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Mathematics, technics, and courtly life in Late Renaissance Urbino

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Abstract The present article seeks to provide an overview of the general characteristics of the cultural and scientific climate in the Duchy of Urbino. Three of the Duchy's milieus seem to have been particularly important for scholars who were engaged in the study of mathematics: the so-called "School of Urbino", the environment of the court, and the world of the technicians and engineers. While the Urbino School has already been the object of previous studies, the other two milieus and their effect on the mathematicians' work have been rather neglected and are, consequently, addressed here. The paper's final section presents some documents that attest the importance of Renaissance scholars' interaction with these environments to their actual scientific activity, taking the case of Guidobaldo dal Monte as an example.

1 The mathematicians of Urbino and the context of their scientific work

Urbino was an important centre of Renaissance culture and, in particular, it was home to two influential sixteenth-century mathematicians, Federico Commandino and Guidobaldo dal Monte, as well as other similarly interesting figures like Bernardino Baldi, Muzio Oddi or Giulio da Thiene. A better knowledge of their cultural milieu and the technical and scientific ideas that circulated in their environment is essential for a fair interpretation of their own scientific activity. Admittedly, we already know who

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the "great" scholars with whom they maintained scientific relations were: practitioners of mathematics like Benedetti, Clavius, Galileo, Maurolico etc. It is comparatively difficult, however, to gain even a rough idea about their everyday scientific environment: the present article seeks to address this problem. Its point of departure lies in the overview provided by the important investigations of P.L. Rose and E. Gamba, V. Montebelli into the scientific dimension of the Duchy of Urbino's academic world (Rose 1975; Gamba and Montebelli 1988).

As far as the cultural and scientific background of Duchy's mathematicians is concerned, existing literature tends to place an exclusive emphasis on the importance of the lectures held by Commandino and the subsequent establishment of the "Urbino School".[1] These lessons were indeed so influential for Commandino's later work and his disciples' education that important aspects of the technical and scientific environment that similarly conditioned their work have been sidelined. This is clear from recent research into Guidobaldo dal Monte's life (Frank 2011/2012); numerous documents have been uncovered which testify to his close relationship with two other milieus: he assumed a remarkably high position at the court of the Duke of Urbino and was continually in touch with the world of architect–engineers and technicians. A similar situation also holds true for Commandino and Baldi. Moreover, the argument about the importance of the courtly milieu and the technicians' environment could be extended to include the works of other Renaissance mathematicians outside the Duchy of Urbino, like Giovanni Battista Benedetti in Turin.[2]

Commandino's late scientific work was notably influenced by Francesco Maria II della Rovere—his teaching activity in Urbino and publication of Euclid's *Elements* (1572) both occurred at the behest of the Duke (Frank 2013c). A similar, perhaps even more evident example of patronage can be seen in Guidobaldo's case—who will generally serve as a point of reference in the present article on account of the fact that the aforementioned studies have given us a relatively detailed knowledge of his scientific activity. Francesco Maria II repeatedly exerted direct influence on his work:[3] he instructed him to write a treatise on the calendar reform (*De ecclesiastici calendarii restitutione*, Pesaro, 1580), to revise Commandino's translation of Pappus's *Collectiones Mathematicae* (Pesaro, 1588) and to compose a treatise on the workings of a sundial.[4]

1 For further information on the "School of Urbino" see Rose (1975) and Gamba and Montebelli (1988). A proposal for a new way of dating of Commandino's lectures is contained in Frank (2013c). Research into the organisation and contents of Commandino's lectures by the present author is similarly underway.

2 A study of Giovanni Battista Benedetti's work in the light of his connection with the court of Savoy is forthcoming. The general problem of the interaction between the courtly milieus and Renaissance mathematicians, work was recently discussed at fifth congress of European Society of History of Science (Athens, 1–3 november 2012), symposium "Humanities, mathematics and technics at Renaissance courts", organised by Gavagna (Salerno) and the present author.

3 As the aforementioned studies into his life attest, Guidobaldo was essentially the Duke of Urbino's "court mathematician". In this context, it is important to note that the dal Monte family was one of the most influential in the duchy at this time. In keeping with his family's stature Guidobaldo was one of the most renowned members of the Urbino court, as is evidenced by the "payrolls" of the court—for further reading see Frank (2011/2012, Part A, I.2 and Appendix I, 1.4.4).

4 As far as *De ecclesiastici calendarii restitutione* is concerned, the Duke wrote to his agent in Rome [Biblioteca Oliveriana Pesaro (referred to henceforward as BOP), ms 458, fol. 17r]: "dipoiché per Vostre

Further, Guidobaldo was entrusted by his lord with the supervision of a whole series of construction projects: he was consulted for a hydraulic problem connected with work on the fountain in front of the ducal palace in Pesaro, and for a similar issue connected with renovation work on the ducal Villa Mirafiore (Menchetti 2013; Frank 2011/2012, see particularly Appendix I, I.3.2). He also appears to have supervised works at the port of Pesaro; and is known to have worked as an architect in the construction of the Santa Maria degli Angeli church in Pesaro (Frank 2011/2012; see particularly Appendix I, I.4.1 and I.4.3), and as a military engineer for the Grand Duke of Tuscany and the Dukes of Mantua and Urbino (Menchetti 2013; Frank 2011/2012, see particularly I.4.1 e I.4.3). Finally, he was commissioned by the Duke to ensure the precision of mechanical clocks that were fabricated by craftsmen in the latter's service and that were valuable instruments used in diplomacy.[5].

Such duties, exemplified here by the case of Guidobaldo, but testified also for other mathematicians such as Muzio Oddi[6] (one of the Duke's architects from around 1597) and Giulio da Thiene[7], led to the exchange of ideas with collaborators at various construction sites: architects and engineers like Girolamo Arduini, Carlo Macigni or Francesco Paciotti, and craftsmen, members of the so-called "intermediate cultural layer",[8] like Simone Barocci (the head of a famous workshop producing scientific instruments),[9] "mastro Lazzaro" (the

Footnote 4 continued
lettere s'intese che al Papa saria piacciuto in ogni modo che si fosse fatta di qua ancora qualche fatica sopra la riforma del Calendario, ci risolvemmo di dar questo assunto al Sig.r Guid'Ubaldo de' Marchesi del Monte, il quale avea fatto in ciò quel tanto che gl'è stato concesso dalla poca salute che ha (...)." Whilst referring to his involvement in the revision of the *Collectiones Mathematicae*, Baldi wrote in his *Life of Commandino*: "*<l'incarico di curare l'edizione delle Collectiones Mathematicae>* fu dato (...) a Guidobaldo (...) che lo fece stampare nella città di Pesaro." As for Guidobaldo's composition of a treatise on sundials at the behest of the Duke, see the following letter of the Duke's adviser Giulio Cesare Mamiani to Guidobaldo on 1 July 1587 (BOP, ms 211, f. 102r/v): "ho fatto vedere a S.A. la scrittura che V.S. Ill.ma ha fatto sopra l'orologio che va nel calamaro a fiume, et l'Alt. Sua è restata molto sodisfatta. Ma desidera che vi si aggiungi l'informatione (...). Sia dunque contenta V.S. di far tutto questo conforme alla mente di S.A. et il tutto più chiaro che si può, et mandarla quando avrà commodità." A more detailed description of these cases is contained in Frank (2011/2012, Appendix I, I.3.1 and I.4.1).

[5] For further information on the construction of mechanical clocks in the Duchy of Urbino in the Renaissance, their use in diplomacy and the mathematicians' involvement in the fabrication process, see Frank (2011/2012, 2013a,b).

[6] For further information on Muzio Oddi see Gamba and Montebelli (1988) and Marr (2011).

[7] Count Giulio da Thiene (died in 1588) was active as a diplomat, military captain and architect in the service of the Dukes of Urbino. The Venetian mathematician Francesco Barozzi referred to Thiene as "Illustrissimo Comiti Iulio Thiene, viro praestantissimo, omnibus scientiis, arteque militari egregie versato" in his description of an instrument for drawing the hyperbola invented by Thiene, in his *De admirandum illud geometricum Problema* (Venice, 1586, pp. 29–31). As far as his role in the scientific environment of the Duchy of Urbino goes, we know that Baldi had originally had the intention of dedicating his translation of Heron's *Automata* (Venice, 1589) to Thiene, if the latter had not died 1 year before the book's publication.

[8] The notion of "*strato culturale intermedio*" was introduced by Carlo Maccagni. For further reading see Maccagni (1993, 1996).

[9] For further information on the Barocci family, in particular Simone, and their artistic and technical work, see the proceedings of the international congress "I Barocci a Urbino tra arte e scienza", 5–6 October 2012 (http://urbinoelaprospettiva.uniurb.it/barocci.asp), forthcoming.

construction manager of Villa Mirafiore) or the clockmaker, Pietro Griffi (see Sect. 3).

The interaction with the representatives of this milieu offered fruitful stimuli for the mathematicians' scientific work: testimonies of this fact are constituted by Muzio Oddi's *Fabrica et uso del compasso polimetro* (Milano, 1633) and *De gli Horologi Solari* (Venezia, 1638). Another example, Guidobaldo's manuscript *Meditatiunculae*, will be examined in Sect. 3.

But another category of interlocutors also appears to have left signs on the scholars' work: as the following section documents, the cultural milieu at court was characterised by a profound interest in philosophical questions, with particular attention paid to the philosophy of Aristotle. Thus, in keeping with Guidobaldo, Baldi and Oddi's close ties to the court, some of their most important interlocutors were philosophers: Jacopo Mazzoni, Federico Bonaventura and Cesare Benedetti. And in fact, certain arguments addressed in their writings seem to reflect discussions on natural philosophy they had had with them or appear to be developments of such exchanges of ideas (see Sect. 4).

Thus, *indirect* impact on their works can also be made out—aside from the Duke's *direct* influence on the mathematicians' work—in particular in the interaction they had with their technical collaborators and their philosophical interlocutors.

These few examples may suffice to illustrate the need to consider the Urbino scholars' scientific work against the background of their cultural, scientific and technical environment. Correspondingly, the present article seeks to augment our knowledge of these background conditions to their scientific activity: Sect. 2 provides a description of the courtly environment, while Sect. 3 outlines important traits of the engineers and technicians' milieu. The final part Sect. 4 seeks to illustrate, as a way of example, the impact that Guidobaldo's interaction with his environment had on several aspects of his scientific work.

2 The courtly environment

There are essentially two aspects of the sixteenth-century della Rovere court that are relevant for our purposes: the great deal of attention given to philosophy, in particular to Aristotle's work; and secondly a profound interest in mathematics in its broader sense, including mechanics and fortifications.

2.1 The interest in mathematics, mechanics and fortifications

The study of mathematics was not unusual at Renaissance courts. A particular importance was attributed to it at the Urbino court, already from the time of Federico da Montefeltro onwards. Aside from this general cultural trend, there were also concrete reasons that urged the Dukes of Urbino to encourage the study of mathematics. The dukes were traditionally military captains variously in the service of the Venetian Republic, the Pontifical State and the Spanish King.[10] The activities that this role entailed were often fraught with mechanical challenges, such as the movement of

[10] The most authoritative tome on the history of the Duchy of Urbino remains Dennistoun (1851).

heavy loads (e.g. cannons), the construction of stable walls or the calculation of the trajectory of projectiles.[11]

These historical preoccupations remained prominent in the courtly milieu of the little duchy, even though the political situation in Italy had increasingly calmed down over the course of the sixteenth century. Indeed, the Venetian ambassador, Federico Badoer, remarked in 1547 of the counts and associates of Duke Guidobaldo II that "all of them live off warfare".[12]

This is the context in which Prince Francesco Maria and Guidobaldo dal Monte grew up together. Correspondingly, both of them were prepared for a life in the military. Guidobaldo was the Prince's page from the age of seven, and had the privilege of enjoying the same education as Francesco Maria II della Rovere, which included, *inter alia*, the study of mechanics (and, more generally, of mathematics). Several Venetian ambassadors describe the form that the Prince's education took. Lazaro Mocenigo wrote in 1571:

> [Il Principe Francesco Maria II] si dà molto alli essercizi del corpo, come al giocar della palla, all'andar a caccia, a piedi ed altri simili essercizi, per abituarsi alli incommodi della guerra, disegnando Sua Eccellenza di seguir anch'egli il mestier dell'armi (...). Studia, è intelligente delle matematiche e delle fortificazioni, e insomma si diletta di tutte quelle cose che veramente sono appartenenti ad un principe.[13]

Some years later (in 1575), another ambassador, Matteo Zane, was to confirm Mocenigo's report:

> [Duca Francesco Maria II] è studioso e litterato assai, e fa profession soprattutto d'arme e d'esser soldato.[14]

So, this "intelligent" young Prince took pleasure in "mathematics and fortifications" even to the extent that he made Commandino teach him it. Baldi writes in *Commandino's Life*:

> Attendeva [Commandino] egli adunque a condurre a fine molte opere già da lui cominciate, quando Francesco Maria, figliuolo di Guidubaldo nostro Duca, giovane d'animo eroico, sapendo quanto quelle scienze stiano bene a chi è per

[11] In this context it is interesting to recall that Niccolò Tartaglia's *Nova Scientia* (Venice, 1537) is dedicated to the Duke of Urbino Francesco Maria della Rovere: revealingly, the work is devoted to the very problem of identifying the trajectory of projectiles; further, his successor Guidobaldo II della Rovere figures as one of Tartaglia's interlocutors in the *Quesiti et Inventioni diverse* (Venice, 1546).

[12] All passages of the Venetian ambassadors' reports to the senate cited in the present section are published in Segarizzi (1913); see p. 171: "Vi è poi de' conti ed altri signori temporali, in numero 18, parte de' quali pagano feudo al Duca (...) e tutti vivono alla guerra."

[13] See Mocenigo's report (1571) in Segarizzi (1913), p. 189: "[Prince Francesco Maria] devotes himself a great deal to physical exercise, such as playing with a ball, hunting, walking and other similar exercises, in order to accustom himself to the inconveniences of war, as His Excellency plans to undertake the profession of arms himself (...). He studies, is proficient in mathematics and fortifications and in any case, he takes pleasure in all those things that are becoming of a prince."

[14] See Zane's report (1575) in Segarizzi (1913), p. 212: "[Duke Francesco Maria II] is studious and really well-read, and devotes himself particularly to matters of war and to be a soldier."

sostenere il carico del governo ed è per dar opera all'arti militari, non comportò che Federico se ne stesso rinchiuso fra le mura della casa paterna, ma propostogli onoratissimi partiti, volle, come aveva già fatto il Padre, chiamarlo ai suoi servizi. Nello quale entrato leggendo a quel Principe gli *Elementi* d'Euclide apportava lui molta sodisfazione nell'interpretarli.[15]

This account is confirmed by Francesco Maria II's autobiography.[16] Along with the Prince, other members of his court like Guidobaldo, in all probability his brother Francesco Maria dal Monte, Alderano Cybo, Giulio Giordani and other passing guests, like Torquato Tasso, frequented these lessons as well.

The enthusiasm with which the young Prince attended these lessons is impressively borne out by another document from a few months after Duke Francesco Maria II's ascension to the throne in 1574. A member of the court, Almerigo Almerici, wrote to his son, Virginio:

Il S.r Duca sta bene ma occupatissimo sempre ne' negotii et nelle audienze publiche che sonno ogni giorno indeffessamente. Et sin qui fa reuscita di gran prencipe et dice che il maggior scontento che abbia il non poter continuare li suoi studii.[17]

2.2 The interest in philosophy at court

Preoccupation with war and an interest in certain mathematical disciplines were not the only traits that characterised the courtly environment: above all, throughout the fifteenth and sixteenth centuries the Duchy of Urbino was an important cultural centre. Indeed, famous artists, architects and men of letters like Raffaelo Sanzi, Piero della Francesca, Leon Battista Alberti, Luca Pacioli, Francesco di Giorgio Martini, Pietro Aretino or Torquato Tasso were connected with its court. The ambassador Badoer stressed this point:

La corte del Duca e di tutta quella casa, come per una consuetudine, è stata sempre onorevole, percioché in ogni tempo, e nell'armi e nelle lettere, ella ha avuto de' più segnalati uomini d'Italia.[18]

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[15] Nenci (1998): "Thus, [Commandino] expected to complete many works that he had already started when Francesco Maria, son of our Duke Guidobaldo [II], a young man of heroic mind, refused to allow Federico to remain closed between the walls of his paternal house, knowing how much the sciences are of use to any who must bear the responsibility of power and attend to matters of war. So, with an offer of lucrative remuneration, the Prince wanted to call him into his service as his father had done before. Having entered this service, Commandino lectured the Prince on the *Elements* of Euclid whose interpretation brought him much satisfaction."

[16] See footnote 21.

[17] BOP, ms 1577, letter number 35 (10 January 1575): "The Duke is well but always very busy with affairs and public audiences that go on unremittingly every day. And hitherto he succeeds in his role as a great prince and says that his only sorrow is that of not being able to persevere with his studies."

[18] See Badoer's report (1547) in Segarizzi (1913), p. 165: "The court of the Duke and of this entire house, as if by custom, has always been honourable: in fact, at all times it hosted the most notable men in both warfare and letters from the whole of Italy."

This interest in letters and humanities extended also to philosophy. An extant description of Carnival 1574 is particularly interesting in this regard, as it allows us to get an idea of the general intellectual climate at the court, and especially of its members' interest in philosophy (the reader should note, for instance, the presence of the Prince (and of the Duke) at the time of the following discussions):

> Abbiamo goduto ancora molti ragionamenti (...) passati fra molti begli intelletti come dire il Mazzone da Cesena, (...), il Tasso, il Pino da Cagli e ms. Cesare Benedetti (...). Intesi primieramente che presò ragionamento innanzi il Principe alla venuta del Mazzone (...) e fu fra il Mazzone e ms. Cesare sopra la differenza ch'è fra Platone et Aristotile intorno alla Reminicenza, dove il Mazzone cercò di diffendere l'opinione di Platone e de' seguaci e ms. Cesare vi sosteneva quella d'Aristotile.
> (...) Un'altra volta s'attaccarono in festa mentre si ballava il Tasso et il Mazzone, et io mi trovai presente. Fra gli altri a una parte della contesa ch'era allora cioè che il Tasso teneva ch'Epicuro ponesse tutto il sommo bene ne' piaceri del corpo e che fosse cattivo, et il Mazzone pareva che tenesse ch'egli avesse avuto sempre buona opinione nelle cose morali e che però egli non fosse tale quale si trova descritto da Cicerone e da Plutarco (...), sopracché contesero un pezzo dov'io conobbi veramente che quel Mazzone era d'una gran lettione e di grandissima memoria e dottrina più che mediocre, et il Tasso avvertito molto et accorto ragionatore.[19]

If confirmation were needed of the Duke's interest in philosophy, Torquato Tasso (who had lived for a certain period at the Urbino court), called Francesco Maria II della Rovere a "Prince educated as a philosopher".[20] This statement is confirmed by the latter's autobiography (written in the third person):

[19] BOP, ms 390, fols. 92r-97v: "We have had the pleasure of many other discussions involving remarkable intellects like that of [Jacopo] Mazzoni of Cesena, [Torquato] Tasso, Pino of Cagli and squire Cesare Benedetti. (...) I first heard that a discussion had begun in front of the Prince upon the arrival of Mazzoni. It was between Mazzoni and the squire Cesare and about the difference that exists between Plato and Aristotle with regard to Reminiscence; Mazzoni tried to defend the opinion of Plato and his followers, and master Cesare took up Aristotle's. Another time, during the festivities and the dances, Tasso and Mazzone had a dispute and I was present. One part of the contention was that Tasso held that Epicure considered the carnal pleasures as the greatest good and that he was evil; Mazzoni appeared to argue that he had always had a high opinion of moral matters and that, therefore, he was not as described by Cicero and Plutarch (...); they argued about this a while, and I truly recognised that this Mazzoni was very well-read, had a most tenacious memory and was of a more than average refinement, and that Tasso is a very sagacious and perspicacious thinker." A part of this passage is presented in Gamba and Montebelli (1988, p. 31).

[20] See the letter written by Tasso to Francesco Maria II in 1578 (Guasti 1852, vol. I, pp. 279–280): "E s'avessi così a parlar con Vostra Altezza come ho a scrivere, non senza molto rossore potrei ragionare: ma la scrittura non arrossa; e con Vostra Altezza posso laudar me stesso, senza noiar Lei in alcuna parte: percioché Ella è così ricca de l'eccellenze e de le laudi convenevoli a principe, ed a principe formato di filosofo, che udendo le laudi de' privati, non ha che invidiare o di che rammaricarsi."

> [Il Principe] ritornò alli suoi studii tralasciati mentre era stato fuori d'Italia, li quali furono prima di matematica lettagli da Federico Comandino, poi di filosofia da Cesare Benedetti, Giacomo Mazzone e Cristofero Guarimone.[21]

The importance attributed to (Aristotelian) philosophy by Francesco Maria II must have been considerable, as we can deduce from his diary:

> A' 25 <gennaio 1585>: detti fine di vedere tutte l'opere d'Aristotele, nelle quali mi ci sono affaticato non meno di 15 anni, essendomi state lette da messer Cesare Benedetti da Pesaro per la maggior parte.[22]

By contrast, his studies of the Bible lasted <4 years.[23] This is even more revealing when one considers that Francesco Maria II was deeply religious.

In short, the Duke's profound interest in Aristotle's philosophy must have had tangible consequences for other members of the courtly milieu as well: in fact, the leading philosophers of the Duchy, Federico Bonaventura and Cesare Benedetti, were proponents of Aristotelian thought.[24]

2.3 The interests of the mathematicians and their interlocutors: common ground with the cultural milieu of the court

Information about Prince/Duke[25] Francesco Maria II's interests in mathematics/mechanics and philosophy might seem, at first sight, hardly relevant to a better understanding of the Urbino mathematicians' scientific environment. In reality, however, the combination of interests in philosophy and mathematics seems to have rubbed off on Guidobaldo, Baldi and Oddi's contacts as well, such as the Count of Carpegna, Curzio Ardizi, Jacopo Mazzoni, Omero Tortora and Pier Matteo Giordani (all of whom were members of the court at the same time).

Tommaso, Count of Carpegna, a regular participant in diplomatic missions on behalf of the Duke, was interested in philosophy and got encouraged to study Aristotle's philosophical writings by his brother-in-law, the aforementioned Federico Bonaventura. At the same time, he was equally interested in mechanics: even though the provenance of Baldi's *Exercitationes* remains obscure, (Baldi 2010; see particularly its introduction). the funeral oration in his honour attests that it was the Count of Carpegna who

[21] BOP, ms 386, fol. 223r/v (written in the third person): "he returned to his studies that were interrupted during his absence from Italy; firstly in the field of mathematics, taught to him by Federigo Commandino, and afterwards in philosophy with Cesare Benedetti, Giacomo Mazzoni and Cristofero Guarimone." Interestingly, the Duke deleted the name "Felice Paciotto" before "Cesare Benedetti". The reason for this deletion remains unclear.

[22] Sangiorgi (1989): "25 [January 1585]: I finished looking over all the works of Aristotle. I have struggled with them for no <15 years, having had them read to me mainly by Cesare Benedetti."

[23] Sangiorgi (1989): "18 [agosto 1587]: Finii di vedere tutta la Bibbia con diversi argomenti, nel quale studio vi posi il tempo di tre anni e dieci mesi."

[24] Furthermore, the Duke's library also reflects the Duke's pronounced interest in philosophy; see Mei, Paoli (2008).

[25] Francesco Maria II became Duke of Urbino in September 1574, succeeding his father Guidobaldo II della Rovere.

asked Baldi to write (what was potentially the first draft of) the commentary on the *Quaestiones Mechanicae*.

Curzio Ardizi, a man of letters and close friend of Baldi and Torquato Tasso's, must have also enjoyed an education in mechanics and/or fortification: a recently discovered document reveals that he was commissioned by Duke Guidobaldo II della Rovere to go to Tunis in order to draw up maps of several fortifications and send back information about their surroundings.[26]

Jacopo Mazzoni, an important interlocutor of Galileo's and famous philosopher of his time who had spent much of 1574/5 at the Urbino court, must have been notably influenced by the climate and interests of the court, as his work *De triplici hominum vita* (finished and printed in 1576, Cesena) reveals: in the preface he thanks Francesco Maria II della Rovere and the dal Monte family for their support. The existence of a 40-pages treatise devoted to fortifications and warfare, in spite of the philosophical character of the work, can be interpreted as a reflection of the influence exerted on him during his period at the della Rovere court.[27]

Omero Tortora, a historian and the author of the *Historia di Francia* (Venezia, 1619) appears to have been similarly influenced by this courtly environment with its orientation towards warfare and mechanics: in his work about French history, he discussed amongst other things the link between cannonball's potential for destruction and on the materials of which they are made. Interestingly, Tortora's reasoning was to become one of the topics of the controversy involving Grassi/Sarsi and Galileo.[28]

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[26] Archivio di Stato Firenze (referred to henceforward as ASF), Ducato di Urbino, I, 127, fol. 1047r: "ho preso ardire di darne anco segno all'Ecc.za V.ra et mandarLe, come fo, il dissegno di Tunesi, della Goletta, del Forte nuovo, et di tutta quella riviera di Barbaria, che designai da quei luoghi più eminenti del Porto Farina, in quel miglior modo che ho saputo (...)". The entire letter is published in Frank (2011/2012, p. 667).

[27] Parts of this treatise are transcribed in Frank (2011/2012, pp. 687–690).

[28] One should not exclude the possibility that Galileo became aware of this text through Guidobaldo himself, who might have spoken to him about the work of Tortora, his disciple. For further information about the controversy, see Galileo (1977, pp. 213–216). Sarsi had written: "Anzi io so che talvolta le palle di piomo lanciate da grandi bombarde si liquefanno nell'aria. Omero Tortora, come modernissimo così diligentissimo scrittore delle cose galliche, dice che talvolta fu inutile la gran forza delle palle lanciate dalle macchine belliche a distruggere le mura perché, essendo prima piccole e di ferro, erano state poi ingrandite con piombo fatto cadere sopra loro: ≪esplodendo infatti, dice, contro le mura, poiché il piombo si liquefaceva nell'aria, solo il piccolo globo interno di ferro, grande quanto un nocciolo, perso l'involucro, arrivava al muro.≫ " Galileo replied in *Il Saggiatore*: "Ma perché non punto deroga di fede né di dignità all'istorico l'arrecare d'un effetto naturale vero una ragione non vera, essendo che all'istorico appartiene il solo effetto, ma la ragione è officio del filosofo; però credendo io al Signor Omero Tortora che le palle d'artiglieria, per essere state incamiciate di piombo, facesser poco effetto nel batter la muraglia nemica, piglierò ardire di negargli la ragione ch'egli, ricevendola dalla commune filosofia, n'adduce; (...) Credo dunque al Signor Tortora, che le palle di ferro covertate di piombo nella batteria di Corbel facesser poco effetto, e che di loro si ritrovasser l'anime di ferro spogliate di piombo; e questo è tutto quello ch'appartiene all'istorico: ma non credo già l'altra parte filosofica, cioè che il piombo si liquefacesse, e che perciò si trovasser nude le palle di ferro; ma credo che giungendo con quello estremo impeto che dal cannone veniva cacciata la palla sopra la muraglia, la coverta di piombo in quella parte che rimaneva compressa tra'l muro esterno e l'interior palla di ferro si ammaccasse e sbranasse, e che l'istesso o poco meno facesse anco l'altra parte del piombo opposta, schiacciandosi sopra il ferro, e che tutto il piombo, dilaniato e trasfigurato, saltasse in diverse bande, il quale poi, imbrattato da calcinacci e perciò simile ad altri fragmenti della ruina, malagevolmente si ritrovasse, e forse anco per avventura non fusse con quella diligenza ricercato, ce richiederebbe la curiosità di chi volesse venire in cognizione s'ei si fusse strutto o pur dilacerato" (Italian translation by F. Flora). I would like to thank Prof. P. D. Napolitani for this suggestion.

And finally, Pier Matteo Giordani, a close friend and interlocutor of Guidobaldo, Baldi and Oddi's, was also interested in both mathematics—it is he who was approached by Baldi for the revision of the *In mechanica Aristotelis problemata Exercitationes*—[29] and philosophy.[30]

So, despite the diversity of professions pursued by the aforementioned mathematicians' friends and correspondents, they have one point in common: their interest in mechanics/fortifications and often, an accompanying interest in philosophy or the humanities. The poet Ardizi who designed maps of fortifications in Tunisia; the historian Tortora who pondered the material properties of cannonballs; the philosopher Mazzoni who included questions related to fortifications in his philosophical writing; the diplomat and enthusiast of philosophy, the Count of Carpegna, who simultaneously entertained an interest in mechanics: all of these sundry individuals and their recurring interest in the same subjects offer a valuable insight into the cultural spirit of the Urbino court.

3 The world of the technicians in the Duchy of Urbino

Several documents testify that the Urbino mathematicians, along with their close ties to the court, were also strongly connected with the engineers and technicians of the duchy.

It is known that the Duchy of Urbino was an important centre of military architecture. In the course of few decades it had generated and hosted several eminent representatives of this field, such as Francesco Maria della Rovere, Francesco di Giorgio Martini, Gerolamo Genga, Giovanni Battista Belluzzi, Francesco Paciotti etc. Like other contemporary architects, the mathematicians Giulio da Thiene, Muzio Oddi and Guidobaldo were also repeatedly employed in their capacity as military engineers.

This aside, various (large-scale) construction projects relating to civil architecture were undertaken in the duchy at the end of the sixteenth and the beginning of the seventeenth century: the construction of the S. Maria degli Angeli church, the *Vecchio Teatro di Corte* (the theatre inside the ducal palace), renovation work at the port of Pesaro, the construction of the ducal Villa Vedetta and modifications at the Imperiale and Mirafiore villas of Duke Francesco Maria II.

A particularly important technical branch in the Duchy was the fabrication of instruments of precision—the workshops in which these were produced were nationally, if not even internationally renowned.[31] In particular, the devices they produced com-

[29] BOP, ms 430, fol. 59r/v, published in Frank (2011/2012). For further evidence of his interest in mathematics, see his correspondences with Guidobaldo, Baldi and Oddi which are conserved in BOP, ms 413, 426 and 430.

[30] An important testament to Giordani's interest in philosophy is provided by the letters he exchanged with Guidobaldo about Aristotle's *Politics*, and the problem of the incorruptibility of the heavens on the occasion of the 1604 nova, published in G. Arrighi, *Un grande scienziato italiano: Guidobaldo dal Monte in alcune carte inedite della Biblioteca Oliveriana di Pesaro*, in "Atti dell'Accademia Lucchese di Scienze, Lettere ed Arte", XII 2 (1965). Further, Giordani was asked by Fabio Albergati to oversee the revision of the latter's *Dei discorsi politici libri cinque* (Roma, 1602), in which Albergati attacked Jean Bodin's *Les six livres de la Rėpublique* (Paris, 1576) and defended Aristotle; see BOP, ms 402, fols. 30–49.

[31] For more on this, see footnote 5.

prised mechanical clocks—which were even offered as gifts to Popes, Cardinals, Kings and Dukes—but also scientific instruments, like planispheres, high precision balances or proportional compasses. It was no coincidence that Galileo had part of his military compasses put together in no other place than Urbino.

In order to give an idea of what the mathematicians' relationship with this environment was like, it is worth tracing the correspondences that Guidobaldo kept in 1583 in which the engineering works that were being conducted to improve the water supply of Villa Mirafiore, and oversight of the fabrication of clocks by craftsmen in the Duke's service were discussed.

On 1 September 1583 the ducal architect, Girolamo Arduini, wrote to the Duke's adviser, Giovanni de' Tommasi:

> Siamo stati il S.r Guid'Ubaldo et io et mastro Lazzaro al Barchetto. Et infatti mastro Lazzaro assicura che l'acqua montarà sicuramente et gettarà la mettà a Mirafiore, et l'altra parte sopra il terrapieno per il mezo di quelle chiavi come dissi a Sua Alt.z Ser.ma. Et abbiamo anco livellato che dal dado del ponte ove si dee pigliar l'acqua sino al piano del terrapieno l'acqua deve ascendere piedi 22 et averà di caduta da detto piano sino al piano del barchetto nanti la casa piedi 30.[32]

The enlargement of a fish tank in the park of Villa Mirafiore had engendered problems with the water supply. After months of vain endeavours to resolve the problem, Guidobaldo was summoned to offer an expert's inspection. As the preceding letter shows, Guidobaldo was accompanied by Arduini and the construction manager of Mirafiore, "master Lazaro".

Guidobaldo must also have been encouraged to report back to Count de' Tommasi, as he wrote to him on the very same day:

> Questa mattina siamo stati al Barchetto, il Cavaliere Arduino e Mastro Lazaro et io. Et ci siamo risoluti che l'acqua potrà andar sul terraglio vicin'alla Porta del Ponte, che se ben il terraglio è più alto che non è la fonte di Mirafiore, nondimeno l'acqua ci andarà. (...)
> Siamo poi venuti a ragionamento tutti tre del sito di far una conserva dove l'acqua si possa radunare accioché la fonte di Mirafiore possi gettar altratanto più acqua, cioè farla gettare in 12 ore quello che la getterebbe in 24. Tutti siamo d'accordo che chi la facesse vicino alla fonte che sarebbe meglio, ma perché bisognerebbe far <la conserva in alto> com'un campanile e ci vorebbe gran spesa, però questa si lascia da parte. Dicevamo per questo che volevamo far detta conserva o in quella possessione dei Frati (credo che siano di Sant'Agostino) per esser in alto il luogo, overo farla più in qua vers'il ponte. Et io gli ho detto che non la faria in nessun di questi luoghi, ma che la farei là su dove è la conservetta che è il principio dove l'acqua comincia a entrar nelli cannoni; perché quello è il più alto luogo che ci sia, e tutti i luoghi sono più bassi di quello. E così l'acqua di lì averà la maggior caduta ch'ella possi aver. Et anche credo che la si farà con minor spesa poi che ne è fatta una parte, et a Mastro Lazaro è piaciuta questa

[32] BOP, ms 434, fol. 58r/v.

> opinione. Dice poi il Cavaliere che mi ha da mostrar non so che altro a Mirafiore che, come io l'avrò veduto, ne darò conto a V.S.[33]

The letter attests to the discussions Guidobaldo had with his collaborators, in particular about the construction of the water reservoir and the water conduits ("cannoni"). One can see that the reduction of financial costs was a clear priority; furthermore, the on-site inspection was no isolated incident ("dice poi il Cavaliere che mi ha da mostrar non so che altro a Mirafiore").

Count de' Tommasi's accepted the proposals of the joint inspection and instructed Arduini on what measures were to be taken:

> Poiché il S.or Guidobaldo, V.S. et mastro Lazzaro dicano che l'acqua montarà, bisogna mo' pensare che'l S.or Duca vuoli che si facci il casino del Barchetto (...). Ho visto quello che V.S. mi dice in materia delli condotti di legno li quali, se fanno danno, è necessario a farli accomodare, né intorno a ciò bisognava aspettar altri commandamenti. V.S. lo facci dunque et quanto prima.
> Nelle cose del condotto di piombo che si ha da fare nel ponte, V.S. potrà essere col S.r Guidobaldo et con il campanaro per sapere la quantità di piombo che si arà da far venire di Venetia (...).[34]

The problem of choosing the appropriate material for the water conduits (wood or lead) dominated the following series of letters. It emerges that Guidobaldo had to accompany Arduini to the manufacturer of the lead conduits ("campanaro", as such also the manufacturer of bells), as the following letter of the latter to Tommasi of 5 September attests:

> Oggi dovevamo essere il S.r Guid'Ubaldo et io con il campanaro et non si è potuto per essere ito fuori, domattina non mancaremo et rissolvere il tutto et anco che fornisca il cannone di piombo per far l'isperienze delle conserve (...).[35]

It is probable that Guidobaldo was also involved in the experiments to see if the leaden conduits transported a sufficient quantity of water ("isperienze delle conserve").

The extant sources also reveal that, at the time of his various inspections at Villa Mirafiore, Guidobaldo also had duties of a different nature to fulfil at the request of the Duke, namely the supervision of the clockmakers' labour. Again, besides Guidobaldo, Count de' Tommasi was involved as the Duke's intermediary, as well as G. Arduini and some clockmakers. In a letter of 30 August, Arduini wrote to de' Tommasi about the difficulty involved in obtaining a certain watchcase in the form of a tortoise ("tartaruca") from a clockmaker called Tortorino:

> Giunto a Urbino parlai al Tortorino, il quale mi disse che non vorebbe manco tempo di un mese a fornire la tartaruca ancorché la vista non gli serviva molto, et che non può per otto dì mettersi a lavorare: perché era lì in procinto di mettere

33 BOP, ms 426, fol. 155r/v. Published by Gamba (1995, pp. 104–105).

34 BOP, ms 434, fol. 98r/v: the letter is not dated, but it clearly refers to the joint inspection of Guidobaldo, Arduini and master Lazzaro.

35 BOP, ms 434, fol. 65r/v.

per monaca una sua nepote; et questo è quanto ho potuto cavare, oltre li prieghi et commandamenti dettoLi in nome di Sua Alt.a Ser.ma.[36]

Faced with this difficulty, Arduini seems to have approached the famous clock-maker, Pietro Griffi of Pesaro. The following letter corroborates that Guidobaldo had the task of controlling the precision of the fabricated clock:

Ho mandato a dire a mastro Pietro orologiero, che dia l'orologio della tartaruca al S.r Guid'Ubaldo, acciò veda se è giusto; il mastro me ha detto che subbito lo portarebbeno et vi sono poi stato io, et ho ritrovato che mastro Pietro è andato a Imola, et non sarà qui prima che domani.[37]

Guidobaldo's afore-cited letter of 1 September informs us that the mathematician did indeed find something in the clock that had to be mended:

Ho poi tenuto la toretta da che Vostra Signoria mi scrisse l'altra Sua, ma non Glene voglio dir altro per adesso perché come torna mastro Pietro gli farò accomodar alcune cosette e poi scriverò in che modo vadano le ore. E Gli bascio le mani. Di Pesaro al primo di settembre del 1583.

Work on the clock then suffered a delay, as we learn from the following letter written at the end of September, again by Arduini to Count de' Tommasi:

Io ho lassato la cura al S.r Guid'Ubaldo che manda l'orologio, il quale è stato fornito questa sera. Infatti l'orefice non può dare fornito le tazze prima delli diece di ottobre, et le sono più che posso al pelo ancorché egli mena le mani et lavoraria no di notte ma m.s Franc.co non li vol dare due libre di candele.[38]

On a similar occasion the following note was sent to Guidobaldo at the behest of the Duke ("Sua Altezza", "S.A.") with a series of instructions on how to construct two clocks, one that needed to be wound every 30 h and another which was destined for Spain (the royal court?) that operated for 26–28 h. It emerges that Guidobaldo was personally instructed by the Duke ("avendo inteso apresso poco l'intentione di S.A.") and that he had to engage even with technical details ("l'assicuri che tirano quelle ore che si delibera").

Che l'orologio senza l'ore et i quarti, et che sveglia, et che se i quarti sonassero con due campane come il tamburo vecchio, gli piacerà assai, che vuol che sia con la [spinala] che sia lavorato all'antica, senza inventione, che tiri 30 ore. Quanto al modello del tamburo, ha da essere come V.S. giudicherà ch'abbi gusto. Et inviarlo a Urbino e in man mia, avertendo ch'è pericoloso il giustar il svegliatolo come è insieme, dove si mostra l'ore et i quarti; nel mandar inanzi et in dietro che non facci sonar l'ore o i quarti senza preaviso. V.S. dunque potrà far una poliza, avendo inteso apresso poco l'intentione di S.A.

[36] BOP, ms 434, fol. 52r.

[37] BOP, ms 434, fol. 57r/v; G. Arduini to Count de' Tommasi, 1 September 1583.

[38] BOP, ms 434, fol. 70r–71r; 27 September 1583.

L'altro <orologio> di Spagna ha da tirar per il manco le 26 hore, ma se fusse 28 saria meglio, perché Mastro Pietro non vuole dire il vero, et però V.S. l'assicuri che tirano quelle ore che si delibera. Quanto alla gente, questo Guem è tanto caro che bisognerà che proviamo a chi ci fa meglio partito, ma che mastro Alessandro non ci impedischi perché Gline torneria male.[39]

In conclusion, the preceding letters demonstrate that Guidobaldo, in spite of being a mathematician by vocation, was entrusted with duties of a technical nature by the Duke. The autumn of 1583, with the repeated inspections of the water supply at Villa Mirafiore and the regular checks on the precision of clocks that characterised the autumn of 1583 do not appear to have been particularly exceptional. Fortune has it that the letter book of the aforementioned Duke's close adviser, Giulio Cesare Mamiani, has been conserved. It contains his letters of 1587 and permits other interesting insights into the tasks Guidobaldo was commissioned with by the Duke: modification of the Villa Mirafiore, supervision of the construction at the port of Pesaro, revisions of Commandino's translation of the *Collectiones Mathematicae* and the composition of a treatise on a particular sundial ("la scrittura che V.S. Ill.ma ha fatto sopra l'orologio"[40])

The resulting engagement with questions of a practical nature and interaction with technical collaborators stimulated his scientific work. This is easily evidenced by an analysis of his manuscript *Meditatiunculae*.[41] Apart from using the manuscript to further his studies of perspective, astronomy and gnomonics, Guidobaldo also addressed practical topics: how to target with a cannon, the advantages and disadvantages of certain kinds of mechanical machines, the maximal and minimal inclination of roofs, and the water intake of a mill. In view of what is known about his practical activities, these reflections appear aimed at an elaboration of Guidobaldo's everyday observations and discoveries as a civil and military engineer and inventor of scientific instruments.

On pages 39–40, Guidobaldo deals with the question of how to target a specific point on a wall with a cannon. Interestingly, he supposes that the cannonball will follow a straight line as its trajectory. On this occasion, however, Guidobaldo is not so much interested in the actual trajectory as in a practical concern he must have encountered himself as a result of his activity as a military engineer: if the cannon is adjusted badly and the cannonball therefore ends up at point f instead of e (see Fig. 1), how can this problem be corrected?

Without going into details, Guidobaldo's suggested solution also attests the close relationship this question had to a practical dilemma: a straw (!) would have to be applied at the aperture of the cannon in order to bring the straw's top g in line with the eye point a and the impact point f. By moving the cannon in such a way that g (taken as an adjustment point) comes to lie on the line ae, the second shot will hit e (Fig. 1).

The entry on pp. 135–136 of the *Meditatiunculae* discusses the advantages and disadvantages of wheels that are vertically or horizontally placed. It shows a different

[39] BOP, ms 430, fol. 217r; the folio reports the recipient (Guidobaldo), but unfortunately neither the date nor the sender.

[40] BOP, ms 211, fols. 108r, 102 r/v, 131r–132v.

[41] Bibliothèque Nationale de France (Paris), ms. Latin 10246; for a transcription and analysis of the *Meditatiunculae* see Tassora (2001).

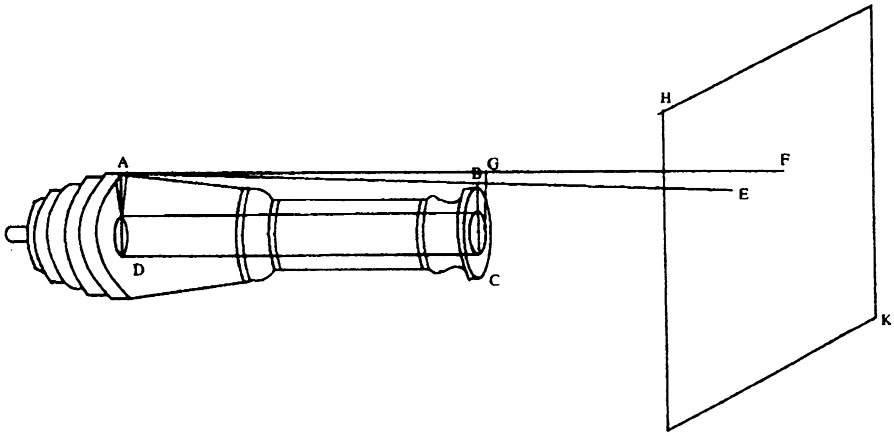


Fig. 1 The problem of a badly adjusted cannon: pp. 39–40 of the *Meditatiunculae* (this and the following figure of *Meditatiunculae* are taken from Tassora 2001)

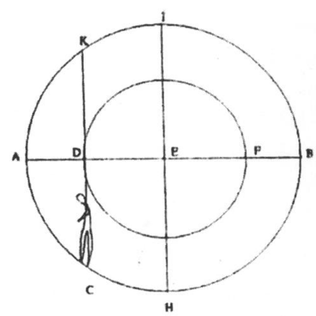


Fig. 2 The representation of a running wheel with a men operating it in C, found on p. 135 of the *Meditatiunculae*

Guidobaldo compared to the one we know from the *Mechanicorum Liber*: instead of dealing abstract geometrical questions to do with machines, he is interested here in the functional and financial aspects of different types of *real* machines. Thus, he discusses the disadvantages of vertical wheels (pointing out the higher cost required to ensure their stability, their shorter lever arm, the impossibility of using animals to power them, etc.) compared to horizontal ones (Fig. 2).

Other manuscript entries (pp. 59–61) show Guidobaldo's considerations of how to reduce the friction that occurs during the operation of machines like pulleys or winches, or pulleys systems (p. 147). This is another question that is not addressed in the *Mechanicorum Liber*, but which is clearly a typical engineering concern.

4 The impact of Guidobaldo's environment on his scientific work

How was the scientific work of the Urbino mathematicians influenced by their interaction with their surroundings? The present section seeks to shed more light on this question, in addition to the information presented in the introductory part of this article. In particular, the recent research into the circumstances surrounding Guidobaldo's work provides us with a detailed example of how these contacts with the Urbino milieu influenced the scholars' scientific activity. For this reason, I shall focus on Guidobaldo in the present section, in particular on three aspects of his work that will help to illuminate the question at hand: the integration of problems raised by his interlocutors into his writings; his use of precision instruments in both theory and practice; and his engagement with questions relating to natural philosophy in his works.

4.1 Guidobaldo's scientific writings: the consideration of problems discussed in his environment

The present paragraph presents two cases that show how Guidobaldo addressed problems that were discussed by those around him in his work.

A particularly interesting entry for our purposes is found on page 6 of the *Meditatiunculae*: it deals with a geometrical problem which consists in showing that two certain lines in a rectangular triangle are equal (see Fig. 3). In the upper left corner Guidobaldo wrote: "Problem proposed by Count Giulio da Thiene". There are some grounds for supposing that Guidobaldo's solution to this question goes back to the early 1570s.[42]

Interestingly, at a later time—probably in the first half of the 1590s—he commented at the bottom of the page:[43] "This problem helps a lot in perspective: if the eye is in a and if one sees the line db, one can find the line fg which appears to have the same size as db, being the section equidistant to de". This comment is very telling: with all probability, Guidobaldo realised the relevance of this problem to perspective when he was working on the *Perspectivae Libri sex* (Pesaro, 1600). Thus, when he addressed the question of how to construct his main work on perspective, he recalled the problem that had been raised several years before by Count Giulio da Thiene and, so, integrated it into his treatise:[44] in fact, Proposition 13 of the *Perspectivae Libri sex*'s first book repeats the very same problem with slightly different wording (Fig. 4).[45]

A second example of Guidobaldo's integration of problems discussed in his social environment is found at the end of the first book of the *Paraphrasis* where he deals

42 Frank, (2013d) *A Proposal for a New Dating of Guidobaldo dal Monte's* Meditatiunculae, in Bollettino di Storia delle Science mathematiche, forthcoming.

43 The use of Italian (as opposed to Latin which is used in the main body of the text) and of different ink to write these lines indicate that this comment did not form part of the initial entry. Given the comment's reference to perspective it seems reasonable to date it to the period when Guidobaldo was studying this branch of mathematics. As his correspondences show, this period can be located in the early 1590s.

44 Note that a consistent part of the *Meditatiunculae* is devoted to studies of the rules of perspective. Guidobaldo simply had to browse through the earlier manuscript pages to find "Count Giulio's problem", while he worked on perspective in the second part of the notebook.

45 The first book of the *Perspectivae Libri sex* is devoted to the question of when certain lines appear bigger than or equal to others in perspective. In the thirteenth proposition Guidobaldo discusses the conditions of the lines' equal appearance; "Problema Propositio XIII" (p. 23) states: "Oculo dato, dataque linea terminata in subiecto plano existente, planum autem per lineam et oculum transiens sit subiecto plano erectum; sectionem subiecto plano erectam invenire, in qua apparens linea datae lineae aequalis appareat et aequalis existat." There he proves that "dico sectionem per EF ductam subiecto plano erectam esse, lineamque EF in sectione ipsi BC et aequalem apparere et aequalem esse."

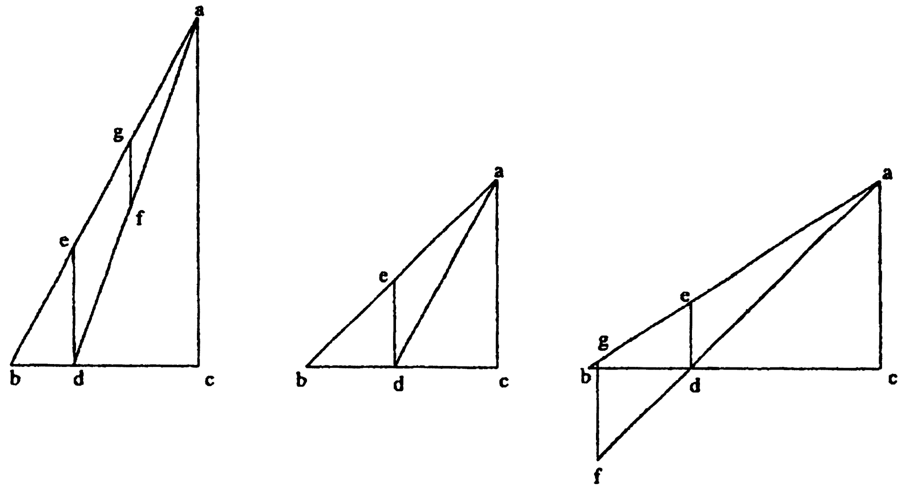
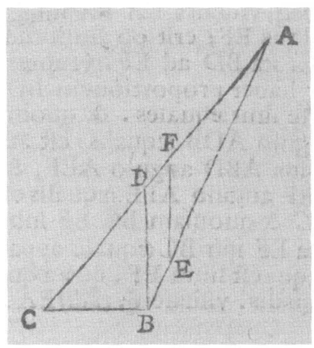


Fig. 3 Guidobaldo shows in the first and third case that gf is equal to bd and in the second that ed is equal to bd (*Meditatiunculae*, p. 6)

Fig. 4 The diagram of Prop. 13 in the first book of the *Perspectivae Libri sex*. Guidobaldo shows that EF is equal to CB, analogously to the first case of Fig. 3



with the following problem: is a plane figure necessarily divided into two parts of equal area by a line passing through its barycentre? He proves that generally the parts after the intersection do *not* have the same area. Leaving aside the wording, the same proof can also be found on p. 116 of the *Mediatiunculae* (Figs. 5).

An idea about the provenance of this problem—one of the rare own theorems that Guidobaldo includes in his commentary on the Archimedean propositions of the *Equilibrium of Planes*—is given by the following letter from Francesco Guerrini, Guidobaldo's disciple, to Clavius, few months after the death of his teacher:[46]

> After the death of the most Illustrious Guido dal Monte, may God rest his soul, several gentlemen of Pesaro asked me to show them the practice of the *Mechanicorum Liber* of the aforesaid Sir, as I am doing. We already finished the first

[46] Archivio della Pontificia Università Gregoriana, 529, fols. 112r–113v; published in Chr. Clavius, *Corrispondenza*, critical edition by U. Baldini and P.D. Napolitani, 7 vols., Pisa, Edizioni del Dipartimento di Matematica dell'Università di Pisa, 1992.

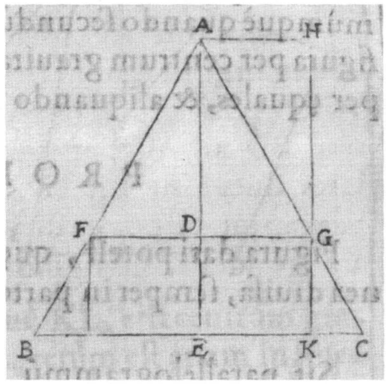


Fig. 5 Guidobaldo shows in the last proposition of the *Paraphrasis*' first book that AFG and $BCGF$ do not have the same area (D being the triangle's barycentre and FG the intersecting line)

chapter *Della Libra* and at the beginning there has been a great controversy about the definition of the centre of gravity, about these words:
≪In fact, if a plane is drawn through this centre, intersecting the figure in an arbitrary way, so it always divides it into equiponderating[47] parts.≫
And if one wanted to insist on the wording "intersecting in an arbitrary way", it would seem that the two parts, after the section, would weigh equally, but in reality the contrary can be proven. (...) I beg You to tell me Your opinion which would be of great use for me (...).

Thus, Guerrini's letter (and the discussions among the "gentlemen of Pesaro") concerns exactly the same problem as addressed by Guidobaldo in the last proposition of the *Paraphrasis*' first book. Formulated in terms of modern physics (in the three-dimensional case), it is equivalent with the question: does the intersection of a body by a plane passing through its barycentre create two parts of equal *weight*, or rather of equal *moment*?

In the light of the latter's activity as a teacher of future engineers, the double occurrence of this problem does not seem coincidental: since the distinction between weight and a kind of (proto-)moment ("aequeponderare") is one of the basic problems of Archimedes' barycentre theory, it appears probable that Guidobaldo encountered this problem at the time of his lessons during which he taught, *inter alia*, the contents of his *Mechanicorum Liber* (and thus, perforce, the foundations of the Archimedean theory as well). In this book, he had cited Pappus' definition of the barycentre, which is closely connected with the problem cited by Guerrini. Thus, as a result of the *Mechanicorum Liber* being taught by Guidobaldo, this problem appears to have come up repeatedly: both when Guidobaldo was working on the *Paraphrasis*—with the result that it was included in his book—and then years later, when Guidobaldo was already dead, during Guerrini's lessons.

[47] This is a neologism to translate the Latin word "aequeponderare", one of the basic notions of the Archimedean barycentre theory. Both the translations "equal" and "of equal moment" would distort the sense of the sentence. For further information in this regard see Frank (2011/2012, pp. 348–349).

4.2 Guidobaldo's use of high precision instruments in theory and practice

We know that Guidobaldo was in close contact with several excellent craftsmen, like the aforementioned Pietro Griffi, one of the clockmakers in the Duke's service, or Simone Barocci, the reputed constructor of scientific instruments (and head of a renowned workshop in Urbino).[48] In fact, Muzio Oddi reports in his *Fabrica et Uso del Compasso polimetrico* (Milano, 1633, proem, pp. 3–4) that while attending Commandino's lessons, Guidobaldo "was often at the place where [Simone] Baroccio worked", and it was there that he developed his new proportional compass, parting from Commandino's model.[49]

Oddi also recounts—this time in *De gli Horologi Solari* (Venezia, 1638, pp. 99–100)—that his teacher had invented a new type of sundial, working with refracted rays of light, and again had enlisted the help of Barocci in its construction.[50]

Nevertheless, Guidobaldo's close relationships with expert craftsmen impacted on more than just his invention of new scientific instruments. Even a crucial element of his mechanical theory seems to be strongly connected with the availability of precision instruments: that is his description of the "Simple Machines" and, in particular, his theory of the indifferent equilibrium.

In fact, in 1580 Guidobaldo exchanged several letters with Giacomo Contarini and Filippo Pigafetta concerning the *Mechanicorum Liber*: the latter two did not succeed in reproducing the ratios between weights and 'forces' that had been predicted by Guidobaldo's geometrical theory of Simple Machines. Thus, he explained to Contarini:

> You should know that, before writing anything about the *Mechanicorum Liber*, I never wanted (so as not to make errors) to consider anything, irrelevant as it may have been, if first I had not seen that the experiment (*esperienza*) agreed exactly with the proof; and of very little thing I have made the experiment. (...) In any case, it is most sure that practice and theory always agree and do not differ from each other. And I tell you even this: the proofs have taught me much about how to make the experiments, regarding which many things have to be considered: firstly, the instruments should be small rather than big; as for example the pulleys with their wheels: if possible they should be made out of brass with very thin, iron axes; and the wheels should be well turned so that they do not waggle round the axes; but if possible, it would be very good if they turned around with just a blow of air.
>
> In fact, the big pulleys which are able to lift heavy weights, are not that adept at telling apart details, as the balances clearly show: in order to distinguish every little detail, one has to use those small ones for weighing coins, and not those

[48] For further information on the Barocci family and its members, see footnote 9.

[49] For further information on the topic of the proportional compass see, *inter alia*, Gamba (1994).

[50] M. Oddi, : "Ben so de' moderni, che l'anno 1572 l'Illustrissimo Signor Guidobaldo de' Marchesi del Monte ne fece fare uno da Simone Baroccio, eccellente artefice, in una mezza sfera d'ottone, e hollo avuto nelle mani molto tempo (...)."

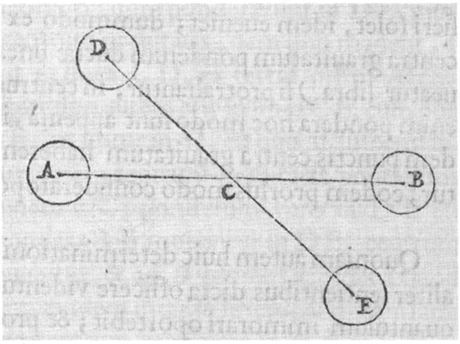


Fig. 6 The abstraction of an isostatic balance, presented in the fourth proposition of the *Mechanicorum Liber*

big ones, with which large objects are weighed like meat or similar things, even if they are precise.[51]

Two aspects of this passage deserve emphasis. The first is the close relation between Guidobaldo's theoretical studies on the one hand and the 'experiences' with his devices on the other ["the proofs have taught me much about how to make the experiments"; "before I have written anything (...) I never wanted (so as not to make errors) to determine anything (...) if first I had not seen that the experiment (*esperienza*) exactly agreed with the proof"]. Secondly, the instruments considered by Guidobaldo were snot the everyday devices used at the marketplace (balances) or at building sites (pulleys): rather, he referred to special instruments, of small dimensions, made out of brass with "very thin" iron axes to reduce friction to a minimum, so that a blow would suffice to make them turn around.[52]

It is interesting to note that even Contarini who was, *inter alia*, a collector of mechanical machines and surely not inexperienced with their working,[53] appears to have been unable to recreate Guidobaldo's precision instruments. On the contrary, one might plausibly suppose that Guidobaldo's instruments—or at least the know-how for producing them—came from Barocci's workshop, just as was the case for the proportional compass and the sundial of refracted rays.

A special—and highly relevant—example of a mechanical precision instrument considered by Guidobaldo is the isostatic balance (whose rotation point lies exactly on its beam, see Fig. 6). In the fourth proposition of the *Mechanicorum Liber* he had (correctly) demonstrated that the equilibrium shown on this balance is indifferent—thus contradicting the theories established by authorities of mechanics like Jordanus and Tartaglia. Since Guidobaldo had perfectly understood the importance of this

[51] Biblioteca Nazionale Marciana (Venezia), It. IV 63 (=Ven. 259); 9 October 1580; published in A. Favaro, *Due lettere inedite di Guidobaldo del Monte a Giacomo Contarini*, in "Atti del Reale Istituto Veneto di scienze, lettere ed arti", LIX 2 (1899–1900), pp. 307–310.

[52] For further information on this topic see Gamba (1995)

[53] For further information on G. Contarini see Rose (1976). On p. 121, Rose writes about a contemporary of Contarini's who praised the latter in these terms: "Of all your distinctions your most wonderful is in mathematical speculations, whereof the rarest and most estimable instruments, so worthily commissioned by you, are to be seen in your house." On the following page, Rose reports an entire list of instruments that were in Contarini's possession.

question—for postulating the impossibility of indifferent equilibrium would imply the invalidity of the Archimedean approach to mechanics based on the concept of the *centre of gravity*—he defended his theory tooth and nail against criticisms and made it a central element of his mechanical theory.[54]

Guidobaldo explained the empirical grounds for his theory in the same letter to Contarini, referring to the scepticism of those who had not had any possibility of testing it with a real model of the isostatic balance:

> I have constructed a balance which most verily shows me that, if its rotation point is to be found in its middlepoint, then it stays still where it was left, in any position to which it was moved, as the fourth proposition *De Libra* in my *Mechanicorum Liber* states. This troubles many scholars who were unable to reproduce it physically.[55]

These isostatic balances were extremely difficult to fabricate, for a minimal divergence from the original would be sufficient to disturb the indifferent equilibrium; Guidobaldo, however, was able to have them fabricated thanks to the specialisation of the workshops in Urbino and indeed, he even sent exemplary models to several of his correspondents who were apparently unable to construct them, so as to convince them of the correctness of his theory.[56]

4.3 Mechanics and natural philosophy in Guidobaldo's writings

Section 2.2 traced an important trait of Guidobaldo's environment: a pronounced interest in philosophy. In keeping with this, some of his closest friends and correspondents where also philosophers (C. Benedetti, F. Bonaventura) who were interested in questions relating to natural philosophy. Not surprisingly, certain passages of Guidobaldo's mechanical writings contain arguments that seem to be influenced by the discussions he had with them.

Guidobaldo's conception that mathematics was *not* sufficient to describe mechanics can already be found in the *Mechanicorum Liber* (1577). He argued that mechanics would need to include elements of natural philosophy alongside mathematics, for instance in the context of the "true motion" of weights:

> In fact, there are some keen mathematicians of our time who assert that mechanics can only be considered either mathematically, or physically; as if mechanics could sometimes be considered either without geometrical demonstrations or without the true motion.[57]

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54 For further information on Guidobaldo's discovery and his strategies for defending it against critics see Frank (2011/2012, Part B, chapter I).

55 "Dove ho anco fatto una libra la quale mi mostra verissimamente che avendo il centro nel mezzo di essa, mossa la libra dove si vuole, sta ferma dove si lascia, come dice la quarta proposizione *De Libra* nel mio libro delle *Mechaniche*, che è cosa che dà fastidio a molti che non l'hanno saputa far materialmente."

56 Again, see Frank (2011/2012, Part B, chapter I).

57 *Mechanicorum Liber*, preface, p. 8*: "Reperiuntur enim aliqui nostraque aetate emunctae naris mathematici, qui mechanicam tum mathematice seorsum, tum phisice considerari posse affirmant; ac si aliquando vel sine demonstrationibus geometricis, vel sine vero motu res mechanicae considerari possint."

Another reference to the connections between mathematics/mechanics and natural philosophy can be found in the *Cochlea* (Venice, 1615). In the posthumously published work Guidobaldo emphasised that mathematics and mechanics shed light on phenomena which at first glance seem to be in contradiction with common sense: like the fact that heavy loads can be moved by exiguous forces.[58]

As for the cochlea, it seems incredible that water, while descending in the screw, in reality moves upwards. Guidobaldo's reference to his philosopher friends is manifest here:

> Who has heard a greater contrariety, not to say contradiction? How can it be accepted that a heavy body freely moves downwards and for this reason goes upwards? Which natural philosopher will ever agree with that? Does not this immediately seem opposed to the senses, to reason, and to nature itself?[59]

Guidobaldo attacks earlier treatments of this subject on account of their failure to explain the *cause* of the water's ascent.[60] The mere (mathematical) description of the cochlea's workings did not suffice in his opinion—a point of view that is echoed in the *Paraphrasis*.

It is mainly in this commentary on Archimedes' *Equilibrium of Planes* that Guidobaldo illustrates his view on the interrelation of aspects relating to mathematics and natural philosophy in the discipline of mechanics. In the preface to the *Paraphrasis* the conception of mechanics as composed of two parts is emphasised and considerably developed. The first one is described as a "natural" one, as mechanics refers to phenomena set in nature ("*naturalia*"), whilst the other is a "mathematical" one, as it makes recourse to notions like *distance* or *ratio* belonging to mathematics. The two fields would be represented by their major exponents—Aristotle as the authority on natural philosophy and Archimedes as the most excellent mathematician.

But Guidobaldo's conception of the relation between Archimedes and Aristotle goes beyond attributing to them an equal equipollent status: he presents a kind of concordism, claiming that in his axioms Archimedes followed what Aristotle had shown and that they agree also in their perception of mechanics as subdivided in

58 Other examples concerning geometry that are cited include: the existence of greater and smaller magnitudes than another given magnitude would suggest that it is possible to find a magnitude equal to the latter. As geometry shows, however, this does not necessarily hold true, as in the case of an angle formed by a line and a circumference: it is either bigger or smaller than a right angle, but never equal to it; or consider two entities that steadily get nearer to each other: it would seem intuitive that they would meet after a certain time. Yet, as the hyperbole and its asymptotes show, this is not true.

59 *Cochlea*, p. 2: "Quis maiorem repugnantiam, ne dicam contradictionem intellixit? Quomodo concedi potest grave aliquod sponte deorsum moveri, et ob id sursum tendere? Quis unquam naturalis philosophus concedet hoc? Non ne statim hoc sensui, rationique repugnare, atque ipsimet naturae contrarium esse videtur?"

60 *Cochlea*, p. 4: "Nam fateor quidem omnes de hac cochlea multa dixisse, sed praecipua quaedam, quae ad instrumenti huius cognitionem perfectam pertinere videntur, omnino praetermisisse. Etenum docent quidem, sive potius tantum affirmant (hoc enim sensu percipitur) hoc instrumentum aquam sursum attolli: qua vero ratione id contingat non docent. (...) nemo unquam hanc cochleam, ut eius cognitio expostulat, declaraverit; ac non solum: non declaraverit, sed (quod ipse viderim) nec artificium, quod in ipsa inest, cognoverit, nullum enim prospectum habetur, eius ignorata causa."

the two fields. Consequently, although Guidobaldo was undoubtedly a follower of Archimedes' mechanical theory, he did not consider Aristotle's approach as inferior:

> In fact, at the beginning of the *Quaestiones Mechanicae*, Aristotle gave many extraordinary clues for discerning the *causes* of mechanical phenomena. In his writings, Archimedes followed him and brought to light the principles of mechanics more clearly, making them even more intelligible. But Aristotle is not diminished in stature for this reason: in fact, he masterfully explained the *causes* behind the problems that he had presented and discussed. (...) for example, Aristotle asks why we move heavy weights with a lever. And he replies that the *cause* is the greater length of the law on the side of the force: and he certainly is right.[61]

This passage might shed some light on Guidobaldo's surprising[62] evaluation of the respective importance of Archimedes and Aristotle to mechanics: it seems that he interpreted Aristotle's approach as an explanation of the *causes*[63] of the mechanical phenomena—correspondingly to the "task" of philosophy: the search of the causes. Archimedes, in turn, would have dealt with the mathematical description of the phenomena, and reached a more complete formalisation in this regard.

Another highly interesting argument in the *Paraphrasis*' preface is the explanation of the barycentre's properties. In his reasoning, Guidobaldo makes recourse to the Aristotelian conception of the cosmos (see above: the "true motion"), in order to justify his integration of the Pappian definition of the *center of gravity* into his own mechanical theory.[64]

Thereby, the basic assumption is that a *heavy body* is at rest in the centre of the world. Thus, all the parts must have equal moments with respect to the point that coincides with the centre of the world. Otherwise, one part would outbalance the other and produce movement, thus contradicting the hypothesis of the body's rest at the centre of the world. And this point is, according to Commandino's definition, the centre of gravity.[65] So, saying that a body moves to the centre of the world *naturali*

[61] *Paraphrasis*, p. 4: "Aristoteles enim in principio *Quaestionum Mechanicarum* multa, eaque praecipua ad *causas* rei mechanicae dignoscendas aperuit. Quem secutus Archimedes in his libris mechanica principia explicatius patefecit eaque planiora reddidit. Nec propterea Aristoteles diminutus exstitit: etenim eorum quae ab ipso proposita et explicata fuere, problematum *causas* egregie patefecit. (...) Aristoteles enim (gratia exempli) quaerens cur vecte magna movemus pondera. *Causam* esse ait longitudinem vectis maiorem ad partem potentiae: et recte quidem." The emphases are mine.

[62] Guidobaldo's conception might at first sight seem surprising, if one considers that his mechanical work shows a profound engagement with the central Archimedean concepts while his references to Aristotle seem to be limited to some scattered citations of the *Quaestiones Mechanicae*.

[63] Note the triple occurrence of the word "*causa*" in this short passage.

[64] Pappus' definition, presented at the beginning of the *Collectiones Mathematicae*'s eighth book and reported by Guidobaldo both in the *Mechanicorum Liber* and *Paraphrasis*, reads: "Centrum gravitatis uniuscuiusque corporis est punctum quoddam intra positum, a quo, si grave appensum mente concipiatur, dum fertur, quiescit; et servat eam, quam in principio havebat positionem, neque in ipsa latione circumvertitur."

[65] Commandino's definition of the centre of gravity, contained in his *Liber de centro gravitatis solidorum* (Bologna, 1565), reads: "Centrum gravitatis uniuscuiusque solidae figurae est punctum illud intra positum, circa quod undique partes aequalium momentorum consistunt. Si enim per tale centrum ducatur planum figuram quomodocunque secans semper in partes aequeponderantes ipsam dividet."

propensione, this also means that the body wants to make coincide the centre of the world and its centre of gravity. Since it is gravity that generates the natural propensity and the movement of a heavy body towards the centre of the world, and since, in fact, it is the body's centre of gravity that really unifies with the centre of the world, it can be said that all bodies possess weight exclusively in their own centres of gravity. Now, if then an arbitrary body is held in its centre of gravity, it has to stand still insomuch as the reason for its movement, namely gravity, does not act under these circumstances. And hence the statement of the Pappian definition of the barycentre, derived as it is from central elements of Aristotle's cosmology. Guidobaldo could have been sure that this authority gave the necessary credibility to the definition, so crucial in his theory of the isostatic balance.

According to Guidobaldo, Archimedes also agreed with the conception of mechanics as composed of a mathematical part and a natural one, and so the explanation of the barycentre's properties in cosmological/philosophical terms was permissible, if not necessary:

> In fact, the aspects that have to be considered mathematically have been proved by Archimedes by means of geometry: like distances, ratios and so on. In contrast, what is related to nature (*naturalia*), has been treated by him in a way that is appropriate to nature (*naturaliter*): such as the arguments which concern the centre of gravity, or the objects that have to move upwards or downwards, and so on.[66]

The cosmological explanation of the barycentre had implications that were important for far more than just its centre of gravity: since this is the basic notion of Archimedes's theory, its reliance on the general conception of the cosmos and of natural philosophy meant above all: that Archimedes's mechanics as part of the Aristotelian cosmos. In this way, Guidobaldo brought into effect his conception of mechanics as a composite discipline.[67]

Another curious argument in this regard is Guidobaldo's reference to *light bodies* (in the Aristotelian sense) in the preface. Though it obviously lacks any real application in mechanics, it is perhaps the best example of how his considerations of philosophical concepts are integrated with his study of mechanics.

The reference is made as part of a discussion of the main subject of Archimedes's *Equilibrium of Planes*, namely *plane figures*, i.e. objects without gravity, mathematical abstractions. As the preceding paragraph has shown, however, Guidobaldo did not agree with a conception of mechanics that confined itself to the consideration of geometrical problems. So, these objects and their relation to reality constituted a

66 *Paraphrasis*, pp. 4–5: "Nam quae mathematice sunt consideranda, geometrice <Archimedes> demonstravit, ut sunt distantiae, proportiones et alia huiusmodi. Quae vero sunt naturalia, naturaliter quoque consideravit; ut ea, quae ad gravitatis centrum spectant, et quae sursum et quae deorsum moveri debent et cetera huius modi."

67 It must be noted that in Archimedes's extant writings there is no trace of such a connection between mechanics and natural philosophy. As on other occasions, Guidobaldo interpreted the Archimedean theory according to his *own* preconception and consequently used it, to a certain extent, for his own purposes. Thus, as much as Guidobaldo's approach to Archimedean mechanics can be characterised as close to the text, it has to be admitted that the interpretation Guidobaldo gave to it changed its meaning.

serious problem for Guidobaldo, since mechanics had to refer to objects with a precise meaning in natural philosophy.

He found an ingenious solution for this dilemma: in keeping with their lack of gravity, plane figures could be treated similarly to *light bodies*. A centre of *lightness*—analogous to the centre of *gravity* for *heavy bodies*—could be assigned to these objects: an idea that would have found support in Aristotle and Ptolemy who would have attributed *moments* to *heavy* and to *light bodies* alike. In keeping with Pappus' definition of the barycentre, light bodies held in their centre of lightness would stand still. The geometrical treatment of the centre of lightness would be analogous to that of the centres of gravity. By extension, also the treatment of plane figures does not differ from that of real bodies with a barycentre.

In conclusion, as the present section has striven to show, several central aspects of Guidobaldo's scientific activity were clearly influenced by his interaction with the cultural and scientific environment of the Duchy of Urbino; and research into his social surroundings contributes to a better understanding of his mathematical work.

As the introductory part sought to show, analogous situations hold true for other mathematicians who were active in the duchy as well, such as Federico Commandino, Bernardino Baldi and Muzio Oddi. Furthermore, studies I am conducting on Giovanni Battista Benedetti's scientific work at the court of Turin show a similar framework: that of a close relationship between mathematics, technics and courtly life in the late Renaissance.

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