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Eclipse theory in the Jing chu li: Part I. The adoption of lunar velocity

Yuzhen Guan

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Abstract This paper investigates the methods of eclipse prediction in China before the fourth century AD, with a detailed example of the eclipse theory in the *Jing chu li* (Luminous Inception System 景初曆). As the official calendar of the Jin dynasty and the Kingdom Wei during the three kingdoms period, the *Jing chu li* was used for more than 200 years after it was adopted in 237 AD. From the *San tong li* (Triple Concordance System 三統曆) of the Western Han to the *Jing chu li*, methods for predicting eclipses developed in three important ways: (i) from predicting only lunar eclipses to the prediction of both solar and lunar eclipses; (ii) from relying only on the mean periods of the sun and the moon to taking into consideration the variation in lunar velocity; and (iii) from estimating only a rough date to predicting the exact time of eclipses. This paper addresses two questions: first, how did ancient Chinese astronomers use cycles to predict eclipses in the Han dynasty? Second, how did astronomers such as Liu Hong 劉洪 and Yang Wei 楊偉 revise early eclipse prediction methods? The original text of the *Jing chu li* is analyzed to show how Yang Wei combined lunar velocity theory with the traditional method of predicting eclipses using cycles.

1 Introduction

Eclipse theory is one of the most important and difficult issues early Chinese astronomers faced in their astronomical and astrological practices. Eclipses held great astrological significance in ancient China. A change in the appearance of a heavenly body or variations in its motions held specific meanings in Chinese astrology. These

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Y. Guan (✉)
Department of Egyptology and Assyriology, Brown University, 2 Prospect Street, Box 1899, Providence, RI 02912, USA
e-mail: Yuzhen_Guan@brown.edu

meanings often related to politics. Unexpected or unpredicted astronomical phenomena were seen as signals sent by heaven to the governors of the country. Eclipses, especially solar eclipses, were recognized as a warning to the emperor that his governance is not satisfactory.[1] Not surprisingly, astronomers were asked to pay particular attention to the interpretation and prediction of eclipses. As an astronomical topic, the complexity of solar and lunar motions and other technical issues relating to the prediction of eclipses made them a crucial test of an astronomical system. This article investigates the development of eclipse theory in the *Jing chu li* (Luminous Inception System[2]), in particular the adoption of lunar velocity theory within eclipse theory. Two questions will be addressed: first, how did ancient Chinese astronomers use cycles to predict solar and lunar eclipses? Second, how did astronomers of the second and third century AD such as Liu Hong and Yang Wei revise early eclipse prediction methods up to the time of the official adoption of the *Jing chu li* in AD 237. The original text of the *Jing chu li* will be analyzed so as to show how Yang Wei combined lunar velocity theory with the traditional method of predicting eclipses using cycles.

The texts of most Chinese *li* are recorded in the treatises of temperament and calendar 律曆志 in the dynastic histories. This tradition begins in the earliest dynastic history, the *Shi ji* (史記, the book of history, compiled by Sima Qian 司馬遷, ?145 BC–?86 BC) and is followed in subsequent histories including the *Han shu* (漢書, the book of the Han dynasty, compiled by Ban Gu 班固, 32 AD–92 AD), the *Hou Han shu* (後漢書, the book of the Eastern Han dynasty, compiled by Fan Ye(范曄, 398 AD–455 AD), the *San guo zhi* (三國志, the history of the three kingdoms period, compiled by Chen Shou 陳壽, 233 AD–297 AD) and the *Jin shu* (晉書, the book of the Jin dynasty, compiled by Fang Xuanling 房玄齡, 579 AD–648 AD).The *Shi ji*

[1] For more details on Chinese astrological divination, see Pankenier (2005), Sun (2000), and Loewe (1994). For translations of astrological treatises, see Ho Peng (1966) and Major (1993).

[2] I adopted the translations of the names of *li* from Sivin (2009). A list of Chinese *li* can be found in the section "Astronomical reforms" in the second chapter of the book. In the same section, as well as in his article "Mathematical astronomy and the Chinese calendar," in Sivin (2011), Sivin discussed the distinct meanings of the word *li* and explained why he uses "astronomical system" as the translation to the word *li*. The traditional translation "calendar" only represents the use of *li* as ephemerides in almanacs. However, other meanings include a system for predicting astronomical phenomena, a treatise on this system and a method of computing an ephemeris for the following year. In Sivin's view, simply using the word "calendar" as a translation of the Chinese *li* is partial and misleading. The difficulty in translating the Chinese word *li* lies in finding one English word corresponds to all these distinctive meanings. I therefore suggest using the word *li* as the English translation of Chinese *li*. This is similar to the use of the term *zij* directly in English as the translation of Islamic astronomical handbooks. Historians either straightforwardly call it *zij* in English context or use the word "table" as a representative. Similarly, I also call Chinese *li* "system" in short for "calendrical systems." I adopted the term "system," and I prefer "calendrical system" rather than "calendar" or "astronomical system" as the representative of the word *li*. "Calendrical system" is close to the literal meaning of the Chinese word *li*. Both "Astronomical system" and "Calendrical system" are clear enough to indicate the complexity, function, and the systematic structure of the Chinese *li* and show the most important similarity between Chinese, Islamic, and western theoretical astronomy in that they all attempt to study and make predictions of significant astronomical phenomena. I agree that the official systems are all astronomical systems. However, there are other systems in China that are astronomical systems but not related to the calendar. For example, the system in the *Zhou bi suan jing* is an astronomical system, but it is not a calendrical system and cannot be used as an official calendar. The structure of the system in the *Zhou bi suan jing* is unlike other Chinese official *li*. I also use the original Chinese word *li* in the name of a system rather than attempting to translate it. For example, I use *Jing chu li* rather than *Jing chu* system. In this article, I use calendar to refer to the Velocity Calendar.

contains an early form of a Chinese *li*, the *Li shu jia zi pian* 曆術甲子篇. Other early systems to be discussed here include the *San tong li* (Triple Concordance System, adopted in 104 BC) in the *Han shu*, the *Si fen li* (Quarter Remainder System 四分曆, adopted in 85 AD) in the *Hou Han shu*, the *Qian xiang li* (Supernal Emblem System 乾象曆, adopted in 206 AD) and the *Jing chu li* (Luminous Inception System 景初曆, adopted in 237 AD) in the *Jin shu* and *Song shu*. These preserved works of calendrical astronomy preserve most of our evidence for early Chinese mathematical astronomy; nonofficial treatises such as the *Zhou bi suan jing* which deal with other aspects of mathematical astronomy, especially the mathematics of the gnomon, provide some additional information (Cullen 1996). Although other *li* from the Western Han to the Jin are mentioned in the histories, the texts of these systems themselves are not preserved. For example, Han Yi 韓翊 constructed the *Huang chu li* (Yellow Inception System 黃初曆) around the time of the *Qian xiang li* and *Jing chu li* but it was not officially adopted by any kingdom and no texts are preserved.

All *li* have a similar structure. They always begin with a preface explaining why the system is built and how good it is. The second section is usually a list of constants used in the calculation. This is followed by a series of procedures to calculate astronomical phenomena, including, for example in the *Jing chu li*, solar theory, lunar theory, intercalation, eclipse theory, and planetary theory. Earlier systems sometimes have fewer sections and simpler methods but their structure and, more importantly, the function of the systems are similar.

2 The adoption of the *Jing chu li*

There are a total of around fifty *li* officially adopted in China. The procedure by which a new system is adopted is the same throughout most of Chinese history: the emperor orders for a new *li* to replace the existing one. Officials in the astronomical bureau will follow the call and work on the new system. After testing the system against astronomical observations, the system will be submitted and accepted. However, the astrological significance and the astronomical complexity of eclipses complicate the replacement of *li* (Niu 2004). According to Sivin, four situations could happen. First, the officials do their work well and produce an exceptional system. Second, the new system does not have improvement on the existing one. Third, the new system is not adopted, even though it is better than the existing one. Fourth, it is also possible that an astronomer, usually an official, develops new theories and realizes the current system is not accurate enough; the ability to predict astronomical phenomena is limited. He presents and succeeds in persuading the emperor and other officials to adopt the new *li* and replace the old one (Sivin 2011).

The adoption of the *Jing chu li* belongs to the fourth circumstance. When the system being used is not accurate enough, Yang Wei presented his system and claimed that his system was better than any other ones. The circumstances surrounding the adoption of the *Jing chu li* is discussed in several early dynastic histories, including the *Hou Han shu*, the *Jin shu*, and the *Song shu*. The report from the *Song shu* is both clear and brief:

> During the Era of Guang He (光和, Glorious Harmony, 178 AD to 184 AD, the third reign period used by the Ling 靈 emperor in the Eastern Han), the Observer

of the *Gucheng* Gate 穀城門候 [3] Liu Hong was the first to realize that the *Si fen li* is off the heavenly motions. He built the method of *Qian xiang* with 589 as Era divisor and 145 as *Dou* Fraction (the fraction of the Lodge *dou*). He built the Velocity Calendar (*Chi ji li* 遲疾曆) to project the lunar motion. It is similar to the *Tai chu li* and the *Si fen li*, while finer and more accurate. During the Era of Huang chu (黃初, 220 A.D. to 226 A.D., the first reign period used by the Wei kingdom) of the Wen 文 emperor in the Wei, the Assistant Grand Astronomer 太史丞 Han Yi believed the *Qian xiang li* cut down the *Dou* Fraction too much, and it will be ahead of the heavenly motions as time goes on. He built the *Huang chu li* with 4883 as Era divisor and 1205 as *Dou* Fraction. Later, the Prefect of the Masters of Writing 尚書令 Chen Qun 陳群 presented a memorial, which said, "it is difficult to get a clear understanding on the *li* and the numbers. Scholars from earlier dynasties argued a lot. The inauguration of the *Huang chu li* is because the *Si fen li* is aged and inaccurate. The mandate has come to Wei, it is a good time to correct the *li* and ascertain the seasons. Han Yi first built the *Huang chu li*, he was afraid it was not accurate and used the *Qian xiang li* to compare and adjust. During three years, even more opinions were brought up on right or wrong. (They) neglect the essential (issues) and concern the minor ones. To argue on the size, (they) question the measures. This took a long time, and there is no result. According to the yi-debate by the three high officials (*san gong* 三公, three high officials who are responsible for important affairs), both (systems) synthesize and explicitly made arguments, they are heading to one place from different roads. It is better to let them use instruments, to practice their theories. In the time of one year, the advantages and disadvantages will certainly be judged. The matters will be agreed." (The emperor) agreed. During the Era of the Ming 明 emperor, Gentleman of the Masters of Writing 尚書郎 Yang Wei built the *Jing chu li*, it has been used through the Jin and the Song.

光和中，穀城門候劉洪始悟《四分》於天疏闊，更以五百八十九為紀法,百四十五為斗分，造《乾象法》。又制遲疾曆以步月行，方于《太初》、《四分》，轉精微矣。魏文帝黃初中，太史丞韓翊以為《乾象》減斗分太過，後當先天，造《黃初曆》，以四千八百八十三為紀法，一千二百五為斗分。其後尚書令陳群奏，以為"曆數難明，前代通儒多共紛爭。黃初之元，以《四分曆》久遠疏闊，大魏受命，宜正曆明時。韓翊首建《黃初》，猶恐不審，故以《乾象》互相參校。歷三年，更相是非，舍本即末，爭長短而疑尺丈，竟無時而決。按三公議，皆綜盡曲理，殊塗同歸，欲使效之璿璣，各盡其法，一年之間，得失足定，合於事宜。"奏可。明帝時，尚書郎楊偉制《景初曆》，施用至於晉、宋。

(Shen 1974)[4]

[3] I take the translations of official titles from Bielenstein (1980). However, here I translate 穀城門候 as "the Observer of the *Gucheng* Gate" instead of "the Captain of the *Gucheng* Gate." I take the Chinese character 候 for an observer who keeps watching sky rather than a captain of a guard team.

[4] *Jin shu* recorded the historical events in more detail; see Morgan (2013). The quote of Chen Qun is a little different in the *Jin Shu*; I adopted several ideas from Morgan's translation into my own translation.

This passage reviews the development of *li* from the end of the eastern Han dynasty to the establishment of the Jin dynasty. During the Era of Guang He, Liu Hong developed his system *Qian xiang li*.[5] In the first year of Huang Chu of the Wen emperor in the Wei, the Assistant Grand Astrologer Han Yi built the *Huang chu li*. This system is named after the new reign period Huang Chu. It is not preserved in any dynastic histories or other textual source and was not officially used in any kingdom in the three kingdoms period either. At that time, the kingdom of Shu 蜀 adopted the *Si fen li* from the Eastern Han and the kingdom of Wu 吳 used Liu Hong's *Qian xiang li* as its official system. The kingdom of Wei is the strongest power at the time but had not officially adopted a new system to replace the *Si fen li*. Officials such as Chen Qun argued that the kingdom needs a new *li* since "the mandate has come to Wei, it is a good time to correct the *li* and ascertain seasons." At the time that Chen Qun wrote, the *Huang chu li* and *Qian xiang li* had already been discussed for three years but no conclusion had been reached when comparing the two systems. Chen Qun argued in favor of stopping the theoretical discussion and making predictions with both systems that could then be compared with observations. The *Jin shu* recorded the results of this competition

> Among the predictions of five solar and lunar eclipses, the *Qian xiang* has four further, the *Huang chu* has one closer.[6]
>
> 凡課日月蝕五事，《乾象》四遠，《黃初》一近。
>
> Among fifteen appearances and disappearances of four planets, *Qian xiang* made seven closer and two correct, *Huang chu* has five closer and one correct.
>
> 凡四星見伏十五；《乾象》七近二中，《黃初》五近一中。

(Li 1974).

One way to know which system is better, *Huang chu li* or *Qian xiang li*, is to see which system makes better predictions according to the observations of the astronomical phenomena, for example, eclipses, in the coming year.[7] Morgan (2013) has discussed the Huang chu debate and observation tests in detail. Here, the results of the eclipse predictions are divided into two groups, closer (*jin* 近) and further (*yuan* 遠), depending on which prediction is more accurate. The *Qian xiang* exceeds the *Huang chu* four times and *Huang chu* has one prediction closer to the observation. The results of the planetary phenomena predictions are divided into two groups, correct and closer. The other predictions by the *Qian xiang li* and the *Huang chu li* are not brought up in the text, but these should be just further or be regarded as missed

[5] According to He Chengtian 何承天, the author of the *Yuan jia li* in 443 AD, Liu Hong, produced the *Qian xiang li* between 178 AD to 184 AD. According to Chen (1986), Liu Hong first developed a draft of the *Qian xiang li* few years after 184 AD, and after testing against observations for more than 10 years, he finally finished the system in 206 AD.

[6] The preserved text says "Qian xiang has four further". But according to the results of the tests, Qian xiang has four closer. See Morgan (2013).

[7] Eclipses were often seen to be the crucial test of a system. For example, in the (much later) Song (1976), treatise of calendar *元史・曆志*, we read "the accuracy of *li* is checked when eclipses happen 曆法疏密，驗在交食.

than the ones which are correct and closer. The number of the other predictions can be easily deduced using simple arithmetic. In the planetary predictions, the *Qian xiang* missed six times and was closer to the observation seven times. The text claims that the *Qian xiang li* successfully predicted the planetary phenomena twice, although it does not say how accurate the prediction needs to be considered as successful. On the other side, the *Huang chu li* missed nine times and was closer five times. It also has one successful prediction. Considering both the eclipse and planetary predictions, both systems have correct and missed predictions. The *Qiang xiang li* took a little advantage over the *Huang chu li*, but the lead is limited. At the same time, the number of missing predictions was a hint that new methods still need to be studied; a new *li* still needed to be built. In the first year of the Jing Chu of the emperor Ming 明, Yang Wei built the *Jing chu li*. In the preface of the *Jing Chu li*, Yang Wei claimed his system is better than any previous systems:

> The *Jing chu li* I built uses brief and accurate methods and precise numbers. It is accurate to implement and use, efficient to regulate, and easy to learn. Even if we again ask Yan and Sang to mentally calculate, Li shou to operate using *chou*, Zhong and Li to observe with gnomon, Xi and He to measure the shadow.[8] Even ask them to observe the heavenly motion, predict and verify the motions of the sun and the moon to a highly accurate extent and to the extreme of the calculations. However, all of these are not as good as mine. As a result, the systems and the numbers all became inaccurate after dynasties. From the Yellow Emperor on, it has been frequently reformed with no stop.

臣之所建《景初曆》，法數則約要，施用則近密，治之則省功，學之則易知。雖複使研桑心算，隸首運籌，重黎司晷，羲和察景，以考天路，步驗日月，究極精微，盡術數之極者，皆未能並臣如此之妙也。是以累代曆數，皆疏而不密，自黃帝以來，常改革不已。

(Li 1974).

Before 237 AD, there had been considerable discussion over the official system in the kingdom of Wei, but the *Si fen li* was able to keep its official status because of the lack of clarity over what system was better. This situation finally changed when Emperor Ming decided to adopt the *Jing chu li* as the official system. The *Jing chu li* continued to be the official system in the Wei, Jin, Song, and Northern Wei for more than two hundred years (237 AD–443 AD).

3 The use of cycles in eclipse prediction before the *Jing chu li*

Visible planetary and lunar motions appear repeatedly with their own periods. Ancient Chinese scholars use time intervals as the units in calculating with these cycles in their

8 Yan, Ji Ran 計然, who has another name Ji Yan. He is a counsellor in the Chunqiu Period. Sang, Sang Hongyan 桑弘羊, 152 BC–80 BC, a politician in the Han Dynasty. *Chou* 籌, counting rods, traditional Chinese mathematical tool. Numbers were carved on bamboo strips. Mathematicians use the strips in calculation. Zhong, Li, Xi, and He are early astronomers.

li. In the *Jin chu li*, Yang Wei uses six units: the *Yuan*, Era, Rule, year, month, and day. In order to simplify calculation between different cycles, terms with the name "divisor" such as the Day Divisor (*Ri fa* 日法), the Rule Divisor (*zhang fa* 章法), the Era Divisor (*Ji fa* 紀法), and the Yuan Divisor (*Yuan fa* 元法) are used as the denominators in fractions. After a certain period, usually a periodic cycle, the fraction disappears and we are left with an integer. For example, the length of a tropical year in the *Jing chu li* is 365 455/1843 days, where 1843 is known as the Era Divisor. One Era has 1843 years, which is a total of 673150 ($365 \times 1843 + 455$) days, a whole number rather than a number with a fraction of days.[9]

The earliest eclipse theories in China developed during the Western Han and the Eastern Han. It is believed that the first *li* to contain an eclipse theory is the *San tong li*. The following system, the *Si fen li*, uses the same eclipse period, 23 eclipse possibilities in 135 months (Chen 2003).

> To calculate the lunar eclipses, use the remainder (after taking out) the Meeting Year, multiply the total number of months by 23. If it is over 135, divide it (by 135). If it is not enough, add 23 to reach a month. If it is over 135, take the whole number out of the result. Count out (the last month), it is the month of the eclipse. This time is on the full moon's day. The sun is on the opposite of the synodic point.
>
> 推月食，置會餘歲積月，以二十三乘之，盈百三十五，除之。不盈者，加二十三得一月。 盈百三十五，數所得，起其正，算外，則食月也。加時在望，日冲辰。

(Ban 1962)

In the *San tong li*, the Meeting Year (*hui sui* 會歲) is a period of 513 years, which equals 6345 months. Liu Xin uses this huge number for a numerical reason. 6345 is the least common multiple of 135, the number of months in one eclipse period and 235, the number of months in one Rule, the 19 years intercalary period. This means there are 47 135-month eclipse periods in one Meeting Year. After 6345 months, the cycle of eclipse period and the cycle of intercalation meet again in the same place. The total number of months is calculated from the epoch, when all astronomical periods in the *li* have a uniform starting point. The method uses the remainder after taking whole numbers of the Meeting Year out of the months from the epoch to the year we are calculating. According to the *San tong li*, there are 23 eclipses in 135 months. After each month, the eclipse period has passed 23/135 eclipses to the next whole number, when there is another eclipse (the text reads simply "to calculate the lunar eclipses," but it is clear that it is referring to eclipse possibilities). We therefore multiply the total number of months by 23 and divide it by 135, which means dividing the number by 135/23, the time between two eclipses. The fraction in the result shows where we are within that 135-month eclipse period. The number of months away from the end of this eclipse period is also how far away from the next eclipse.

For example, if we calculate using 4 months as the remainder of the number of months after taking out the Meeting Year, we multiply 4 by 23 to find 92. Since 92 is

[9] For a more detailed introduction on the use of cycles in early Chinese mathematical astronomy, see Sivin (1969).

less than 135, no eclipse has happened in the past 4 months. To find the next eclipse, we need to go beyond 135 and so we must add on two multiples of 23 to get to 138, which is larger than 135. Thus, there will be an eclipse in the second month of that year. If we calculate using 100 months as the remainder of the number of months after taking out the Meeting Year, we will find 100 times 23 equals 2300, which is greater than 135. We therefore need to take out multiples of 135, which can be done by dividing 2300 by 135 to find 17 5/135, from which we can remove the integer part to be left with 5/135. Next, we subtract 5 from 135 and divide the result by 23 to give 5 15/23, meaning that there will be an eclipse in the fifth month of the year.

It is unclear whether the eclipse period in the *San tong li* is intended to be used for both solar and lunar eclipses or only for lunar eclipse. Recently, Shi Yunli and Xing Gang recalculated the lunar and solar eclipses with the *San tong li* method and argued that the *San tong li* was only used in lunar eclipse prediction. (Shi and Xing 2005)

Later, in the three kingdom period and during the Jin dynasty, several new techniques were incorporated with, in particular, the consideration of lunar velocity. In addition, in the *Qian xiang li*, Liu Hong improved the traditional method by the invention of eclipse limits. Eclipse limits show the distance between where an eclipse possibility happens and the intersection of the sun and the moon's path. If the distance is outside of a certain range, an eclipse will not happen. Liu Hong uses less than 14 1115/1457 du and greater than 158 (790+232/1882)/1457 du as the eclipse limits.[10] In the *Jing chu li*, Yang Wei also considered the magnitude of the eclipse and the direction of impact of the shadow.

4 The general eclipse prediction method in the *Jing chu li*

Three issues are discussed in the eclipse section in the *Jing chu li*: first, the general eclipse method; second, related issues in eclipse prediction, for example the magnitude of the eclipse, the relative position of the moon to the sun's path, and the direction of obscuration when eclipse happens; and third, the adoption of lunar velocity theory in the eclipse theory. Similar to the earlier systems, the *Jing chu li* also uses an eclipse period and other cycles in its general eclipse method. The section reads as follows:

> The method of calculating the conjunction at the new moon, the Cross Coincidences, and lunar eclipse is use the Lunar Accumulated Fen of that Era, add the number of the Draconitic-Synodic Differential Value of that Era to it, take the Conjunctional Connect out of it, the remainder is the Draconitic Value at the new moon of the eleventh month of the year. Add the Connect Number to it, take the Conjunctional Connect out when it reaches Conjunctional Connect, the remainder is the Draconitic Value at the new moon of the next month. Add the Syzygy Conjunction Constant to the Draconitic Value of the new moon of one month, take the Conjunctional Connect out when it reaches Conjunctional Connect, the remainder is the Draconitic Value at the full moon of the month. If the Draconitic Value at the new or full moon is smaller than the Syzygy Con-

10 See Chen (1991) for the numbers and calculation of the eclipse limits.

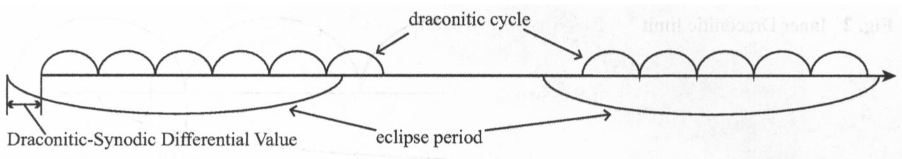

Fig. 1 Draconitic cycles, eclipse periods, and the Draconitic-Synodic Differential Value

junction Constant or bigger than the Inner Draconitic Limit, there will be Cross Coincidence at the new moon, lunar eclipse at the full moon.

推合朔交會月蝕術曰：置所入紀朔積分，以所入紀下交會差率之數加之，以會通去之，餘則所求年天正十一月合朔去交度分也。以通數加之，滿會通去之，餘則次月合朔去交度分也。以朔望合數各加其月合朔去交度分，滿會通去之，餘則各其月望去交度分也。朔望去交分，如朔望合數以下，入交限數以上者，朔則交會，望則月蝕。

(Li 1974).

Even though the first sentence of the text indicates it only predicts lunar eclipse, it is quite clear that this method is also intended to be used in the prediction of solar eclipse, given that both the new moon's day and the full moon's day are mentioned. The general eclipse method is to calculate the differences between one cycle and another, here, between the draconitic cycle and the eclipse cycle. In Fig. 1, the arc above the line shows the draconitic cycle starting from the beginning of the Era. The arc below the line shows the eclipse cycle, which starts a little earlier than the beginning of the Era. The reason it starts earlier is because Yang Wei invented a term Draconitic-Synodic Differential Value to mark the difference at the beginning of the Era between the draconitic cycle and the synodic cycle. This is the first time calculations in *li* are designed to start from multiple points rather than from the *Li* Epoch (*li yuan* 曆元) only. The Draconitic-synodic differential value for *Jia shen* Era 甲申紀 is 620139

The principle of this method is to calculate the Draconitic Value, the period between the moment of the beginning of the draconitic cycle and the moment of conjunction. This shows the difference in time between the moment of conjunction and the moment when the lunar latitude is zero. If they are close enough, an eclipse might happen. If they are not close, the distance between the moon and the sun will be too far for the shadow to cause the eclipse. The way to judge whether it is close enough is to use eclipse limits, which are named the Syzygy Conjunction Constant and the Inner Draconitic Limit in the *Jing chu li*. The term Lunar Accumulated Fen refers to the fen at the beginning of one particular month and equals the number of synodic months from the beginning of that Era to the month we are calculating multiplied by the Connect Number, *tongshu* 通數 134630, the length of a synodic month in the unit of the Day Divisor. The Chinese character fen shows the period related to the Lunar Accumulated Fen is also in the unit of Day Divisor, 1/4559 of one day. To calculate the Draconitic Value, one should take multiples of the Conjunctional Connect, 790110, out of the sum of Lunar Accumulated Fen and Connect Number to get the result. The term Inner Draconitic Limit shows how close is the end of one draconitic cycle to the end of one eclipse period: if the result is smaller than 67315 or bigger than 722795,

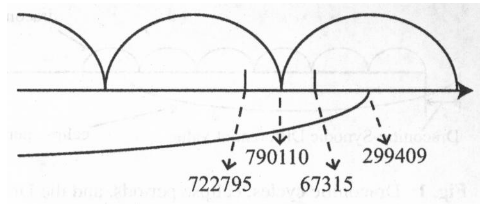

Fig. 2 Inner Draconitic limit

there will be either a solar eclipse possibility at the new moon or a lunar eclipse at the full moon (recall that all the calculations are in the unit of the Day Divisor). See Fig. 2 for situation around the end of the cycles. The arc above the line shows the draconitic cycle, and the arc below the line shows where one eclipse cycle ends.

For example, if we want to calculate the first year of Jing Chu 景初 237 AD According to the first sentence in the *Jing chu li*, "from (the year of) *Ren Chen* to the first year of the Jing Chu, there is a total of 4046 years." 壬辰以來，至景初元年丁巳歲，積四千四十六，算上。 (Li 1974) Each Era has 1843 years and so the first year of Jing Chu will be the 360th year of the third Era, *Jia Shen* 甲申 Era. We now only need to consider the situation in the *Jia Shen* Era in the general eclipse theory. To calculate the total number of months before the first year of Jing Chu, we multiply 360 by 235 and divide by 19 to find the result 4452 12/19. This means there are a total of 4452 months before the eleventh month of the first year of Jing Chu. To calculate the Lunar Accumulated Fraction, we multiply the accumulated months 4452 by the Connective Number, 134630. The result of the Lunar Accumulated Fraction is 599372760. To calculate the Draconitic value, we add the Draconitic-Synodic Differential Value for the *Jia Shen* Era 620139 to it, take multiples of Conjunctional Connect, 790110, out, and the result 299409 is the Draconitic Value of the new moon's day of the eleventh month of the year. It is bigger than 67315 and smaller than 722795, so there is neither a Cross Coincidence at the new moon nor a lunar eclipse possibility at the full moon.

5 The adoption of lunar velocity theory in eclipse theory

One significant feature in the *Jing chu li* is the consideration of lunar velocity in the calculation of the time of an eclipse. Earlier systems only used the mean synodic month to calculate eclipse times. The *Qian Xiang li* is the first system in China to consider lunar velocity theory (Cullen 2002). In the *Jing chu li*, lunar velocity is used to calculate the time of the eclipse. This is explained in the next section of the *Jing chu li*:

> The method of calculating a lunar eclipse in the Velocity Calendar at the conjunction at the new moon, the Cross Coincidences, says first add the Velocity Differential Value of that Era to the Lunar Accumulated Fen of that Era, take (multiples of) the Connect Cycle out, what is left is in days if it reaches the Day Divisor, the remainder is the Day Remainder. Count out the starting day, it is the moment of the conjunction of the eleventh month of the to-be-calculated year in the Velocity Calendar.

To calculate the next month, add one to day and 4450 to the Day Remainder. To calculate the full moon's day, add 14 to day and 3489 to the Day Remainder. Convert the remainder to days when it reaches the Day Divisor and take out the days when it reaches 27. Take the remainder from the Circuit Day Remainder. If the remainder is not enough to subtract, take one day from it and add the Cycle Vacancy to it.

推合朔交會月蝕入遲疾曆術曰：置所入紀朔積分，以所入紀下遲疾差率數加之，以通周去之，餘滿日法得一日，不盡為日餘，命日算外，則所求年天正十一月合朔入曆日也。

求次月，加一日，日餘四千四百五十。求望，加十四日，日餘三千四百八十九。日余滿日法成日，日滿二十七去之。又除余如周日餘，日餘不足除者，減一日，加周虛。

(Li 1974)

This section shows how to calculate the conjunction at the new moon, the Cross Coincidences, and eclipses according to the Velocity Calendar. To calculate the conjunction, we first calculate the difference between the numbers in synodic month and anomalistic month. The start point of all calculation is the beginning of that Era. First, add the Lunar Accumulated Fen, which corresponds to the length of time from the beginning of that Era to the first day of the to-be-calculated month, to the Velocity Differential Value, which corresponds to the difference between the beginning of that Era to the moment of the moon's closest perigee. Take multiples of the Connect Cycle 125621, the length of anomalistic month, out of it. The result gives the difference between synodic months and anomalistic months. Accordingly, the result shows the difference between the synodic cycle and the velocity cycle. This section is a preparative step for the calculation afterward. All calculations are in the unit of the Day Divisor.

To calculate the next month, add the difference between one synodic month and one anomalistic month: one synodic month is 134630/4559 and one anomalistic month is 125621/4559; thus, the difference is (134630–125621)/4559 = 9009/4559 = 1 4450/4559. According to the text, "to calculate the next month, add one to day and 4450 to the Day Remainder." The full moon's day is half a month from the beginning of one month. Half a month is (134630/2)/4559 that equals 67315/4559, which is also 14 3489/4559. According to the text, "to calculate the full moon's day, add 14 to day and 3489 to the Day Remainder."

This method is calculating the situation at the beginning and midpoint of a month based on the mean period of synodic month and anomalistic month. The following sections contain the Velocity Calendar, a table showing the velocity of the moon in different days in one month, and an explanation of how to calculate the true position of the moon based on the lunar velocity in any day. The table is named *Chi ji li* 遲疾曆 (the three characters *Chi* 遲, *ji* 疾 and *li* 曆 literally mean slow, fast, and calendar) and is summarized in Table 1.[11]

[11] The velocity table in the *Jing chu li* is based upon the one devised by Liu Hong in the *Qian xiang li* but calibrated to work the *Jing chu li* Day Divisior.

Table 1 Velocity Calendar (lunar velocity table) in the *Jing chu li*

1 Day	2 The velocity 月行遲疾度 Unit: *du*	3 *Fen*1 (1/19 *du*)	4 Velocity Difference 損益率 *Fen*1	5 Regulation Accumulative *fen* 盈縮積分 *Fen*2	6 Lunar traveling *fen* 月行分 *Fen*1
1	14	14	More 26	The beginning of +	280
2	14	11	More 23	+ 118534	277
3	14	8	More 20	+ 223391	274
4	14	5	More 17	+ 314571	271
5	14	1	More 13	+ 392074	267
6	13	14	More 7	+ 451341	261
7	13	7	Less	+ 483254	254
8	13	1	Less 6	+ 483254	248
9	12	16	Less 10	+ 455900	244
10	12	13	Less 13	+ 410310	241
11	12	11	Less 15	+ 351043	239
12	12	8	Less 18	+ 282658	236
13	12	5	Less 21	+ 200596	233
14	12	3	Less 23	+ 104857	231
15	12	5	More 21	The beginning of –	233
16	12	7	More 19	– 95739	235
17	12	9	More 17	– 182360	237
18	12	12	More 14	– 259863	240
19	12	15	More 11	– 323689	240
20	12	18	More 8	– 373838	243
21	13	3	More 4	– 410310	246
22	13	7	Less	– 428546	250
23	13	12	Less 5	– 428546	259
24	13	18	Less 11	– 405751	265
25	14	5	Less 17	– 355602	271
26	14	11	Less 23	– 278099	277
27	14	12	Less 24	– 173242	278
Circuit Day	14° 13′626		Less 25 and 626	– 63826	279 626/4559

The table shows the velocity of the moon in an anomalistic month. There are six parameters in the table: the day, the speed of the moon in *du* (degree), and *fen* (1/19 of a *du*), the Velocity Difference in *fen*, the Regulation Accumulative *Fen* in the Day divisor, and the Lunar Traveling *Fen*. The Velocity Calendar in the *Qian xiang li* has five columns (see Table 2),[12] the velocity columns (*ri zhuan du fen* 日轉度分) are similar to columns 1–3 in the Velocity Calendar in the *Jing chu li*. The Rate (*lie cui*

[12] For a detailed discussion of the velocity table in the *Qian xiang li*, see Cullen (2002).

Table 2 First five days of the Velocity Calendar in the *Qian xiang li*

1 Day	2 The velocity Unit: *du*	3 *Fen* (1/19 *du*)	4 Rate	5 Velocity Difference *Fen*	6 Velocity Accumulation	7 Lunar traveling fen *Fen*
1	14	10	Lag − 1	More 22	The beginning of + (Ying 盈)	276
2	14	9	Lag − 2	More 21	+ 22	275
3	14	7	Lag − 3	More 19	+ 43	273
4	14	4	Lag − 4	More 16	+ 62	266
5	14	0	Lag − 4	More 12	+ 78	262

列衰) column only appears in the *Qian xiang li.* It shows the changing of the moon's day-to-day speed. The Velocity Difference (*sun yi lü* 損益率) column, the Velocity Accumulative fen (*ying suo ji fen* 盈縮積分) column, and the Lunar Traveling Fen (*yue xing fen* 月行分) column are similar to the third, fourth, and fifth column here. The Velocity Accumulative fen in the *Jing chu li* appears in the unit of Day Divisor after multiplying 4559 to the Velocity Accumulation.

In the first column, days are sequentially listed from 1 to 27. The Circuit Day in the last line refers the fraction of the day in an anomalistic month after 27 whole days. This fraction is 2528/4559 in the *Jing chu li.* In columns 2 and 3, the speed of the moon is listed day by day in *du* and fen_1, 1/19 *du*. In the fourth column, the Velocity Difference compares the speed of the moon on one day and the average daily speed. From day 1 to day 14, less means add the speed by a certain number to get the average speed; more means subtract the speed by a number to reach the average speed. From day 15 to the Circuit Day, less means subtract the speed by a number to reach the average speed and more means add the speed by a number to get the average speed. The line of the Circuit Day listed the distance travelled by the moon with 14 *du*, 13 fen_1, and 626. The number 626 has a denominator 2526. It is for one day rather than for the fraction of one day, this makes the total of the velocity from day 1 to day 28 is 374 *du* 17 fen_1 and 626/2526. It is more than the circle of 365 1/4 *du* (Liu 2003). We are able to calculate the numbers in column 4 by calculating the difference between the value in columns 2 and 3 and the average speed of the moon, 13 *du* 7 fen_1. For example, the speed of the moon in the 11th day is 12 *du* 11 fen_1. We subtract 12 11/19 from 13 7/19 to get 15/19 and put 15 in column 4, day 11. The result corresponds to the "Less 15" in column 4, day 11. In the column of the Regulation Accumulative Fen, the value shows the total of the Velocity Difference in *Fen* beginning from the first day. The positive and negative sign shows the difference in the daily speed of the moon compare to the average speed of the moon. The numbers are calculated from column 2 and conversed to the unit of fen_2 by multiplying the Day Divisor 4559. Yang Wei uses the same Chinese character *fen* 分 for both the unit fen_2 and fen_1. 1 du equals 19 fen_1 and 1 fen_1 equals 4559 fen_2. As a result, 1 fen_1 equals 86621 fen_2. The number 86621 comes from the Rule divisor, 19, multiplied by the Day Divisor 4559. We are able to calculate the data in column 5 from column 4. For example, day 1 in column 4 is 26. This means the moon travels 26 *fen*, or 1 7/19 *du* per day. In order to calculate the value for the third day in the fifth column, we multiply 1 7/19 by 86621 to get 118534. The Velocity Difference for day 2, column 4 is 23. This means the moon travels 23 *fen*1, or 1 4/19 du per day. In order to calculate the value for the third day in the fifth column, we multiply 1 4/19 to 86621, the result is 104857. The total of the Velocity Difference for day 1 118534 and day 2 104857 is 223391, which gives us the value in day 3, column 5. The value for Day n in column 5 is the total of the Velocity Difference in column 4 from day 1 to day n-1 multiply by 4559, the result of 86621/19. Column 6 has the number showing the distance of the moon in fen_1 for each day. It was calculated by multiplying 19 to the value of lunar velocity from column 2 and 3.

> To calculate the Velocity Major and Minor remainder of lunar eclipse at the new moon and the Cross Coincidences, multiply the fraction of day in the (Velocity) Calendar by the Velocity Difference in the Calendar, subtract or add (the result) from the Regulation Accumulative Fen to get the Velocity Accumulation fen. Subtract the Rule Year from the Lunar Traveling Fen according to the Velocity Calendar, divide the remainder from it (the Velocity Accumulation fen), subtract the positive or add the negative result to the minor remainder. If it reaches the Day Divisor, the time of the Cross Coincidence is on the following day. If it is not enough to subtract, the Cross Coincidence is on the preceding day. Follow the Velocity Major and Minor Remainder to set the moment of the lunar eclipse. If the moment is in the Circuit Day of the (Velocity) Calendar, multiply the fraction of day in the Circuit Day by the Negative Accumulative Fen to get the Velocity Accumulation Fen. Multiply the Decreasing Ratio by the fraction of day in the (Velocity) Calendar, then multiply it by the fraction of day in the Circuit Day, add it to [the multiplication of the remainder of the day according to the Calendar and][13] the minor *fen* of the Circuit Day, subtract it from the Velocity Accumulation Fen, the remainder is the Post Velocity Accumulation Fen. Subtract the Rule Year from the Lunar Traveling fen in the Circuit Day, multiply the remainder by the fraction of day in the Circuit Day, add it to the du and the minor fen of the Circuit Day, divide it from the Post Velocity Accumulation Fen, add the result to the Minor Remainder, as the earlier method.
>
> 推合朔交會月蝕定大小餘：以入曆日餘乘所入曆損益率，以損益盈縮積分，爲定積分。以章歲減所入曆月行分，餘以除之，所得以盈減縮加本小餘。加之滿日法者，交會加時在後日；減之不足者，交會加時在前日。月蝕者，隨定大小餘爲日加時。入曆在周日者，以周日日餘乘縮積分，為定積分。以損率乘入曆日餘，又以周日日餘乘之，以[入曆日余乘]周日日度小分，並之，以損定積分，餘為後定積分。以章歲減周日月行分，余以周日日餘乘之，以周日度小分並之，以除後定積分，所得以加本小餘，如上法。

(Li 1974)

[13] The phrase "[multiplication of the remainder of the day according to the Calendar and 入曆日余乘]" is added to the text according to the explanations on the method from Wang Yingwei 王應偉 (1998) and Liu Hongtao (2003).

The *Jing chu li* is able to calculate not only the date of the Cross Coincidences and lunar eclipses, but also the exact moment. In earlier sections, we calculated the conjunction at the new moon, the Cross Coincidences, and lunar eclipses based on the mean speed of the moon. This section allows us to calculate the actual position of the moon based on the lunar velocity table in the preceding section and the mean position of the moon at the beginning of the month. This method is based upon predicting eclipses based on the Velocity Calendar in the earlier section. In order to make an accurate prediction, Yang Wei needs to adopt the lunar velocity theory in the general eclipse theory with mean lunar motion. The difficult part is to consider where the moment of the eclipse is in the Velocity Calendar and how much the moon has moved compared to its mean position. In order to calculate the exact time, Yang Wei calculates the Minor Remainder of the moment. He calls this the Velocity Minor Remainder. He does not calculate the Velocity Major Remainder because it is affected by the result of the Minor Remainder. If the moon's velocity is big enough, he adds or subtracts one day the mean major remainder, if not, the major remainder remains and is used as the Velocity Major Remainder. The starting point of all calculations is the beginning of that Era. All calculations are in the unit of the Day Divisor.

There are two parts in the text, one explaining how to calculate the Velocity Minor Remainder from day 1 to day 27 in the Velocity Calendar and the other explaining how to calculate the Velocity Minor Remainder in the Circuit Day. The first step for both methods is to calculate the Velocity Accumulative fen (accumulative velocity fen). For day 1 to day 27,

$$\text{V.A.F.} = \text{R.A.F} \pm \text{F.V.C.} \times \text{V.D.}$$

Then, the *Jing chu li* calculates the Velocity Minor Remainder,

$$[\text{m.r.} \pm \text{V.A.F.}/(\text{L.T.F.} - \text{R.Y.})]/\text{D.D.}$$

where

m.r.	minor remainder calculated earlier for the Cross Coincidence
R.A.F	Regulation Accumulative Fen
F.V.C.	the fraction of day in the (Velocity) Calendar
V.D.	Velocity Difference
V.A.F.	Velocity Accumulation fen
R.Y.	Rule Year, 19
L.T.F.	Lunar Traveling Fen
D.D.	Day Divisor, 4559

The reason to divide the Day Divisor is because the numbers of the m.r., R.A.F., and F.V.C. are all in the unit of the Day Divisor. We only need to discuss the astronomical meaning of m.r. ± V.A.F. / (L.T.F. – 19). Yang Wei begins the calculation from m.r., the result from earlier section and adds the regulation V.A.F. / (L.T.F. – 19) to it. According to the Velocity Calendar, the V.A.F. shows the regulation from the beginning of the month to the preceding day. Yang Wei wants to get the actual position of the moon by means of i) adding the velocity of the moon from the beginning of the month to the preceding day by adding the V.A.F. to the m.r. and ii) adding the velocity of the moon on the calculating day by multiplying F.V.C. to the V.D., which marks the change in the speed of the moon on that day. The result shows how far the moon moved in the unit of fen_1 (the R.A.F. is in the unit of fen_2, the multiplication of *fen*1 and the D.D., V.D. is in the unit of fen_1), Yang Wei divides it by the difference between the sun and the moon's motion. The speed of the sun is 1 *du* per day, which equals 19 fen_1 per day. L.T.F. means the moon travels 231–280 fen_1 in one day. This changes the result from having a distance unit to fraction without unit. Then, Yang Wei adds the result to the minor remainder to get the Velocity Accumulation fen. If it is bigger than the Day Divisor, add one to the major remainder, which marks the day of the Cross Coincidence. Then, the time of the Cross Coincidence will be on the following day. If the result is negative, subtract one to the major remainder and the time of the Cross Coincidence is in the preceding day. If the result is between 0 and 1, there is no more regulation on the major remainder and the Cross Coincidence is on the day calculated from the mean periods. Following the method from this section, the time of lunar eclipse should be based on the Velocity Minor Remainder, that is to say, the lunar velocity theory.

The second part to calculate the Velocity Minor Remainder in the Circuit Day is mathematically more complicated but astronomically the same. Yang Wei uses a Post Velocity Accumulation Fen. This method can be converted to the same method used to calculate the Velocity Minor Remainder from day 1 to day 27 in the Velocity Calendar (Liu 2003).

$$\begin{aligned}&\text{Velocity Accumulation Fen} = \text{F.C.D.} \times \text{R.A.F.}\\&\text{Post Velocity Accumulation Fen} = \text{F.V.C} \times \text{R.A.F.}\\&\quad - (\text{V.D.} \times \text{F.V.C.} \times \text{F.C.D.} + \text{m.f.C.D.})\end{aligned}$$

F.C.D. the fraction of day in the Circuit Day in the (Velocity) Calendar
m.f.C.D. the minor *fen* of the Circuit Day

> To calculate the time, multiply the Velocity Minor Remainder by 12, convert it to double hours when it reaches the Day Divisor. The counting begins from *Zi* 子, count out. The result is the moment of oppositions in double-hour. If there is a remainder (of double hours), multiply it by 4 and divide it by the Day Divisor. one (1/4) is small (*shao* 少), two is half (*ban* 半), three is larger (*tai* 太). If there is again a remainder, multiply it by three and divide it by the Day Divisor. one names strong (*qiang* 強). If (the remainder) is more than one half (1/2), adopt it. If it is less than one half, take it out. Add strong and small (1/12 + 3/12) to get strong small (4/12). Add strong and half (1/12 + 6/12) to get strong half (7/12). Add strong and large (1/12 + 9/12) to get strong large (10/12). Two strong (1/12) make it a weak small (2/12), add it to small (3/12) to get weak half (5/12), add it to half (6/12) to get weak large (8/12), add it to large (9/12) to get weak (11/12) in one double-hour. Name it with the related double-hour and get the small, large, half, strong, and weak. If the full moon's day when the eclipse happens is no more than 4 days away from one solar term, see the Limit Number. If the full moon's day is more than 5 days away from the solar term, see the Interval Limit

Table 3 Use of the twelve branches in double-hour reckoning

Zi	*Chou*	*Yin*	*Mao*	*Chen*	*Si*	*Wu*	*Wei*	*Shen*	*You*	*Xu*	*Hai*
子	丑	寅	卯	辰	巳	午	未	申	酉	戌	亥
1	2	3	4	5	6	7	8	9	10	11	12

Table 4 Twelve divisions of a double-hour

qiang	shao ruo	shao	shao qiang	ban ruo	ban	ban qiang	tai ruo	tai	tai qiang	ruo
強	少弱	少	少強	半弱	半	半強	太弱	太	太強	弱
Strong	Weak small	Small	Strong small	Weak half	Half	Strong half	Weak large	Large	Strong large	Weak
1/12	2/12	3/12	4/12	5/12	6/12	7/12	8/12	9/12	10/12	11/12

> Number. If the Velocity Minor Remainder is less than the Interval Limit Number or the Limit Number, count in the preceding day.
>
> 推加時：以十二乘定小餘，滿日法得一辰，數從子起，算外，則朔望加時所在辰也。有餘不盡者四之，如日法而一爲少，二爲半，三爲太。又有餘者，三之，如日法而一爲強，半法以上排成之，不滿半法廢棄之。以強并少爲少強，并半爲半強，并太爲太強。得二強者爲少弱，以之并少爲半弱，以之并半爲太弱，以之并太爲一辰弱。以所在辰命之，則各得其少、太、半及強，弱也。其月蝕望在中節前後四日以還者，視限數；在中節前後五日以上者，視間限。定小餘如間限、限數以下者，以算上爲日。

(Li 1974).

This section shows how to converse the result of the time in the last section to the 12 double-hour civil time reckoning. Twelve branches are used as the names of the 12 double hours (see Table 3). Twelve divisions are used in the exact time reckoning within double hours (see Table 4). Strong and weak are adjectives used on the divisions and whole numbers. Strong means "1/12 more" and add 1/12 to the divisions. Weak means "1/12 less" and subtract 1/12 from the divisions.

6 Conclusion

As one of the most significant astronomical phenomenon in Chinese mathematical astronomy and astrology, eclipses and eclipse theories are important parts in Chinese *li*. The adoption the lunar velocity theory in the *Qian xiang li* and *Jing chu li* is a significant improvement on eclipse theory. In Chinese dynastic histories, there is a tradition of making comments on people and their work in earlier periods. In an evaluation paragraph on the early *li* makers found in the *Song shu*, Shen Yue 沈約 aptly sums up the importance of Yan Wei's work,

> Among the ancient *li* makers, Deng Ping was able to revise the existing method and establish new ones based on it. Liu Hong is the first to criticize *Sifen li*; he also

worked on lunar velocity. After careful considerations, following the intention of examining the differences, Yang Wei set a parameter called difference based on the accumulated fen at conjunction and used it to predict lunar eclipse. These three people are *li* masters in the Han and Wei; however, Hong's lunar velocity table cannot be used to check the dates in the *Chun qiu* and Wei's planetary theory is significantly wrong with time passes. This is because Hong has not paid enough attention; Wei is limited by putting *Shang Yuan* in the day *Ren chen*.

古之為曆者，鄧平能修舊制新，劉洪始減《四分》，又定月行遲疾，楊偉斟酌兩端，以立多少之衷，因朔積分設差，以推合朔月蝕。此三人，漢、魏之善曆者，然而洪之遲疾，不可以檢《春秋》；偉之五星，大乖於後代，斯則洪用心尚疏，偉拘於同出上元王辰故也。

(Shen 1974).

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Appendix: Technical terms in the *Jing chu li*, *San tong li*, and *Qian xiang li*

1. *Yuan*, 元, a cycle of 11058 years.
2. Era (*Ji* 紀), a cycle of 1843 years. Era Divisor (*ji fa* 紀法), 1843.
3. Rule (*Zhang* 章), a cycle of 19 years, the intercalary period. Rule Year (*Zhang sui* 章岁), 19, the year of one Rule. Rule Month (*Zhang yue* 章月), 235, the month of one Rule. Rule Divisor (*Zhang fa* 章法), 19.
4. Day Divisor (*ri fa* 日法), 4559.
5. Meeting Year (*hui sui* 會歲). In *San tong li*, it is a period of 513 years, which equals 6345 months. 6345 is the least common multiple of 135, the number of months in one eclipse period and the month of one Rule. This means there are 47 135-month eclipse period in one Meeting Year.
6. Dou Fraction (*dou fen* 斗分), the fraction of the Lodge dou.
7. Velocity Calendar (*chi ji li* 遲疾曆). It looks like a calendar but it is a table appeared in the format of a calendar showing the speed of the moon in different days in one month.
8. Velocity Differential Value (*chi ji cha lü* 遲疾差率). The difference between the moon's daily speed to its average daily speed.
9. Connect Number (*tong shu* 通數), 134630, the length of one synodic month in the unit of the Day Divisor 4559. Each synodic month has 29 2419/4559 days.
10. Connect Cycle (*tong zhou* 通周). 125621, the cycle of anomalistic month.
11. Day Remainder (*ri yu* 日餘). The remainder of a day, usually produced in calculation.

12. Cycle Vacancy (*zhou xu* 周虛). The difference between the Day Divisor and the Circuit Day Remainder is 2031, 2528 + 2031 = 4559. Yang Wei names it Cycle Vacancy, which is for mathematical use rather than astronomical. The other part of the remainder of day.
13. Cross Coincidences (*jiao hui* 交會), jiao, crossing, refers to the intersection of the sun's path and the moon's path, or to the transit of the sun or moon. *Hui* refers to conjunctions. *Jiao hui* implies the situation that when eclipse happens, the conjunction is close to the lunar node.[14]
14. Lunar Accumulated Fen (*shuo* 朔積分). Shuo means the beginning of one month, *ji* means accumulated, fen is the unit used in calculation. Lunar Accumulated Fen refers to the *fen* at the beginning of one month. The value equals the number of synodic months from the beginning of that Era to the calculating month multiply by Connective Number, 134630, the length of one synodic month.
15. Draconitic-Synodic Differential Value (*jiaohui chalü* 交會差率). *Jiao* 交 refers to "related to draconitic cycle." *Hui* 會 refers to "related to synodic cycle." The value shows the difference between two cycles when a particular astronomical phenomenon, such as eclipse, happens.
16. Conjunctional Connect (*hui tong* 會通 790110). Conjunctional Connect is the eclipse period in the unit of day divisor 4559. If we divide Conjunctional Connect by Connect Number, the length of one synodic month, the result is 790110/134630, which equals 5 116960 / 134630 months.
17. Draconitic Value (*qu jiao du fen* 去交度分) means the time between the calculating time to the happening of conjunction. The term is also in the unit of day divisor. To calculate the Draconitic value, one should take Conjunctional connect out of the sum of Lunar accumulated fen and Connect Number to get the result.
18. Syzygy Conjunction Constant (*shuowang heshu* 朔望合數) is 67315. The term presents half of the synodic month. The number is half of the Connect number 134630.
19. Inner Draconitic Limit (*rujiao xianshu* 入交限數), 722795. It is the difference between the Conjunctional Connect 790110 and the Syzygy Conjunction constant 67315. The term shows how close is the end of one draconitic cycle to the end of one eclipse period.
20. *Li* Epoch(*li yuan* 曆元), also called Grand Epoch (shang yuan 上元).
21. Circuit Day Remainder (zhou ri ri yu 周日日餘). The remainder of a day after a complete cycle, usually produced in calculation.
22. Ratio of Increasing and Decreasing (*sun yi lü* 損益率). The data showing the difference in the actual lunar speed to the average daily speed (Chen 1995). Two ratios are included in the column, Decreasing Ratio 損率, showing the values slower than the average speed and Increasing Ratio 益率, the values faster than the average speed.
23. Regulation accumulated fen (*sun yi ying suo ji fen* 損益盈縮積分).
24. Positive Accumulative Fen (*ying ji fen* 盈積分).
25. Negative Accumulative Fen (*suo ji fen* 縮積分).

[14] Sivin, *Granting the Seasons* (ref. 3).

26. Velocity Accumulation Fen (*ding ji fen* 定積分).
27. Post Velocity Accumulation Fen (*hou ding ji fen* 后定積分).
28. Lunar Travelling Fen (*yue xing fen* 月行分). The distance the moon travelled in fen according to the Velocity Calendar.
29. Velocity Major remainder (定大餘).
30. Velocity Minor remainder (定小餘).
31. Limit Number (*xian shu* 限數). Limit Number shows the length of the night on one solar term day.
32. Interval Limit Number (*jian xian* 間限). The Interval Limit Number shows the length of the night in the middle of two consecutive solar term days. The *Jing chu li* has a table showing the Limit Number and the Interval Limit Number for the 24 solar terms.

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