS F: 91° – 97°
[c] MS F: 90° – 93°

etary equations and, in MS F, although not in MS M, we are also given the planetary nodes.

The planetary equations in the zij of Abu l-Ḥasan ᶜAlī al-Qusanṭīnī are in the Sindhind tradition rather than the Ptolemaic tradition. In fact, while these tables for the planetary equations are not otherwise found in extant Arabic zijes, they agree essentially with those in the Latin version of al-Khwārizmī's zij (Kennedy and King 1982, pp. 4, 11).

The parameters in the model for Venus in the Middle Ages were often borrowed from those for the Sun, notably the apogee and the eccentricity (or, equivalently, their maximum equations of center). In the *Almagest* the solar apogee is 65;30° whereas the apogee for Venus is 55° in Ptolemy's own time (*Almagest* III.4 and X.2; Toomer 1984, pp. 155 and 470). In other words, Ptolemy does not link these parameters for Venus with those for the Sun. Nevertheless, it was common practice in the Middle Ages to set the apogees of the Sun and Venus at the same longitude such that when the apogee of the Sun was changed for any reason, the apogee for Venus was changed as well, to maintain agreement. Similarly, when the maximum equation of center for the Sun was changed for any reason, the maximum equation of center for Venus was changed as well, generally without any explanation. For example, in the zij of al-Battānī the solar apogee is 82;15° (or 82;14°) and the maximum solar equation is 1;59,10° (Nallino 1903–1907, 1:72 and 2:81), while the apogee of Venus is equal to that of the Sun and the maximum equation of center for Venus is 1;59° (Nallino 1903–1907, 1:114 and 2:129). Although al-Battānī offers no explanation, al-Bīrūnī (d. 1048) gives us the historical background:

> Venus's equation with reference to the center of its equant sphere is equal, according to Ptolemy, to the solar equation, and it was in the *Zīj al-Shāh* that the corrected solar [longitude] is the corrected argument of Venus. This is impossible unless their apogees and equations are equal, and this is how they are in [the *Zīj al-Shāh*]. This opinion was transferred by [Ḥabash] to the principles of Ptolemy, so he set the apogee of Venus which is [also] the apogee of the Sun according to the moderns, and its equation for the argument, equal to those of the Sun.

Thus, according to al-Bīrūnī, the decision to make the parameters of Venus equal to those of the Sun was taken by Ḥabash al-Ḥāsib (ninth century), and he depended on the *Zīj al-Shāh*, composed in Persia ca. 450 and revised in the sixth and seventh centuries, which has models that belong to an Indian tradition and are non-Ptolemaic (Goldstein and Sawyer 1977, pp. 167–168).

In *al-Muqtabis*, the apogee of Venus (76;45,21°) is the same as that of the Sun, but the maximum equations are different, for Ibn al-Kāmmad left the maximum equation of center for Venus at 1;59° as in al-Battānī, while adopting a more recent value (1;52,44°) for the maximum solar equation of center. The same approach is found in Ibn al-Raqqām's *Mustawfī* zij, where the maximum of center for Venus is 1;51°, whereas that of the Sun is 1;49,7° (Samsó 2014, p. 318).

As was the case for the Sun and the Moon, in the tables for the planetary equations in the zij of Juan Gil (MS G 123b–138a) the argument is given at intervals of 0;30°. There are no significant discrepancies between the entries in these tables in the zij of Juan Gil and those in the zij of Ibn al-Kammād.

In the Tables of Barcelona, as was the case for the lunar equations, for each planet there are two tables: one to determine its position on the deferent and the other for its position on the epicycle (Millás 1962, pp. 200–219, and Chabás 1996a, pp. 496–499). All parameters are the same as those used by Ibn al-Kammād, but for minor differences. In the first table, the argument is no longer the mean center, but the mean longitude, which results after adding to the mean center the value for the longitude of the apogee (see Table 15). The two columns represent the equation of center and the minutes of proportion. In the second table, we are given the equation of anomaly and two corrections to adjust the equation depending on the distance of the planet to apogee or perigee (last three columns in Table 19).

The tables for the planetary equations by Ibn Waqār (MS W 39a–50a) take the same approach as the Tables of Barcelona but, as was already the case for the solar and lunar equations, they introduce new elements and some variants. In the first table for each planet, the entries for the equation of center are similar to those in the Tables of Barcelona with the same maximum values; however, the apogees have been rounded and, as a result, the entries are slightly different from those in the Tables of Barcelona. For example, the apogee of Saturn in the Tables of Barcelona is 8s 1;23°, whereas in MS W it is exactly 8s 1°. As in the Tables of Barcelona, in MS W there are also entries in the first table for the minutes of proportion (here called minutes of the equation), but they vary from 0 to 120 rather than 0 and 60. The reason is to avoid subtraction, for the values in the Tables of Barcelona are sometimes positive and sometimes negative; as a rule, the entries in MS W are 60 greater than those in the Tables of Barcelona, taking into account the negative values there. In the standard tables for the planetary equations, one enters with the mean center to find the minutes of proportion, but in MS W, instead of the mean center, one enters the first table with the mean longitude to find the minutes of proportion. In the second table for each planet, the equation of anomaly is presented in two or more columns, one for minimum values and another for maximum values (headed 0 and 120, respectively), instead of three columns as in the Tables of Barcelona, with a mean value for the equation of anomaly and two corrections, one additive and another subtractive. Since only a few values of the minutes of proportion are represented in the headings of the columns, interpolation is necessary for intermediate values. The entries in the two columns labeled 0 and 120 are clearly related to those in the Tables of Barcelona and yield the same values for the equation of anomaly. For Saturn and Jupiter, there are just the columns for 0 and 120 (representing the minimum and maximum equations of anomaly for a given value of the corrected anomaly), whereas for Mars there are columns at intervals of 12 minutes from 0 to 120 (to reduce the need for interpolation between values that are far apart); similarly, for Venus there are columns at intervals of 40 minutes from 0 to 120, and for Mercury at intervals of 24 min from 0 to 120. To interpolate for intermediate values of the minutes of proportion, that is, between the values in the headings of the columns, there is a set of interpolation tables to be applied to the preceding tables for the equation of anomaly (MS W 50b–51a). Although the presentation is new, the underlying parameters have not changed.

## 6.4 Nodes

MS F displays longitudes of the planetary nodes in the headings for the planetary equations (see Table 21); they are not found in MS M: see MS F 42r (Saturn), 44r (Jupiter), 45r (Mars), 47r (Venus), 48r (Mercury).

The values for the planetary nodes derive from Indian sources, and they are found in al-Khwārizmī's zij: Saturn 3s 13;12°, Jupiter 2s 22;1°, Mars 0s 21;54°, Venus 1s 29;27°, and Mercury 0s 21;10° (Suter 1914, pp. 15, 64–65; Neugebauer 1962a, p. 42: the only variant in MS F is for Saturn). The values in al-Khwārizmī's zij also appear in the Toledan Tables (Toomer 1968, p. 45). Al-Bīrūnī reports that these values for the planetary nodes depend on an Indian tradition (Wright (ed. and trans.) 1934, p. 105; cf. Pingree 1978, p. 568). Using the rules in Indian astronomy for determining the positions of the planetary nodes, Pingree (1968b, p. 100) computed the following values for June 16, 632, the epoch of era Yazdegird, which was used in the original (lost) version of al-Khwārizmī's zij (Neugebauer 1962a, p. 83; van Dalen 1996, p. 198): Saturn 3s 13;12°, Jupiter 2s 22;1°, Mars 0s 21;54°, Venus 1s 29;47°, and Mercury 0s 21;11°. The only variant is for Mercury where the computed value differs from the values in the medieval texts by 1 min.

These values, computed for June 16, 632, were applied by Maslama in his version of al-Khwārizmī's zij, where the epoch is the Hijra, July 15, 622 (about 10 years earlier). There is no table for the motion of the planetary nodes for, as Ibn al-Muthannā tells us in his commentary on al-Khwārizmī's zij, "the nodes of a planet deviate from their positions by only a small amount which is barely perceptible even after many years" (Goldstein 1967a, pp. 27, 153). In spite of this, some manuscripts give values which are labeled, "the 'mean motions' of the planetary nodes," but they are not real mean motions, for they are equal to the complements in 360° of the respective longitudes of the nodes (e.g., Vatican, MS Ottob. 1826, f. 38v).

The Almanac of 1307 has also a table for the planetary nodes (Madrid, Biblioteca Nacional, MS 3349, f. 54r, and MS 17961, ff. 61r and 90v):

| | | |
|---|---|---|
| Saturn | Cnc 13;13° = 103;13° | |
| Jupiter | Gem 12; 2° = 72; 2° | (al-Khwārizmī: 2s 22;1° = 82;1°) |
| Mars | Ari 21;55° = 21;55° | |
| Venus | Tau 29;40° = 59;40° | (al-Khwārizmī: 1s 29;27° = 59;27°) |
| Mercury | Ari 21;12° = 21;12° | |

Yet, in another Latin copy of the Almanac of 1307 (Paris, Bibliothèque nationale de France, MS 7403, f. 28v), the nodes for Jupiter and Venus are given as Gem 22;2° and Tau 29;49°, respectively, whereas the three others agree with the entries in the other two manuscripts. The reading for Jupiter in this copy is to be preferred, for it agrees closely with the value in al-Khwārizmī. In sum, this almanac has essentially the

**Table 21** Planetary Nodes in MS F

| | Nodes |
|---|---|
| Saturn | 3s 13;12° [a] |
| Jupiter | 2s 22; 1° |
| Mars | 0s 21;54° |
| Venus | 1s 29;27° |
| Mercury | 0s 21;10° |

[a] MS F: 6s

**Table 22** Planetary Nodes for 550 A.H. in MS M

| | (s) | (°) | (′) | (″) |
|---|---|---|---|---|
| Saturn | 3 | – | 38 | 48 |
| Jupiter | – | 0 | 21 | 18 |
| Mars | 1 | – | 41 | 38 |
| Venus | – | 18 | 45 | 39 |
| Mercury | 0 | – | 11 | 18 |

values for the planetary nodes found in the zij of al-Khwārizmī and in *al-Muqtabis*, except in the case of Venus, where some error in transmission (affecting the minutes) may have happened. There is also an increase in one or two minutes, for which we have no explanation.

Among the tables in MS M that do not belong to *al-Muqtabis*, there is one with the title, "Capita draconum planetarum in anno quingentesimo et quinquagesimo ab annis seductionis" (f. 66r). The year 550 A.H corresponds to 1155–56 A.D. Unfortunately, the table has no headings and only half of its entries are displayed (see Table 22), making it impossible to draw any conclusions.

## 7 Velocities of the Sun and Moon

MS M 51v; MS F 57v–58r

Except for copying errors, the entries in both manuscripts agree with those in al-Khwārizmī's table (Suter 1914, pp. 175–180). The entries are given at intervals of 1°, and the extremal values are as follows:

$$v_s(1^\circ) = 0;2,22^\circ/\text{h}, \text{ and } v_s(180^\circ) = 0;2,34^\circ/\text{h (with MS F; MS M: } 0;2,24^\circ/\text{h}),$$
$$v_m(1^\circ) = 0;30,12^\circ/\text{h}, \text{ and } v_m(180^\circ) = 0;35,40^\circ/\text{h}.$$

For an edition and a recomputation of the table for lunar velocity, see Chabás and Goldstein (1994), pp. 10–13, reprinted in Chabás and Goldstein (2015), pp. 190–194. Table 23 displays entries of the hourly solar velocity.

The same two tables are also found in the tables of Juan Gil (MS G 91a) and the Tables of Barcelona (Millás 1962, pp. 232–233; Chabás 1996a, p. 508), as well as in a manuscript containing the Tables of Toulouse (Paris, Bibliothèque nationale de France, MS Lat. 16658, ff. 90v–93r; on the Tables of Toulouse, see Poulle 1994), and in Ibn al-Raqqām's *Mustawfī* zij (Samsó 2014, p. 321). The Toledan Tables (Toomer 1968, p. 82; Pedersen 2002, pp. 1409–1412) and the Almanac of Azarquiel (Millás

**Table 23** Hourly Solar Velocity in MSS M and F (excerpt)

| Argument (°) | (°) | 0 ('/h) | 1 ('/h) | 2 ('/h) | 3 ('/h) | 4 ('/h) | 5 ('/h) |
|---|---|---|---|---|---|---|---|
| 1 | 29 | 2;22 | 2;23 | 2;24 | 2;28 | 2;31 | 2;33 |
| 2 | 28 | 2;22 | 2;23 | 2;24 | 2;28 | 2;31 | 2;33 |
| ... | | | | | | | |
| 10 | 20 | 2;22 | 2;24 | 2;26 | 2;29 | 2;32 | 2;33 |
| ... | | | | | | | |
| 20 | 10 | 2;23 | 2;24 | 2;27 | 2;30 | 2;32 | 2;33 |
| ... | | | | | | | |
| 30 | 0 | 2;23 | 2;25 | 2;28 | 2;31 | 2;33 | 2;34 [a] |
| | | 11 | 10 | 9 | 8 | 7 | 6 |

[a] With MS F; the last 10 entries in this column in MS M are 2;24 instead of 2;34

1943–50, p. 174) have tables for solar and lunar velocities that agree with those in al-Battānī's zij, but differ from those presented here.

# 8 Latitude

## 8.1 Moon

MS M 35v and 53r ("Tabula latitudinis lune iudicate"); MS F 40r and 60r ("Table of adjusted [*meduyyaq*] lunar latitude")

There are two tables for the lunar latitude in *al-Muqtabis*, which exemplify the two astronomical traditions that coexisted in the Iberian Peninsula during the Middle Ages. Both tables give the lunar latitude, $\beta$, as a function of the argument of lunar latitude, $\omega$; in both, the argument is given at intervals of 1°, and the entries are given in degrees and minutes. One table has a maximum of 5;0°, which is the value used by Ptolemy, al-Battānī, Azarquiel, among many others, and is the standard table in medieval astronomy. The entries in this table can be recomputed by means of the modern formula

$$\beta = \arcsin(\sin i \times \sin \omega),$$

where $i$ is the inclination of the lunar orb to the ecliptic, taken here as 5;0°.

The other table (see Table 24) is based on a maximum of 4;29°, and the latitude displayed is frequently called "adjusted," "precise," or "equated." This table is also found, with minor variants, in al-Khwārizmī's zij (Suter 1914, pp. 132–134, column 6; cf. Neugebauer 1962a, pp. 95–98), where the maximum value is 4;30°. A similar table, with the same maximum value, appears in the zij of Yaḥyā ibn Abī Manṣūr (Kennedy 1956a, p. 146). In our opinion, the fact that *al-Muqtabis* reads 4;29° instead of 4;30° does not suggest a different parameter; rather, it should be interpreted as a

**Table 24** Adjusted Lunar Latitude in MS M (excerpt)

| Argument (°) | | 0 / 8 (°) | 1 / 7 (°) | 2 / 6 [a] (°) |
|---|---|---|---|---|
| 1 | 29 | 0; 5 | 2;19 | 3;56 |
| 2 | 28 | 0; 9 | 2;22 | 3;58 |
| 3 | 27 | 0;14 | 2;27 | 4; 0 |
| ... | | | | |
| 10 | 20 | 0;47 | 2;53 | 4;13 |
| ... | | | | |
| 20 | 10 | 1;32 | 3;27 | 4;26 |
| ... | | | | |
| 28 | 2 | 2; 6 | 3;48 | 4;29 |
| 29 | 1 | 2;11 | 3;51 | 4;29 |
| 30 | 0 | 2;15 | 3;54 | 4;29 |
| | | 5 / 11 | 4 / 10 | 3 / 9 |

[a] The sequence should be: 0/6, 1/7, 2/8, as in MS F

variant reading of 4;30°. Kennedy and Ukashah (1969, pp. 95–96) have shown that the entries in this table were computed according to the "method of sines" by means of the formula:

$$\beta = 4;30 \times \sin \omega .$$

This table for the adjusted lunar latitude is of Indian origin (see Sengupta (trans.) 1934, p. 32; cf. Neugebauer 1962a, p. 98) and, together with other tables related to the theory of eclipses in the *Muqtabis* zij, it had a considerable impact on subsequent astronomers. Indeed, this table for adjusted lunar latitude is found in several other sets of tables compiled in the Iberian Peninsula.

The same table, with a maximum of 4;29°, is found in the Tables of Barcelona, where the Catalan term "endressade" (fixed, corrected) in Ripoll, MS 21, and the Hebrew term *metuqqan* (corrected)—both in the canons and the table in Vatican, MS Heb. 356, f. 62a—are used to refer to the lunar latitude (Millás 1962, p. 234; Chabás 1996a, p. 504). We note that the Tables of Barcelona display another table with a maximum latitude of 5;0°, as is also the case in *al-Muqtabis*. In contrast, the zijes of Juan Gil (MS G 89b) and Ibn Waqār (MS W 55a) only have tables for lunar latitudes with a maximum of 5;0,0°.

A table for eclipses in *al-Muqtabis* is also extant in Vatican, MS Pal. lat. 1414, headed "Tabula latitudinis lune verificate," with a maximum of 4;29° (f. 143r). Three of its entries differ from those in the corresponding table by Ibn al-Kammād. Of these, two agree with the entries in the zij of al-Khwārizmī.

The value of 4;30° for the maximum lunar latitude was also used by Abu l-Ḥasan ᶜAlī al-Qusanṭīnī (Kennedy and King 1982, p. 21) as well as by Levi ben Gerson. In the latter case, it is independent of the Indian tradition, for it is based on Levi's own observations (Goldstein 1974, pp. 132–134 and 212–217). Various authors followed

**Table 25** Latitude of the Superior Planets in MS M (excerpt)

| Argument | | Saturn North | Saturn South | Jupiter North | Jupiter South | Mars North | Mars South | Min. prop. |
|---|---|---|---|---|---|---|---|---|
| (°) | (°) | (°) | (°) | (°) | (°) | (°) | (°) | (°) |
| 6 | 354 | 2; 4 | 2; 4 | 1; 7 | 1; 5 | 0; 7 | 0; 4 | 59;36 |
| 12 | 348 | 2; 5 | 2; 4 | 1; 8 | 1; 6 | 0; 9 | 0; 4 | 58;37 |
| ... | | | | | | | | |
| 84 | 276 | 2;27 | 2;26 | 1;26 | 1;26 | 0;46 | 0;43 | 6;24 |
| 90 | 270 | 2;30 | 2;30 | 1;30 | 1;30 | 0;52 | 0;49 | 0; 0 |
| 96 | 264 | 2;33 | 2;33 | 1;33 | 1;33 | 0;59 | 0;56 | 6;24 |
| 102 | 258 | 2;36 | 2;36 | 1;36 | 1;36 | 1; 6 | 1; 4 | 12;24 |
| ... | | | | | | | | |
| 174 | 186 | 3; 2 | 3; 4 | 2; 4 | 2; 7 | 4;14 | 6;36 | 59;36 |
| 180 | 180 | 3; 2 | 3; 5 | 2; 5 | 2; 8 | 4;21 | 7; 7 | 60; 0 |

Levi in this respect, including Jacob ben David Bonjorn and Abraham Zacut. It is worth noting that Zacut's *Almanach Perpetuum* has two tables for lunar latitude, one each for maximum entries of 4;29° and 5;0°, in the tradition of *al-Muqtabis*. Both tables are also found in the *Tabule Verificate* for Salamanca with epoch January 1, 1461 (Chabás and Goldstein 2000, p. 32 and 130–131).

## 8.2 Planets

MS M 45r–v; MS F 67v–68r

The table for the latitude of the planets is presented as two sub-tables, one for Saturn, Jupiter, and Mars, and one for Venus and Mercury. The one for the latitude of the superior planets (see Table 25) is the same, but for scribal errors, as that in the *Almagest* XIII.5 and in al-Battānī (Nallino 1903–1907, 2: 140 (columns 1–3) and p. 141 (column 4)). The structure of this table differs greatly from the corresponding one in the *Handy Tables*, and in the zij of al-Khwārizmī. Toomer listed some MSS associated with the Toledan Tables that contain such a table, but concluded that it is not part of the original Toledan Tables (Toomer 1968, pp. 71–72).

In contrast to the superior planets, the latitude table for the inferior planets (see Table 26) does not conform to the pattern of the *Almagest*, the zij of al-Battānī, the zij of al-Khwārizmī, or the tables associated with the Toledan Tables. Rather, this table reproduces, with variant readings, the entries which are multiples of 6° in the *Handy Tables* (Stahlman 1959, pp. 331–334, where the entries are given at 3°-intervals). In particular, the maximum values for the mean latitude of Mercury (3;52°) agrees in both sets of tables, but those for the mean latitude of Venus differ (8;35° in MS M and 8;51° in the *Handy Tables*). However, chapter 16 (f. 10va) of the canons to *al-Muqtabis* gives 4;18° and 8;36° as the values for the maximum latitude of Mercury

**Table 26** Latitude of the Inferior Planets in MS M (excerpt)

| Argument | | Venus | | | | | Mercury | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Prop. | Max. | Mean | Min. | Prop. | Prop. | Max. | Mean | Min. | Prop. |
| (°) | (°) | (′) | (′) | (°) | (′) | (′) | (′) | (′) | (°) | (′) | (′) |
| 6 | 354 | 60 | 1 | 0;28 [a] | 1 | 60 | 60 | 11 | 1;46 | 5 | 60 |
| 12 | 348 | 59 | 1 | 0;30 | 1 | 59 | 58 | 11 | 1;47 | 5 | 59 |
| ... | | | | | | | | | | | |
| 84 | 276 | 5 | 4 | 0;58 | 4 | 6 | 25 | 18 | 2;13 | 10 | 6 |
| 90 | 270 | 2 | 4 | 1; 2 | 4 | 0 | 38 | 18 | 2;18 | 11 | 0 |
| 96 | 264 | 7 | 5 | 2;13 | 5 | 6 | 48 | 20 | 2;23 | 12 | 6 |
| 102 | 258 | 13 | 6 | 2;22 | 6 | 12 | 50 | 22 | 2;25 | 13 | 12 |
| ... | | | | | | | | | | | |
| 174 | 186 | 59 | 33 | 8;24 | 38 | 60 | 40 | 40 | 3;47 | 29 | 60 |
| 180 | 180 | 60 | 36 | 8;35 [b] | 40 | 60 | 40 | 40 | 3;52 | 30 | 60 |

[a] 1;28° in the *Handy Tables*. Moreover, all entries for argument from 6° to 90° differ from those in the *Handy Tables*, where they are 1;30° (for 12°), ...1;58° (for 84°), and 2;5° (for 90°).
[b] 8;51° in the *Handy Tables*; 8;55° in MS F

and Venus. Kennedy (1956a, p. 173) reports maximum values for Mercury (4;18°) and Venus (8;56°) and associates the *Mumtaḥan* zij and Ibn Hibintā with them.

Nevertheless, the outstanding feature here is the juxtaposition of different Ptolemaic tabular material: the *Almagest* for the superior planets, and the *Handy Tables* for the inferior planets. The source for such a mixed approach has not been determined.

We note that in the tables for planetary latitudes associated with *al-Muqtabis* no values are given for the longitudes of the planetary nodes, although MS M 66r has a list of them.

In addition, MS F 49v–54r has another table for planetary latitudes, corresponding to the one in al-Khwārizmī's zij (Suter 1914, pp. 139–167, columns 7–8). In the headings of the sub-tables for each planet, we find values for the ascending nodes: 8s 16;15° (Saturn), 9s 7;59° (Jupiter), 11s 8;6° (Mars), 10s 0;33° (Venus), and 11s 8;50° (Mercury). These nodes differ from those displayed in Table 21 and, as far as we can determine, they do not occur in any other source.

## 9 Stations

MS M 46r; MS F 54v

For each planet, we are only given four values: the positions of the first and the second stationary points for arguments of 0° and 180° (see Table 27).

The tabulated values for the same argument add up correctly to 360°. In all cases, they agree with those found in the Toledan Tables (Toomer 1968, pp. 60–68; Pedersen 2002, pp. 1265–1305), and the zij of al-Khwārizmī (Suter 1914, pp. 138–167, tables 27–56), both zijes displaying tables for each integer degree of the argument. Nearly,

**Table 27** Planetary stations in MS M

| | Saturn | Jupiter | Mars | Venus | Mercury |
|---|---|---|---|---|---|
| 1st station at apogee | 3s 22;44 | 4s 4; 5 | 5s 7;28 | 5s 15;51 | 4s 27;14 |
| 2nd station at apogee | 8s 7;16 | 7s 25;55 | 6s 22;32 | 6s 14; 9 | 7s 2;46 |
| 1st station at perigee | 3s 25;30 | 4s 7;11 | 5s 19;15 | 5s 18;21 | 4s 21;42 |
| 2nd station at perigee | 8s 4;30 | 7s 22;49 | 6s 10;45 | 6s 11;39 | 7s 5;18 |

the same values are also found in the zij of al-Battānī (Nallino 1903–1907, 2:138–139), which is of Ptolemaic origin. However, *Almagest* XII.8 and the *Handy Tables* (Stahlman 1959, pp. 335–339) have slightly different tables for the stations: in the latter, the argument was modified, as well as the interval for the calculation of the entries (6° in the *Almagest*, 3° in the *Handy Tables*): cf. Neugebauer 1975, pp. 202–206, 1005–1006.

The same table is found in Hyderabad, Andra Pradesh State Library, MS 298, where it is attributed to Ibn al-Kammād in his lost zij *al-Kawr ᶜalā l-dawr* (Mestres 1996, p. 422, and Mestres 1999, p. 53).

The tables for the planetary stations in Juan Gil's zij are on MS G 139a with the heading: Table for the stations, retrograde and direct, for the five planets on their epicycles. The entries are at 6°-intervals in two columns: from 0s 6° to 3s 0° in the first column and their complements in 6s 0° in the second column. In the case of Saturn, the first entries for first and second stations, respectively, are: 3s 22;43° for 0s 6°, and 8s 7;17° for 11s 24° (*Almagest* XII.8: 112;45° and 247;15°). But for Jupiter, the entries in MS G agree with those in the *Almagest*.

Ibn Waqār has a short table for the planetary stations (MS W 35b): there are only two values for each planet, and they agree with those in *al-Muqtabis* for first and second stations at apogee except for Mercury where MS W has 4s 24;42° and 7s 5;18°, respectively. In addition, there are lengthy tables for the planetary stations (MS W 84a–87b) that differ from those in any other set of tables known to us, but the values reported in their respective headings are those listed in *al-Muqtabis*.

MS M 58v has a completely different table for retrogradation (see Sect. 16.8).

## 10 Parallax

Parallax is defined as the difference between the zenith distance of a celestial object as seen by an observer on Earth and the zenith distance that would be observed from the center of the Earth: see Fig. 2. The maximum value of parallax takes place at the local horizon, and it is called "horizontal parallax." In general, one seeks the components of parallax in celestial longitude and latitude in order to correct the position of a celestial body at a given time for an observer at a given location. In the Middle Ages, parallax was ignored for the fixed stars because they are so far away. In fact, only solar and

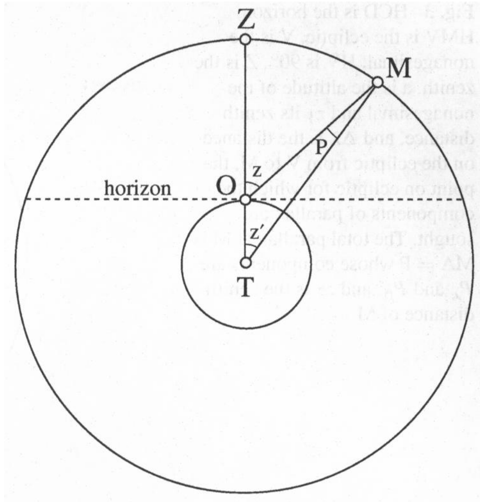


**Fig. 2** Parallax as a function of zenith distance: T is the center of the Earth, O is the observer, Z is the zenith, M is the point whose zenith distance observed from O is z where angle ZTM is z′. The total parallax P is the difference between z and z′

lunar parallax were tabulated and, for purposes of calculating the conditions of a solar eclipse, only the difference between lunar and solar parallax was considered.

The tables for parallax in longitude and latitude in Ibn al-Kammād's zij derive from an Indian tradition and are very different from those in the *Almagest* and the *Handy Tables* that were widely used in the Middle Ages: for the tradition of the *Handy Tables* see, e.g., Toomer (1968, pp.97–112). The sources for the Indian astronomical tradition concerning parallax that influenced Islamic astronomers are the *Sūrya-Siddhānta* and the *Khaṇḍakhādyaka*, as noted in Neugebauer (1962a, pp.122–124); cf. Burgess (trans.) [1860] 1935, pp. 149–151, and Sengupta (trans.) 1934, pp. 98, 104. The basic insight of this Indian tradition is that the latitudinal component of parallax is the same for all points on the ecliptic at a given moment, an approach which was not properly articulated until Kepler did so (Neugebauer 1962a, p. 124).

## 10.1 Latitude

MS M 53r; MS F 59v

This table displays the adjusted parallax in latitude, $P_\beta$, in minutes and seconds of arc, where "adjusted parallax" means the difference between the lunar and the solar parallax (see Kennedy 1965b, p. 35). Its maximum value is 0;48,32° at 90°. A similar table appears in al-Khwārizmī's zij (Suter 1914, pp. 191–192, column headed "Diversitas respectus in latitudine"), but the values tabulated there differ in all cases, e.g., the entry for 90° is 0;48,45°. This specific table was discussed by Neugebauer (1962a, pp. 121–123), who showed that this value corresponded to the definition of adjusted horizontal parallax given in the *Khaṇḍakhādyaka* (cf. Kennedy and Faris 1970, p. 26). The same table is extant in Hyderabad, Andra Pradesh State Library, MS 298 (Mestres 1996, p. 423; 1999, p. 83).

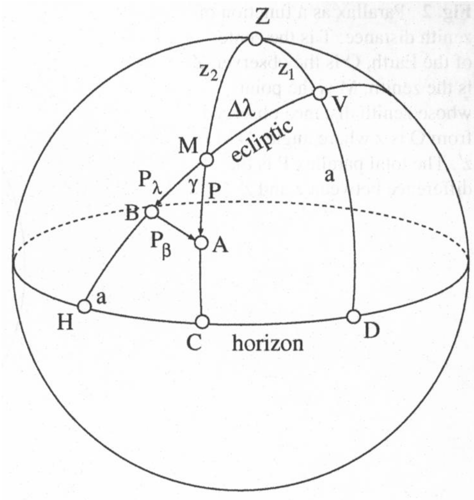


**Fig. 3** HCD is the horizon, HMV is the ecliptic, V is the nonagesimal, HV is 90°, Z is the zenith, a is the altitude of the nonagesimal and $z_1$ its zenith distance, and $\Delta\lambda$ is the distance on the ecliptic from V to M, the point on ecliptic for which the components of parallax are sought. The total parallax at M is MA = P whose components are $P_\lambda$ and $P_\beta$, and $z_2$ is the zenith distance of M

Among the eclipse tables attributed to Yaḥyā ibn Abī Manṣūr, analyzed by Kennedy and Faris (1970), there is one entitled "table for the solar latitude" *(jadwal ᶜarḍ al-shams)*, preserved in El Escorial, MS Ar. 927, ff. 10v and 71v, which agrees with the corresponding table in *al-Muqtabis*. According to these two authors (p. 26), the calculator "did an extraordinarily bad job"; a "reasonably accurate approximation" of the entries for the latitudinal component of the adjusted parallax is given by the modern formula:

$$P_\beta = 0;48,32 \times \sin z_1,$$

where $z_1$ is the zenith distance (see Fig. 3). The value 0;48,32° is to be understood as the horizontal parallax (at 90° from the zenith) and arc $z_1$ is the distance from the zenith to the nonagesimal on the ecliptic (the highest point on the ecliptic at a given moment in time), where the ecliptic is perpendicular to the altitude circle and the longitudinal component of parallax is zero. This value of $P_\beta$ then applies to all points on the ecliptic at that moment in time. We note that 0;48,32° probably derives from a Ptolemaic parameter for the obliquity of the ecliptic, for $2 \times \sin 23;51,20° = 0;48,31,55°$, rather than from an obliquity of 24°, for $2 \times \sin 24° = 0;48,48,30°$. Table 28 displays selected values from both *al-Muqtabis* and the zij of al-Khwārizmī.

An analogous table, with a maximum of 0;48,34° (instead of 0;48,32°), is found in the Tables of Barcelona (Millás 1962, p. 235; Chabás 1996a, pp. 509–510), sharing the inconsistencies of its antecedent in *al-Muqtabis*. The table in *al-Muqtabis* is also found in Vatican, Biblioteca Apostolica, MS Pal. lat. 1414, f. 142v.

As regards Ibn al-Waqār, his table for parallax for the latitude of Toledo (MS W 54a) does not follow the format used by Ibn al-Kammād in his zij; rather, it is similar to those appearing in the *Handy Tables* and the Toledan Tables, among others (see Chabás and Goldstein 2012, p. 132, Table 12.1B).

**Table 28** Parallax in Latitude in MS M and the Zij of al-Khwārizmī (excerpts)

| z (°) | MS M (′) | al-Khw. (′) | z (°) | MS M (′) | al-Khw. (′) |
|---|---|---|---|---|---|
| 1 | 0;50 [a] | 0;51 | 50 | 36; 8 | 37;21 |
| ... | | | ... | | |
| 10 | 8;23 [b] | 8;28 | 60 | 40;44 [c] | 42;13 |
| ... | | | ... | | |
| 20 | 16;41 | 16;40 | 70 | 44;43 | 45;49 |
| ... | | | ... | | |
| 30 | 23;35 | 24;22 | 80 | 47;21 | 48; 1 |
| ... | | | ... | | |
| 40 | 31;38 | 31;20 | 90 | 48;32 | 48;45 |

[a] MS F: 0;0,10°
[b] MS F: 8;27
[c] MS F: 40;54

Juan Gil has a table with the heading "Table of the degrees of parallax in longitude and latitude" (MS G 90a). The argument, "Degrees of the longitude and latitude of cities," ranges from 1° to 90° at intervals of 1°. The columns for the entries are labeled "Minutes and seconds of longitude and latitude of the Moon." The entry for 1° is 0;50′, for 10° is 8;27′, and for 90° is 48;32′, as in MSS M and F, Table 28 (despite the headings in MS G).

The canons to the tables of Judah ben Asher (d. 1391) are extant in a unique copy in a poor state of preservation in Vatican, MS Heb. 384, ff. 284a–341b (cf. Richler 2008, p. 329; see now Goldstein and Chabás 2015). In the long discussion devoted to parallax, there are references to tables for parallax in longitude and in latitude that seem to describe the two columns for parallax in the zij of al-Khwārizmī. Moreover, in Chapter 40 of these canons, two methods for computing parallax are discussed, where the second involves computation according to the Indian procedure (see Goldstein and Chabás 1999, pp. 190–191).

## 10.2 Longitude

MS M 53v; MS F 59v

This table (see Table 29) gives the longitudinal component of the adjusted parallax, $P_\lambda$, in hours and minutes, as a function of the argument, given in time from 0;15h to 9h, at intervals of 0;15h. The entries reach a maximum of 1;36h and can be easily derived from the column with the heading "Horae diversorum/diversitatis respectuum lunae [in longitudine]" in al-Khwārizmī's zij (Suter 1914, pp. 191–192; see also Neugebauer 1962a, pp. 125–126). However, in the zij of al-Khwārizmī the argument is not given in time, but in degrees, and the parallax in longitude is given to seconds. Kennedy (1965b, pp. 49–50) has shown that al-Khwārizmī's table can be computed by means of the following modern formula:

**Table 29** Parallax in Longitude in MS M

| Arg. (h) | $P_\lambda$ (h) | Arg. (h) | $P_\lambda$ (h) | Arg. (h) | $P_\lambda$ (h) |
|---|---|---|---|---|---|
| 0;15 | 0;10 | 3;15 | 1;30 | 6;15 | 1;26 |
| 0;30 | 0;19 | 3;30 | 1;33 | 6;30 | 1;24 |
| 0;45 | 0;34 | 3;45 | 1;34 | 6;45 | 1;22 |
| 1; 0 | 0;41 | 4; 0 | 1;35 | 7; 0 | 1;19 |
| 1;15 | 0;49 | 4;15 | 1;35 | 7;15 | 1;16 |
| 1;30 | 0;58 | 4;30 | 1;36 | 7;30 | 1;12 |
| 1;45 | 1; 5 | 4;45 | 1;35 | 7;45 | 1;10 |
| 2; 0 | 1;11 | 5; 0 | 1;35 | 8; 0 | 1; 6 |
| 2;15 | 1;17 | 5;15 | 1;34 | 8;15 | 1; 2 |
| 2;30 | 1;21 | 5;30 | 1;32 | 8;30 | 0;58 |
| 2;45 | 1;26 | 5;45 | 1;30 | 8;45 | 0;54 |
| 3; 0 | 1;28 | 6; 0 | 1;29 | 9; 0 | 0;50 |

$$P_\lambda = 1;36 \times \sin\theta(t),$$

where $\theta$ is the solution of the equation

$$t = \theta - (\varepsilon \times \sin\theta),$$

that is solved by an iterative process, and $\theta$, the argument (in degrees), meets the condition $0° \leq \theta \leq 150°$. The coefficient $1;36 = 0;4 \times 24 = 24/15$ includes the standard Indian value for the obliquity of the ecliptic, 24°.

Analogously, in the case of Ibn al-Kammād's table, the entries can be recomputed by means of the equation

$$P_\lambda = 1;36 \times \sin\theta(t),$$

where

$$t = \theta - (24 \times \sin\theta)/15,$$

for all $0° \leq \theta \leq 135°$. We note that $135° = 9\text{h} \times 15°/\text{h}$, and 9h is the maximum value of the argument, expressed in time.

The formula for computing the longitudinal component of parallax depends on solving a plane right triangle (as an approximation of the spherical triangle) two of whose sides are known: the hypotenuse is the total parallax at a given point on the ecliptic, and one side is the latitudinal component of latitude (already computed). The third side of this triangle is the longitudinal component that is sought: see Fig. 3.

The formula, as given in the *Khaṇḍakhādyaka*, is equivalent to:

$$P_\lambda = P_0 \sin \mathrm{a} \times \sin \Delta\lambda,$$

where $P_0$ is the horizontal parallax. As Neugebauer (1962a, p. 123) proves in detail, this formula can be derived by solving for MB ($= P_\lambda$) in right triangle ABM, using the Pythagorean theorem, where MA = P and BA = $P_\beta$:

$$P = P_0 \sin z_2$$

and

$$P_\beta = P_0 \sin z_1.$$

One also has to solve spherical right triangle MCH (Fig. 3) for angle a ($= \angle$MHC). Since ZH $= 90° =$ VH, H is the pole of arc ZVD and angle a is equal to arc $VD = 90° - z_1$. Then, in right spherical triangle MCH, HM $= 90° - \Delta\lambda$, MC $= 90° - z_2$, and

$$\sin a = \sin \mathrm{MC}/\sin \mathrm{HM} = \cos z_1 = \cos z_2/\cos \Delta\lambda.$$

Very few tables of this kind are known. In the East, there is the table of al-Khwārizmī and the one discovered by Kennedy (1965b, p. 48) in the zij of Ibn al-Shāṭir (ca. 1350), explicitly paraphrasing an early Islamic source that has not been identified (Kennedy and Faris 1970, pp. 20–21 and 33–38). In the West, besides this table in the zij of Ibn al-Kammād, we know of one in the Tables of Barcelona (Millás 1962, p. 234; Chabás 1996a, p. 508–509) and another in Vatican, MS Pal. lat. 1414, f. 143r, whose entries all agree with those in *al-Muqtabis*. An analogous table is extant in the recension of Ibn-Isḥāq's zij in the Hyderabad manuscript (Mestres 1999, p. 82).

Juan Gil has a table for parallax (MS G 90a) with the title, "Table for the lunar parallax in longitude according to increasing [lit.: higher] degrees on the ecliptic." The argument, with the heading "Degrees of the Moon's distance [in longitude] from mid-heaven" ranges from 1° to 90° at 1°-intervals. Here, "mid-heaven" refers to the nonagesimal (Neugebauer 1962a, p. 71 and 124). The entry for 1° is 0;2h, for 30° it is 0;48h, and for 90° it is 1;36h. However, despite the title and the heading, the table assumes that the horizontal parallax is 1;36h and the entries are the total parallax, P, where the argument is the zenith distance, rather than the lunar distance from the nonagesimal. The parameter 1;36h is the same as the maximum entry in the table in *al-Muqtabis* (see Table 29, above), but the structure of Juan Gil's table is different. As is clear from the entry for 30° which is half that for 90°, the table was computed by means of the formula

$$P = 1;36 \sin z_2.$$

A table that has the same maximum 1;36h (for argument 66°), with the argument in degrees from 0;30° to 120°, is found in Ibn al-Raqqām's *Mustawfī* zij (Samsó 2014, pp. 321–322).

**Table 30** Table of elongation and a half of its sixth in MS M (excerpt)

| Elongation (°) | Sun (°) | Moon (°) |
|---|---|---|
| 0;30 | 0; 2,30 | 0;32,30 |
| 1; 0 | 0; 5, 0 | 1; 5, 0 |
| 1;30 | 0; 7,30 | 1;37,30 [a] |
| ... | | |
| 6; 0 | 0;30, 0 | 6;30, 0 |
| ... | | |
| 11; 0 | 0;55, 0 | 11;55, 0 |
| 11;30 | 0;57,30 | 12;27,30 |
| 12; 0 | 1; 0, 0 [b] | 13; 0, 0 |

[a] With MS F; MS M reads 1;36,30
[b] With Vatican, MS Pal. lat. 1414, f. 142v; MSS M and F read 0;0,0

# 11 Syzygies

## 11.1 Elongation

MS M 53v; MS F 57r

In this table, the argument is the elongation between the Moon and the Sun, given in degrees and minutes, from 0;30° to 12;0°, at intervals of 0;30°. The two other columns are headed "the amount for the Sun" and "the amount for the Moon" and represent the solar longitude ($L_s = e/12$) and the lunar longitude ($L_m = 13e/12$), respectively, from which it follows that $e = L_m - L_s$: see Table 30.

Exactly the same table, without any variants, is found in Vatican, MS Pal. lat. 1414, f. 142v. A similar, but more extensive table, appears in the Tables of Barcelona (Millás 1962, p. 229; Chabás 1996a, pp. 506–507), where the elongation is given at intervals of 0;6° instead of 0;30°, and its range is from 0° to 13;12° instead of from 0° to 12;0°. No such table is found in the zij of Ibn Waqār.

MS G displays two tables, one for the Sun (91b), headed "Table for the elongation of the two luminaries and the motion of the Sun, and it is a table for its elongation and a half of a sixth of it," and one for the Moon (92a), headed "Table for the elongation of the two luminaries and the motion of the Moon, and it is a table for its elongation and half of a sixth of it." In both cases, they are double-argument tables where the vertical argument ranges from 1' to 60' and the columns are labeled 0° to 11° at 1°-intervals. The entries are in degrees and minutes; it is strange that, despite the fact that there are many more entries, the precision is only to minutes.

## 11.2 Time from mean to true syzygy

MS M 52r; MS F 58v

This is a double-argument table to determine the time from mean to true syzygy as a function of the elongation between the Sun and the Moon, $\eta$, and their relative hourly velocity, $v_m - v_s$ (Table 31). The vertical argument, the elongation, is displayed from

**Table 31** Time from mean to true syzygy in MS M (excerpt)

| [η] | $[v_m - v_s]$ 27;30 (h) | 28; 0 (h) | ... | 33; 0 (h) | 33;30 (h) |
|---|---|---|---|---|---|
| 0;30 | 1; 6 | 1; 4 | | 0;54 | 0;53 |
| 1; 0 | 2;11 | 2; 8 | | 1;49 | 1;47 |
| 1;30 | 3;17 | 4;12 | | 2;43 | 2;41 |
| ... | ... | ... | | ... | ... |
| 6; 0 | 13; 5 | 12;51 | | 10;45 [a] | 10;45 |
| ... | ... | ... | | ... | ... |
| 11;30 | 25; 6 | 24;39 | | 20;36 | 20;46 [b] |
| 12; 0 | 26;11 | 25;43 | | 21;30 | 21;50 [b] |

[a] Read 10;54

[b] These values seem to belong to the column labeled 33;0, and the values there to this column

0;30° to 12;0° at intervals of 0;30°, and the horizontal argument, the velocity in elongation, from 0;27,30°/h to 0;33,30°/h at intervals of 0;0,30°/h. The entries, given in hours and minutes, are obtained by dividing the elongation by the relative hourly velocity, but the results are rather crude. They represent the time that the Moon takes to travel the longitudinal arc between the Sun and the Moon at mean syzygy, i.e., the time interval from mean to true syzygy. For an extensive study of the approaches used by medieval astronomers to determine this time interval, see Chabás and Goldstein 1997 (reprinted in Chabás and Goldstein 2015, pp. 40–56). Although not displayed in the excerpt given in Table 31, in MS M the columns for 0;32,0°/h and 0;32,30°/h are surprisingly identical, and the second halves of the last two columns seem to be interchanged. The entries in MS F are in agreement with MS M, and the errors in common suggest that they were already in the Arabic archetype.

Table 31 is not among those found in Vatican, MS Pal. lat. 1414. The zij of Juan Gil contains a table very similar in structure and presentation, except for the fact that it displays many more entries, for the vertical argument is given at intervals of 6° from 0s 6° to 12s 0°, and the entries were computed more accurately (MS G 92b–93b).

Another similar, but not quite identical, table is found in the Tables of Barcelona (Millás 1962, pp. 230–231; Chabás 1996a, pp. 507–508). In this case, the vertical argument is given from 0;10° to 6;40°, at intervals of 0;10°, whereas the horizontal argument ranges only from 0;28°/h to 0;33,30°/h, although the same interval, 0;30°/h, is maintained. Also, here the entries for columns 0;32,0°/h and 0;32,30°/h are suspiciously alike (in fact, they are identical 35 times out of 40). The zij of Ibn Waqār addresses the issue of the time from mean to true syzygy, but the approach is slightly different. It is a two-step procedure requiring the use of two double-argument tables: in the first one, a time interval is obtained for each pair of values of the elongation and the lunar velocity, and this time interval serves as arguments in the second one, together with the solar velocity (Chabás and Goldstein 2015, pp. 44–45).

# 12 Eclipses

## 12.1 Solar eclipses

MS M 52v: "Tabula rectitudinum ad eclipses solares"; MS F 60r: Table of *samt* (lit.: Table of the zeniths [*nokhḥiyyot ha-ro'sh*] for solar eclipses; supra: Table of the zeniths [*ha-qodqodim*])

This table in 7 columns gives the declination of upper mid-heaven, i.e., the intersection above the horizon of the ecliptic and the local meridian, in degrees and minutes, as a function of the longitude of the ascendant, i.e., the intersection of the ecliptic and the eastern horizon, in degrees. For an edition of it, see Chabás and Goldstein (1994), p. 15, reprinted in Chabás and Goldstein (2015), p. 196. This table is found in the zij of Yaḥyā ibn Abī Manṣūr (Kennedy and Faris 1970, pp. 21–24) with the heading "The table of *samt* for determining solar eclipses" (*jadwal al-samt li-ᶜilm kusūf al-shams*). Note that the Arabic term *samt* means "direction," "azimuth," and "zenith." The table was compiled with a value of 23;51° for the obliquity of the ecliptic, as used by Ptolemy in the *Handy Tables*, and for a latitude of 35;55,48° (to seconds!), as indicated in the manuscript containing this table (El Escorial, Biblioteca del Monasterio, MS Ar. 927), corresponding to Yaḥyā's native Ṭabaristān, as reported by Kennedy and Faris.

In MSS M and F, the declination reaches a maximum of +23;50° at Vir 28°–Lib 2° and a minimum of −23;51° at Psc 29°–Ari 1°. It is worth noting that the geographical latitude associated with Ibn al-Kammād is 38;30° (Córdoba) and the parameter appearing in his table for solar declination is 23;33° (MS M 35v, MS F 40r).

Vatican, MS Pal. lat. 1414, f. 141v, has the same table, with minor variants: e.g., the minimum (−23;51°) occurs at Psc 30° = Ari 0°.

A similar table is found in Madrid, Biblioteca Nacional, MS 3349, f. 2v, in a fourteenth-century manuscript for the most part in Portuguese (Chabás and Goldstein 2010, pp. 200–201; see also Chabás and Goldstein 2012, p. 179) with the heading "Tavoa dos çomutes en Burgos", where the Portuguese term *çomutes* is probably a rendering of the Arabic *sumūt*, the plural of *samt*. Although the extreme values are also +23;51° at Vir 29°–Lib 1° and –23;51° at Psc 29°–Ari 1°, the rest of the entries differ. In this case, the table seems to have been compiled for an obliquity of the ecliptic of 23;51° and a geographical latitude of 42°–43°. The same table is displayed in MS G 94a, with the heading "Table of the zeniths (*ha-qodqodim*) for Burgos," where the Hebrew term *qodqodim* (heads) corresponds to the Arabic *sumūt* (the Arabic expression for "zenith" is often *samt al-ra's:* "the direction of the head"). As far as we can determine, there are no other tables of *samt* in medieval Hebrew manuscripts. Finding this table in the zij of Juan Gil of Burgos clarifies its presence in the Portuguese manuscript.

No such table is found among the Tables of Barcelona or in MS W.

**Table 32** Solar eclipses in MS M (excerpt)

| [lat.] (′) | [magn.] (digits) | $[v_m - v_s]$ | 27;30 (h) | 28;30 (h) | ... | 32;30 (h) | 33;30 (h) |
|---|---|---|---|---|---|---|---|
| 34;13 | 0;15 | | 0; 6 | 0; 6 | ... | 0; 6 | 0; 6 |
| 33;17 | 0;30 | | 0;13 | 0;12 | ... | 0;11 | 0;11 |
| 32;20 | 0;45 | | 0;19 | 0;18 | ... | 0;18 | 0;16 |
| 31;23 | 1; 0 | | 0;26 | 0;25 | ... | 0;23 | 0;22 |
| 30;28 | 1;20 | | 0;29 | 0;28 | ... | 0;27 | 0;26 |
| 29;31 | 1;40 | | 0;36 | 0;31 | ... | 0;30 | 0;29 |
| ... | | | | | | | |
| 22;57 | 4; 0 | | 0;54 | 0;51 | ... | 0;46 | 0;46 |
| ... | | | | | | | |
| 16;19 | 6; 0 | | 1; 4 | 1; 2 | ... | 0;54 | 0;53 |
| ... | | | | | | | |
| 10;46 | 8; 0 | | 1;10 | 1; 6 | ... | 1; 1 | 1; 0 |
| ... | | | | | | | |
| 1;12 | 11; 0 | | 1;17 | 1;12 | ... | 1; 5 | 1; 4 |
| 1; 1 | 11;20 | | 1;17 | 1;12 | ... | 1; 5 | 1; 4 |
| 0; 0 | 11;40 | | 1;17 | 1;12 | ... | 1; 5 | 1; 4 |

### MS M 54r: "Tabula eclipsium solarium"; MS F 60v

This second table for solar eclipses in *al-Muqtabis* is for determining the half-duration of an eclipse (Table 32). Actually, it is a combination of two tables. On the one hand, column 2 for the magnitude of the eclipse, given in linear digits and minutes, is the vertical argument of a double-argument table including all columns to the right, where the horizontal argument is the hourly relative velocity of the Sun to the Moon, from $0;27,30°/h$ to $0;33,30°/h$ at intervals of $0;1°/h$; on the other hand, columns 1 and 2 form a different table, where column 1 lists the adjusted latitude (at conjunction) of the Moon, ranging from $0;34,13°$ to $0°$. The entries in MS F in all columns agree with those in MS M, but for minor variants. As was the case for some previous tables, this one seems to derive from Yaḥyā ibn Abī Manṣūr (El Escorial, MS Ar. 927, f. 13r). Kennedy and Faris (1970, pp. 27–30) transcribed and analyzed this table. They restored the first three entries in column 2 to 0;0, 0;20, and 0;40 and recomputed the entries in the table with "only rough agreement" (p. 29) from rules described by Brahmagupta, known to early Islamic astronomers as part of the Sindhind tradition. These rules are also preserved in Ibn al-Muthannā's commentary on al-Khwārizmī's zij (Goldstein 1967a, pp. 104–108, 226–230).

The analogous table in Vatican, MS Pal. lat. 1414, f. 142r, agrees with that in Table 32 except for variant readings, and it agrees for the first three values of column 2. However, the entries in column 1 differ substantially in the seconds, for in most cases they read 20 or 48.

In MS G 96b–97a, there is a similar table for solar eclipses arranged in the same way (with columns labeled from 27;30 to 33;30), but the rows in the column for the magnitude in digits are labeled at intervals of 0;20 instead of 0;15, as in Ibn Kammād's zij. Moreover, the entries are different where the argument is the same.

Table 33 Lunar eclipses in MS M

| Arg. lat. | Magnitude | Duration | Totality |
|---|---|---|---|
| 12 | 0;55 | 0; 6, 0 | 0; 0, 0 |
| 11 | 1;35 | 0;14, 0 | 0; 0, 0 |
| 10 | 3;50 | 0;50,40 | 0; 0, 0 |
| 9 | 4;16 | 1; 6,50 | 0; 0, 0 |
| 8 | 6;47 | 1;43, 9 | 0; 0, 0 |
| 7 | 8;19 | 1;57,50 | 0; 0, 0 |
| 6 | 10;29 | 2; 0, 9 | 0; 0, 0 [a] |
| 5 | 12; 0 | 2; 0,50 | 0; 0,48 |
| 4 | 12; 0 | 2; 0, 0 | 1; 2,40 [b] |
| 3 | 12; 0 | 2;20, 0 | 1; 6,48 |
| 2 | 12; 0 | 2;25, 0 | 1; 8,48 |
| 1 | 12; 0 | 2;30, 0 | 1;10,12 |

[a] MS F: 0;0,40
[b] MS F: 0;0,40

No such table is found among the Tables of Barcelona or in MS W. The two tables for solar eclipses in Ibn Isḥāq's zij differ from those presented here and were taken from Ibn al-Kammād's *al-Kawr* (Mestres 1996, p. 423; 1999, p. 103).

## 12.2 Lunar Eclipses

MS M 52v; MS F 61r

This table in 4 columns (see Table 33) displays the magnitude of a lunar eclipse, in digits and minutes of a digit (column 2, here called *prima porta*); the duration of the eclipse, in hours, minutes, and seconds (column 3, here *secunda porta*); and the duration of totality, also in hours, minutes, and seconds (column 4, *tercia porta*), as functions of the argument of latitude of the Moon, given in integer degrees.

It is noteworthy that in this zij there is only a single table for lunar eclipses, whereas other zijes display two distinct tables, one for greatest and the other for least distances. Similarly, in Vatican, MS Pal. lat. 1414, f. 143r, we find the same table as in MS M; and so it is in the Tables of Barcelona (Millás 1962, p. 238, and Chabás 1996a, p. 512), both with variants.

The table in MS G 97a has the same format, but the argument is given at intervals of 0;30° and the entries, very faint in the copy of the manuscript available to us, seem to be different.

Among the material in MS M that comes after the tables associated with *al-Muqtabis*, is a table of the same type (f. 57v), with the title: "Hec tabula est quam extraxit et composuit Alkemed in eclipsibus lunaris in canone suo que est extracta a canone Ebi Iusufi cognoscitur Byn Tarach, que est ualde uerax" (see Table 34). This author is probably the late eighth-century astronomer Yaᶜqūb ibn Ṭāriq, a collaborator of al-Fazārī in Baghdad, whose lost zij was based on the Sindhind tradition (cf. Pingree 1968b and 1970). Comparison between Tables 33 and 34 shows that many of

**Table 34** Lunar eclipses according to Ya$^{c}$qūb ibn Ṭāriq in MS M

| Arg. lat. | Magni-tude | Duration | Totality |
|---|---|---|---|
| 12 | 0;54 | 0; 6, 0 | 0; 0, 0 |
| 11 | 1;35 | 0;54, 6 | 0; 0, 0 |
| 10 | 3;50 | 1;50,40 | 0; 0, 0 |
| 9 | 4;56 | 2; 6,50 | 0; 0, 0 |
| 8 | 6;47 | 2; 2, 9 | 0; 0, 0 |
| 7 | 8;39 | 2;36,50 | 0; 0, 0 |
| 6 | 10;29 | 3; 0,20 | 0; 0, 0 |
| 5 | 11; 0 | 3; 0,50 | 0; 0,40 |
| 4 | 12; 0 | 3; 0, 0 | 0; 2,40 |
| 3 | 12; 0 | 4;20, 0 | 1; 6,48 |
| 2 | 12; 0 | 4;40, 0 | 1; 8,43 |
| 1 | 12; 0 | 4;50, 0 | 1;10,42 |

the entries are corrupt; see especially those for the duration of the eclipse, with close similarities in the minutes and seconds, but significant disagreements in the hours.

Ibn Isḥāq's zij contains two sets of tables for lunar eclipses taken from Ibn al-Kammād's *al-Kawr* (Mestres 1996, p. 422; Mestres 1999, pp. 110–111). For another table of lunar eclipses in MS M, see Sect. 16.3, below.

## 12.3 Colors of eclipses

MS M 52v: "*Colores*"; MS F 61r: Colors of eclipses

The discussion of colors of eclipses derives from Indian astronomy, and tables for them appear in some medieval astronomical texts: for the Indian tradition, see, e.g., *Sūrya-Siddhānta* vi.23, in Burgess (trans.) [1860] 1935, p. 185; cf. Petri (1968). The table in the zij of Ibn al-Kammād is presented in 6 columns. The first four correspond strictly to a table of colors for both solar and lunar eclipses: Column 1 displays the argument of lunar latitude, in degrees; column 2 gives the color of solar eclipses as a function of it; column 3 displays lunar latitude, in minutes; and column 4 gives the color of lunar eclipses in terms of lunar latitude. For a reproduction and discussion of this part of the table, see Chabás and Goldstein (1994), pp. 18–19, Goldstein (2005), and Chabás and Goldstein (2012), pp. 176–179. Columns 5 and 6 concern the areas of solar eclipses and are usually found in a separate table, displaying the magnitudes of solar eclipses in area digits as a function of the magnitude of the eclipse in linear digits (see Table 35).

As far as we can determine, there is no other medieval text where the information is presented in a single table of six columns. Indeed, these two topics correspond to different traditions. The table for the magnitude of solar eclipses is already found, with variants, together with an extra column for the magnitude of lunar eclipses, in Ptolemy's *Almagest* VI.8 (Toomer 1984, p. 308) and his *Handy Tables* (Stahlman

**Table 35** Magnitude of solar eclipses in MS M

| *Digiti circuitus* | *Digiti corporis solis* |
|---|---|
| 1 | 0;20 |
| 2 | 1; 0 |
| 3 | 1;45 |
| 4 | 2;40 |
| 5 | 3;40 |
| 6 | 4;40 |
| 7 | 5; 0 |
| 8 | 7;50 |
| 9 | 8;20 |
| 10 | 9;40 |
| 11 | 10;51 |
| 12 | 12; 0 |

1959, p. 258), in al-Khwārizmī's zij (Suter 1914, p. 190, columns 6–7), al-Battānī's zij (Nallino 1903–1907, 2: 89), the Toledan Tables (Toomer 1968, p. 113; F. S. Pedersen 2004, pp. 1448–1452), and the Almanac of Azarquiel (Millás 1943–50, p. 233), to name but a few.

The zij of Ibn al-Bannā' includes a similar list, restricted to the colors of the two luminaries during eclipses, and a separate table for the digits of eclipse (Madrid, Museo Naval, MS Arabic (unnumbered), f. 37a; cf. Vernet 1951, p. 134). The corresponding list for the colors of eclipses in the zij of Abu l-Ḥasan ᶜAlī al-Qusanṭīnī is essentially the same as that in the zij of Ibn al-Bannā' (Kennedy and King 1982, p. 21). The Tables of Barcelona present the information on colors and magnitudes of eclipses given by Ibn al-Kammād, but displayed in two separate tables (Millás 1962: cols. 1–4 in table 51, p. 238, and cols. 5–6 in table 48, p. 228; see, e.g., Ripoll, MS 21, ff. 148r and 147v, respectively). In Vatican, MS Pal. lat. 1414, f. 143r, there is only the table for the colors of eclipses. The zij of Ibn Isḥāq also has two sub-tables for the colors of lunar and solar eclipses (Mestres 1996, p. 419). For an outstanding example of the persistence of the Indian tradition on colors of eclipses, see the table for this purpose included in the *editio princeps* of the Parisian Alfonsine Tables (Ratdolt 1483, f. 18v), soon followed by the second edition (Venice 1492, f. k2v) and the third edition (Venice 1518, f. 116v). It should also be kept in mind that the *Libro de las Taulas Alfonsies*, explaining the use of the Castilian Alfonsine Tables, has a chapter describing such a table, with the title "De qué color sera ell eclipsy": see Chabás and Goldstein 2003, pp. 70, 196–199.

For the related Hebrew tradition of colors of eclipses, see Goldstein (2005), where editions of the tables by Juan Gil and Isaac al-Ḥadib, among others, are presented.

## 13 Fixed stars

MS M 47r; MS F 61v–62r

This is a list of 30 stars, and it displays the following information for each star: magnitude, name, ecliptic coordinates (longitude and latitude), and the planets associated with it (for astrological purposes). Kunitzsch (1966), pp. 99–102, edited this list under his type XV based on the only two known Latin manuscripts containing it: MS M and Vienna, Nationalbibliothek, MS 5311, f. 129v. The ecliptic longitude of each star is derived from that in Ptolemy's star catalogue by adding 6;38°, thus indicating that the epoch of this star list is the Hijra. A third such list has recently been identified in Paris, Bibliothèque nationale de France, MS 7324, but only the names of the stars and the associated planets are displayed, not the coordinates (Chabás 2015). Besides these three Latin manuscripts, Ibn al-Kammād's star list is preserved in at least four Arabic manuscripts and seven Hebrew manuscripts (see Goldstein and Chabás (1996); reprinted in Chabás and Goldstein (2015), pp. 373–388, with an edition of the list in Arabic in MS Vatican Heb. 379, f. 195b, and in Hebrew in MS Vatican Heb. 356, f. 65b).

MS F has only 29 stars, not 30: most of the names are in Hebrew, but some are in Arabic in Hebrew characters. In contrast to MS M which lists two stars in Pisces, MS F has only one star in that zodiacal sign. The Hebrew terms are not the same as those in MS Vatican Heb. 356, that is, Solomon Franco's translation into Hebrew of the star names in Arabic is an independent witness of this tradition. Moreover, the stars are not in the same order as in MS M, and there are variants for the numerical entries.

MS G 149a has a list of 32 stars: in addition to the name, longitude, latitude, and associated planets, there are columns headed "degrees with which they cross the 9th sphere," "degrees with which they cross the meridian," "distance from the equator," and "direction with respect to the equator" (i.e., north or south): cf. Chabás and Goldstein 2015, pp. 378–379, where this manuscript was designated "J."

MS W 59a has the same list of 30 stars (as in Ibn al-Kammād), with their longitudes and latitudes.

The Tables of Barcelona have no specific list of stars. Nevertheless, the fact that some of the Hebrew manuscripts displaying Ibn al-Kammād's list also contain copies of the Tables of Barcelona, and the fact that in Paris, Bibliothèque nationale de France, MS 7324, this list follows another one explicitly associated with the astronomers who worked on the compilation of the Tables of Barcelona, lead us to think that Ibn al-Kammād's star list was included in this set of tables.

Zacut's *Ḥibbur* has a list of 61 stars, whose longitudes differ by 6;38° from those stars listed in Ptolemy's *Almagest*, and is thus intended for the Hijra, as is also the case for Ibn al-Kammād's star list. For an excerpt of Zacut's list, see Chabás and Goldstein (2000), p. 71. The *Almanach Perpetuum* based on Zacut's tables has a list of coordinates of 56 stars, very similar to that in the *Ḥibbur*, but here the names are missing, making the list unusable (see Chabás and Goldstein 2000, pp. 145–150).

## 14 Geographical list

MS M 54v; MS F 62v

This table consists of a list of 30 geographical places both in MS M and in MS F, for each of which we are given its longitude and latitude, in degrees and minutes (see

**Table 36** Geographical coordinates in MS M

| Climate | Place | Long. (°) | Lat. (°) | Climate | Place | Long. (°) | Lat. (°) |
|---|---|---|---|---|---|---|---|
| 5 | *Senterin* | 23;40 | 40;15 | 4 | *Tripoli Africana* | 41;40 | 32; 0 |
| 4 | *Tange* | 24;10 | 35;15 | 5 | *Sicilia* | 45;20 | 37;30 |
| 4 | *Fes* | 24; 0 | 33; 0 | 4 | *Alexandria* | 63; 0 | 31; 0 |
| 4 | *Sebte* | 25;40 | 35;20 | 3 | *Egyptum* | 64;50 | 29;55 |
| 5 | *Sibilia* | 25;40 | 37;15 | 4 | *Askalona* | 65; 0 | 33; 0 |
| 5 | *Maleca* | 26;22 | 37; 0 | 4 | *Filisten* | 66;15 | 32;30 |
| 5 | *Corduba* | 27; 0 | 38;30 | 4 | *Jerusalem* | 69; 5 | 38;40 |
| 5 | *Granata* | 27;30 | 37;30 | 5 | *Chimis* | 69;35 | 34;10 |
| 5 | *Almerie* | 28; 0 | 36;30 | 6 | *Antiochia* | 72; 0 | 21;30 |
| 5 | *Toletta* | 28; 0 | 40; 0 | 2 | *Aliememe* | 72; 0 | 21;30 |
| 5 | *Mursie* | 29;34 | 37;30 | 5 | *Ruccat elbaida* | 73;15 | 36; 0 |
| 3 | *Sarcusta* | 30;55 | 41;30 | 3 | *Ithrib* | 75; 0 | 25; 0 |
| 5 | *Balensia* | 30;20 | 37;20 | 2 | *Macca* | 77; 0 | 21;40 |
| 4 | *Cartagen*(?) | 33;45 | 32; 0 | 4 | *Alcufe* | 79:30 | 31;50 |
| 4 | *Africa* | 36;31 | 33; 0 | 4 | *Baldacca* | 80; 0 | 33; 9 |

Table 36). The list in MS F is very similar to that in MS M, but has only 24 places in common with MS M; there are also differences in the order of the geographical places, their coordinates, and the spelling of their names. In the margin of MS F, Juan Gil is mentioned together with two additional places: Burgos, with coordinates 27° and 42°, and Valencia, for which only the latitude, 39°, is given. Note that this city already appears in MS M, with different latitude.

The list in MS M was edited in Laguarda (1990), p. 103. The longitudes of the cities differ from those in the Toledan Tables, where the shore of the Western Ocean seems to have been used as the meridian of reference for most longitudes (see Toomer 1968, p. 136). However, the prime meridian in this list is located about 17;30° west of the shore of the Western Ocean and was called "the meridian of water" for it fell in the middle of the ocean (Laguarda 1990, p. 75; Comes 1994). For a general discussion of the prime meridian in Islamic sources, see Kennedy and Kennedy (1987), p. xi. As an example, let us consider the case of Córdoba: the entry for its longitude is 27;0° and that for its latitude 38;30°. The same values for the coordinates of Córdoba are found in some other Islamic sources, notably in a work by Abu l-Ḥasan ᶜAlī al-Marrākushī (see Kennedy and Kennedy 1987, p. 95). By contrast, in the Toledan Tables, the longitude of Córdoba is given as 9;20° and its latitude as 38;30° (Toomer 1968, p. 134; Pedersen 2002, p. 1512). The difference in longitude is 17;40°. Similarly, the coordinates for Toledo in Ibn al-Kammād's list are 28;0° and 40;0°, whereas in the Toledan Tables they are given as 11;0° and 40;0°. In this case, the difference in longitude is 17°.

It is worth noting that the meridian of water was also used in Hyderabad, Andra Pradesh State Library, MS 298: Ibn Isḥāq's original set of radices was computed for the longitude of Toledo, given as 28°, but the anonymous compiler of the zij computed new radices for Tunis, whose longitude is given as 41;45° (Mestres 1999, p. 26).

**Table 37** Modern names of the geographical places

| MS M | Modern names | Diff. | MS M | Modern names | Diff. |
|---|---|---|---|---|---|
| *Senterin* | **Santarém, Portugal** | | *Tripoli Africana* | Tripoli, Libya | 1;40° |
| *Tange* | Tangier, Morocco | 17;40° | *Sicilia* | Sicily, Italy | 9;20° |
| *Fes* | Fes, Morocco | | *Alexandria* | Alexandria, Egypt | 11;40° |
| *Sebte* | Ceuta, Spain | 17;40° | *Egyptum* | Egypt | 9;50° |
| *Sibilia* | Sevilla, Spain | | *Askalona* | Asqalan, Ashkelon | 9;20° |
| *Maleca* | **Málaga, Spain** | | *Filisten* | Palestine | |
| *Corduba* | **Córdoba, Spain** | 17;40° | *Jerusalem* | Jerusalem | 13; 5° |
| *Granata* | Granada, Spain | | *Chimis* | Damascus, Syria | 9;35° |
| *Almerie* | **Almería, Spain** | | *Antiochia* | Antiochia | |
| *Toletta* | Toledo, Spain | 17; 0° | *Aliememe* | Carmel | 16;20° |
| *Mursie* | Murcia, Spain | | *Ruccat elbaida* | Raqqah, Syria | –0;18° |
| *Sarcusta* | Zaragoza, Spain | | *Ithrib* | Medina, Saudi Arabia | 9;40° |
| *Balensia* | Valencia, Spain | | *Macca* | Mecca, Saudi Arabia | 10; 0° |
| *Cartagen(?)* | Cartagena, Spain | | *Alcufe* | Kufa, Iraq | 10; 0° |
| *Africa* | Africa | | *Baldacca* | Baghdad, Iraq | 10; 0° |

Table 37 displays the modern names of the places in Table 36. In the 17 cases where these places are also found in the standard table for geographical coordinates (62 items) in the Toledan Tables (see Toomer 1968, pp. 134–135, and Pedersen 2002, pp. 1512–1513), we present the difference between the longitude given in the Toledan Tables and that in Ibn al-Kammād's list. Note that Antioquia and Aliememe (Carmel) are given the same coordinates, but located in different climates.

A glance at the names shows that Ibn al-Kammād's list is much more oriented to al-Andalus than that in the Toledan Tables, and that faraway places like *Insula Tule* in the Arctic Ocean and *Albeyt* in Tibet do not appear in it. While keeping to the tradition, Ibn al-Kammād probably tried to make this list more useful to his fellow astronomers in al-Andalus. We also note that, contrary to the list in the Toledan Tables, Ibn al-Kammād's list in MS M is organized according to increasing longitudes, which is not the case for the last 8 entries in MS F.

MS G 148a displays a list of 45 geographical places, many of which agree with those in MS M (especially at the beginning of the list). Some places have been added, including (among others) Burgos, Paris, Tunis, Rome, and Armenia. MS W 59a has a list of 30 places, but it seems to be unrelated to the list in Ibn Kammād. Among the places listed in MS W are Burgos and Lisbon.

The Tables of Barcelona include a list of 29 geographical places, partially derived from that of Ibn al-Kammād: eleven cities are given in the same order and with the same coordinates, but for variant readings (Millás 1962, p. 238, and Chabás 1996a, pp. 515–516). For the rest, while retaining cities such as Alexandria, Jerusalem, and Mecca, the author included Barcelona and other places close to it, such as Girona, Montpellier, Majorca, and Minorca.

## 15 Astrology

### 15.1 Excess of revolution

MS M 54v; MS F 61r

The "excess of revolution" is the difference between a tropical or sidereal year and 365d. As can be seen from the entries for one year in Tables 38 and 39, this quantity exceeds 6 hours or 90 time degrees (such that 1h = 15°), indicating that the entries are sidereal and refer to a sidereal year. This quantity was used in an astrological context to determine the ascendant of the anniversary of someone's nativity.

MS M has two sub-tables with the heading "Reuoluciones annorum mundi et natiuitatum." The entries in one are given as an angle, in degrees and minutes, and in the other as time, in hours and minutes. MS F has both sub-tables: the first one is headed "Table for the cycles of the years of the nativities and the years of the world (supra: and some call it the table for the ascensional cycles)," and its entries are expressed in degrees, minutes, and seconds. The second sub-table in MS F is headed "excess of the *peraṭim*," where the usage of *peraṭim* (sing. *peraṭ*) is unusual: it generally means "details," but elsewhere in this manuscript *peraṭ* has the meaning of "era" or "calendar" (see Sect. 1, above; cf. Goldstein (2013), p. 176). The meaning in this context is unclear. Tables 38 and 39 show one sub-table from each manuscript. In the first sub-table, extant in both MSS but with greater precision in MS F, the entry for 100 years (or for 1 year) results in a year-length of 365;15,36,0,30d = 365d 6;14,24,12h. In the second sub-table, the entry for 100 years implies a length of the solar year of 365;15,35,58,30 days, a value close, but not equal, to the parameter derived from the previous sub-table.

The table for the excess of revolution in the zij of Ibn al-Kammād differs from that in al-Khwārizmī (Suter 1914, p. 230), where the entries for 1 year are 93;2;15° and 6;12h. However, in MS M 66r, we find a copy, but for scribal errors, of al-Khwārizmī's table, with the title, *Residuum ascensionum ad reuoluciones annorum solarium secundum Muhad Arcadius*. Millás (1942, p. 246) identified Muhad Arcadius with Ibn Muᶜādh al-Jayyānī, from Jaén (11th century). The entries in this table correspond to a sidereal year of 365;15,30,22 days, a parameter already found in an Indian astronomical text, Brahmagupta's *Brahmasphuṭasiddhānta* (ca. 625 A.D.): Pingree (1968b), p. 99, and Kennedy (1958), p. 261; see also Neugebauer (1962a), p. 131, and Goldstein (1967a), p. 242.

According to Mestres (1999, p. 15, n. 2), Hyderabad, Andra Pradesh State Library, MS 298, preserves the table for the excess of revolution in Ibn al-Kammād's first zij, *al-Amad ᶜalā l-Abad*, and it yields a year-length of 365;15,23,30 days.

The zij of Joseph of Ibn Waqār gives the excess of revolution only as an angle, to minutes (MS W 57b), and the relevant parameter is 92;36°, the same as in MS M.

**Table 38** Excess of revolution in MS F

| Years | Angle (°) | Years | Angle (°) |
|---|---|---|---|
| 10 | 216; 0,30 | 1 | 93;36, 3 [a] |
| 20 | 72; 1, 0 | 2 | 187;12, 6 |
| 30 | 288; 1,30 | 3 | 280;48, 9 |
| 40 | 144; 2, 0 | 4 | 14;24,12 |
| 50 | 0; 2,30 | 5 | 108; 0,15 |
| 60 | 216; 3, 0 | 6 | 201;36,18 |
| 70 | 72; 3,30 | 7 | 295;12,21 [b] |
| 80 | 288; 4, 0 | 8 | 28;48,24 |
| 90 | 144; 4,30 | 9 | 122;24,27 |
| 100 | 0; 5, 0 | 10 | 216; 0,30 |

[a] MS M: 92;36
[b] MS M: 294;12

**Table 39** Excess of revolution in MS M

| Years | Time (h) | Years | Time (h) |
|---|---|---|---|
| 10 | 14;23 | 1 | 6;14 |
| 20 | 4;47 | 2 | 12;28 |
| 30 | 19;11 | 3 | 18;43 |
| 40 | 9;35 | 4 | 0;57 |
| 50 | 23;59 | 5 | 7;12 |
| 60 | 14;23 | 6 | 13;26 |
| 70 | 4;47 | 7 | 19;40 |
| 80 | 19;11 | 8 | 1;55 |
| 90 | 9;35 | 9 | 8; 9 |
| 100 | 23;58 | 10 | 14;23 [a] |

[a] MS F: 14;24

However, the rest of the entries indicate that this is an error for 92;26° (e.g., the entry for 10 years is 204;16°). Moreover, in El Escorial, MS Árabe 873, copied by, among others, Judah Ibn Waqār (a member of Joseph's family), in part in Toledo in 1379 and in part in Guadalajara in 1387 and 1388, we find the same table, given to seconds. In this case, the entry for one year is 92;25,46° and that for 10 years 204;15,40° (see Castells 1991, pp. 44–48). Thus, Joseph's table seems to be a rounding of Judah's, with no direct relation to the table by Ibn al-Kammād.

In the Tables of Barcelona (Millás 1962; p. 238), the entries for one year (6;1h and 90;15°) also seem to be unrelated to those in the zij of Ibn al-Kammād.

## 15.2 Ruling powers of the planets

MS M 65r; MS F 63r

The interpretation of a horoscope depends for the most part on three astrological systems: (1) dignities, (2) astrological houses, and (3) aspects. The dignities are assigned to fixed points or intervals on the zodiac, and there are five categories: domiciles, exaltations, triplicities, terms, and faces. Opposite the domicile is the detriment, and opposite the exaltation is the dejection (or fall). Astrological houses are tied to the local horizon, and they constitute a 12-fold division of the zodiac, beginning with the ascendant (the intersection of the eastern horizon with the ecliptic at a given time). Aspects refer to the relative positions of the planets at a given time. Ibn al-Kammād has a table for dignities, but in his tables he does not address the other two systems. According to Kūshyār Ibn Labbān (tenth century: see Yano (ed. and trans.) 1997, pp. 54–55), the five categories of dignities are "essential" (*dhātīya*), whereas the astrological houses and aspects are "accidental" (*$^c$araḍīya*), in so far as they depend on positions that vary over time (cf. Lilly 1647, pp. 101–104). To be sure, in an astronomical context *dhātī* (*lit.* essential) means "sidereal" in contrast to *ṭabī$^c$ī* (*lit.* natural) which means "tropical": see Samsó (2007), essay 9, pp. 107–108. But for the tables discussed in this section, the context is astrological.

MS M has four sub-tables, and the information they provide generally agrees with that in the zij of al-Khwārizmī (Suter 1914, p. 231; Neugebauer 1962a, p. 132), but comparison is made difficult by the fact that in MS M every other entry was copied (in red) while the rest of them, probably intended to be copied in blue ink, were left blank.

The first sub-table is for the dignities of the planets in the zodiacal signs. The table consists of five columns and seven rows. The first column displays the seven planets (in the Middle Ages, this included the five planets visible to the naked eye, the Sun, and the Moon). The other four columns are for the domiciles, the exaltations, the detriments, and the dejections (here called *casus*). This sub-table has a different presentation from that in the zij of al-Khwārizmī.

The second sub-table in MS M displays the lords of the triplicities, both diurnal and nocturnal. The table consists of seven columns and four rows. The first column displays four groups of three signs: Aries, Leo, and Sagittarius; Taurus, Virgo, and Capricorn; Gemini, Libra, and Aquarius; and Cancer, Scorpio, and Pisces. Each group of three signs is associated with three planets for daytime and three others for nighttime.

The third sub-table is for the faces. The table consists of four columns, of which the first lists the 12 zodiacal signs. Each of them is divided into three equal parts, called "faces" or "decans," according to a well-established descending order: Mars, Sun, and Venus for Aries, then Mercury, Moon, and Saturn for Taurus, then Jupiter, Mars, and so on, until the last third of Pisces (Mars). For a transcription, see Pedersen (2002), p. 1593.

The fourth sub-table presents the "Egyptian terms" (which are called "Persian" in al-Khwārizmī's zij). The table consists of six columns, of which the first lists the 12 zodiacal signs. The principle is the following: each sign is divided into 5 unequal parts, called "terms," and each term is assigned to one of the 5 planets and is given an integer number between 2 to 12, in such a way that the sum of these numbers for each zodiacal sign must equal 30°. In *Tetrabiblos* I.20–21, Ptolemy lists terms in two systems, one associated with the Egyptians and another introduced by Ptolemy in his own name. For a transcription, see Robbins 1940, p. 97, and Pedersen (2002), p. 1594.

MS F 63r displays a single table, entitled "Table for the essential lordship of the planets in the zodiac." The Hebrew term corresponding to "essential" (*$^c$aṣmit*) suggests that the underlying Arabic term is *dhātīya* (or, *dhātī* if the noun it modified in Arabic was masculine): for a similar construction in Arabic, see the discussion of MS W 58a, below. The term "essential" here refers to the fact that the lords of the dignities are associated with specific points or intervals on the zodiac that are independent of time. For discussion of the astrological dignities in a medieval Hebrew text, see Sela 2013, pp. 91, 101, 222–223, and 239.

The entries in the table in MS F 63r are mostly too faint to read (at least in the copy available to us), but enough of them are legible to reconstruct the table, given similar information in other texts. There are eight columns with the following headings: domicile, detriment, exaltation, dejection, diurnal triplicities, nocturnal triplicities, terms according to the Egyptians, and faces. There are twelve rows, one for each zodiacal sign. The entries display the names of one or more planets. For a list of the domiciles and detriments (each zodiacal sign is governed by a planet), see, e.g., Wright (ed. and trans.) (1934), pp. 256–257; for a list of exaltations and dejections (a specific degree in one of the zodiacal signs for each planet), see, e.g., Wright (ed. and trans.) (1934), p. 258. For a list of the diurnal and nocturnal triplicities, see, e.g., Suter 1914, p. 231; for a list of the terms, see, e.g., Suter (1914), p. 231; and for a list of the faces, see, e.g., Suter (1914), p. 231. For an overview of the dignities, see Chabás and Goldstein (2012), pp. 215–218.

The same data on the astrological dignities (together with other data) are found in MS W 58a; however, the presentation of the astrological data is different from, and probably unrelated to, what is found in MS M or MS F. In MS W, the heading is "Table for the rulers of the dignities in the zodiac according to the essential system" (*jadwal aṣḥāb al-ḥuẓūẓ fī l-burūj ᶜalā madhhab al-dhātī*), where the manuscript has *aṣḥab* (instead of *aṣḥāb*). The term in Arabic for dignities (*ḥuẓūẓ;* sing. *ḥaẓẓ*) generally means "shares," but *ḥaẓẓ* is translated *dignitas* in Adelard of Bath's medieval Latin translation of Abū Maᶜshar's *Introduction to Astrology*: see Burnett et al. (1994), pp. 14–15, and 94–95.

In MS G 99a, the information on the astrological dignities is displayed in a diagram consisting of a series of concentric circles. Much the same information is found in Madrid, Biblioteca Nacional, MS 3349 (Chabás and Goldstein 2010, pp. 206–210) and in Munich, Staatsbibliothek, MS Heb. 109, containing Abraham Zacut's *Ḥibbur* (Chabás and Goldstein 2000, pp. 86–89).

## 16 Others tables in MS M (not in MS F)

### 16.1 Two tables for Azarquiel's solar theory

MS M 55r: "Tabula motus centri circuli exeuntis centrum in longitudine longiori et propinquiori a centro terre"; MS 55v–56r: "Tabula directionis composite centri circuli solis exeuntis centrum de diuersitate centri eiusdem in longitudine propinquiori et longiori a circulo diuersitatis motus centri morantis tempus"

These two tables present Azarquiel's solar model and facilitate computation of the solar equation. The text in MS M 19rb ascribes them explicitly to Azarquiel and

**Table 40** Lunar Crescent Visibility in MS M

| | [Climate] | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1st (°) | 2nd (°) | 3rd (°) | 4th (°) | 5th (°) | 6th (°) | 7th (°) |
| Aries | 11;24 | 11; 4 | 11;19 | 10; 6 | 10; 7 | 9; 9 | 9;38 |
| Taurus | 11;11 | 11;24 | 10;13 | 10;21 | 10;12 | 8;13 | 9; 8 |
| Gemini | 11; 2 | 11;51 | 10; 8 | 10;12 | 10;29 | 9;24 | 9; 3 |
| Cancer | 11;14 | 11;14 | 11;38 | 14;12 | 12;25 | 12;46 | 12; 9 |
| Leo | 13;10 | 13;13 | 13;50 | 15; 0 | 16; 7 | 16;17 | 13;15 |
| Virgo | 14;27 | 16; 9 | 17; 2 | 17;10 | 23;27 | 23;21 | 24;50 |
| Libra | 14; 2 | 16;50 | 18;19 | 19;14 | 21;23 | 22;24 | 24; 1 |
| Scorpio | 14;12 | 14;32 | 16;19 | 16;16 | 18; 4 | 19;42 | 21; 2 |
| Sagittarius | 14; 0 | 13;39 | 13;18 | 42;42 | 13;31 | 14; 2 | 12;31 |
| Capricorn | 11;10 | 11;15 | 11;21 | 27;27 | 11; 0 | 11; 9 | 11;45 |
| Aquarius | 11; 3 | 11;30 | 10; 2 | 4; 4 | 9;15 | 9; 7 | 9;15 |
| Pisces | 11;24 | 11;11 | 11;20 | 19;19 | 9;18 | 9; 0 | 8;14 |

gives instructions for their use. Since Toomer (1969, p. 325) reproduced excerpts and explained both tables, we only describe them briefly.

The first table gives the solar mean motion of center for Arabic years (hundreds, tens, and units) and months. The radix is 2s 23;40,31°, and the entry for one year is 0;6,16°. The second table has four columns. One is for the argument at intervals of 1° from 1° to 360°. The second and third columns display the solar equation (with a maximum of 1;45,36° at 91°–92°) and the difference in the solar equation (with a maximum of 0;37,18° at 91°–92°). The argument for both these columns is the solar mean center. The fourth column is for the minutes of proportion, for which the argument is the solar mean motion of center found in the first table.

## 16.2 Lunar crescent visibility

MS M 56v: "Tabula uisuum lunarium post occasum solis in climatibus septem"

The entries in this table represent the difference in longitude between the Sun and the Moon for which the difference in setting times is 12° (see Table 40). The entries, given in degrees and minutes, are displayed in a double-argument table for the beginning (?) of each zodiacal sign and for each of the seven climates.

In his analysis of various early Islamic tables for determining the lunar crescent visibility, King describes 14 tables like the one presented here and suggests that they have an early Andalusian origin (King 1987, p. 197). A similar table is found in the *Almanac of Azarquiel*, but beginning in Libra rather than in Aries (Millás 1943–50, p. 228). The table presented here also appears in zijes by Maghribi astronomers including Ibn Isḥāq, Ibn al-Raqqām, Ibn ᶜAzzūz al-Qusanṭīnī, and Abu l-Ḥasan ᶜAlī al-Qusanṭīnī (King 1987, pp. 200 ff., cf. Kennedy and King 1982, p. 20), as well as

**Table 41** Lunar eclipses in MS M (excerpt)

| (′) | digits | (h) | (′) | digits | (h) |
|---|---|---|---|---|---|
| 1 | 21 | 2; 0 | 31 | 12 | 1;10 |
| ... | | | ... | | |
| 5 | 19 | 2; 0 | 35 | 9 | 1; 8 |
| ... | | | ... | | |
| 10 | 18 | 1;14 | 40 | 6 | 1; 5 |
| ... | | | ... | | |
| 15 | 16 | 1;13 | 45 | 5 | 1; 2 |
| ... | | | ... | | |
| 20 | 15 | 1;13 | 50 | 3 | 0;14 |
| ... | | | ... | | |
| 25 | 13 | 1;11 | 55 | 2 | 0; 9 |
| ... | | | ... | | |
| 30 | 12 | 1;10 | 60 | 0 | 0; 3 |

in the early sixteenth-century Arabic version by Moses Galiano (Ar. Mūsā Jalīnūs) of the *Almanach Perpetuum* (King 1987, pp. 202–203, El Escorial, MS Ar. 966, f. 192v; on Galiano, see Chabás and Goldstein (2000), p. 163, Samsó 2002–2003, p. 68, and Morrison 2011). The tables examined by King contain so many corrupt entries that he was led to conclude: "This table takes the prize as the most corrupt table in the known medieval sources" (p. 207). For an analysis of other Islamic tables for the visibility of the lunar crescent, see Hogendijk (1988).

## 16.3 Lunar eclipses

MS M 57r: "Tabula eclipsis lune et quot digiti eclipsantur ex ea et hore dimidii temporis eclipsis" (see Table 41)

The arguments in this table are the minutes of lunar latitude. The other columns represent the digits of the eclipsed disk and the duration of the eclipse (in hours and minutes). In most zijes (e.g., al-Khwārizmī's zij), the argument is the distance of the Moon from the node. Normally, the minimum lunar latitude in a table for lunar eclipses is $0'$, i.e., when the Moon is at a node on the ecliptic. In al-Battānī's zij (Nallino 1903–1907, 2: 90), the argument is the lunar latitude (as in Table 41): for argument 0;0° the digits of eclipse are 21;31,30d for greatest lunar distance and 21;36d for least lunar distance, which could be rounded to 21d, as in Table 41 for $1'$. The corresponding half-duration of the eclipse in al-Battānī's zij varies from 0;53h (= 0;29,30h + 0;23,30h) to 1;4h (= 0;35,20h + 0;28,56h), or a duration of about 2h, as in Table 41. In al-Battānī's table 0d corresponds to a lunar latitude that varies from about $53'$ to $63'$ (Table 41 has $60'$), 10d from about $28'$ to $34'$ (Table 41 has $33'$–$34'$), and 20d to a latitude of about $4'$ (Table 41 has $3'$–$4'$), that is, the entries in both tables are similar and the trend is the same. We are unaware of any other copy of Table 41.

MS M 57v also has a table for lunar eclipses according to Yaᶜqūb ibn Ṭāriq (see Sect. 12.2, Table 34).

**Table 42** Excess of revolution according to Azarquiel

| Years | Angle (°) | Years | Angle (°) | Years | Time (h) | Years | Time (h) |
|---|---|---|---|---|---|---|---|
| 10 | 204; 0 | 1 | 92;24 | 10 | 13;36 | 1 | 6; 9 |
| 20 | 48; 0 | 2 | 184;48 | 20 | 3;13 | 2 | 12;19 |
| 30 | 252; 0 | 3 | 277;12 | 30 | 16;50 | 3 | 18;29 |
| 40 | 96; 0 | 4 | 9;36 | 40 | 6;28 | 4 | 0;38 |
| 50 | 300; 0 | 5 | 102; 0 | 50 | 20; 5 | 5 | 6;48 |
| 60 | 144; 0 | 6 | 194;24 | 60 | 9;42 | 6 | 12;57 |
| 70 | 348; 0 | 7 | 286;48 | 70 | 23;19 | 7 | 19; 7 |
| 80 | 192; 0 | 8 | 19;12 | 80 | 12;56 | 8 | 1;17 |
| 90 | 36; 0 | 9 | 111;36 | 90 | 2;33 | 9 | 7;26 |
| 100 | 240; 0 | 10 | 204; 0 | 100 | 16; 9 | 10 | 13;36 |

## 16.4 Excess of revolution according to Azarquiel

MS M 57v: " Hec tabula composita est secundum opinionem ebyeshac ezartal"

In Table 42, the entries are based on a yearly excess of 92;24°, corresponding to a sidereal year of 365;15,24d = 365d 6;9,36h, which is shorter than that given Ibn al-Kammād (see Sect. 15.1).

## 16.5 Latitudes of the seven climates

MS M 57v has a table for the maximum values of geographical latitudes in the seven climates (see Table 43).

Longest daylight, M, which depends on the geographical latitude, has served as a way to define zones of latitude which, historically, were called climates. The seven traditional climates have their midpoints located at values of M from 13h to 16h, in steps of 1/2 hour. In *Almagest* II.13, the corresponding values for the geographical latitude are 16;27° (13h), 23;51° (13½h), 30;22° (14h), 36° (14½h), 40;56° (15h), 45;1° (15½h), and 48;32° (16h). The extremes of the climates are defined by the values of M which are 0;15h greater or smaller than those defining the midpoints. For example, in the zij of al-Battānī, the latitudes corresponding to the upper limit of the first six climates are 20;28° (13¼h), 27;28° (13¾h), 33;37° (14¼h), 38;54° (14¾h), 43;25° (15¼h), and 47;12° (15¾h): Nallino 1903–1907, 2: 65–66. Table 43 gives a somewhat different list of the upper limits of all seven climates.

## 16.6 Tables for chronology in al-Khwārizmī's zij

MS M 58r: "Tabula cuius est inter annos gentium et alios annos preter illos ad inuicem"; MS M 58v: "Numeri dimissi per 28, 28 secundum annos romanorum et egyptiacum"

These two sub-tables are clearly related to Table 1 in the zij of al-Khwārizmī (Suter 1914, p. 109), which is also presented as two sub-tables. The first one displays the

**Table 43** Maximum Latitudes of the Seven Climates

| Climate | (°) |
|---|---|
| End of first climate | 20;30 |
| End of second climate | 27;30 |
| End of third climate | 33;40 |
| End of fourth climate | 39; 0 |
| End of fifth climate | 43; 0 |
| End of sixth climate | 47;15 |
| End of seventh climate | 50;35 |

time intervals between events considered the beginning of chronological eras. The table opens with the time interval between the Flood and the beginning of the Yazdegird (or Persian) era, and it is given as 1.1.1.4.35.10.23, indicating that the first day of the Flood was "the day of Jupiter" (i.e., Thursday). The date for the Flood, corresponding to the Indian era of the Kaliyuga, is February 17, –3101 (it was indeed a Thursday), and that for the Persian era is June 16, 632 (see e.g., Chabás and Goldstein 2012, p. 15). Since their respective Julian day numbers are 588465 and 1952063, the difference between them is 1,363,598 days, which corresponds exactly to the entry in the zij of al-Khwārizmī: 3735 Egyptian years, 10 months, and 23 days. And indeed $3735 \times 365 + 10 \times 30 + 23 = 1{,}363{,}598$. Therefore, the beginning of the entry in MS M, 1.1.1.4.35, has to be understood as 3735 years. We also note that the entry for the time interval between the Incarnation and the Hijra is given as 621 years, 4 months, and 15 days. Since the Hijra took place in July 622, this entry means that the Julian year here begins in March (see Sect. 16.9, below). The second sub-table gives a list of the multiples of 28 (years) up to 2352, whereas in the zij of al-Khwārizmī it only gets to 2100. According to Millás (1942), p. 245, both sub-tables derive from Maslama rather than from al-Khwārizmī.

## 16.7 Mighty years

MS M 58v: "Circuitus planetarum magni in sectis et divinationibus"

The title refers to the "Mighty Years" of the planets, used in an astrological context. In Table 44, the entries of MS M are compared with those of Abū Ma[c]shar as preserved by al-Sijzī (Pingree 1968a, p. 64). It is readily seen that the order of the planets does not conform to the standard one. This seems to be a specific feature of this list, for other analogous tables have the standard order: see, e.g., al-Bīrūnī (Wright (ed. and trans.) 1934, p. 255), where the entry for Mercury is given as 461. See also Sect. 16.11.

## 16.8 Retrogradation of the planets

MS M 58v: "Tabula dierum prouenientium in retrogradationibus planetarum et directionibus eorum" (see Table 45).

**Table 44** "Mighty Years" of the Planets

| Planet | MS M | Abū Ma$^c$shar |
|---|---|---|
| Sun | 1461 | 1461 |
| Venus | 1151 | 1151 |
| Mercury | 480 | 480 |
| Moon | 420 | 520 |
| Saturn | 625 | 265 |
| Jupiter | 567 | 427 |
| Mars | 684 | 284 |

**Table 45** Retrogradations of the Planets

| | [1] 1st direct motion (d) | [2] Retrogradation (d) | [3] 2nd direct motion (d) | [4] Sum (d) | [5] Sum (d) | [6] Retrogradation (°) |
|---|---|---|---|---|---|---|
| Saturn | 121 | 136 | 121 | 398 and 2 h | 242 | 7 |
| Jupiter | 141 | 113 | 141 | 395 less 2 h | 282 | 10 |
| Mars | 393 | 71 | 393 | 797 | 726 | 20 |
| Venus | 273 | 42 | 273 | 588 | 546 | 20 |
| Mercury | 47 | 22 | 47 | 116 | 94 | 15 |

The Toledan Tables have a table for the retrogradation of the planets that gathers the information provided in words in *Almagest* XII.2–6: the retrograde arcs and their corresponding durations for three positions (maximum, mean, and minimum distances) of the five planets. However, the table presented here differs substantially from it. Of the 6 numerical columns in Table 45, columns 2 and 6 have entries corresponding to, although with some differences, the retrograde arcs (in degrees) and the associated times (in days) when the planets are at mean distance. Columns 1 and 3 give the amount of direct motion from conjunction to first station (labeled here "to the left of the Sun") and from second station to conjunction ("to the right of the Sun"), before and after its retrograde motion, respectively. Column 4 displays the time it takes for the planet to complete a full loop from one phenomenon to the next phenomenon of the same kind, and its entries are the sums of the entries in cols. 1, 2, and 3, whereas column 5 represents the time the planet has progressed in direct motion.

The entries in Table 45 are not internally consistent in all cases and do not always agree with parameters used in Indian astronomy (which are very close to those derived from Ptolemy's data in the *Almagest*: see Pedersen 1974, pp. 425–429). For example, the entries in column 4, which are the periods of anomaly of the planets, that is, the time needed to return to the same velocity, can be compared with those derived from the *Sūrya-Siddhānta* (Burgess (trans.) [1860] 1935, p. 69): 378d 2h (Saturn), 398d 21h (Jupiter), 779d 22h (Mars), 583d 22h (Venus), and 115d 22h (Mercury). Clearly, for Saturn the text should read "378d and 2h," for 121d + 136d + 121d = 378d. In the

**Table 46** Years of the Planets

| | Sun | Moon | Saturn | Jupiter | Mars | Venus | Mercury |
|---|---|---|---|---|---|---|---|
| Great | 120 | [108] | 57 | [79] | 66 | [82] | 76 |
| Mean | [391/2] | 381/2 [a] | [431/2] | 451/2 | [401/2] | 45 | [48] |
| Least | 19 | [25] | 30 | [12] | 15 | [8] | 20 |

[a] al-Bīrūnī: 391/2

case of Mars, 393d + 71d + 393d = 857d, which is neither the entry in column 4 nor the value in the *Sūrya-Siddhānta*. From the value in column 5 for Mars, the entries in cols. 1 and 3 should be 363d (instead of 393d), which would be consistent with the entry in column 4.

## 16.9 Conversion from the Arabic to the Julian calendar

MS M 62r

There are four sub-tables. The first gives the number of collected Julian years as a function of the number of Arabic years from 90 to 900, at intervals of 90 years. The radix (the Hijra) is given as 621 years 4 months 15 days (as in Sect. 16.6), indicating that the Julian year begins in March. The second sub-table displays the number of expanded Julian years as a function of the number of Arabic years from 1 to 30, at intervals of 1 year; entries for 60 and 90 years are also given. The two other sub-tables are for the Arabic and Julian months (beginning in March).

## 16.10 Duration of pregnancy

MS M 62v–64r: "Tabula extractionis annorum quantitatis durationis creature in uentre matris per longitudinem lune a gradu occidentis"

This table is certainly part of *al-Kawr* ${}^{c}$*alā l-dawr*. It deals with the astrological determination of the duration of pregnancy and has been discussed in Vernet (1949), pp. 273–300. For the rich variety of tables concerning the gestation period, see Chabás and Goldstein (2012), pp. 223–226.

## 16.11 Years of the Planets

MS M 64v: "Tabula circuituum annorum planetarum in natiuitatibus"

Table 46, the years of the planets, is related to Table 44. As was the case with previous tables in this manuscript, the entries that were filled in are all written in red ink, but the rest of the entries, probably intended to be written in blue ink, were left blank. Entries in square brackets complete the table, taking the data from al-

Bīrūnī's *Elements of the Art of Astrology* (Wright (ed. and trans.) 1934, p. 255). The years associated with the planets are astrological, although some have an astronomical basis. Al-Bīrūnī offers the explanation that "the years of the planets are of four degrees, least, mean, great, and greatest [or mighty]. The last are only used for marking certain time cycles, although some people say that in ancient days the planets granted such long years of life. Astrologers of the present day only use the three former degrees for determining the length of life at a nativity, and the numbers which they thus elicit must not be interpreted literally as years, but freely, for sometimes they represent years but sometimes months, weeks, days, or hours" (Wright (ed. and trans.) 1934, p. 239).

## 16.12 Seasons, astrological terms, the length of the solar year, and the length of the synodic month

MS M 65v: "Mediatus cursus solis in descensu eius ad quartas circuli secundum probationem huius canonis"

Here, we find data for 480 A.H. (1087–88 A.D.), most likely referring to the beginning of the seasons and their duration. On one side we find Aries (11.28.10.0 = 11s 28;10,0°), Cancer (3.0.21.0 = 3s 0;21,0°), Libra (6.1.50.0 = 6s 1;50,0°), and Capricorn (8.29.38.0 = 8s 29;38,0°), on the other what seem to be three durations expressed in days: 93.3.12.50, 185.12.0.0, and 279.13.28.0.2. The positions corresponding to the four zodiacal signs refer to the mean positions of the Sun when the true Sun is at the beginning of each season. For example, when the true Sun is at Aries 0° (= 0s 0°), the mean Sun is at 11s 28;10°, where the solar equation is 1;50°. When the true Sun is at Cancer 0° (= 3s 0°), the mean Sun is at 3s 0;21°, where the solar equation is −0;21°. If the solar apogee is taken to be near Gemini 20°(= 2s 20°) and the maximum solar equation about 1;53°, then according to the tables in *al-Muqtabis*, the solar equation at 11s 28;10° is about 1;52° and at 3s 0;21° it is about −0;19° (see Chabás and Goldstein 2015, pp. 184 and 187). For values of the solar apogee near 2s 20°, see Sect. 4.2, above. The lengths of the seasons in days seem to be 93;3,12,50d for spring, 92;8,47,10d (= 185;12,0,0d − 93;3,12,50d) for summer, and 94;1,28,0,2d (= 279;13,28,0,2d − 185;12d) for autumn. The seasonal lengths should be the interval in mean longitude from the beginning of one season to the next divided by the daily mean solar motion (≈ 0;59,8°/d), but the numbers in the text are not familiar to us and we have no explanation for them.

On the same page, there is a table for the astrological terms, headed "Tabula terminorum ciuium Babillonie ueteris qui sunt magistri ymaginum," where again every other entry was left blank. Separating the two halves of this astrological table horizontally, two astronomical parameters are inserted: the length of the solar year "according to Ibn al-Kammād" (365;15,36,19,35,32 days), for which see Sect. 15.1, and the length of the synodic month (29;31,50,5,1 days).

**Table 47** Visibility of the planets

| | | (°) | (′) |
|---|---|---|---|
| Saturn | – | 11 | 13 |
| Jupiter | – | 5 | 8 |
| Mars | – | 11 | 14 |
| Venus | – | 5 | 6 |
| Mercury | – | 10 | 11 |

## 16.13 Table by Maslama

MS M 66r: "Tabula extracta per misilme de eo quod confirmatum extitit per ciues huius artis yspanenses super diuisionem Yspanie per signa duodecim 12 12 12 12."

This table associates the 12 zodiacal signs with cities, all of them in the Iberian Peninsula, including Zaragoza, Córdoba, Granada, Toledo, Valencia, Tortosa, and Sevilla. It was transcribed by Millás (1942, p. 246) who suggested that "misilme" stands for "Maslama."

## 16.14 Visibility of the Planets

MS M 66r: "Tabula uisuum planetarum et absconsionum eorum sub radiis solis"

This table refers to the visibility of the planets (see Table 47), and for each of them it gives the solar depression angle (also called *arcus visionis*), that is, the distance from the Sun to the horizon on an arc through the local zenith when the planet is at its first or last visibility on the horizon. This quantity depends on the local horizon, but in *Almagest* XIII.7 Ptolemy restricted his data to an intermediate latitude, where the longest daylight is 14 1/4h and gave the following results: 11° for Saturn, 10° for Jupiter, 11 1/2° for Mars, 5° for Venus, and 10° for Mercury. But for Jupiter, these numbers agree with those in Table 47 (see Toomer 1984, pp. 636–640; Neugebauer 1975, pp. 234–238). In the *Handy Tables*, Ptolemy disregarded this restriction and enlarged his computations to the seven climates (Neugebauer 1975, pp. 256–261; Stahlman 1959, pp. 159–165). Later on, in his *Planetary Hypotheses*, Ptolemy further refined the data (Goldstein 1967b, p. 9), but in all cases they were given to degrees or half-degrees. Moreover, in the *Planetary Hypotheses* Ptolemy remarked that at acronychal risings of the outer planets (i.e., when the planet rises as the Sun sets), the *arcus visionis* is about half that for first visibility: this may explain the discordant value in the text for Jupiter (5;8° in Table 47, instead of 10° in the *Almagest*). For visibility tables in Islamic astronomy, see Kennedy and Agha (1960), pp. 134–140.

**Acknowledgments** We are most grateful to Rosa Comes (Barcelona) for her assistance with the Latin text and to Sholomo Sela (Tel Aviv) and Julio Samsó (Barcelona) for their help in clarifying some astrological terms in Hebrew and Arabic.

## References

Beaujouan, G. 1969. *L'astronomie dans la péninsule ibérique à la fin du moyen âge*. Coimbra.
Burgess, E. (trans.) [1860] 1935. *Translation of the Sūrya-Siddhānta, A Text-Book of Hindu Astronomy*. Translated by E. Burgess, edited by P. Gangooly, with an introduction by P. Sengupta, Delhi.
Burnett, C., Yamamoto, K., and Yano, M. (eds. and trans.). 1994. *Abū Ma$^{c}$shar: the abbreviation of the introduction to astrology together with the Medieval Latin Translation of Adelard of Bath*. Leiden and New York.
Castells, M. 1991. Notas astrológicas y astronómicas en el manuscrito medico árabe 873 de El Escorial. *Al-Qantara* 12: 19–59.
Castells, M. 1996. Una tabla de posiciones medias planetarias in el Zīŷ de Ibn Waqār (Toledo, ca. 1357). In *From Baghdad to Barcelona: Studies in the Islamic Exact Sciences in Honour of Prof Juan Vernet*, eds. Casulleras, J., and J. Samsó, 445–452. Barcelona.
Castells, M., and Samsó, J. 1995. Seven Chapters of Ibn al-Saffar's Lost Zij. *Archives internationales d'histoire des sciences*, 45: 229–262; reprinted in Samsó 2007, Essay 3.
Casulleras, J. 2010. *La astrología de los matemáticos. La matemática aplicada a la astrología a través de la obra de Ibn Mu$^{c}$āḏ de Jaén*. Barcelona.
Chabás, J. 1991. The astronomical tables of Jacob ben David Bonjorn. *Archive for the History of Exact Sciences* 42: 279–314.
Chabás, J. 1992. *L' astronomia de Jacob ben David Bonjorn*, Barcelona.
Chabás, J. 1996a. Astronomía andalusí en Cataluña: Las Tablas de Barcelona. In *From Baghdad to Barcelona: Studies in the Islamic Exact Sciences in Honour of Prof Juan Vernet*, eds. Casulleras, J., and J. Samsó, 477–525. Barcelona.
Chabás, J. 1996b. El Almanaque Perpetuo de Ferrand Martines (1391). *Archives internationales d'histoire des sciences* 46: 261–308.
Chabás, J. 2015. A list of stars 'correcte cum 2 magnis armillis' in 1362. *Journal for the History of Astronomy* 46: 206–217.
Chabás, J., and B.R. Goldstein. 1994. Andalusian astronomy: *al-Zīj al-Muqtabis* of Ibn al-Kammād. *Archive for History of Exact Sciences*, 48: 1–41; reprinted in Chabás and Goldstein 2015, 179–226.
Chabás, J., and B. R. Goldstein. 1997. Computational astronomy: Five centuries of finding true syzygy. *Journal for the History of Astronomy* 28: 93–105; reprinted in Chabás and Goldstein 2015, 40–56.
Chabás, J., and B.R. Goldstein. 2000. *Astronomy in the Iberian Peninsula: Abraham Zacut and the transition from manuscript to print*. Transactions of the American Philosophical Society, 90.2. Philadelphia.
Chabás, J., and B.R. Goldstein. 2003. *The Alfonsine Tables of Toledo*. Archimedes: New Studies in the History and Philosophy of Science and Technology, 8. Dordrecht and Boston.
Chabás, J., and B.R. Goldstein. 2004. Early Alfonsine astronomy in Paris: The tables of John Vimond (1320). *Suhayl* 4: 207–294.
Chabás, J., and B.R. Goldstein. 2010. Astronomical activity in Portugal in the fourteenth century. *Journal for the History of Astronomy* 41: 199–212.
Chabás, J., and B.R. Goldstein 2012. *A survey of European astronomical tables in the late middle ages*. Leiden.
Chabás, J. and B.R. Goldstein. 2015. *Essays on medieval computational astronomy*. Leiden.
Comes, M. 1994. The 'Meridian of Water' in the tables of geographical coordinates of al-Andalus and North Africa. *Journal for the History of Arabic Science* 10: 41–51.
Comes, M. 2001. *Ibn al-Hā'im's trepidation model. Suhayl* 2: 291–408.
Comes, M. 2007. Ibn al-Kammād: Abū Ja$^{c}$far Aḥmad ibn Yūsuf ibn al-Kammād. In *The biographical encyclopedia of astronomers*, ed. T. Hockey, et al., 559–560. New York: Springer Reference.
Debarnot, M.-T. 1987. The zīj of Ḥabash al-Ḥāsib: A survey of MS Istanbul Yeni Cami 784/2. In D.A. King and G. Saliba (Eds.), *From deferent to equant: A volume of studies in the history of science in the ancient and medieval near east in honor of E.S. Kennedy, Annals of the New York Academy of Sciences* 500: 35–69.
Goldstein, B.R. 1964. On the theory of trepidation. *Centaurus* 10: 232–247.
Goldstein, B.R. 1967a. *Ibn al-Muthannā's Commentary on the astronomical tables of al-Khwārizmī*. New Haven and London.
Goldstein, B.R. 1967b. *The Arabic Version of Ptolemy's* planetary hypotheses. Transactions of the American Philosophical Society, NS 47.4. Philadelphia.

Goldstein, B.R. 1974. *The Astronomical Tables of Levi ben Gerson*. Transactions of the Connecticut Academy of Arts and Sciences, 45. New Haven.

Goldstein, B.R. 1977. Ibn Mu<sup>c</sup>ādh's treatise on twilight and the height of the atmosphere. *Archive for History of Exact Sciences* 17: 97–118.

Goldstein, B.R. 2001. The astronomical tables of Judah ben Verga. *Suhayl* 2: 227–289.

Goldstein, B.R. 2003. An anonymous Zij in Hebrew for 1400 A.D.: a preliminary report. *Archive for History of Exact Sciences* 57: 251–271.

Goldstein, B.R. 2005. Colors of eclipses in medieval hebrew astronomical tables. *Aleph* 5: 11–34.

Goldstein, B.R. 2011. Solomon Franco on the zero point for trepidation. *Suhayl* 10: 77–83.

Goldstein, B.R. 2013. Preliminary remarks on the astronomical tables of Solomon Franco. *Aleph* 13: 175–184.

Goldstein, B.R., and J. Chabás. 1996. *Ibn al-Kammād's star list. Centaurus* 38: 317–334.

Goldstein, B.R., and J. Chabás. 1999. An occultation of venus observed by Abraham Zacut in 1476. *Journal for the History of Astronomy* 30: 187–200.

Goldstein, B.R., and J. Chabás. 2015. Three tables for the daily positions of the moon in a fifteenth-century hebrew manuscript. *Aleph* 15: 319–341.

Goldstein, B.R., and F.W. Sawyer, III. 1977. Remarks on Ptolemy's equant model in islamic astronomy. In *Prismata*, ed. Y. Maeyama and W. G. Saltzer. Wiesbaden, pp. 165–181; reprinted in B.R. Goldstein. 1985. *Theory and observation in ancient and medieval astronomy*. London, Essay 7.

González Llubera, I. 1953. Two old Portuguese astrological texts in Hebrew characters. *Romance Philology* 6: 267–272.

Haddad, F.I., E.S. Kennedy, and D. Pingree. 1981. *The book of reasons behind astronomical tables by <sup>c</sup>Alī ibn Sulaymān al-Hāshimī*. Delmar.

Hermelink, H. 1964. Tabulae Jahen. *Archive for History of Exact Sciences* 2: 108–112.

Heller, J. (ed.) 1549. *De elementis et orbibus coelestibus*. Nuremberg.

Hogendijk, J.P. 1988. Three Islamic Lunar Crescent Visibility Tables. *Journal for the History of Astromomy* 19: 29–44.

Kennedy, E.S. 1956a. A survey of Islamic astronomical tables. *Transactions of the American Philosophical Society*, NS 46.2. Philadelphia.

Kennedy, E.S. 1956b. Parallax theory in Islamic astronomy. Isis 57: 33–53; reprinted in Kennedy, E.S. 1983. *Studies in the Islamic exact sciences*, 164–184. Beirut.

Kennedy, E.S. 1958. The Sasanian Astronomical Handbook *Zīj-i Shāh* and the astrological doctrine of 'Transit' *(mamarr)"*. *Journal of the American Oriental Society* 78: 246–262; reprinted in Kennedy 1983, 319–335.

Kennedy, E.S., and M. Agha. 1960. Planetary visibility tables in Islamic astronomy. *Centaurus* 7: 134–140; reprinted in Kennedy 1983, 134–140.

Kennedy, E.S., and N. Faris. 1970. The solar eclipse technique of Yaḥyā b. Abī Manṣūr. *Journal for the History of Astronomy* 1: 20–38; reprinted in Kennedy 1983, 185–203.

Kennedy, E. S., and M. H. Kennedy. 1987. *Geographical coordinates of localities from Islamic sources*. Frankfurt.

Kennedy, E.S., D.A. King. 1982. Indian astronomy in fourteenth century Fez: The versified Zīj of al-Qusunṭīnī. Journal for the History of Arabic Science 6: 3–45; reprinted in D.A. King.. 1986. *Islamic Mathematical Astronomy*, 8. London: Essay.

Kennedy, E.S., and W. Ukashah. 1969. Al-Khwārizmī's planetary latitude tables. *Centaurus* 14: 86–96; reprinted in Kennedy, E.S. 1983. *Studies in the Islamic exact sciences*, 125–135. Beirut.

King, D.A. 1987. Some early Islamic tables for determining lunar crescent visibility. In *From deferent to equant: A volume of studies in the history of science in the ancient and medieval near East in Honor of E.S. Kennedy, Annals of the New York Academy of Sciences*, eds. D.A. King and G. Saliba, Vol. 500, 185–225.

King, D.A. 2004. *In Synchrony with the Heavens*. Vol. 1: *The Call of the Muezzin*. Leiden and Boston.

King, D.A., J. Samsó, and with a contribution by B.R. Goldstein. 2001. Astronomical handbooks and tables from the Islamic World (750–1900): An Interim report. *Suhayl* 2: 9–105.

Kunitzsch, P. 1966. *Typen van Sternverzeichnissen in astronomischen Handschriften des zehnten bis vierzehnten Jahrhunderts*. Wiesbaden.

Laguarda, R. A. 1990. *La Ciencia Española en el Descubrimiento de América*. Valladolid.

Langermann, Y.T. 1984. Two astronomical treatises by Solomon Franco. *Kiryat Sefer* 59: 637–638. [in Hebrew].

Langermann, Y. T. 1993. Some astrological themes in the thought of Abraham ibn Ezra. In *Rabbi Abraham ibn Ezra: Studies in the writings of a twelfth-century Jewish Polymath*, eds. I. Twersky and J. M. Harris. Cambridge, MA; reprinted in Y.T. Langermann. 1999. *The Jews and the Sciences in the Middle Ages*. Aldershot, Essay 3.

Lesley, M. 1957. Bīrūnī on rising times and daylight lengths. *Centaurus* 5: 121–141; reprinted in Kennedy, E.S. 1983. *Studies in the Islamic exact sciences*, 253–273. Beirut.

Lilly, W. 1647. *Christian astrology*. London.

Mancha, J.L. 1998. On Ibn al-Kammād's table for trepidation. Archive for History of Exact Sciences 52: 1–11; reprinted in J.L. Mancha. 2006. *Studies in Medieval Astronomy and Optics*, 9. Essay: Aldershot.

Mercier, R. 1976–1977. Studies in the medieval conception of precession. *Archives internationales d'histoire des sciences* 26: 197–220, and 27: 33–71.

Mercier, R. 1996. Accession and recession: Reconstruction of the parameters. In eds. Casulleras and Samsó, 299–347.

Mercier, R. 1987. Astronomical tables in the twelfth century. In ed. Ch. Burnett, 87–118. *Adelard of bath, an english scientist and arabist of the early twelfth century*, London.

Mestres, A. 1996. Maghribī astronomy in the 13th century: A description of manuscript Hyderabad Andra Pradesh State Library 298. In *From Baghdad to Barcelona: Studies in the Islamic Exact Sciences in Honour of Prof Juan Vernet*, eds. Casulleras, J., and J. Samsó, 383–443. Barcelona.

Mestres, A. 1999. *Materials Andalusins en el Zīj d'Ibn Isḥāq al-Tūnisī*. University of Barcelona. Ph.D. dissertation.

Millás, J.M. 1942. *Las traducciones orientales en los manuscritos de la Biblioteca Catedral de Toledo*. Madrid.

Millás, J.M. 1943–50. *Estudios sobre Azarquiel*. Madrid-Granada.

Millás, J.M. 1949. *Estudios sobre historia de la ciencia española*. Barcelona.

Millás, J.M. 1962. *Las Tablas Astronómicas del Rey Don Pedro el Ceremonioso*. Madrid-Barcelona.

Millás Vendrell, E. 1963. *El comentario de Ibn al-Muṯannà a las Tablas Astronómicas de al-Jwārizmī, Estudios y edición crítica del texto latino de la versión de Hugo Sanctallensis*. Madrid-Barcelona.

Morrison, R. 2011. An astronomical treatise by Mūsā Jālīnūs alia Moses Galeano. *Aleph* 11: 385–413.

Nallino, C.A. 1903–1907. *Al-Battānī sive Albatenii Opus Astronomicum*, 2 vols. Milan.

Neugebauer, O. 1962a. *The astronomical tables of al-Khwārizmī*. Copenhagen.

Neugebauer, O. 1962b. Thābit Ben Qurra 'on the solar year' and 'on the motion of the eighth sphere'. *Proceedings of the American Philosophical Society* 106: 264–299.

Neugebauer, O. 1975. *A history of ancient mathematical astronomy*. Berlin and New York.

Neugebauer, O. and Pingree, D. (eds. and trans.) 1970. *The Pañcasiddhāntikā of Varāhamihira*, 2 vols. Copenhagen.

Neugebauer, O., and O. Schmidt. 1952. Hindu astronomy at Newminster in 1428. *Annals of Science* 8: 221–227.

Pedersen, F.S. 2002. *The Toledan tables: A review of the manuscripts and the textual versions with an edition*. Copenhagen.

Pedersen, O. 1974. *A survey of the Almagest*. Odense.

Petri, W. 1968. Colours of lunar eclipses according to Indian tradition. *Indian Journal of History of Science* 3: 91–98.

Pingree, D. 1963. Astronomy and astrology in India and Iran. *Isis* 54: 229–246; reprinted in 2014. *Pathways into the study of ancient sciences: Selected essays by David Pingree*, eds. I. Pingree, and J.M. Steele, 161–178, Philadelphia.

Pingree, D. 1965. The Persian 'Observation' of the Solar Apogee in ca. A.D. 450. *Journal of Near Eastern Studies* 24: 334–336.

Pingree, D. 1968a. *The Thousands of Abū Ma$^{c}$shar*. London.

Pingree, D. 1968b. The fragments of the works of Ya$^{c}$qūb ibn Ṭāriq. *Journal of Near Eastern Studies* 27: 97–125; reprinted in eds. I. Pingree and J. M. Steele. 2014. *Pathways into the study of ancient sciences: Selected essays by David Pingree*, 283–311. Philadelphia.

Pingree, D. 1970. The fragments of the works of al-Fazārī. *Journal of Near Eastern Studies* 29: 103–123; reprinted in eds. I. Pingree and J.M. Steele. 2014. *Pathways into the study of ancient sciences: Selected essays by David Pingree*, 313–333. Philadelphia.

Pingree, D. 1972. Precession and trepidation in Indian astronomy before A.D. 1200. *Journal for the History of Astronomy* 3: 27–35; reprinted in eds. I. Pingree and J.M. Steele. 2014. *Pathways into the study of ancient sciences: Selected essays by David Pingree*, 179–187. Philadelphia.

Pingree, D. 1976. The Indian and Pseudo-Indian passages in Greek and Latin astronomical and astrological texts. *Viator* 7: 141–195; reprinted in eds. I. Pingree and J.M. Steele. 2014. *Pathways into the study of ancient sciences: Selected essays by David Pingree*, 393–447. Philadelphia.

Pingree, D. 1978. History of mathematical astronomy in India. *Dictionary of Scientific Biography* 15: 533–633.

Pingree, D. 1996. Indian astronomy in medieval Spain. In eds. Casulleras and Samsó, 39–48; reprinted in eds. I. Pingree, and J.M. Steele. 2014. *Pathways into the study of ancient sciences: Selected essays by David Pingree*, 241–250. Philadelphia.

Poulle, E. 1994. *Un témoin de l'astronomie latine du XIII$^{e}$ siècle: les tables de Toulouse, 55–81*. In Comprendre et maîtriser la nature au moyen âge: Mélanges d'histoire des sciences offerts à Guy Beaujouan. Geneva and Paris.

Ragep, F.J. 1996. Al-Battānī, cosmology, and the early history of Trepidaton in Islam. In *From Baghdad to Barcelona: Studies in the Islamic Exact Sciences in Honour of Prof Juan Vernet*, eds. Casulleras, J., and J. Samsó, 267–298. Barcelona.

Richler, B. et al. 2008. *Hebrew manuscripts in the Vatican Library: Catalogue*. Studi e Testi, 438. Vatican City.

Rius, M. 2007. Ibn al-Samḥ: Abū al-Qāsim Aṣbagh ibn Muḥammad ibn al-Samḥ al-Gharnāṭī. In *The biographical encyclopedia of astronomers, Springer reference*, ed. T. Hockey, et al., 568. New York: Springer.

Rubió y Lluch, A. 1908. *Documents per l'Historia de la Cultura Catalana Mig-eval*, Vol. 1. Barcelona.

Sachau, C.E. (trans.) 1879. *Albīrūnī: Chronology of ancient nations*. London.

Salam, H., and E.S. Kennedy. 1967. Solar and Lunar tables in early Islamic astronomy. *Journal of the American Oriental Society* 87: 492–497.

Samsó, J. 1980. Notas sobre la trigonometría esférica de Ibn Mu$^{c}$ād. Awrāq 3: 60–68; reprinted in Samsó, J. 1994. *Islamic Astronomy in Medieval Spain*, Essay 7. Aldershot.

Samsó, J. 1994. Ibn Isḥāq al-Tūnisī and Ibn Mu$^{c}$ādh al-Jayyānī on the Qibla. In Samsó, J. 1994. *Islamic Astronomy in Medieval Spain*, Essay 6. Aldershot.

Samsó, J. 1996. al-Bīrūnī' in al-Andalus. In *From Baghdad to Barcelona*: *Studies in the Islamic Exact Sciences in Honour of Prof Juan Vernet*, eds. Casulleras, J., and J. Samsó, 583–612. Barcelona.

Samsó, J. 1997. Andalusian astronomy in 14th century Fez: *al-Zīj al-Muwāfiq* of Ibn $^{c}$ Azzūz al-Qusanṭīnī. *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 11: 73–110; reprinted in Samsó 2007, Essay 9.

Samsó, J. 2002–2003. In pursuit of Zacut's Almanach Perpetuum in the eastern Islamic World. *Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften* 15: 67–93; reprinted in Samsó 2007, Essay 16.

Samsó, J. 2007. *Astronomy and Astrology in al-Andalus and the Maghrib*. Aldershot.

Samsó, J. [1992] 2011. *Las Ciencias de los Antiguos en al-Andalus*. 2nd ed., with addenda and corrigenda by J. Samsó and M. Forcada. Almería.

Samsó, J. 2014. Ibn al-Raqqām's *al-Zīj al-Mustawfī* in MS Rabat National Library 2461. In *From Alexandria, Through Baghdad: Surveys and Studies in the Ancient Greek and Medieval Islamic Mathematical Sciences in Honor of J. L. Berggren*, eds. N. Sidoli, and G. Van Brummelen, 297–328. Berlin and Heidelberg.

Samsó, J., and E. Millás. 1998. The computation of planetary longitudes in the zīj of Ibn al-Bannā'. *Arabic Sciences and Philosophy* 8: 259–286; reprinted in Samsó 2007, Essay 8.

Sédillot, J.-J., and L.-A. Sédillot. 1834. *Traité des instruments astronomiques des Arabes*. Paris; reprinted Frankfurt 1984.

Sengupta, P. C. (trans.) 1934. *The Khaṇḍakhādyaka: An astronomical treatise of Brahmagupta*. Calcutta.

Smith, A.M., and B.R. Goldstein. 1993. The Medieval Hebrew and Italian versions of Ibn Mu$^{c}$ādh's 'On Twilight and the Rising of Clouds'. Nuncius: Journal of the. *History of Science* 8: 611–643.

Stahlman, W.D. 1959. *The astronomical tables of Codex Vaticanus graecus 1291*. Brown University Ph.D. dissertation.

Steinschneider, M. 1863. Intorno al libro *Saraceni cuisdam de eris*, stampato nel 1549, ed al libro *Tabule Jahen* tradotto da Gherardo Cremonese. In *Intorno ad alcuni matematici del medio evo*, ed. B. Boncompagni, 9–20. Rome.

Steinschneider, M. 1893. *Die hebraeischen Übersetzungen des Mittelalters*. Berlin.

Suter, H. 1914. *Die astronomischen Tafeln des Muḥammad ibn Mūsā al-Khwārizmī*. Copenhagen.

Toomer, G.J. 1968. A survey of the Toledan Tables. *Osiris* 15: 5–174.

Toomer, G.J. 1969. *A history of errors. Centaurus* 14: 306–336.
Toomer, G.J. 1984. *Ptolemy's Almagest.* New York.
Van Brummelen, G. 2009. *The Mathematics of the Heavens and the Earth: The Early History of Trigonometry.* Princeton and Oxford.
van Dalen, B. 1996. Al-Khwārizmī's astronomical tables revisited: Analysis of the equation of time. In eds. Casulleras and Samsó, 195–252.
Vernet, J. 1949. Un tractact d'obstetricia astrològica. *Boletín de la Real Academia de Buenas Letras de Barcelona* 22: 69–96; reprinted in J. Vernet, *Estudios sobre Historia de la Ciencia Medieval.* Barcelona-Bellaterra, 273–300.
Vernet, J. 1951. *Contribución al Estudio de la Labor Astronómica de Ibn al-Bannā'.* Tetuán.
Vernet, J. 1956. "Las Tabulae Probatae", Homenaje a Millás Vallicrosa, Barcelona, 2: 501–522; reprinted in J. Vernet, Estudios sobre Historia de la Ciencia Medieval. *Barcelona-Bellaterra* 1979: 191–212.
Villuendas, M.V. 1979. *La trigonometría europea en el siglo XI. Estudio de la obra de Ibn Mu$^{c}$ād̲, El Kitāb maŷhūlāt.* Barcelona.
Wright, R.R. (ed. and trans.) 1934. *The Book of Instruction in the Elements of the Art of Astrology by Abu'l-Rayḥān Muḥammad ibn Aḥmad al-Bīrūnī.* London.
Yano, M. (ed. and trans.) 1997. *Kūshyār Ibn Labbān's Introduction to Astrology.* Tokyo.