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Displaced tables in Latin: the Tables for the Seven Planets for 1340

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Abstract The anonymous set of astronomical tables preserved in Paris, Bibliothèque nationale de France, MS lat. 10262, is the first set of displaced tables to be found in a medieval Latin text. These tables are a reworking of the standard Alfonsine tables and yield the same results. However, the mean motions are defined differently, the presentation of the tables is unprecedented, and some new functions are introduced for computing true planetary longitudes. The absence of any instructions as well as unusual technical terms in the headings make it difficult to appreciate the cleverness that went into the construction of these tables that are extant in a unique copy. In this article we provide a detailed analysis of these tables and their underlying parameters.

The displaced tables are typical of a pervasive tendency in Islamic science to provide extensive and elegant numerical tables for the convenience of practitioners. The underlying astronomical theory is neither questioned nor affected.[1]
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[1] Kennedy (1977, p. 16).

1 Introduction

In 2012 we published *A Survey of European Astronomical Tables in the Late Middle Ages* in which we discussed a wide range of tables, many of which were previously known. In this monograph we mentioned an unusual set of tables, whose significance had not been appreciated hitherto, that depends on a principle of displacement to eliminate subtractions in the course of computing planetary positions. This principle was employed in some zijes in the Islamic world, but until now there was nothing comparable in the Latin West. As we will see, this set of tables is a most ingenious reworking of the Parisian Alfonsine Tables, rather than a translation of an Arabic zij (where the term zij is used in Arabic to refer to a set of astronomical tables with instructions for their use). What makes this set of tables so unusual is that, for example, the mean planetary motions are defined differently from those in the standard Alfonsine tables, and some of the functions for computing true planetary longitudes from their mean motions also differ noticeably from those in the standard Alfonsine tables; nevertheless, computation with these displaced tables yield the same results as computation with the standard Alfonsine tables. Moreover, the tables for first station are presented in a completely different way from those we have seen in the Ptolemaic tradition, although the underlying parameters are unchanged. Given the absence of instructions, it was not an easy task to unravel the cleverness of the construction of these tables. We had some helpful guidance from a paper by Kennedy (1977), but it was not sufficient for uncovering various subtleties in these tables for which there are no counterparts in the tables that Kennedy analyzed.

Paris, Bibliothèque nationale de France, MS lat. 10262, is a fifteenth-century manuscript containing two sets of astronomical tables: an anonymous set for 1340 (ff. 2r–46v); and another set, called *Tabule frequentine* (ff. 47v–71r), composed by Melchion de Friquento of Naples in 1438 (f. 46v: 1437 *completus*).[2] Neither set has previously been examined. In this paper we focus on the first set, entirely composed of tables with no accompanying text. At the bottom of the last table (f. 46v), we read: *Expliciunt tabule de septem planetis et de veris locis eius* (Here end the tables for the seven planets and their true positions). Therefore, we shall refer to this set as the Tables for the Seven Planets (where the Sun and the Moon are considered planets). The yearly tables begin in March, and we are told that the tables are valid for 1340 *completus*, or 1340 (complete), meaning that the epoch is noon of February 28, 1341, the last day of the year, counting from March 1, 1340.[3]

The name of the author of these tables is not given in the manuscript, and no locality is mentioned. However, on f. 9r, at the bottom of a table for the mean motion of the Sun, we are told that the radix for the Sun, *anno domini 1340 completo*, is 8s 16;41,49° (=256;41,49°). In contrast to the use of physical signs of 60° in the Parisian

[2] For a description of this manuscript, see Thorndike (1957), especially pp. 144–145. Richard L. Kremer informs us that he is preparing a study of the *Tabule frequentine*, tentatively called "Melchion de Friquento's eclipse tables of 1437, a revised Latin version of Immanuel Bonfils's Six Wings".

[3] The last day of February belongs to 1340 according to the convention of this text (in fact, it is the last day of that year), but to 1341 according to our modern convention (it is the 59th day of this year). The same convention applies to all subsequent years in this text. Note also that the common astronomical practice in the Middle Ages was to take noon as the beginning of the day.

Alfonsine Tables (PAT), we note the use here of zodiacal signs of 30°. If we compute the mean motion of the Sun using the PAT for Toledo, we find its mean longitude to be 346;20,53° and its mean argument of center, 256;43,54°.[4] Therefore, the given radix refers to the mean argument of center of the Sun. The difference in the mean argument of center, 0;2,5°, is the amount traveled by the Sun in about 0;50h, which corresponds to a difference in geographical longitude of about 12;40° east of Toledo. This figure agrees, although not exactly, with the longitude of the meridian of Paris, taken as 12;0° east of Toledo in many astronomical tables of the fourteenth and fifteenth centuries.[5]

The Tables for the Seven Planets give multiple examples of displaced tables. A table is said to be displaced with respect to another when its entries are the same as those in the standard table after adding a constant to its argument (horizontal displacement), or derive from the entries in the standard table by adding a constant (vertical displacement). This can also be expressed in algebraic terms: if $y = f(x)$ is the function underlying a given table, then the function embedded in the displaced table is $y = f(x + kh)$, for a displacement on the X-axis, or $y = f(x) + kv$, for a displacement on the Y-axis. Of course, both displacements can occur at the same time, leading to an equation such as

$$y = f(x + kh) + kv. \tag{1}$$

The purpose of displaced tables is to avoid subtractions, that is, the use of complicated rules for handling negative numbers before they were available to astronomers who computed planetary positions by means of astronomical tables.[6]

This problem was already felt by astronomers in the ninth century, who called the standard table *aṣlī* and the displaced table *waḍ*^{c}*ī*.[7] Displaced tables were clever computational devices, with implications for the method of computation of astronomical quantities, but did not challenge either the parameters or the models on which the original tables were based. Kennedy (1977) presented an explanation of displaced planetary tables in the medieval Islamic world, and demonstrated the equivalence of computations using one such set with computations using Ptolemy's tables. Kennedy focused on the tables of Ibn al-A^{c}lam (d. 985) and called attention to the fundamental relation of three displacements for each planet (see Eq. 26, below).[8] Byzantine astronomers apparently depended on their Muslim predecessors for displaced tables:

[4] According to this computation, 89;36,59° is the longitude of the solar apogee, λ_A, which is the difference between the solar longitude and the solar mean argument of center at a given time.

[5] See, e.g., Kremer and Dobrzycki (1998).

[6] It is important to distinguish between displaced tables and shifted tables. A shifted table contains the same information as the standard table (the same columns and rows, and the same entries) but the first row is not that for argument 0° or 1° but for some other convenient number, to stress the fact that the entry reaches a specific value such as a maximum or a minimum for that particular value of the argument. Shifts occur, for instance, for the planetary latitudes of certain planets in John of Murs's Tables of 1321 (Chabás and Goldstein 2009, see esp. pp. 308–309).

[7] Salam and Kennedy (1967, p. 497). See also Debarnot (1987, p. 43 (Table 9)) and Jensen (1971).

[8] Displaced tables in Islamic astronomy are also discussed in Jensen (1971), Saliba (1976), Saliba (1977), Mercier (1989), and Van Brummelen (1998). For displaced tables in the Maghrib, see Samsó and Millás (1998), and Samsó (2003).

see Tihon (1977–1981, espec. 68:76 and 110). As far as we can determine, none of the displaced tables compiled by astronomers in the Islamic World and Byzantium were known in the West. Two Jewish astronomers in Provence, Levi ben Gerson (d. 1344) and Immanuel ben Jacob Bonfils of Tarascon (*fl.* 1350), composed zijes in Hebrew in which they used the principle of displaced tables for the times and longitudes of syzygies (but not for the motion of the planets); however, there is no evidence to suggest that they depended on Islamic or Byzantine sources.[9] In astronomy written in Latin the tables described here are the first to use displaced tables systematically, although we know of an earlier use of this type of table by John Vimond around 1320, limited to his table for trepidation (or "access and recess").[10]

As mentioned above, the Tables for the Seven Planets have no accompanying text; therefore, the following comments are based entirely on the information provided by the tables themselves. As will be seen, the terminology in these tables often differs from that commonly used at the time, and many of the tables have a presentation that diverges from other tables with the same parameters that are based on the same model.

There follows a table of contents, arranged by section number.

Displacements are discussed in Sects. 4, 6, 8, 10, 11, and 12.

[9] For Levi ben Gerson, see Goldstein (1974, pp. 136–146, 229–241); for Bonfils, see Solon (1970, pp. 3–4, 11) and Kremer (see n. 2). It is noteworthy that the *Tabule frequentine* in this very same manuscript contain an adaptation of Bonfils's tables, where displaced tables occur.

[10] On Vimond, see Chabás and Goldstein (2003, pp. 275–277) and Chabás and Goldstein (2004, pp. 265–267).

2 Multiplication table

The first table in this manuscript (ff. 2r–7v) is a multiplication table for base 60 arithmetic, with entries for each integer from 1 to 60. This table, presented as a 60 × 60 square matrix, is quite common in manuscripts containing sets of astronomical tables (see, e. g., Chabás and Goldstein 2012, p. 227). The fact that f. 8r–v is blank makes it uncertain if this multiplication table belongs to the set examined here, for it could very well have been inserted by the copyist to facilitate computation.

3 Mean motion of the Sun

Folio 9r contains five tables for the mean motion of the Sun. The first lists the *Radices ad 32 annos post annum 1340*, i.e., the values of the mean motion of the Sun at the beginning of each year for a period of 32 consecutive years. The entry for year 1 is 8s 16;26,51°, and it corresponds to the mean argument of center of the Sun at noon of February 28, 1342; the entry for year 32 is 8s 16;36,12°. It should be emphasized that a period of 32 years for the motion of the Sun is very unusual; in any case, we have not found it in the previous literature, where periods of 20, 24, and 28 abound (see Chabás and Goldstein 2012, p. 54). We also note that the usual phrase *anni expansi* (expanded years, that is, those within a cycle, in contrast to *anni collecti* for the years at the interval of a cycle) does not appear here or in the headings of the other tables.

The other four tables for the Sun have a general title, beginning *Tabula porcionis solis ad annos radicum...*, where *porcio* has to be understood here as "argument of center".[11] The second table on this folio gives the mean motion of the Sun for accumulated months; its first entry corresponds to March (1s 0;33,18°) and the last one to February (11s 29;45,39°). These entries agree exactly with those given by John of Lignères in his *Tabule magne* for a year beginning in January (see Chabás and Goldstein 2012, Table 5.1B, pp. 55–56), who also used zodiacal signs of 30°. The third table is headed *ad annos perpetuacionis*, from 1372 to 1852, at 32-year intervals, and again we see the use of non-standard terminology, for the phrase *anni collecti* is totally absent from this set of tables. The fourth and fifth tables are, respectively, for the mean motion of the Sun for days from 1 to 31, and for hours and fractions of an hour from 1 to 60. The entry for argument 1d is 0s 0;59,8°.

As mentioned above, at the bottom of the folio we are told that the *Radix porcionis solis ad anno domini 1340 completo* is 8s 16;41,49°. From this value and the entry for year 32 (8s 16;36,12°) we derive a mean motion in the argument of center of 0;59,8,13,32,49°/d. As indicated previously, the entries in these tables represent the

[11] We know of no prior usage of *porcio* for the argument of center of the Sun. There are, however, examples where it means "anomaly" in the case of the planets. On *porcio* (or *portio*) in the sense of "anomaly", see Goldstein et al. (1994, pp. 63–64) (see also Nallino 1903–1907, vol 2, p. 328), where a reference is given to John of Lignères's use of this word in the canons to his tables, whose incipit is *Priores astrologi motus corporum celestium* (1322). The Arabic for anomaly is *ḥiṣṣa*, which means "portion" and one of the Hebrew terms for anomaly, *manah*, also means "portion". Both the Latin and Hebrew terms apparently come from Arabic. The fact that in our text the argument of center of the Sun is considered the "anomaly" indicates that the author had in mind an epicyclic model for the Sun, which is equivalent to the eccentric model according to Apollonius's theorem (see *Almagest* III.3; Toomer 1984, pp. 141–153).

mean argument of center of the Sun, $\bar{\kappa}$, and this is another peculiar characteristic of the Tables for the Seven Planets, because in other sets the mean solar motion, that is, $\bar{\lambda}$, is usually tabulated. The two quantities are related by means of λ_A, the longitude of the solar apogee, for $\bar{\lambda} = \bar{\kappa} + \lambda_A$. A small table, found later in the text (f. 17r), gives the motion of the solar apogee. It is entitled *Motus augium equatus* and the entries are yearly values, probably corrected for precession. The entry for 32 year is 0;19,42,35°, which implies a daily motion of 0;0,0,6,4°/d. By adding this value to the one derived for the mean motion in the argument of center of the Sun, the result is 0;59,8,19,37°/d, in good agreement both with the value used by Vimond (0;59,8,19,37,4°/d) and the value in the PAT (0;59,8,19,37,19,13,56°/d) for the mean solar motion in longitude.[12] It would thus seem that the Tables for the Seven Planets belong to this tradition. We will show that the rest of the tables provide additional evidence of this relationship.

4 Solar equation

The solar equation is given on ff. 9v–10r. The title of the corresponding table (*Tabula equacionum solis cum auge eius equata*) indicates that the value for the solar apogee has already been taken into consideration. The argument, which we call $\bar{K}$, in parallel with the standard usage of $\bar{\kappa}$,[13] begins at 0°, not at 1° as was the common practice at the time. The entry for 0° is 2s 29;37,9° (=89;37,9°), and corresponds to the longitude of the solar apogee at epoch, 89;36,59° (see n. 4). The minimum 2s 27;27,9° is reached at 92°–94°, and the maximum 3s 1;47,9° at 267°–269°. Subtracting algebraically either of these two values from the entry for 0°, one gets 2;10,0°, which is the standard Alfonsine parameter for the maximum solar equation.[14] Therefore, all entries for the solar equation, $C(\bar{K})$, are displaced upwards by 89;37,9°, compared to the entries in the standard tables in the Alfonsine corpus, $c(\bar{\kappa})$, to make all entries positive, thus allowing the user to avoid dealing with a set of complicated rules for adding and subtracting various terms:

$$C(\bar{K}) = c(\bar{\kappa}) + 89;37,9^\circ. \tag{2}$$

This is the first example of a displaced table in this set, where the argument is the mean argument of center, $\bar{K}$, and the entries represent $C(\bar{K})$, which is the sum of the Alfonsine solar equation (whether positive or negative), $c(\bar{\kappa})$, and the longitude of the solar apogee, λ_A. In this case the displacement is vertical (see Fig. A).

The canons to this table, which are not extant, should have had an instruction indicating something like "to find the true position of the Sun, enter the table with the

12 For Vimond, see Chabás and Goldstein (2004, p. 221); for the PAT, see Ratdolt (1483, f. d5r). For a modern edition of these tables, based on Ratdolt (1483), see Poulle (1984).

13 Throughout this paper capital letters are used to represent variables and functions in the Tables of the Seven Planets.

14 As in the tables of John Vimond (Chabás and Goldstein 2004, p. 223), the solar equation—with the same maximum of 2;10°—is not explicitly given. For the maximum solar equation, 2;10°, in the PAT, see Ratdolt (1483, f. e3r).

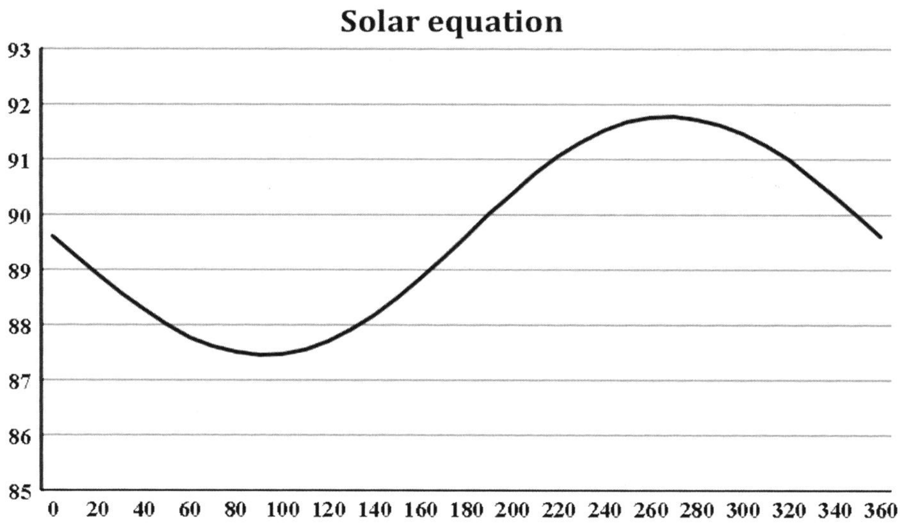

Fig. A The solar equation displaced vertically

mean argument and add what you find to it" or, in algebraic terms,

$$L = \bar{K} + C(\bar{K}). \tag{3}$$

And, indeed, in Ptolemy's model for the equation of the Sun, λ, the true longitude of the Sun is obtained adding its mean longitude, $\bar{\lambda}$, to the solar equation: $\lambda = \bar{\lambda} + c(\bar{\kappa})$. Now the solar equation is also the difference between the true argument of center of the Sun, κ, and its mean argument of center, $\bar{\kappa}$: it is negative if $0° \leq \bar{\kappa} \leq 180°$, and positive if $180° \leq \bar{\kappa} \leq 360°$. The Tables of the Seven Planets yield the same result as that deduced from Ptolemy, for

$$\begin{aligned}\lambda &= \bar{\lambda} + c(\bar{\kappa}) \\ &= (\bar{\kappa} + \lambda_A) + c(\bar{\kappa}) \\ &= \bar{\kappa} + (\lambda_A + c(\bar{\kappa})) \\ &= \bar{K} + C(\bar{K}) = L,\end{aligned}$$

provided that $\bar{K} = \bar{\kappa}$, which is the case here. The two terms in L are positive and both are explicitly tabulated on ff. 9v–10r.

5 Length of daylight, diurnal seasonal hours, and the equation of time

On ff. 10v–11v there is a table with a column for the argument from 1° to 30°, and three other columns for each of the twelve zodiacal signs, beginning with Aries. The headings are *Hora equalis, tempus horarum*, and *equatio dierum*. The first entries for Aries 1° are 12;3h, 15;4°, and 8;28 min, respectively. The entries in the first column, under *Hora equalis*, display the length of daylight (i.e., the time interval from sunrise to sunset) for a given locality; it reaches a maximum, 15;10h, at Gem 28° – Cnc 2°, and a minimum, 8;50h, at Sgr 28° – Cap 2°. If we consider the obliquity of the ecliptic, ε, to be 23;33°, the resulting geographical latitude, φ, for which the table is valid is $\varphi = 42;44°$, which agrees fairly well with the parallel through Toulouse (rather than with that of Paris, where the longest daylight is 16;0h). The geographical longitude (see above) and latitude that we derive from the tables do not yield good agreement with any place where astronomy was practiced in the fourteenth century, but a locality approximately fitting both computed coordinates is Perpignan.

The entries in the second column, under *tempus horarum*, display the length of a diurnal seasonal hour (i.e., a twelfth of the length of daylight); it reaches a maximum, 18;57h, at Gem 27° – Cnc 3°, and a minimum, 11;3h, at Sgr 27° – Cap 3°. These two columns are mutually consistent.

The third column displays the equation of time (i.e., the difference between apparent and mean time where apparent time is counted from true noon, that is, the moment that the true Sun crosses the meridian, and mean time is counted from mean noon). The argument is the solar longitude expressed in degrees and the entries are given in minutes and seconds of an hour. The extremal values are 0;21,24h (at Tau 26°), 0;11,16h (at Leo 5°), 0;31,48h (at Sco 8°), and 0;0,0h (at Aqr 20°–23°). When converting these values into time-degrees, that is, multiplying each entry by 360°/24h, we obtain 5;21°, 2;49°, 7;57°, and 0,0°, respectively, which are the characteristic values found in the equation of time ascribed to Peter of Saint Omer and used by John of Lignères, among others (see, e. g., Chabás and Goldstein 2012, pp. 37–40).

6 Mean motions of the Moon

The mean motions of the Moon are addressed in five tables on ff. 12r–13r. In addition to columns for the various arguments, each table has three columns headed *centrum lune* (here meaning double elongation), *porcio lune* (argument of lunar anomaly), and *medius locus lune* (mean longitude of the Moon). Again, for the first two quantities this is not the standard terminology.

The first table lists the radices at the beginning of each consecutive year from 1342 for a period of 32 years. The entries for year 1 are 6s 3;36,2° (double elongation), 3s 17;36,22° (anomaly), and 8s 10;16,38° (longitude). The second table gives the mean motion of the Moon for the months; the first entries, for March, are 1s 5;49,36° (double elongation), 1s 15;0,53° (anomaly), and 1s 18;28,6° (longitude). The third and fourth tables are, respectively, from 1372 to 1852, at 32-year intervals, and for hours and parts of an hour, from 1 to 60. The fifth table is for days from 1 to 31, and the first entries for argument 1d for the three variables are 0s 24;22,54° (double elongation), 0s 13;3,54° (anomaly), and 0s 13;10,35° (longitude), confirming that the first column is indeed the double elongation, for 0s 24;22,54° $= 2 \cdot$ (0s 13;10,35° – 0s 0;59,8°).

As was the case for the Sun, at the bottom of f. 12v we are given the radices for the three variables, *anno Christi 1340 perfecto*: 9s 14;21,15,14° (double elongation), 0s 18;53,8,0° (anomaly), and 4s 0;53,34,57° (longitude). We note that the values are given here to thirds and that the last digit for the anomaly is “0”, indicating that the

Table 1 Recomputation for the Moon at epoch: noon, February 28, 1341

	Text	Computation	Text − comp.
Mean argument of center of the Sun	8s 16;41,49°	256;41,46°	0; 0, 3°
Double elongation	9s 14;21,15,14°	284;20,13,41°	0; 1, 1,33°
Mean argument of anomaly	0s 18;53, 8, 0°	32; 1,34,53°	−13; 8,26,53°
Lunar mean longitude	4s 0;53,34,57°	128;29, 2, 0°	−7;35,27, 3°

Table 2 More detailed recomputation for the Moon at epoch: noon, February 28, 1341

	Text	Computation	Text − comp.
Mean argument of center of the Sun	8s 16;41,49°	256;41,49°	0; 0, 0°
Double elongation	9s 14;21,15,14°	284;21,14,39°	+0; 0, 0,35°
Mean argument of anomaly	0s 18;53, 8, 0°	32; 2, 7,32°	−13; 8,59,32°
Lunar mean longitude	4s 0;53,34,57°	128;29,34,57°	−7;36, 0, 0°

author was sensitive to accuracy. From the values of the radices and those for year 32 in the first table for the mean motions of the Moon, we derive the following values for the mean motions: 24;22,53,23,16°/d (double elongation),[15] 13;3,53,57,30°/d (anomaly),[16] and 13;10,35,1,15°/d (longitude).[17] All of them are typical parameters of the PAT (Ratdolt 1483, f. d5v–d7r).

We have recomputed the radices using the PAT for noon of February 28, 1341 for the meridian of Paris, 12° east of Toledo, that is, at 0;2d before noon in Toledo. The results are presented in Table 1.

The agreement is good for the double elongation and the mean argument of center of the Sun, but much better results are obtained when recomputing the radices for 0;1,57,30d before noon in Toledo, that is, for a locality 11;47,45° east of that Spanish city. The results are presented in Table 2.

From Table 2 it follows that the mean argument of anomaly of the Moon tabulated here is diminished by 13;9° from that obtained from the PAT for a locality near the meridian of Paris, and that the mean lunar longitude is diminished by exactly 7;36°. As will be seen in Sect. 8, the displacements applied to the two quantities correspond, on the one hand, to the maximum value of the lunar equation of center in the PAT (13;9°) and, on the other hand, to the sum (7;36°) of two parameters, the maximum

15 The value for year 32 given in the text is 4s 15;17,11°, and the difference between this value and that of the radix is 210;55,56°. Thus the amount of the double elongation in 32 years of 365;15 days is 210;55,56° and 791 complete revolutions, leading to a daily mean motion of 24;22,53,23,16°/d.

16 The value for year 32 given in the text is 12s 22;28,24° (read 2s 22;28,24°), and the difference between this value and that of the radix is 63;35,6°. Thus the amount traveled by the Moon in the argument of anomaly in 32 years of 365;15 days is 63;35,6° and 424 complete revolutions, leading to a daily mean motion of 13;3,53,57,30°/d.

17 The value for year 32 given in the text is 1s 16;35,39°, and the difference between this value and that of the radix is 285;42,4°. Thus the amount traveled by the Moon in longitude in 32 years of 365;15 days is 285;42,4° and 427 complete revolutions, leading to a daily mean motion of 13;10,35,1,15°/d.

values for the equation of anomaly (4;56°) and for the increment (2;40°). So, if we call $\bar{A}$ the mean argument of anomaly in the Tables of the Seven Planets, and $\bar{\alpha}$ that in the PAT, we have

$$\bar{A} = \bar{\alpha} - 13;9^\circ. \tag{4}$$

Similarly, if $\bar{L}$ is the mean lunar longitude in the Tables of the Seven Planets, and $\bar{\lambda}$ that in the PAT, we have

$$\bar{L} = \bar{\lambda} - 7;36^\circ. \tag{5}$$

7 Lunar latitude

The table for the lunar latitude (f. 13r) has a maximum of 5;0,0° at 90°; it is the standard table found in many zijes, including those used by Alfonsine astronomers (see Chabás and Goldstein 2012, pp. 103–104).

8 Lunar equations

The lunar equations are presented in three separate tables. The author, well aware of Ptolemy's second lunar model (see Fig. B), has split the treatment of the lunar equations, c_3 and c, according to the two independent variables involved: the double elongation, 2η, and the true argument of anomaly, α. The true longitude of the Moon is the sum of four positive terms, all of which are tabulated (see Eq. 14, below), whereas in the standard Alfonsine tables the true longitude of the Moon is the algebraic sum of three terms (see Eq. 8, below). Despite the differences in these procedures, we show that the results are the same.

The longitude of the Moon, λ, is found by applying an equation, c, to its mean longitude, $\bar{\lambda}$, which is a linear function of time: $\lambda = \bar{\lambda} + c$. Moreover, the mean argument of anomaly, $\bar{\alpha}$, is also a linear function of time. In Ptolemy's second lunar model (*Almagest* V.8), the lunar equation, c, depends on two independent variables: the true argument of anomaly, α (angle $A_e CL$), and the double elongation, 2η (twice the angular distance between the mean longitude of the Moon and the mean longitude of the Sun). In this context the lunar equation is a combination of functions of one variable represented by the 4th, 5th, and 6th columns of Ptolemy's table in *Almagest* V.8. The vast majority of later sets of tables used Ptolemy's approach, although some of them have slightly modified parameters or rearrangements of the order of the columns. In PAT these three columns correspond respectively to the 6th, 5th, and 4th columns; in the following, we will use the conventions of PAT to identify the columns c_i:

$$c = c_6(\alpha) + c_5(\alpha) \cdot c_4(2\eta) \tag{6}$$

where

$$\alpha = \bar{\alpha} + c_3(2\eta). \tag{7}$$

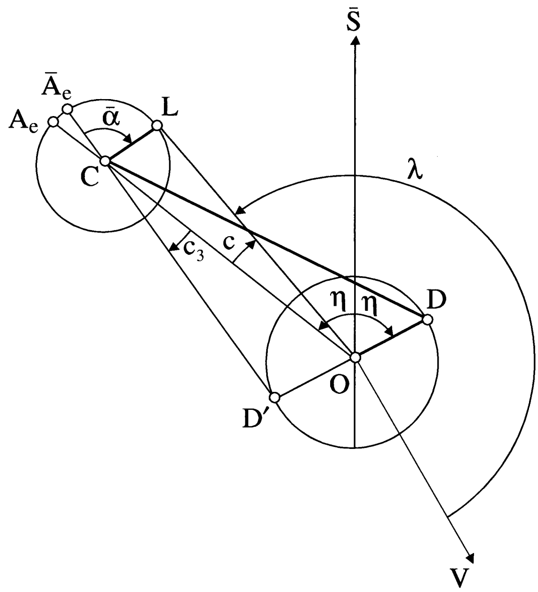

Fig. B Ptolemy's second lunar model. O is the observer, D is the center of the deferent circle whose radius is DC, C is the center of the epicycle, $\bar{S}$ is the direction to the mean Sun, L is the true position of the Moon, V is the direction to the vernal point, and D' is 180° from D on a circle about O with radius OD. The angle η is the elongation of the mean longitude of the Moon from the mean longitude of the Sun, angle $\bar{\alpha}$ is the mean argument of anomaly counted from the mean apogee $\bar{A}_e$, whose position is fixed by the direction $D'C$, A_e is the true lunar apogee, angles c_3 and c are the corrections to the mean lunar longitude, and angle λ is the true longitude of the Moon

In Eqs. 6 and 7, $c_3(2\eta)$ is the equation of center, $c_4(2\eta)$ represent the minutes of proportion, $c_5(\alpha)$ is called the increment, and $c_6(\alpha)$ is the equation of anomaly. Thus, the true longitude of the Moon is given by

$$\lambda = \bar{\lambda} + c_6(\alpha) + c_5(\alpha) \cdot c_4(2\eta). \tag{8}$$

The first table (ff. 13v–14r) is for the equation of center; it has a column for the argument, from 0° to 29°, and then columns for each sign, from 0s to 11s. For each sign and each degree of the argument, we are given entries for the equation of center and the minutes of proportion (c_3 and c_4, respectively, in PAT's arrangement, both variables depending on the double elongation). For the tabulated equation of center, the entry for 0s 0° is 13;9°. It reaches a maximum of 26;18° (twice 13;9°) at 114°–115°, and a minimum of 0;0° at 245°–246°. So the equation of center of the Moon is tabulated with a vertical displacement, for its entries are displaced upwards by 13;9° with respect to the standard Alfonsine table (see Chabás and Goldstein 2012, Table 6.2A, p. 71). If we call $c_3(2\eta)$ the equation of center in the corresponding Alfonsine table, the tabulated entries, $C_3(2\eta)$, for the equation of center in the Tables for the Seven Planets are given by

$$C_3(2\eta) = c_3(2\eta) + 13;9°. \tag{9}$$

It is worth noting that Eq. 7 and the definition of $\bar{A}$ in Eq. 4 imply that $A = \alpha$.[18]

[18] $A = \bar{A} + C_3(2\eta) = (\bar{\alpha} - 13;9°) + C_3(2\eta) = (\bar{\alpha} - 13;9°) + (c_3(2\eta) + 13;9°) = \bar{\alpha} + c_3(2\eta) = \alpha.$

For the minutes of proportion the entry for 0s 0° is 0′. The entries increase monotonically to 60′ at 173°–187°, and decrease to 0′ at 349°–371°. Except for copyist's errors, they agree with those in the PAT. Therefore,

$$C_4(2\eta) = c_4(2\eta). \tag{10}$$

Let us now turn to the third table (ff. 14v–15r), which is for the equation of anomaly; it has a column for the argument, from 0° to 29°, and then columns for each zodiacal sign, from 0s to 11s. For each sign and each degree of the argument, we are given entries for the equation of anomaly and the *dyversitas dyametri proporcionalis*[19] (corresponding to c_6 and c_5, respectively, in PAT's arrangement, both variables depending on the true argument of anomaly). For the equation of anomaly, the entry for 0s 0° is 4;56,0°. It reaches a minimum of 0;0,0° at 91°–99° and a maximum of 9;52,0° (twice 4;56,0°) at 264°, that is, the vertical displacement is 4;56°. This parameter was systematically used by Parisian Alfonsine astronomers, but its origin goes back much earlier (Chabás and Goldstein 2003, pp. 252–253). Thus, if we call $c_6(\alpha)$ the equation of anomaly in the corresponding PAT, the tabulated entries, $C_6(A)$, for the equation of anomaly in the Tables for the Seven Planets are given by

$$C_6(A) = c_6(\alpha) + 4;56°. \tag{11}$$

The first entry in the column for the *dyversitas dyametri proporcionalis*, usually called "increment", is 2;40°; it reaches a minimum of 0;0° at 113°–119° and a maximum of 5;20° (twice 2;40°) at 251°–257°. Except for copyist's errors, the entries agree with those in the PAT, but for the vertical displacement of 2;40°. Again, if we call $c_5(\alpha)$ the increment in the corresponding PAT, the tabulated entries, $C_5(A)$, for the increment in the Tables for the Seven Planets are given by

$$C_5(A) = c_5(\alpha) + 2;40°. \tag{12}$$

In the second table (f. 14v), headed *dyversitas dyametri centralis*, the argument is given at intervals of 1′, from 0′ to 60′, and the entries, $D(M)$, decrease monotonically from 2;40,0° (at 0′) to 0;0,0° (at 60′). Moreover, the entries show a constant decrease of 0;2,40° for each degree of the argument (see Table A); hence,

$$D(M) = 2;40 \cdot (1 - C_4(2\eta)). \tag{13}$$

The tables described above yield the same results as those obtained from the PAT for the true position of the Moon, and were computed with a similar expression (see Eq. 8). If λ and L are the true lunar longitudes in the PAT and the Tables for the Seven

[19] The word *dyversitas*, used in the headings of the lunar and the planetary equations, is often spelled *diversitas*.

Table A *dyversitas dyametri centralis* (excerpt)

Arg. (′)	*Dyv. dyam. centralis* (°)
0	2;40, 0
...	
10	2;13,20
...	
20	1;46,40
...	
30	1;20, 0
...	
40	0;53,20
...	
50	0;26,40
...	
60	0; 0, 0

Planets, respectively, then using Eqs. 5, 8, 10, 11, and 12, λ can be written as

$$\begin{aligned}\lambda &= \bar{\lambda} + c_6(\alpha) + c_5(\alpha) \cdot c_4(2\eta)\\ &= (\bar{L} + 7;36°) + (C_6(A) - 4;56°) + (C_5(A) - 2;40°) \cdot C_4(2\eta)\\ &= (\bar{L} + C_6(A) + 2;40°) + C_5(A) \cdot C_4(2\eta) - 2;40° \cdot C_4(2\eta)\\ &= (\bar{L} + C_6(A) + C_5(A) \cdot C_4(2\eta)) + 2;40° - 2;40° \cdot C_4(2\eta) = L.\end{aligned}$$

The term, $2;40° \cdot (1 - C_4(2\eta)) = D(M)$, is found in the table for the *dyversitas dyametri centralis*, and therefore the expression to compute the true longitude of the Moon is

$$L = \bar{L} + C_6(A) + C_5(A) \cdot C_4(2\eta) + D(M), \tag{14}$$

where all the terms are tabulated and none of them is to be subtracted. Note that there is no counterpart to the function $D(M)$ in the standard Alfonsine tables.

The canons to this set of tables, which are not extant, should have had an instruction more or less in the following terms: "To determine the true position of the Moon, find the mean longitude of the Moon, and keep it; find the double elongation, and enter with it in the table for the equation of center and the minutes of proportion, and keep them. Find the mean argument of anomaly, and add to it the equation of center to obtain the true argument of anomaly, and enter with it in the table for the equation of anomaly and the increment, and keep them. Enter the table for the *dyversitas dyametri centralis* with the minutes of proportion, and keep what you find there. Then add the mean longitude to the true argument of anomaly, and add the result to what is obtained from multiplying the increment by the minutes of proportion. To the value obtained add what you found in the table for the *dyversitas dyametri centralis*."

To illustrate Eq. 14 consider the Moon at epoch (noon, February 28, 1341), when the double elongation, 2η, was 4,45;9,0° (=285;9,0°). The equation of center in the standard Alfonsine tables is $c_3(2\eta) = -10;26°$ and, because of Eq. 9, $C_3(2\eta) =$

2;43°(=−10;26° + 13;9°). The corresponding minutes of proportion are $c_4(2\eta) = 19$ and $C_4(2\eta) = 19$, because of Eq. 10. In this case the term $D(M) = 1;49,20°$. Now, the argument of anomaly, $\bar{\alpha}$, according to the standard Alfonsine tables, is 32;27,43°, and thus $\alpha = 22;1,43°$(=32;27,43°−10;26°). From Eq. 4, $\bar{A} = 19;18,43°$, and thus $A = 22;1,43°$ (=19;18,43°+2;43°). The tabulated values for the equation of anomaly, $C_6(A)$, and the *dyversitas dyametri proporcionalis,* $C_5(A)$, are 3;13,19° and 1;48°, respectively, whereas the values for the equation of anomaly, $c_6(\alpha)$, and the increment, $c_5(\alpha)$, found in the standard Alfonsine tables are −1;42,41° and −0;52°, respectively. We note that Eqs. 11 and 12 hold. According to Eq. 6, using the standard Alfonsine tables the correction to be applied is $c = (-1;42,41°) + (-0;52°) \cdot (19/60) = -1;59,9°$, whereas using the Tables of The Seven Planets $C = 3;13,19° + 1;48° \cdot (19/60) = 3;47,31°$.

With the standard Alfonsine tables the mean lunar longitude at epoch, $\bar{\lambda}$, is computed to be 128;55,23°; thus, with Eq. 8, the true lunar longitude of the Moon at epoch is

$$\begin{aligned}\lambda &= \bar{\lambda} + c_6(\alpha) + c_5(\alpha) \cdot c_4(2\eta)\\ &= 128;55,23° - 1;59,9° \;=\; 126;56,14°.\end{aligned}$$

With the Tables for the Seven Planets we obtain $\bar{L} = 121;19,23°$ (see Eq. 5); thus, with Eq. 14, the true longitude of the Moon at epoch is

$$\begin{aligned}L &= \bar{L} + C_6(A) + C_5(A) \cdot C_4(2\eta) + D(M)\\ &= 121;19,23° + 3;47,31° + 1;49,20° = 126;56,14°,\end{aligned}$$

in agreement with λ.

9 Lunar node

Folio 16r contains five tables for the mean motion of the lunar node. The first lists the values of the mean motion of the node at the beginning of each year, beginning with 1342, for a period of 32 years. The entry for year 1, i.e., 1342, or more precisely, February 28, 1341, is 3s 22;3;17°. The second table gives the mean motion of the Sun for the months; its first entry corresponds to March (0s 1;38,30°) and the last one to February (0s 19;19,42°). The third table lists values from 1372 to 1852, at 32-year intervals. The fourth and fifth tables are, respectively, for the mean motion of the lunar node for days from 1 to 31, and for hours and parts of an hour from 1 to 60. The entry for argument 1d is 0s 0;0,3,11°. As was the case for other mean motions, at the bottom of the folio we are told that the radix at *anno Christi 1340 perfecto* is 3s 2;43,34,43°. From this value and that for year 32 we derive −0;3,10,38,7°/d as the mean motion of the lunar node,[20] in full agreement with the corresponding parameter in the PAT.

[20] The value for year 32 given in the text is 11s 21;39,20°, and the difference between this value and that of the radix is 258;55,45,17°. Thus the amount traveled by the lunar node in 32 years of 365;15 days is 285;42,4° and one complete revolution, leading to a daily mean motion of −0;3,10,38,7°/d.

10 Precession/trepidation

On folios 16v–17r there are various tables for precession/trepidation in the framework of the PAT. In general, the approach found in the Alfonsine corpus is to treat two terms separately: a linear term, usually called "motion of the apogees and the fixed stars", based on a mean motion of 0;0,0,4,20,41,17,12°/d, and presented in a single table; and a periodic term, usually called "motion of access and recess of the 8th sphere", requiring the use of two tables (one for the "mean motion", based on a value of 0;0,0,30,24,49,0°/d, and another for its "equation", found in a separate table with a maximum of 9;0,0°): see Chabás and Goldstein (2012, pp. 48–52).

The author of the Tables for the Seven Planets follows the same approach, uses the same parameters, but gives a different presentation. Folio 16v has four tables under the general title *Tabula motus augium et stellarum fixarum atque 8ª spere*, displaying the two mean motions of the components of Alfonsine variable precession. Each table has two columns, in addition to that for the argument: the mean motion of the apogees and the fixed stars on the one hand, and the mean motion of the 8th sphere on the other. The motion of trepidation is to be applied to the positions of the fixed stars as well as to the positions of the planetary apogees.

The first table gives entries for both quantities at the beginning of each year for a period of 32 consecutive years. The entries for year 1 are 0s 0;51,12° (apogees) and 2s 8;10;51° (8th sphere), and those for year 32 are 0s 1;4,52° (apogees) and 2s 9;46;31° (8th sphere). The second table gives the mean motion of both quantities for accumulated months. The third table, headed *ad annos perpetuacionis*, has entries for years 1372 to 1852, at 32-year intervals, of both quantities, whereas the fourth table is for their mean motions for days from 1 to 31. As was the case for other quantities, at the bottom of the folio we are given the radices for both quantities, *anno Christi 1340 perfecto*: 0s 9;50,45,59,27° (apogees), and 2s 8;7,45,20,10° (8th sphere).[21] From these radices and the entries for year 32 we derive the following mean motions: 0;0,26,28°/y (apogees) and 0;3,5,11°/y (8th sphere), which correspond to about 0;0,0,4,20,51°/d (apogees) and 0;0,0,30,25,12°/d (8th sphere), that is, 1 revolution in 49,000 and 7,000 years, respectively. These are the standard values found in Alfonsine astronomy for precession/trepidation. Combining the mean motion of the 8th sphere and its radix, we find that the argument of the 8th sphere was 0° about 1324 years and 9 months before the epoch of these tables (February 28, 1341), that is, in May 16 AD. This date is explicitly given by Giovanni Bianchini (fifteenth century) as the epoch of the Alfonsine model; see Chabás and Goldstein (2009, p. 32), and the Tables for the Seven Planets offer a justification of it.

It remains to compute the periodic term of Alfonsine trepidation, that is, the equation of the 8th sphere, which is tabulated on f. 17r. Indeed, the entries given in the table are based on the standard Alfonsine equation of the 8th sphere which reaches a maximum of 9;0,0° at 90°, a minimum of −9;0,0° at 270°, and vanishes at 0° and 180° (see also Ratdolt 1483, f. d3v). The corresponding entries on f. 17r are 9;0,0° at 0°, 18;0,0° (maximum) at 90°, 9;0,0° at 180°, 0;0,0° (minimum) at 270°, and 0;0,0°

[21] We note that the radix for the apogees has the wrong number of degrees (probably a copyist's error); to be consistent with the 32 values in the table, it should be 0s 0;50,45,59,27°.

at 360°; thus, the entries are displaced upwards by 9° from the standard table, and can be represented by the modern expression (where y is the entry and x is the argument):

$$y = 9^\circ + \arcsin(\sin 9^\circ \cdot \sin x). \tag{15}$$

This is another case of a vertically displaced table we find in this manuscript. Interestingly, the only example in Latin of a displaced table previously known to us is that of John Vimond (see n. 10), whose table for the equation of the 8th sphere presents both a vertical displacement (8;17°) and a horizontal displacement (113°). However, Vimond's purpose was not to avoid "negative" numbers in his table (otherwise he would have used a vertical displacement of at least 9;0°, as here); rather, he wished to set the origin of the equation of the 8th sphere at the value (8;17°) it had at the time he composed his tables (1320). Apparently, the author of the Tables for the Seven Planets did not have the same goal as Vimond, although he used the same principle.

As mentioned above, on folio 17r there is also a small table entitled *Motus augium equatus ad 32 annos post annum 1340*, listing 32 values. The first entry is 0;0,37,11° and the last entry, for 32 years, is 0;19,42,35°. The entries show a steady increase but the line-by-line differences do not reveal a clear pattern. A complement to this table is found on f. 18r among the tables for the mean motions, where we find one headed *Motus augium equatus ad annos perpetuacionis*, listing entries from 1372 to 1852 at intervals of 32 years. The first entry is 0;19,42,35° but, as in the previous case, the line-by-line differences do not vary in a smooth way. With all these cautions, from the entry for 32 years it is possible to derive a motion of the apogee of 0;0,36,57°/y, or 0;0,0,6,4°/d, which is the amount to be added to the mean motion in solar anomaly to obtain the mean motion in the argument of solar longitude (see Sect. 3). The value we deduce from the table is certainly close to Ptolemy's value for precession of 1° in 100 years, that is 0;0,36° in 1 Egyptian year of 365 days.

11 Planetary mean motions

Next come tables for the mean motions, equations, and stations of the five planets, on ff. 17v–22r (Saturn), ff. 22r–26v (Jupiter), ff. 27r–31r (Mars), ff. 31r–35v (Venus), and ff. 35r–40r (Mercury). In this section we review the mean motions of the planets, for each of which we are given five tables. As we will see, all the mean motions are displaced with respect to those in the standard Alfonsine tables.

The first of these five tables lists the radices of two variables, *centrum* (argument of center) and *porcio* (here meaning the displaced argument of anomaly, as will be seen below), for the beginning of each year after 1340 for a period of 32 consecutive years. The entries for both variables for year 32 are, respectively: 1s 6;7,52° and 1s 14;30,11° (Saturn); 8s 12;53,57° and 8s 18;7,43° (Jupiter); 2s 0;5,0° and 2s 19;44,28° (Mars); 6s 25;36,12° and 7s 22;56,20° (Venus); and 3s 19;22,12° and 2s 4;36,10°(Mercury). The second of these tables gives the mean motion of the planets for accumulated months, beginning in March, for the two variables. The third of these tables is headed *ad annos perpetuacionis* and displays entries for the two variables from 1372 to 1852, at 32-year intervals. The fourth and fifth of these tables are, respectively, for the mean motion of the planets for days from 1 to 31, and for hours and parts of an hour from 1 to 60.

From the values of the radices and those for year 32 in the first table for the mean motions of the planets, we derive the following values for the daily mean motions: 0;2,0,29,13°/d (Saturn, argument of center); 0;4,59,9,23°/d (Jupiter, argument of center); and 0;31,26,32,35°/d (Mars, argument of center). These three parameters differ by 0;0,0,6,4°/d from the standard Alfonsine mean motions, as was the case for the argument of solar anomaly, indicating that the precession of 1° in 100 years was applied to the planets as well as to the Sun. We have also derived the daily mean motions of anomaly for the inferior planets: 0;36,59,27,24°/d (Venus) and 3;6,24,7,43°/d (Mercury), which agree exactly with the corresponding parameters in the PAT.

As was the case for the Sun and the Moon, we are also given the radices *anno christi 1340 completo*, which we display in Table 3.

We have recomputed the mean positions of the planets using the PAT for noon of February 28, 1341 for the meridian of Paris, 12° east of Toledo. The results are presented in Table 4.

From Table 4 it follows that the arguments of center of the five planets given in these tables are diminished, but for a few seconds, by 14°, 18°, 61°, 51°, and 28° from those obtained from the PAT for a locality near the meridian of Paris. Table 4 also indicates that the entries for the argument of anomaly are diminished, but for a few seconds, by 7°, 6°, 12°, 3°, and 4° from those derived from the PAT. As will be shown in Sect. 12, all these integer values correspond to the horizontal and vertical displacements (kh_3 and kv_3, respectively) used in the tables for the equations of the

Table 3 Radices for year 1340 (complete)

	Centrum	*Porcio*
Saturn	0s 4;56,59°	2s 8;46,40°
Jupiter	0s 1;38,17,48°	4s 29;28,59,11°
Mars	1s 25; 6,26, 4°	2s 24;48,42,[..]°
Venus	6s 25;41,48,36°	7s 17; 6,11°
Mercury	3s 19;27,49°	3s 23;27,56°

Table 4 Recomputation for the planets at epoch: noon, February 28, 1341

	Argument of center	Difference text − comp.	Rounded value	Argument of anomaly	Difference text − comp.	Rounded value
Saturn	18;56,58°	−13;59,59°	14	75;46,38°	−6;59,58°	7
Jupiter	19;18,21°	−17;40, 3°[a]	18	155;28,57	−5;59,58°	6
Mars	116; 6,24°	−60;59,58°	61	96;48,44°	−12; 0, 2°	12
Venus	256;41,56°	−51; 0, 7°	51	230; 6, 9°	−2;59,58°	3
Mercury	137;27,46°	−27;59,57°	28	117;27,52°	−3;59,56°	4

[a] The difference is −18;0,3° if we take the radix for the argument of center in Table 3 to be 1;18,17,48°, which is the result one would obtain from a set of standard Alfonsine tables

planets. If we call $\bar{\kappa}$ and $\bar{\alpha}$ the planetary arguments of center and anomaly in the PAT, the corresponding quantities in the Tables for the Seven Planets are defined by:

$$\bar{K} = \bar{\kappa} - kh_3, \tag{16}$$

and

$$\bar{A} = \bar{\alpha} - kv_3. \tag{17}$$

12 Planetary equations and stations

In this section we review the equations and the stations of the five planets, for each of which we are also given five tables, in which the equation of center (one table; see Sect. 12.1) and the equation of anomaly (four tables; see Sects., 12.2 and 12.3) are treated separately. Figure C displays the model for Mars. As we will see, the true longitude of a planet is found by adding four positive terms, all of which are tabulated (see Eqs. 43 and 45, below). These terms are all different from those in the standard

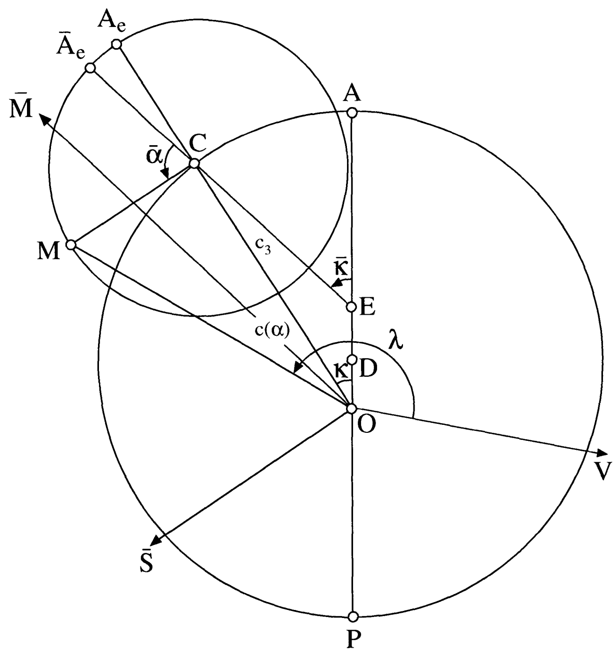

Fig. C Model for Mars, where O is the observer, D is the center of the deferent circle ACP, A is the apogee, P is the perigee, E is the equant point, OV is the direction to the vernal point on the ecliptic, $\bar{S}$ is the direction to the mean Sun, $\bar{A}_e$ is the mean apogee on the epicycle whose center is C, A_e is the true apogee on the epicycle, $\bar{\kappa}$ and $\bar{\alpha}$ are the mean arguments of center and anomaly, respectively, $\bar{M}$ is the mean position of the planet, and M is its true position (where, for an outer planet, CM is parallel to $O\bar{S}$)

Table B *equatio centri*, *equatio porcionis*, and minutes of proportion for Saturn (excerpt)

(°)	*Equatio centri* (°)	*Equatio porcionis* (°)	Minutes of proportion (′)
0	5;30	8;30	0
1	5;24	8;36	0
2	5;17	8;43	1
...			
60	0;48	13;12	37
...			
79	0;29	13;31	58
80	0;29	13;31	59
81	0;30	13;30	1
82	0;30	13;30	2
...			
180	8;39	5;21	60
...			
254	13;31	0;29	0
255	13;31	0;29	0
...			
264	13;25	0;35	2
265	13;23	0;37	1
266	13;22	0;38	59
267	13;21	0;39	58
...			
359	5;36	8;24	0

Alfonsine tables, where the true longitude is found as the algebraic sum of four terms. The author of these tables has considered two cases: (1) the epicyle is near the apogee of the deferent, and (2) the epicycle is near the perigee of the deferent. In case 1 he has the distance of the center of the epicycle vary from its maximum to its mean value, whereas in case 2 he has the distance of the center of the epicycle vary from its mean to its minimum value.

12.1 Equation of center and *equatio porcionis*

In the first table the argument ranges from 0° to 29° for each zodiacal sign, and for each degree we are given three entries: *equatio centri*, *equatio porcionis*, and minutes of proportion. The first two are given in degrees and minutes, and the third entry is given in minutes (see Table B).

Table 5 displays significant values of the equation of center for the five planets. To determine the relation between the equation of center presented here and that in the PAT, let us consider the case of Mars. The entry for 0° is 2;29°; the minimum (0;36°) is reached at 31°–35° and the maximum (23;24°) at 203°–207°. The difference between the two extremal values is 22;48°, that is, twice 11;24°, the standard parameter used for Mars in the PAT. The entries in the table are displaced exactly 12° upwards as compared with the corresponding entries in the PAT, and 12° is the minimum integer for a vertical displacement ensuring that all entries for the equation of center are positive. But the entries for the equation of center have a second displacement, a horizontal displacement, of 61° with respect to the table in the PAT. With the notation used above, $\bar{K} = \bar{\kappa} - 61°$. If we call $c_3(\bar{\kappa})$ the equation of center in the corresponding PAT,[22] the tabulated entries, $C_3(\bar{K})$, for the equation of center in the Tables for the Seven Planets are given by $C_3(\bar{K}) = c_3(\bar{K} + 61°) + 12°$. This is the first example of a horizontal displacement in these tables. For each planet the corresponding displacement applied to $\bar{\kappa}$ (see Table 5) accounts for the difference between the entries given in the Tables for the Seven Planets and the PAT. Thus, the general rule for the equation of center of the planets can be written as

$$C_3(\bar{K}) = c_3(\bar{K} + kh_3) + kv_3, \tag{18}$$

where kh_3 and kv_3 are the horizontal and vertical displacements associated with $c_3(\bar{\kappa})$, the equation of center in the PAT. Once $C_3(\bar{K})$ is known, the true argument of center, K, is defined as:

$$K = \bar{K} + C_3(\bar{K}), \tag{19}$$

on analogy with the expression for the true argument of center, κ, used in the Alfonsine tables,

$$\kappa = \bar{\kappa} + c_3(\bar{\kappa}), \tag{20}$$

where $c_3(\bar{\kappa}) \leq 0°$ when $0° \leq \bar{\kappa} \leq 180°$, as shown in Fig. D.

All the underlying parameters displayed in Table 5 are strictly in the tradition of the Alfonsine corpus. Interestingly, the function for Venus is presented with displacements that differ from that for the Sun (see Sect. 4, above), although they share the same maximum equation, 2;10°.

As for the column labeled *equatio porcionis*, note that for a given argument its entries and those corresponding to them in the equation of center always add up to an integer number of degrees: 14° (Saturn), 12° (Jupiter), 24° (Mars), 6° (Venus), and 8° (Mercury). This is exactly twice the vertical displacement applied to each planet, that is, $2 \cdot kv_3$. Thus, if we call the *equatio porcionis* $E(\bar{K})$, it follows that the author of the Tables of the Seven Planets defined it as

$$E(\bar{K}) + C_3(\bar{K}) = 2 \cdot kv_3. \tag{21}$$

This definition was intended to eliminate subtractions, as will be justified below: there is no counterpart to the function $E(\bar{K})$ in the standard Alfonsine tables. Figure D

[22] In the PAT the equation of center is displayed in the third column in the tables for planetary equations, hence the notation used here.

Table 5 Equation of center for all planets

	Entry for 0°	Minimum equation of center	Maximum equation of center	Underlying parameter	Vertical displacement (kv_3)	Horizontal displacement (kh_3)
Saturn	5;30°	0;29° (76°–80°)	13;31° (252°–256°)	6;31°	7°	14°
Jupiter	4;15°	0; 3° (72°–78°)	11;57° (246°–252°)	5;57°	6°	18°
Mars	2;29°	0;36° (31°–35°)	23;24° (203°–207°)	11;24°	12°	61°
Venus	1;21°	0;50° (37°–47°)	5;10° (211°–221°)	2;10°	3°	51°
Mercury	2;47°	0;58° (65°–69°)	7; 2° (235°–239°)	3; 2°	4°	28°

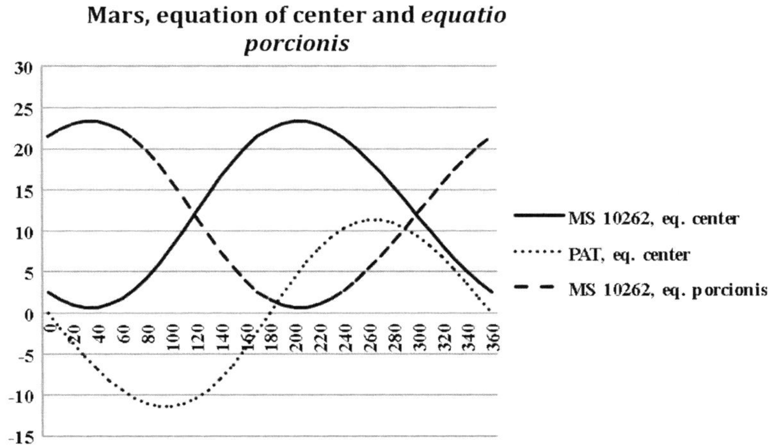

Fig. D Mars, equation of center and *equatio porcionis*

shows the equation of center and the *equatio porcionis* in the case of Mars. The other planets follow the same pattern (see Table 5).

When $E(\bar{K})$ is known, the true anomaly, A, is defined as:

$$A = \bar{A} + E(\bar{K}) \tag{22}$$

on analogy with the expression for the true argument of anomaly, α, used in the Alfonsine tables,

$$\alpha = \bar{\alpha} - c_3(\bar{\kappa}) \tag{23}$$

where, again, $c_3(\bar{\kappa}) \leq 0°$ when $0° \leq \bar{\kappa} \leq 180°$.

In the third column of this table the entries for the minutes of proportion are also dependent upon those in the PAT (see Fig. E for the case Mars) except for two features: a horizontal displacement, different for each planet, and a vertical displacement of 60′, which is limited to the positions when the planet is near its apogee. Note that in the PAT the minutes of proportion for these positions are subtractive, whereas they are additive for the positions near perigee. Consequently, in order to avoid subtractions, the author of the Tables for the Seven Planets only applied a vertical displacement of 60′ to half of the column, that is, where the center of the epicycle is near apogee, and left unchanged the additive part (for details see Table 6). With this approach, if an entry, $c_4(\kappa)$, gives the minutes of proportion in the PAT,[23] the corresponding entry in the Tables for the Seven Planets in the case of Mars is $C_4(K) = c_4(K + 49°) + 60'$, near apogee, and $C_4(K) = c_4(K + 49°)$, near perigee. Thus, the general rule for the equation of center of the planets can be written as

[23] In the PAT the minutes of proportion are displayed in the fourth column in the tables for planetary equations; hence the notation used here. Note, however, that in the corresponding table in Ptolemy's *Almagest* they are found in the eighth column and depend on the mean argument of center, whereas in the PAT they are a function of the true argument of center. For this function in the *Almagest*, see Neugebauer (1975, pp. 185–186).

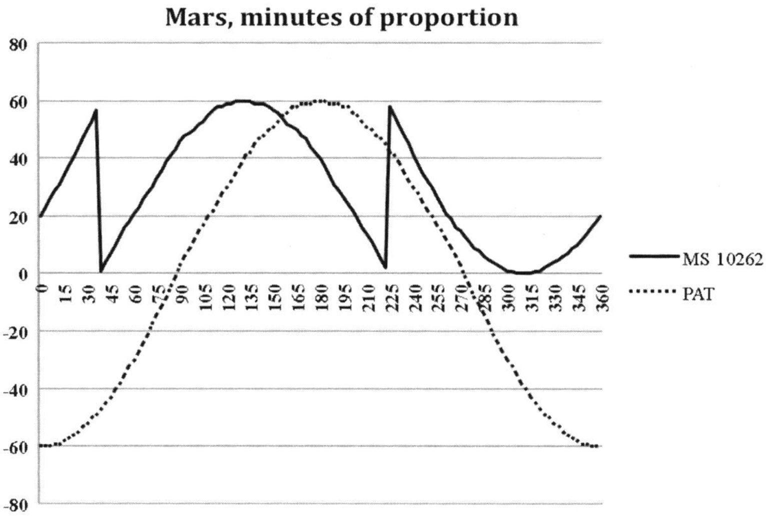

Fig. E Mars, minutes of proportion

Table 6 Displacements for the minutes of proportion

	Entry for 0°	Vertical displacement (kv_4)	Horizontal displacement (kh_4)
Saturn	0	60′ (265°–266° to 360° and 0° to 80°–81°)	7°
Jupiter	0	60′ (259°–260° to 360° and 0° to 77°–78°)	12°
Mars	20	60′ (223°–224° to 360° and 0° to 38°–39°)	49°
Venus	19	60′ (223°–224° to 360° and 0° to 40°–41°)	48°
Mercury	10	60′ (271°–272° to 360° and 0° to 41°–42°)	24°

$$C_4(K) = c_4(K + kh_4) + kv_4, \tag{24}$$

near apogee, and simply

$$C_4(K) = c_4(K + kh_4), \tag{25}$$

near perigee, where kh_4 and kv_4 (=60′) are the horizontal and vertical displacements, respectively, associated with $c_4(\kappa)$, the minutes of proportion. Figure E displays the minutes of proportion for Mars; the other planets follow the same pattern (see Table 6).

As in previous cases, the vertical displacements are intended to avoid cumbersome rules for addition and subtraction corresponding to the simple rules we now give by means of algebraic signs. The horizontal displacements are intended to counterbalance another displacement in a different column or table. That this is indeed the case can be seen from the fact that the horizontal displacements applied to the argument of center (the column on the right side in Table 5) and to the minutes of proportion (the column on the right side in Table 6) add up algebraically to the vertical displacement applied to the equation of center (see Table 5):

$$kh_3 - kh_4 = kv_3. \tag{26}$$

From Eqs. 16, 18, 20, and 26 it follows that

$$K = \kappa - kh_4. \tag{27}$$

Therefore kh_4 is the displacement to be applied to the Alfonsine true argument of center to obtain the true argument of center in the Tables of the Seven Planets.[24]

12.2 Equation of anomaly near greatest distance

For each planet the equation of anomaly is displayed in four tables: two, here called (i) and (ii), associated with greatest distance (i.e., between greatest and mean distance) and two, with exactly the same presentation, associated with least distance (i.e., between mean and least distance: see Sect. 12.3). In the *Almagest* and sets of tables related to it the equation of anomaly is treated separately when the center of the epicycle is near apogee in contrast to the case when the center of the epicycle near perigee. Ptolemy first assumed that the center of the epicycle is at mean distance and computed the equation of anomaly under this condition (Ptolemy's column 6). He then considered the case where the center of the epicycle lies between maximum distance (apogee), where the equation of anomaly is least, and mean distance. Ptolemy's column 5 gives the subtractive difference to be applied to the equation of anomaly at mean distance. This is because at maximum distance the equation of anomaly is least. To interpolate between mean distance and maximum distance Ptolemy applied the minutes of proportion in his column 8 to the subtractive difference; thus, in this case, the total equation is

$$c(\alpha) = c_6(\alpha) - c_5(\alpha) \cdot c_8(\bar{\kappa}) \tag{28}$$

where $\bar{\kappa}$ is near apogee. As Neugebauer (1975, pp. 185–186) demonstrated, Ptolemy used different formulas for $c_8(\bar{\kappa})$ near apogee and near perigee, but put the values in a single column. Between mean distance and perigee Ptolemy computed the entries in his column 7 which is the amount to the added to the value for mean distance to get the equation of anomaly at least distance (perigee) where the equation of anomaly is greatest; thus, in this case, the total equation is

$$c(\alpha) = c_6(\alpha) + c_7(\alpha) \cdot c_8(\bar{\kappa}), \tag{29}$$

where $\bar{\kappa}$ is near perigee. So in some cases one has to subtract and in other cases one has to add. The author of the Tables for the Seven Planets wished to avoid subtractions and he took a different approach without changing the model or the parameters. Instead of

[24] Indeed, $K = \bar{K} + C_3(\bar{K}) = \bar{\kappa} - kh_3 + c_3(\bar{\kappa}) + kv_3 = \bar{\kappa} + c_3(\bar{\kappa}) - kh_3 + kv_3 = \kappa - (kh_3 + kv_3) = \kappa - kh_4$.

first tabulating the values for mean distance with a subtractive difference, he tabulated the equation of anomaly at greatest distance, where the equation is least. Hence, all corrections are positive, and the total equation is

$$C(A) = D(A) + C_5(A) \cdot C_4(K), \tag{30}$$

where $D(A)$ is the equation of anomaly at apogee, $C_5(A)$ corresponds to Ptolemy's $c_5(\alpha)$, and $c_4(K)$ (defined in Eq. 24) corresponds to Ptolemy's $c_8(\bar{\kappa})$; all terms are positive. For values of κ near perigee, the author of the Tables for the Seven Planets adhered more closely to Ptolemy's formula since it did not involve subtraction. Thus, between mean distance and minimum distance, our author used

$$C(A) = C_6(A) + C_7(A) \cdot C_4(K), \tag{31}$$

where $C_6(A)$ corresponds to Ptolemy's $c_4(\alpha)$, $C_7(A)$ corresponds to Ptolemy's $c_7(A)$, and $C_4(K)$ (defined in Eq. 25) corresponds to Ptolemy's $c_8(\bar{\kappa})$. We will see that the entry for mean distance is the same whether one uses the formula near apogee or the formula near perigee.

(i) The argument is the minutes of proportion, as indicated in the heading, displayed here as integers from 0 to 60. There are two other columns, one labeled *diversitas dyametri centralis* and another for the first station near greatest distance (see Table C). The entries corresponding to 0 refer to greatest distance, and the entries corresponding to 60 refer to mean distance.

This table is unprecedented in the astronomical literature and, in the absence of canons explaining its use, it is difficult to interpret the meaning of the expression, *diversitas dyametri centralis*. Table 7 shows the extremal values for all planets.

A close inspection of the entries for the *diversitas dyametri centralis* indicates that they are strongly dependent on the apogees of the planets at epoch (noon, February 28, 1341, Toledo). A comparison of our computation of the planetary apogees for the city of Toledo at that time and the entries for 0 for each planet is displayed in Table 8.

From Table 8 it follows that the entries for 0 tabulated here for the *diversitas dyametri centralis* are obtained by adding a constant to the longitude of the apogee of

Table C *Diversitas dyametri centralis* and first station near greatest distance for Jupiter (excerpt)

Minutes of prop. (′)	*Diversitas dyametri centralis* (s, °)	First station (s, °)
0	5 22;37,40	4 4; 5
1	5 22;37,10	4 4; 7
...		
15	5 22;30,10	4 4;30
...		
30	5 22;22,40	4 4;53
...		
45	5 22;14,40	4 5;17
...		
59	5 22; 8,10	4 5;38
60	5 22; 7,40	4 5;39

Table 7 *Diversitas dyametri centralis* and first station near greatest distance for all planets

	Diversitas dyametri centralis		First station	
	Entry for 0	Entry for 60	Entry for 0	Entry for 60
Saturn	8s 12;42,18°	8s 12;21,36°[a]	3s 22;44°	3s 24; 7°
Jupiter	5s 22;37,40°	5s 22; 7,40°	4s 4; 5°	4s 5;39°
Mars	4s 25;38,49°	4s 20; 0,49°	5s 7;28°	5s 13;11°
Venus	3s 2;48, 9°	3s 1; 6, 9°	5s 15;51°	5s 17; 9°
Mercury	7s 3;50, 9°	7s 0;38, 9°	4s 27;14°	4s 25; 8°

[a] Probably an error for 8s 12;21,18°. For justification, see below. One should also note that the number of seconds in the entries for 0 and 60 in columns 1 and 2 agree for each planet (but for Saturn here), both in this table and in the equivalent, Table 12

Table 8 Planetary apogees near greatest distance

	Computed	Entry for 0 − comp.	Rounded value[a]
Saturn	251;35,19°	1; 6,59°	1; 7 (= 7 − 5;53)
Jupiter	171;31,37°	1; 6, 3°	1; 6 (=12 − 10;54)
Mars	133;23,50°	12;14,59°	12;15 (=49 − 36;45)
Venus	89;37, 0°	3;11, 9°	3;11 (=48 − 44;49)
Mercury	208;51,10°	4;58,59°	4;59 (=24 − 19; 1)

[a] As will be seen below, the rounded value for each planet should result from subtracting the maximum of $c_6 - c_5$ (see Table 10) from kv_3 (see Table 5). In the case of Jupiter, this gives 1;26° (=12 − 10;34), whereas we obtain 1;6° when using the entry for 0 in the table, 5s 22;37,40° (see Table 7), and a set of standard Alfonsine tables. If the author had taken 5s 22;57,40° as the entry for 0, the rounded value would have been fully consistent with those for the other planets

each planet at epoch, λ_0. This constant is specific to each planet and results in turn from the displacements applied to its variables, $kh_4 - \max(c_6 - c_5)$: see Table 10. Therefore, these entries play a crucial role in computing the true planetary longitudes but have no direct astronomical meaning. On the other hand, for each planet the range for the entries is 0;20,42° (=8s 12;42,18° − 8s 12;21,36°) or 0;21° (if we consider 8s 12;21,18° as the intended entry for 60) for Saturn, 0;30° for Jupiter, 5;38° for Mars, 1;42° for Venus, and 3;12° for Mercury. It turns out that these values agree with the vertical displacement applied to the difference at greatest distance, c_5, associated with the argument of anomaly. We have called this quantity kv_5 (see Table 11). In the case of Mars, its value is 5;38°, and the *diversitas dyametri centralis* near greatest distance, D_5, which is a function of the minutes of proportion, M, can be written as $D_5(M) = D_5(0) - 5{;}38 \cdot C_4(K)$. Thus, the general rule for the *diversitas dyametri centralis* near greatest distance is given by

$$D_5(M) = D_5(0) - kv_5 \cdot C_4(K),$$

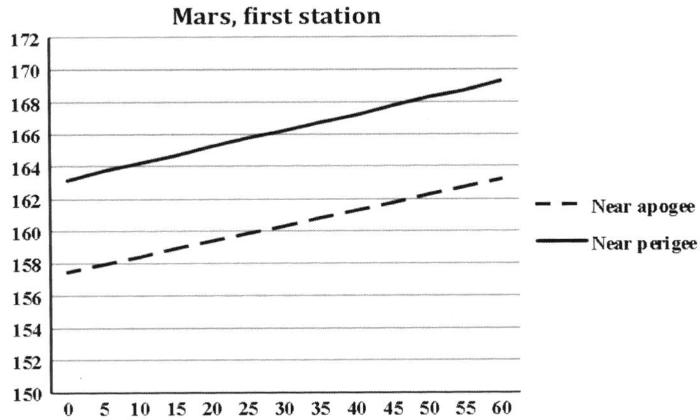

Fig. F First station of Mars

that is,

$$D_5(M) = \lambda_0 + kh_4 - \max(c_6 - c_5) - kv_5 \cdot C_4(K). \tag{32}$$

The third column in this table displays the first station as a function of the minutes of proportion. The entries for the first station represent the true argument of anomaly of the first stationary point, and those for 0 agree with the entries in the *Almagest* (see, e.g., Toomer 1984, p. 588) and sets of tables related to it. They correspond to the greatest distance of the center of the epicycle, when the mean argument of center, $\bar{\kappa}$, is zero. It should be recalled that the table in *Almagest* XII.8 gives the positions of the stationary points on the epicycle as a function of $\bar{\kappa}$ (see Neugebauer 1975, pp. 202–205), not as a function of the minutes of proportion, as is the case here. The entries increase (except for Mercury) monotonically and vary in a range different for each planet: 1;23° (Saturn), 1;34° (Jupiter), 5;43° (Mars), 1;18° (Venus), and −2;6° (Mercury). Figure F displays the first station of Mars, both near apogee (see Table 7) and near perigee (Table 12).

If we call $S_5(M)$ the tabulated argument of anomaly of the first station near apogee, the entries can be computed by means of a linear function of M, the minutes of proportion:

$$S_5(M) = S_5(0) + [S_5(60) - S_5(0)] \cdot C_4(K), \tag{33}$$

which, in the case of Mars, turns into

$$S_5(M) = 157;28° + 5;43° \cdot C_4(K).$$

(ii) In the second table the argument ranges from 0° to 29° for each zodiacal sign. There are two columns, one for the equation of anomaly (the heading has *argumentum* for anomaly) and another for the *diversitas dyametri*. The title indicates that the entries are given for greatest distance of the center of the epicycle (*ad longitudinem* *longiorem*), that is, near apogee. The heading of the entries for the *diversitas dyametri* is generally *diversitas dyametri directus*, and sometimes *diversitas dyametri retrogradus*, a change which is indicated by the insertion of the words *statio*, *prima*, and *secunda*, among the entries. See Table D.

The relevant information for the equation of anomaly is summarized in Table 9, where the "underlying parameter" was derived by taking half the difference between maximum and minimum equation of anomaly.

Table D Equation of anomaly and *diversitas dyametri* for Mars (excerpt)

(°)	Equation of anomaly (°)	*Diversitas dyametri* (*directus* or *retrogradus*) (°)	(°)	Equation of anomaly (°)	*Diversitas dyametri* (*directus* or *retrogradus*) (°)
0	36;45	5;38	180	36;45	5;38
1	37; 7	5;40	...		
...			191	21;19	1;34
90	67;40	8; 5			*Statio*
...			192	20; 3	1;20
125	73;29	9;48	...		
126	73;30	9;52	203	9;15	0; 1
127	73;30	9;55			*Secunda*
128	73;29	9;59			*Directus*
...			204	8;31	0; 0
154	66;22	11;16	205	7;48	0; 0
155	65;42	11;16	206	7; 8	0; 0
156	64;59	11;16	...		
157	64;15	11;15	232	0; 1	1;17
		Statio	233	0; 0	1;21
158	63;29	11;14	234	0; 0	1;24
...			235	0; 0	1;28
169	52;11	9;42	...		
		Prima	270	5;50	3;11
		Retro.	...		
170	50;55	9;26	358	36; 0	5;35
...			359	36;23	5;36

Table 9 Equation of anomaly near greatest distance for all planets

Greatest distance	Entry for 0°	Minimum equation of anomaly	Maximum equation of anomaly	Underlying parameter
Saturn	5;53°	0;0° (261°–266°)	11;46° (92°–97°)	5;53°
Jupiter	10;34°	0;0° (258°–263°)	21; 8° (97°–102°)	10;34°
Mars	36;45°	0;0° (233°–234°)	73;30° (126°–127°)	36;45°
Venus	44;49°	0;0° (225°)	89;38° (135°)	44;49°
Mercury	19; 1°	0;0° (250°–252°)	38; 2° (108°)	19; 1°

Table 10 Maximum values of c_6, c_5 and $c_6 - c_5$ in the PAT

	Maximum of c_6	Maximum of c_5	Maximum of $c_6 - c_5$
Saturn	6;13° (at 94°–99°)	0;21° (at 100°–106°)	5;53° (at 94°–99°)
Jupiter	11; 3° (at 99°–102°)	0;30° (at 107°–117°)	10;34° (at 97°–102°)
Mars	41;10° (at 131°–132°)	5;38° (at 153°–156°)	36;45° (at 126°–127°)
Venus	45;59° (at 135°–136°)	1;42° (at 161°–162°)	44;49° (at 135°)
Mercury	22; 2° (at 111°–112°)	3;12° (at 129°–131°)	19; 1° (at 108°)

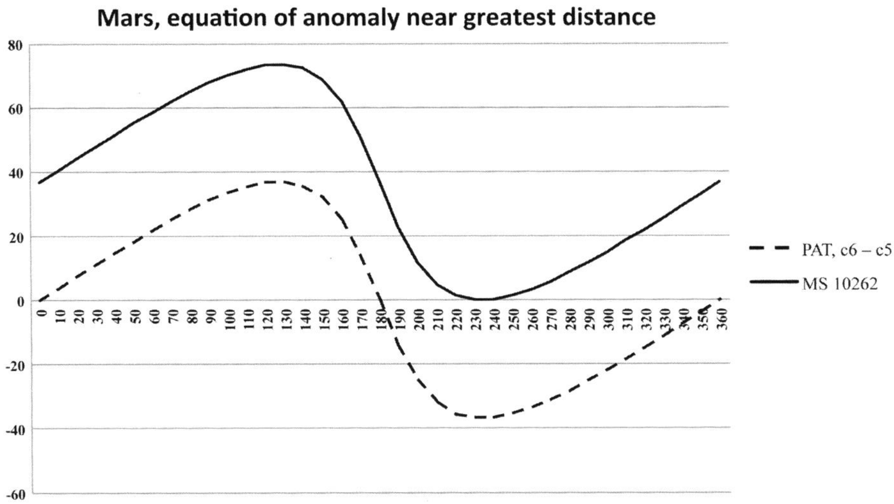

Fig. G Equation of anomaly near greatest distance for Mars

The underlying parameters in this table are the maximum values of the difference $c_6 - c_5$, that is, the maximum difference between the equation of anomaly at mean distance and the subtractive difference in the PAT.[25] All these values are shown in Table 10.

Figure G displays the tabulated values of the equation of anomaly for Mars near greatest distance, here called $D(A)$ and the difference between c_6 and c_5 in the PAT. It is readily seen that $D(A)$ is displaced upwards with respect to $c_6 - c_5$. The other planets follow the same pattern.

In the case of Mars the expression for the equation of anomaly at greatest distance can be written as

$$D(A) = c_6(\alpha) - c_5(\alpha) + 36;45°,$$

[25] In the PAT the fifth and sixth columns in the tables for planetary equations display, respectively, the subtractive difference and the equation of anomaly at mean distance; hence the notation used here.

Table 11 Difference near greatest distance

Greatest distance	Entry for 0°	Minimum difference at greatest distance	Maximum difference at greatest distance	Vertical displacement (kv_5)
Saturn	0;21°	0;0° (253°–261°)	0;42° (99°–104°)	0;21°
Jupiter	0;30°	0;0° (242°–252°)	1; 0° (108°–118°)	0;30°
Mars	5;38°	0;0° (204°–207°)	11;16° (153°–156°)	5;38°
Venus	1;42°	0;0° (198°–199°)	3;24° (161°–162°)	1;42°
Mercury	3;12°	0;0° (229°–231°)	6;24° (129°–131°)	3;12°

which, in the case general, corresponds to

$$D(A) = c_6(\alpha) - c_5(\alpha) + \max(c_6 - c_5). \tag{34}$$

The *diversitas dyametri* tabulated for each planet is the difference near greatest distance. The entries in this column agree with those for the same quantity, $c_5(\alpha)$, in the PAT, but for a vertical displacement, which differs from one planet to another (see Table 11).

In the case of Mars this displacement amounts to 5;38°. In general, if $c_5(\alpha)$ is the difference near greatest distance in the corresponding PAT, that in the Tables for the Seven Planets is given by

$$C_5(A) = c_5(\alpha) + kv_5. \tag{35}$$

We note that $A = \alpha$, which is a consequence of equations 17, 18, 21, and 22.[26] Therefore, the displacements applied by the author of the Tables for the Seven Planets to the standard quantities used in Alfonsine planetary astronomy are such that they keep invariant the argument of anomaly.

As in Eq. 28, the total correction when the planet is near apogee in Ptolemy's notation is

$$c(\alpha) = c_6(\alpha) - c_5(\alpha) \cdot c_8(\bar{\kappa}),$$

which, in standard Alfonsine notation, is equivalent to

$$c(\alpha) = c_6(\alpha) - c_5(\alpha) \cdot c_4(\bar{\kappa}).$$

In the Tables for the Seven Planets the corresponding expression near apogee is

$$C(A) = D(A) + C_5(A) \cdot C_4(K), \tag{36}$$

an expression only involving positive terms and leading to the same true longitude of the planets as the standard Alfonsine procedure.

[26] Indeed, $A = \bar{A} + E(\bar{K}) = (\bar{\alpha} - kv_3) + (2 \cdot kv_3 - C_3(\bar{\kappa})) = \bar{\alpha} + kv_3 - [(c_3(\bar{\kappa}) + kv_3)] = \bar{\alpha} - c_3(\bar{\kappa}) = \alpha$.

Table E *Diversitas dyametri centralis* and first station near least distance for Venus (excerpt)

Minutes of prop. (′)	*Diversitas dyametri centralis* (s, °)	First station (s, °)
0	3 1;38, 9	5 17; 9
1	3 1;36,17	5 17;10
...		
15	3 1;10, 9	5 17;25
...		
30	3 0;42, 9	5 17;43
...		
45	3 0;14, 9	5 18; 4
...		
59	2 29;48, 1	5 18;20
60	2 29;46, 9	4 18;21

Table 12 *Diversitas dyametri centralis* and first station near least distance for all planets

	Diversitas dyametri centralis		First station	
	Entry for 0	Entry for 60	Entry for 0	Entry for 60
Saturn	8s 12;22,18°	8s 11;57,18°	3s 24; 8°	3s 25;27°
Jupiter	5s 22; 8,40°	5s 21;35,40°	4s 5;40°	4s 7;11°
Mars	4s 21;13,49°	4s 13;10,49°	5s 13; 6°	5s 19;15°
Venus	3s 1;38, 9°	2s 29;46, 9°	5s 17; 9°	5s 18;21°
Mercury	7s 0;49, 9°	6s 28;48, 9°	4s 25; 8°	4s 24;28°

12.3 Equation of anomaly near least distance

The following two tables, for least distance (i.e., between mean and least distance), have the same presentation as those for greatest distance (i.e., between greatest and mean distance), reviewed in Sect. 12.2 (i) and (ii).

(i) As in Table C the argument is the minutes of proportion from 0′ to 60′, and there are two other columns, one labeled *diversitas dyametri centralis* and another for the first station near least distance (see Table E). Here the entries for 0′ correspond to mean distance, and the entries for 60′ correspond to least distance.

The extremal values of the two columns (*diversitas dyametri centralis* and first station) for all planets are listed in Table 12.

As was the case for greatest distance, the entries for 0′ tabulated here for the *diversitas dyametri centralis* are obtained by adding a constant to the longitude of the apogee of each planet at epoch, λ_0. This constant is specific to each planet and results in turn from the displacements applied to its variables: $kh_4 - kv_6$ (see Table 13).

On the other hand, the range for the entries is: 0;25° for Saturn, 0;33° for Jupiter, 8;3° for Mars, 1;52° for Venus, and 2;1° for Mercury. The entries for least distance are

Table 13 Planetary apogees at least distance

	Computed	Entry for 0 − comp.	Rounded value[a]
Saturn	251;35,19°	0;46,59°	0;47 (= 7 − 6;13)
Jupiter	171;31,37°	0;37, 3°	0;37 (=12 − 11;23)
Mars	133;23,50°	7;49,59°	7;50 (=49 − 41;10)
Venus	89;37, 0°	2; 1, 9°	2; 1 (=48 − 45;59)
Mercury	208;51,10°	1;57,59°	1;58 (=24 − 22; 2)

[a] As will be seen below, the rounded value for each planet should result from subtracting kv_6 (see Table 14) from kv_3 (see Table 5). In the case of Jupiter, this gives 0;57° (=12 − 11;3), whereas we obtain 0;37° when using the entry for 0 in the table, 5s 22;8,40° (see Table 12) and a set of standard Alfonsine tables. If the author had taken 5s 22;28,40° as the entry for 0, the rounded value would have been fully consistent with those for the other planets. (This is the same situation we noted in Table 8; thus, it would seem that the author either miscopied an entry for Jupiter on which he built up his two tables for the *diversitas dyametri centralis*, or that he had at his disposal a table for Jupiter generating a difference of 0;20°.)

not exactly a continuation of those for greatest distance (Table 7), because they were computed by means of different expressions and different ranges were used. In the case of Mars the vertical displacement applied here, kv_7, is 8;3° rather than $kv_5 = 5;38°$, as was the case at greatest distance. Then the *diversitas dyametri centralis* near least distance, D_7, can be written as $D_7(M) = D_7(0) - 8;3 \cdot C_4(K)$, where M is a value for the minutes of proportion. Thus, the general rule for the *diversitas dyametri centralis* near least distance is given by

$$D_7(M) = D_7(0) - kv_7 \cdot C_4(K)$$

that is,

$$D_7(M) = \lambda_0 + kh_4 - kv_6 - kv_7 \cdot C_4(K). \tag{37}$$

The third column in this table displays the argument of anomaly of the first station near least distance as a function of the minutes of proportion (Table 12). The entries follow the same pattern as those near greatest distance, but we note that the entries for 60′ at greatest distance (see Table 7) do not coincide with the entries for 0′ near least distance (see Table 12) in three cases: Saturn, Jupiter, and Mars. The entries for 60′ essentially agree with those in the *Almagest* and sets of tables related to it, corresponding to the least distance of the center of the epicycle, when the mean argument of center, $\bar{\kappa}$, is 180°. The entries, graphed in Fig. F, increase (except for Mercury) monotonically and vary in a range which is different for each planet: 1;19° (Saturn), 1;31° (Jupiter), 6;9° (Mars), 1;12° (Venus), and −0;40° (Mercury).

If we call $S_7(M)$ the tabulated argument of anomaly of the first station near perigee, the entries are can be computed by means of a linear function of M, the minutes of proportion,

$$S_7(M) = S_7(60) + [S_7(0) - S_7(60)] \cdot (1 - C_4(K)), \tag{38}$$

Table F Equation of anomaly and *diversitas dyametri* for Mercury (excerpt)

(°)	Equation of anomaly (°)	*Diversitas dyametri* (*directus* or *retrogradus*) (°)	(°)	Equation of anomaly (°)	*Diversitas dyametri* (*directus* or *retrogradus*) (°)
0	22; 2	2; 1	180	22; 2	2; 1
1	22;19	2; 2	...		
...			213	5;27	0; 8
90	42;35	3;30			*Secunda*
...			214	5; 7	0; 6
110	44; 3	9;48	215	4;48	0; 4
111	44; 4	9;52			*Statio*
112	44; 4	9;55			*Directus*
113	44; 3	9;59	216	4;30	0; 3
...			...		
130	42;46	4; 1	223	2;38	0; 1
131	42;37	4; 2	224	2;25	0; 0
...			...		
136	41;39	4; 2	229	1;27	0; 0
137	41;26	4; 1	230	1;18	0; 1
...			...		
144	39;34	3;59	247	0; 1	0; 9
		Prima	248	0; 0	0;10
145	39;16	3;58	249	0; 0	0;11
146	38;57	3;56	250	0; 1	0;12
147	38;37	3;54	...		
		Statio	270	1;29	0;32
		Retro.	...		
148	38;16	3;52	358	21;29	1;59
...			359	21;45	2; 0

which, in the case of Mars, turns into

$$S_7(M) = 159;15° - 6;9° \cdot (1 - C_4(K)).$$

(ii) In the second table the argument ranges from 0° to 29° for each zodiacal sign. There are two columns (equation of anomaly and *diversitas dyametri*). The title indicates that the entries are given for least distance of the center of the epicycle (*ad longitudinem propiorem*), that is, near perigee (see Table F). As was the case for Table D, the words *statio, prima*, and *secunda* are inserted among various entries.

The relevant information for the equation of anomaly is summarized in Table 14, where the "underlying parameter" was derived by taking half the difference between maximum and minimum equation of anomaly.

For each planet the underlying parameter agrees with that in the PAT for mean distance, and so do the entries for the equation of anomaly, but for a vertical

Table 14 Equation of anomaly near least distance

Least distance	Entry for 0°	Minimum equation of anomaly	Maximum equation of anomaly	Underlying parameter	Vertical displacement (kv_6)
Saturn	6;13°	0;0° (261°–266°)	12;26° (94°–99°)	6;13°	6;13°
Jupiter	11; 3°	0;0° (258°–261°)	22; 6° (99°–102°)	11; 3°	11; 3°
Mars	41;10°	0;0° (229°)	82;20° (131°)	41;10°	41;10°
Venus	45;59°	0;0° (224°–225°)	91;58° (135°–136°)	45;59°	45;59°
Mercury	22; 2°	0;0° (248°–249°)	44; 4° (111°–112°)	22; 2°	22; 2°

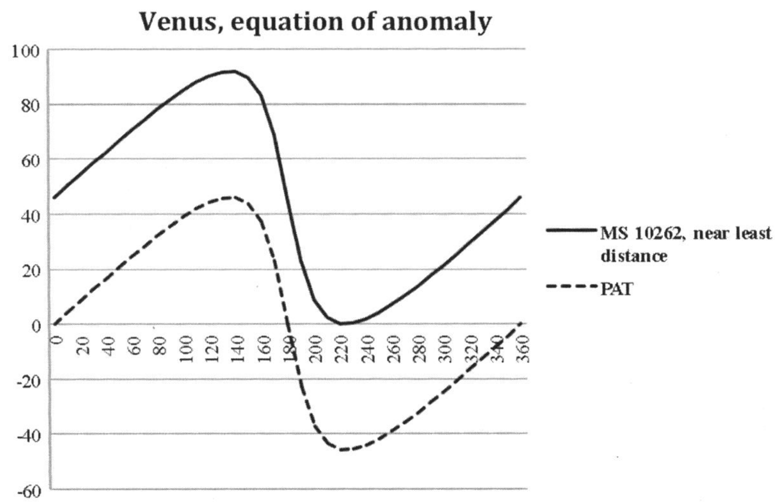

Fig. H Venus, equation of anomaly as a function of the true argument of anomaly

displacement. If $c_6(\alpha)$ represents the equation of anomaly in the corresponding PAT, that in the Tables for the Seven Planets is given by

$$C_6(A) = c_6(\alpha) + kv_6, \tag{39}$$

where $kv_6 = 45;59°$ in the case of Venus.[27] Figure H displays the tabulated values of the equation of anomaly for Venus near least distance in the Tables for the Seven Planets and at mean distance in the PAT. The terminology in the Tables of the Seven Planets may be confusing, but the author sought to maintain a parallel structure in his treatment of the equation of anomaly near apogee and near perigee. The entries near apogee range from greatest distance to mean distance, and those near perigee range from mean distance to least distance. Hence, what is here called "the equation of anomaly near least distance" actually refers to the equation of anomaly at mean distance, to be applied to distances between mean and least distance.

[27] This expression is equivalent to $C_6(A) = c_6(A) + kv_6$, because $A = \alpha$.

Table 15 Difference at least distance

Least distance	Entry for 0°	Minimum difference at least distance	Maximum difference at least distance	Vertical displacement (kv_7)
Saturn	0;25°	0;0° at 249°–258°	0;50° at 102°–111°	0;25°
Jupiter	0;33°	0;0° at 240°–253°	1; 6° at 107°–120°	0;33°
Mars	8; 3°	0;0° at 201°	16; 6° at 159°	8; 3°
Venus	1;52°	0;0° at 197°–199°	3;44° at 161°–163°	1;52°
Mercury	2; 1°	0;0° at 224°–229°	4; 2° at 131°–136°	2; 1°

The *diversitas dyametri* tabulated for each planet is the difference near least distance to be added to the corresponding equation of anomaly at mean distance (see the extremal values in Table 15). The entries in this column agree with those for the same quantity, $c_7(\alpha)$, in the PAT, but for a displacement, which differs from one planet to another. If $c_7(\alpha)$ is the difference at least distance in the corresponding PAT, the difference at least distance in the Tables for the Seven Planets is given by

$$C_7(A) = c_7(\alpha) + kv_7. \tag{40}$$

Note that at least distance of the center of the epicycle, i.e., perigee, the equation of anomaly is greatest.[28]

As indicated previously in Eq. 29, in Ptolemy's notation the total correction when the planet is near perigee is

$$c(\alpha) = c_6(\alpha) + c_7(\alpha) \cdot c_8(\bar{\kappa}),$$

which, in standard Alfonsine notation, is equivalent to

$$c(\alpha) = c_6(\alpha) + c_7(\alpha) \cdot c_4(\bar{\kappa}).$$

In the Tables for the Seven Planets, the corresponding expression near perigee is

$$C(A) = C_6(A) + C_7(A) \cdot C_4(K). \tag{41}$$

This is an expression involving only positive terms and leads to the same true longitude of the planets as the standard Alfonsine procedure.

To find the true longitude, λ, of the planet at that time according to the PAT one has to add the correction $c(\alpha)$ to its mean longitude, that is, to the sum of the longitude of the apogee, λ_0, and the true argument of center of the planet, κ:

$$\lambda = \lambda_0 + \kappa + c(\alpha),$$

where $c(\alpha) = c_6(\alpha) + c_7(\alpha) \cdot c_4(\kappa)$.

[28] These tables bear some similarity with the tables of John Vimond, who tabulated $c_6 - c_5$, but he did not apply any displacements to the planets. Vimond also tabulated $c_5 + c_7$, which has no counterpart in these tables (see Chabás and Goldstein 2004, pp. 248–256).

Now, the author of the Tables of the Seven Planets introduced an analogous expression to find the true longitude of the planet:

$$L = D_7(M) + K + C(A), \tag{42}$$

where $C(A) = C_6(A) + C_7(A) \cdot C_4(K)$: see Eq. 41. The term $D_7(M)$ is tabulated under *diversitas dyametri centralis*.

The general expression for the longitude of a planet near perigee is therefore given as

$$L = D_7(M) + K + C_6(A) + C_7(A) \cdot C_4(K), \tag{43}$$

which can be obtained from

$$\lambda = \lambda_0 + \kappa + c_6(\alpha) + c_7(\alpha) \cdot c_4(\kappa). \tag{44}$$

To be sure,

$$\begin{aligned}\lambda &= \lambda_0 + \kappa + c_6(\alpha) + c_7(\alpha) \cdot c_4(\kappa)\\ &= \lambda_0 + (K + kh_4) + (C_6(A) - kv_6) + (C_7(A) - kv_7) \cdot C_4(K)\\ &= \lambda_0 + kh_4 - kv_6 - kv_7 \cdot C_4(K) + K + C_6(A) + C_7(A) \cdot C_4K) = L,\end{aligned}$$

provided that $D_7(M) = \lambda_0 + kh_4 - kv_6 - kv_7 \cdot C_4(K)$, which is the case: see Eq. 37. All terms appearing in Eq. 43 are positive and are found directly in the tables. Note that $D_5(M)$ and $D_7(M)$ have no counterparts in the standard Alfonsine tables.

To illustrate Eq. 43 consider the position of Mars at epoch (noon, February 28, 1341), when the planet is near perigee. The mean argument of center, $\bar{K}$, and the mean argument of anomaly, $\bar{A}$, are given as 55;6,26° and 84;48,42°, respectively, rounded to the seconds (see Table 3). From Eqs. 16 and 17 it follows that $\bar{\kappa}$ and $\bar{\alpha}$ in the standard Alfonsine Tables are 116;6,26° and 96;48,42°, respectively (where $kh_3 = 61°$ and $kv_3 = 12°$: see Table 5). The corresponding values of the equation of center in both sets of tables are $C_3(\bar{K}) = 1;23°$ and $c_3(\bar{\kappa}) = -10;37°$, and Eq. 18 holds. Thus, the true arguments of center are $K = 56;29°$ and $\kappa = 105;29°$, and Eq. 27 holds, where $kh_4 = 49°$ (see Table 6). The tabulated *equatio porcionis* for $\bar{K} = 55;6,26°$ is $E(\bar{K}) = 22;37°$, and thus $A = 84;48,42° + 22;37° = 107;26°$. The argument of anomaly, α, according to the PAT, is 107;26° (=96;48,42° + 10;37°), and we note that $A = \alpha$. The minutes of proportion corresponding to $K = 56;29°$ and $\kappa = 105;29°$ are $C_4(K) = 17/60$ and $c_4(\kappa) = 17/60$, and we note that $C_4(K) = c_4(\kappa)$, as indicated in Eq. 25, valid near perigee. The tabulated values for the equation of anomaly, $C_6(A)$, and the *diversitas*, $C_7(A)$, are 79;11° and 11;48°, respectively, whereas the values for the equation of anomaly, $c_6(\alpha)$, and the additive difference, $c_7(\alpha)$, found in the PAT are 38;1° and 3;45°, respectively. We note that Eqs. 39 and 40 hold, where $kv_6 = 41;10°$ and $kv_7 = 8;3°$ (see Tables 14, 15). From Eq. 41, $C(A) = 79;11° + 11;48° \cdot (17/60) = 82;32°$, whereas the PAT yield $c(\alpha) = 38;1° + 3;45° \cdot (17/60) = 39;5°$.

Now, the true longitude derived with the standard Alfonsine Tables is $\lambda = 133;23,50° + 105;29° + 39;5° = 277;57,50°$ (see Table 8). On the other hand, from Eq. 37, $D_7(17) = 141;13,49° - 8;3° \cdot 17/60 = 138;56,58°$ (see Table 12). Thus, $L = 138;56,58° + 56;29° + 82;32° = 277;57,58°$, in agreement with the previous result.

Similarly, the general expression for the longitude of a planet near apogee is given as

$$L = D_5(M) + K + D(A) + C_5(A) \cdot C_4(K), \tag{45}$$

and it can be obtained from the standard Alfonsine procedure

$$\lambda = \lambda_0 + \kappa + c_6(\alpha) - c_5(\alpha) \cdot c_4(\kappa). \tag{46}$$

To be sure,

$$\begin{aligned}
\lambda &= \lambda_0 + \kappa + c_6(\alpha) - c_5(\alpha) \cdot c_4(\kappa) \\
&= \lambda_0 + (K + kh_4) + c_6(\alpha) - c_5(\alpha) + c_5(\alpha) + c_5(\alpha) \cdot c_4(\kappa) \\
&= \lambda_0 + (K + kh_4) + D(A) - \max(c_6 - c_5) + c_5(\alpha) + c_5(\alpha) \cdot c_4(\kappa) \\
&= \lambda_0 + kh_4 - \max(c_6 - c_5) + K + D(A) + c_5(\alpha) \cdot (1 + c_4(\kappa)) \\
&= \lambda_0 + kh_4 - \max(c_6 - c_5) + K + D(A) + (C_5(\alpha) - kv_5) \cdot C_4(K) \\
&= \lambda_0 + kh_4 - \max(c_6 - c_5) + K + D(A) - kv_5 \cdot C_4(K) + C_5(A) \cdot C_4(K) \\
&= \lambda_0 + kh_4 - \max(c_6 - c_5) - kv_5 \cdot C_4(K) + K + D(A) + C_5(A) \cdot C_4(K) = L,
\end{aligned}$$

provided that $D_5(M) = \lambda_0 + kh_4 - \max(c_6 - c_5) - kv_5 \cdot C_4(K)$, which is the case: see Table 7 and Eq. 32. All terms appearing in Eq. 44 are positive and are found in the tables.

To illustrate Eq. 44, consider the position of Mars at noon, October 1, 1340, when $\bar{\kappa} = 37;31,6°$, and thus the planet is near apogee. From Eq. 16 it follows that $\bar{K} = 336;31,6°$ (where $kh_3 = 61°$; see Table 5). In the PAT, the corresponding equation of center is $c_3(\bar{\kappa}) = -6;26,40°$, whereas the tabulated *equatio centri* and *equatio porcionis* for $\bar{K} = 336;31,6°$ are $C_3(\bar{K}) = 5;33,20°$ and $E(\bar{K}) = 18;26,40°$, respectively. We note that Eq. 18 holds, where $kv_3 = 12°$ (see Table 5). We also note that Eq. 27 holds, where $kh_4 = 49°$ (see Table 6). Then, $\kappa = 31;4,26°$ because of Eq. 18, and $K = 342;4,26°$ because of Eq. 19. In the PAT, the corresponding minutes of proportion is $c_4(\kappa) = -51/60$, whereas the minutes of proportion tabulated here is $C_4(K) = 9/60$. Therefore Eq. 24 holds, where $kv_4 = 60'$. The mean argument of anomaly of Mars for that date is $\bar{\alpha} = 27;35,24°$, and thus the true argument of anomaly is $\alpha = 34;2,4°$, because of Eq. 23, and $A = 15;35,24°$, because of Eq. 17, where $kv_3 = 12°$. Therefore, $A = 34;2,4°$, because of Eq. 22. We note that $A = \alpha$. In the PAT, the corresponding entries for the equation of anomaly and the difference at greatest distance are $c_6(\alpha) = 13;26°$ and $c_5(\alpha) = 0;48°$, respectively. Thus, the total correction is $c(\alpha) = 13;26° + 0;48°(-51/60) = 12;45,12°$. On the other hand, the tabulated values for the *diversitas dyametri* and the equation of anomaly are $C_5(A) = 6;26°$ (we note that Eq. 35 holds, where $kv_5 = 5;38°$: see Table 11) and $D(A) = 49;23°$. From Eq. 36 it follows that $C(A) = 49;23° + 6;26° \cdot 9/60 = 50;20,54°$.

Table 16 Displacements in the Tables for the Seven Planets

	kv		kv_3	kv_5	kv_6		kv
Sun	89;37,9°	Moon	13; 9°	2;40°	4;56°	Eq. 8th sphere	9°

	kv_3	kh_3	kv_4	kh_4	kv_5	kv_6	kv_7
Saturn	7°	14°	60′	7°	0;21°	6;13°	0;25°
Jupiter	6°	18°	60′	12°	0;30°	11; 3°	0;33°
Mars	12°	61°	60′	49°	5;38°	41;10°	8; 3°
Venus	3°	51°	60′	48°	1;42°	45;59°	1;52°
Mercury	4°	28°	60′	24°	3;12°	22; 2°	2; 1°

Now, the true longitude derived with the standard Alfonsine Tables is $\lambda = 133;23,50° + 31;4,26° + 12;45,12° = 177;13,28°$ (see Table 8). On the other hand, from Eq. 32, $D_5(9) = 145;38,49° - 5;38° \cdot 9/60 = 144;48,7°$ (see Table 7). Thus, $L = 144;48,7° + 342;4,26° + 50;20,54° = 177;13,27°$, in agreement with the previous result.

In Table 16 we present a summary of the values for the displacements, both vertical and horizontal, applied by the anonymous author of the Tables of the Seven Planets to the Sun, the Moon, the 8th sphere, and the planets.

13 Latitudes of the superior planets

Folio 40r displays a table for the latitudes of the three superior planets (the inferior planets are addressed in a very different way on ff. 41r–46v: see Sect. 17, below). This table is in the *Almagest* tradition, and is found in many other sets of tables such as the zij of al-Battānī and the Toledan Tables (see Chabás and Goldstein 2012, Table 9.2B, p. 109). For the zij of al-Battānī, see Nallino (1903–1907); and for the Toledan Tables, see Toomer (1968) and Pedersen (2002).

14 Planetary visibility

On f. 40v there is a table entitled *Tabula visionis et occultationis* for the three superior planets and the two inferior planets. It is a table for the visibility of the planets, also called a table of planetary phases, which is already found in *Almagest* XIII.10, as well as in many other sets of astronomical tables, such as the *Handy Tables*, the zij of al-Battānī, and the Toledan Tables (see Chabás and Goldstein 2012, Table 11.2, p. 125).

15 Possibility of an eclipse

On f. 40v there is a small table entitled *Tabula latitudinis lune in principio medio et fine eclipsis*, an excerpt of which we reproduce below (see Table G). The argument, which

Table G Lunar latitude at eclipse (excerpt)

Argument of latitude (s, °)				Latitude (′)
0 0	6 0	12 0	6 0	0; 0
0 1	5 29	11 29	6 1	5;13
...				
0 12	5 18	11 18	6 12	62;16
0 13	5 17	11 17	6 13	67;23

is the argument of lunar latitude, is presented in four columns and, as indicated in the title, it is restricted to the values for which an eclipse is possible, that is, ±13° from the lunar nodes. The purpose of the table is to show the correspondence between the argument of latitude and the latitude of the Moon, and the entries can be recomputed by means of the modern formula

$$\beta = \arcsin(\sin i \cdot \sin \omega),$$

where ω is the argument of latitude, β is the latitude, and the i is the inclination of the lunar orb to the ecliptic, taken here as 5;0° (the same parameter as in the table for lunar latitude on f. 13r).

16 Eclipsed fraction of the solar and lunar disks

A table for the eclipsed fraction of the solar and lunar disks is also found on f. 40v. The argument ranges from 1 to 12 linear digits (where the diameter of the eclipsed body is 12 digits, and the entries are the corresponding areal digits (where the area of the eclipsed body is 12). It is very common in sets of astronomical tables, such as the zij of al-Battānī, the Toledan Tables, and it is already found in *Almagest* VI.8. However, this was not one of the tables included in the *editio princeps* of the Alfonsine Tables (see Chabás and Goldstein 2012, Table 15.4, p. 175).

17 Latitudes of Venus and Mercury

The tables for the latitudes of Venus and Mercury are presented as six sub-tables for each planet, on ff. 41r–43v and ff. 44r–46v, respectively. Both are double argument tables. The vertical argument (center) is given in four columns at 6°-intervals and altogether there are 30 columns for the horizontal argument (anomaly), also at 6°-intervals. Each of these columns contains entries for the inclination (*latitudo prima*) and the slant (*latitudo secunda*). The deviation (*latitudo tercia*) is presented in another column, which remains invariant in all sub-tables for each planet. Table H displays an excerpt of the table for the latitude of Venus.

The maximum values for Venus and Mercury are, respectively, 7;22° and 4;5° for the inclination, 2;30° and 2;30° for the slant, and +10′ and −13′ for the deviation. This table differs, strongly in presentation and slightly in the basic parameters, from

Table H Latitude of Venus (excerpt)

Center				Anomaly										
(s, °)	(s, °)	(s, °)	(s, °)	0s6°/11s24°		…	3s0°/9s0°		…	5s24°/6s6°		6s0°/6s0°		
				First (°)	Second (°)		First (°)	Second (°)		First (°)	Second (°)	First (°)	Second (°)	Third (′)
0 6	6 6	5 24	11 24	0; 7	0;1	…	0;0	0;12	…	0;38	0; 5	0;46	0;0	1
0 12	6 12	5 18	11 18	0;13	0;2	…	0;0	0;23	…	1;15	0;10	1;30	0;0	2
0 18	6 18	5 12	11 12	0;19	0;2	…	0;0	0;35	…	1;52	0;14	2;14	0;0	3
…														
1 0	7 0	5 0	11 0	0;31	0;4	…	0;0	0;59	…	3; 6	0;24	3;41	0;0	5
…														
2 0	8 0	4 0	10 0	0;54	0;7	…	0;0	1;41	…	5;26	0;42	6;25	0;0	9
…														
2 18	8 18	3 12	9 12	1; 0	0;8	…	0;0	1;53	…	6; 1	0;47	7;10	0;0	10
2 24	8 24	3 6	9 6	1; 1	0;8	…	0;0	1;55	…	6; 8	0;48	7;16	0;0	10
3 0	9 0	3 0	9 0	1; 2	0;8	…	0;0	1;57	…	6;12	0;48	7;22	0;0	10

tables by Parisian astronomers who included the third component of latitude, John Vimond and John of Murs.[29] For the maximum values of deviation, Vimond had $+10'$ and $-45'$ (Ptolemy's values in the *Almagest*), for Venus and Mercury, respectively, whereas John of Murs had $+10'$ and $-23'$, in contrast to $+10'$ and $-13'$ in the tables reviewed here. John of Lignères was aware of the deviation, for he mentions it in the chapters on the latitudes of Venus and Mercury in the canons of his *Priores astrologi motus corporum celestium*,[30] but we do not know of any tables by him similar to those presented here. All in all, the tables for planetary latitudes in the Tables for the Seven Planets, although certainly in the same tradition, are not simply related to those by John Vimond and John of Murs, or by any other known table-maker.

18 Conclusion

Throughout the time from the reception of the Alfonsine Tables in Paris (no later than 1320) to the publication of the *editio princeps* in 1483 in Venice, an intense effort was made to adapt tables in the Alfonsine framework to the needs of practitioners. These adaptations focused on presentation rather than on the parameters underlying the tables of the Alfonsine corpus of tables. The set of tables which we have called the Tables for the Seven Planets for 1340 is an early example of this kind of work, with zodiacal signs of 30° (rather than physical signs of 60°), vacillation in the beginning of the year, and cyclical radices of 32 years, with special characteristics such as displacements, different parameter for the latitude of Mercury, and different terminology.

[29] Chabás and Goldstein (2004, pp. 257–258) and Chabás and Goldstein (2009, p. 309).

[30] An edition of chapters 22 and 23 is found in Saby (1987, pp. 207–211).

The use of double argument tables and the extensive use of displacements, both vertical and horizontal, show a deep insight into planetary astronomy and great skill in producing astronomical tables. This set of tables is also the first example that has been discovered in the Latin world of a systematic use of displaced tables, of which only a few examples were previously known in the medieval astronomical literature. Unfortunately, no name is associated with this set, but the author was an astronomer working around 1340, probably in Southern France (judging by the geographical coordinates underlying these tables), who deserves the highest praise for his skill in providing clever and complex solutions to many problems, and for constructing a compact and consistent set of tables for the planets, building upon the work done by the Parisian group of astronomers in the early 1320s.

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