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Magnification: how to turn a spyglass into an astronomical telescope

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Abstract According to the received view, the first spyglass was assembled without any theory of how the instrument magnifies. Galileo, who was the first to use the device as a scientific instrument, improved the power of magnification up to 30 times. How did he accomplish this feat? Galileo does not tell us what he did. We hold that such improvement of magnification is too intricate a problem to be solved by trial and error, accidentally stumbling upon a complex procedure. We construct a plausibility argument and submit that Galileo had a theory of the telescope. He could develop it by analogical reasoning based on the phenomenon of reflection in mirrors—as it was put to use in surveying instruments—and applied to refraction in sets of lenses. Galileo could appeal to this analogy and assume Della Porta's theory of refraction. He could thus turn the spyglass into a revolutionary scientific instrument—the telescope.

1 The problem

The course of events between July 1609, when Galileo Galilei (1564–1642) first took interest in the telescope, and March 1610, when he published his *Sidereus nuncius*, is

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well known.[1] Yet scholars are still perplexed by the question whether Galileo knew how does the telescope function? Given the extant documents and the recent secondary literature, it appears that this is a moot question.[2] To be sure, the instrument itself is available and the practice is known, for Galileo tells the reader on different occasions what he did with the telescope. The motivating question is, then, how did Galileo transform a toy into a scientific instrument? How did he turn the Dutch tubular spectacle into an astronomical telescope?

In the summer of 1609, Galileo made a telescope that magnified 8 times and by January 1610 he had instruments that magnified 14, 21, and about 30 times. Optical analyses of the telescopes attributed to Galileo show that to get magnification of 14 times he had to produce lenses of 0.75 diopter for the objective and 10.64 diopter for the eyepiece (1330 and 94 mm, respectively). To get magnification of 21 times, Galileo had to produce lenses of 1.02 diopter for the objective and 21 diopter for the eyepiece (980 and 47.5 mm, respectively). And, for the telescope that magnified about 30 times, Galileo manufactured an objective lens of 0.585 diopter and an eyepiece of about 17.5 diopter (1710 and 57 mm, respectively).[3] These types of lenses were not available in the spectacle market;[4] they had to be ordered to specification.[5] How did Galileo come up with these specifications?

It has been argued that the invention of the Dutch telescope became possible only because of the aperture stop.[6] This claim has to be modified. The aperture stop indeed helps remedy the adverse effects of optical aberrations, but only during the final tuning phases of the telescope, i.e., when the instrument is already in hand.[7] More critical is the effect that arises from the intrinsic optical features of the Galilean layout (i.e., convex objective and concave eyepiece). To clarify this point, let us examine the Galilean telescope that magnifies 21 times. The telescope's 1.02-diopter plano-convex objective has a focal length of 980 mm, a central thickness of 2 mm, and an aperture diameter of

[1] See, Van Helden (1974, pp. 39–58; 1975, pp. 251–263; 1977, pp. 3–67; 1981; 2009, pp. 64–69; 2010, pp. 183–203), Biagioli (2006, 2010), Strano (2009), Reeves (1997, 2008), Willach (2008), Ilardi (2007), Dupré (2005, pp. 145–180; 2003, pp. 369–399), Malet (2005, pp. 262–273), Camerota (2004, pp. 143–170), Zik and Van Helden (2003, pp. 173–190), Zik (2001; 1999, pp. 31–67), Shea (1990, pp. 51–76; 1996, pp. 507–526), Westfall (1985, pp. 11–30), and Ronchi (1963, pp. 542–561, 1967, pp. 195–206).

[2] Dupré (2005, pp. 146–148).

[3] Molesini et al. (1993, p. 6220). The original eyepiece of the telescope that magnified about 30 times is lost. Thus, the focal length of the eyepiece of this telescope had to be reconstructed, that is, computed. We thank Giuseppe Molesini of the National Institute of Applied Optics, Arcetri, Florence, for providing us with his detailed computations and interferometric images of the Galilean telescopes and their lenses.

[4] Willach (2008, pp. 51–55) and Ilardi (2007, pp. 64–73, 230–235).

[5] On techniques for the production and application of mirrors and spectacle lenses, see Della Porta (1589, Bk. 17, pp. 271–280) and Sirtori (1618, pp. 18–23, 33–81). See also Willach (2008, pp. 17–25, 56–69), Bedini (1994, Chap. 2, pp. 89–115, Chap. 4, pp. 147–204, Chap. 5, pp. 18–38), and Bryden (1993, pp. 6–11).

[6] Willach (2008, pp. 93–97, 103). Note that before 1604, when Kepler explained the formation of the retinal image, the eye lens (*crystallina*) was assumed to be the organ on which vision occurs. It was common knowledge among opticians at the time that the pupil of the eye, functioning as an aperture stop, improved the acuity of sight: see Della Porta (1593, pp. 68–86, 100, 103–105).

[7] Galileo was the first scholar to systematically apply aperture stops. On the calibration method which Galileo applied in the final tuning of his telescope, see Zik (1999, pp. 46–47, 52–53; 2001, pp. 266–269).

16 mm. The 21-diopter bi-concave eyepiece has a focal length of −47.5 mm, and its central thickness is 1.8 mm. This means that the diameter of the exit pupil is 0.77 mm, while the diameter of the rays crossing the eyepiece is about 4.12 mm. The implication is that the bulk of the rays passing through the eyepiece cross section of the Galilean layout are less liable to interferences caused by the poor quality of the glass and inadequate configuring and polishing techniques of lenses at the time. Aperture stops are usually applied to improve resolution in the tuning phases. This is evident by the fact that the aperture diameter in Galileo's telescope that magnified 14 times was 26 mm, while the telescope that magnified 21 times was stopped at 16 mm.[8]

According to the received view, theories available at the time were not conducive to determining lens specification for some optical design.[9] The implication is that Galileo stumbled upon a way to improve the performance of the telescope by appealing to artisanal ingenuity rather than to theoretical know-how. Galileo could not determine the magnifying power of the telescope from the focal lengths of his concave and convex lenses, so the argument goes, for the simple reason that such concepts and theoretical insights were not available at the time. Galileo thus found a reliable method within the tradition of sixteenth-century optics and practices of the lens makers, the argument continues, that by-passed in some miraculous way the optical theory that we now think underwrites his success.[10] We question this claim and seek an alternative answer.

It is inconceivable that one could simply stumble on the (above) "numbers" by some procedure of elimination, in a trial-and-error method. For obvious reasons, Galileo was not intent on disclosing his theory of the telescope.[11] But, it is clear that quantitative ray tracing method, that is, the law of refraction or, in its place, a geometrical scheme, is crucial for the development of an adequate theory of lenses. We therefore consider *De refractione optices parte* (1593) of Giovan Battista Della Porta (1535–1615) a pioneering essay for it offers a quantitative analysis of both reflection and refraction.[12]

To offer an alternative to the questionable received view, we develop a plausibility argument which is based on the following assumptions:

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[8] Parameters, such as overall lens length of the telescope, its magnification, proper alignment, and quality of the lenses, have an effect on the diameter and shape of the aperture stop required for improving the resolving power of these relatively crude instruments. In a letter of January 7, 1610, [to Antonio de Medici?] Galileo wrote (Favaro (1890–1909, 10: 278, letter 259, our translation, italics added): "It would be better if the convex lens, which is the furthest from the eye were in part covered, and that the opening which is left uncovered be of an *oval* shape, because in this manner it would be possible to see objects much more distinctly." (É bene che il vetro colmo, che è il lontano dall' occhio, sia in parte coperto, et che il pertuso che si lascia aperto sia di figura ovale, perchè così si vedranno li oggetti assai più distintamente.)

[9] Van Helden makes this remark in a comment on Galileo's claim, namely, how he used the "science of refraction" to improve the telescope: see Galileo ([1610] 1989, p. 37, n. 25) and Van Helden (1974, pp. 40–49). See also footnote 42, below.

[10] Van Helden (2009, pp. 66–67), Biagioli (2006, pp. 116, 120), Dupré (2005, p. 148), and Shea (1996, pp. 509, 511).

[11] On the tensions between secrecy and transparency in Galileo's *Sidereus nuncius*, see Biagioli (2006, pp. 14–19, 77–134) and Zik and Van Helden (2003).

[12] On Della Porta's career and influence, see Smith, A. Mark (2010b, 1: xcii–xcvii), Eamon (1994, pp. 194–233), and Shumaker (1979, pp. 109–120). On Della Porta's optical studies, see Lindberg (1984, pp. 142–148) and Ronchi (1963, pp. 542–561; 1957, pp. 24–66).

1. Euclid's ray optics establishes geometrical relation between linear magnitudes and apparent angles. This scheme is not physical so the relation can be maintained independently of causal considerations.
2. Optical magnification which applies to a range of optical systems, including surveying instruments and telescopes, is defined by the ratio between the apparent angle of an object seen through the system and its apparent angle seen by the naked eye. This definition is geometrical; it has nothing to do with image formation, image resolution, or visual perception.
3. The improved performance of any optical system which includes a set of lenses is critically dependent on the phenomenon of refraction.

We claim:

4. The theory of the telescope could have been developed in two steps:
   - 4.1 Reason analogically from magnification in surveying instruments (based on reflection in mirrors) to magnification in telescopes (based on refraction through lenses); and,
   - 4.2 Assume Della Porta's discussion which offers a quantitative analysis of both reflection and refraction by which the burning point of spherical mirrors as well as lenses can be computed and apply to refraction in sets of lenses.

We begin (Sect. 2) with a brief account of the method used for measuring magnitudes of remote objects, and the theory of the military compass published by Galileo in 1606 (this discussion corresponds to step 4.1, above). We continue (Sect. 3) with a study of Della Porta's *De refractione optices parte* of 1593 (this discussion corresponds to step 4.2, above). We then propose (Sect. 4) a solution to the question, How Galileo could theorize about the telescope?

2 Theory and application of surveying instruments

The application of Euclid's optics to surveying instruments is well known.[13] The governing rules of these instruments are based upon two elements:

1. The line of sight of the visual ray marks a reference point on the remote objects;
2. Geometrical elements, such as lines, arcs, and triangles, mediate between the object and the observer.

To get a sense of the application and computation carried out with these instruments, let us examine the following example (Fig. 1): while a remote object AB subtends angle a at point E, an object, KO, of the same height, placed nearer at distance OE, subtends a greater angle b at E. Object KO is seen by the eye placed at E, and is larger

[13] For the history of Greek and Roman surveying instruments, see Lewis (2004). Methods for measuring heights using a mirror are described in many manuscripts on perspective, cosmography, and practical geometry, published throughout the Middle Ages and early Renaissance: see Smith, A. Mark (2001a, 2: 448–457, 475–493; 1996, pp. 15–17, 90–98), Frangenberg (1992), Brownson (1981, pp. 181–188), Kemp (1978), Unguru (1977, pp. 22–40), Lindberg (1972, pp. 50–59, 118–135, 207–213), and Burton (1945, pp. 357–361).

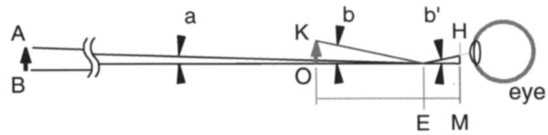


Fig. 1 Theory of surveying instrument

than object AB. If a flat mirror is placed now at E and the eye is moved further away from E to point H, it is straightforward that, as long as angles b and b' remain equal, the manipulation of the parameters OE, EM, and HM determines the visual angle subtended at the eye. The height of object KO is correlated to the other parameters by the proportionality, KO/EO = HM/EM. Thus, the apparent magnitude of object KO is changed in relation to the apparent magnitude of object AB.

It is important to note that plane mirrors, used in these instruments, form virtual images. The observer sees the image of an object positioned "behind" the mirror, because he or she interpolates the visual rays backward along straight lines to the point of origin. The apparent size of an object is measured by the apparent angle it subtends at the point of observation.[14]

Instruments of greater or lesser sophistication were based on these elements: Heron's Dioptra, Ptolemy's Parallactic Quadrant, Levi Ben Gerson's the Jacob Staff, Egnatio Danti's Torqvetto Astronomico, Tycho Braha's Quadrant, and Galileo's Military Compass.

2.1 Galileo's Military Compass (1606): optics and instruments

Galileo built the Geometric and Military Compass to be used as a mechanical computing device, an elevation gauge, and for topographical survey.[15] To get a sense of Galileo's instrument, we follow an example of its operations as described by Galileo himself. Of special importance for our argument is how the change of distances and manipulation of angles of vision made an observed object to be perceived in different apparent angles, thus affecting its apparent magnitude. One can measure the height of an object by raising or lowering the place where the instrument is located (Fig. 2):

> Wishing to measure height AB, hold the Instrument at some place above the ground, as at point F, and sight at point A along side EF, noting the graduations between G and I as cut by the thread to be, say, 65. Then, going down and being vertically under point F, say at point C, we sight the same height along side DC, noting the graduations between L and O, which will be more than before, as say 70. Then, take the difference between those numbers 65 and 70, which is 5, and as many times as this is contained in the larger number (that is, in 70, which will

[14] Hecht (1990, pp. 153–156).

[15] On the sources of Galileo's mathematical knowledge and his familiarity with the technical-artistic tradition of the fifteenth and sixteenth centuries as well as his experimental pursuits, see Pitt (1992, pp. 4–8, 110–138), Settle (1996, pp. 27–28; 1968, pp. 121–126), Bedini (1994, Chap. 1, pp. 256–292, Chap. 2, pp. 89–115), and Machamer (1973, pp. 1–46). For Galileo's engineering skills, see Valleriani (2010, pp. xi–xxii, 27–41, 193–207), Lefèvre (2001, pp. 11–27), and Drake (1999a, pp. 106–125; 1999b).

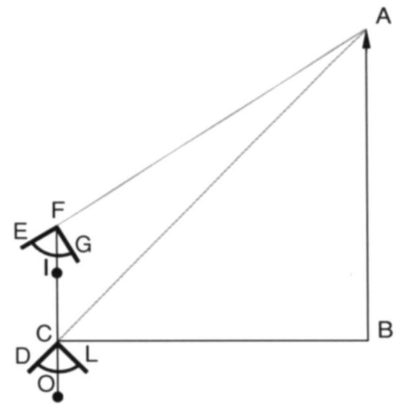


Fig. 2 Measuring the height of an object

> contain 5 fourteen times), that many times will height BA contain distance CF, which latter we shall measure, as we can easily do, and thus we shall come to know the height of AB.[16]

With this set of instructions, Galileo shows how a certain height can be measured from a distance by taking two angle readings on a vertical, parallel to the height in question.

Galileo demonstrated how linear magnitudes (i.e., distances, heights, widths, and breadths) of remote objects could be obtained by applying the rules of measurement by sighting,[17] using an optical device such as the military compass in its capacity as a quadrant.[18] The compass enables the surveyor to measure the apparent angle created between visual rays (lines of sight) reaching the object. The reading of the measured angles is obtained by marking points denoted by the plumb-line on the graduation scale of the instrument. This measuring tool correlates linear magnitudes with angles; thus, by taking two different measurements of the objects, that is, by measuring two different apparent angles of the same object, the surveyor can compute a certain linear magnitude of the remote object. Clearly, the purpose of the instrument is to overcome spatial difficulties, such as the inaccessibility of the object.

Note that the instrument can be manipulated in reverse, that is, in the opposite direction of the original design. In this setup, the geometrical correlation between linear magnitudes and apparent angles is purely logical and so the relation may be maintained irrespective of causal considerations. Now, when one inserts a mirror into the instrument,[19] by Euclid's *Optics*, Prop. 19, the correlation between the linear magnitude and the apparent angle is mediated with the mirror via the virtual image (reflected in the mirror). Because the law of reflection is reflexive, it creates reciprocal relations between, on one hand, the distances of the object and the surveyor from the mirror and,

[16] Galileo ([1606] 1978, p. 83).

[17] Favaro (1890–1909, 2: 415): "Che nelle regole per misurar con la vista ci occorreranno."

[18] Galileo ([1606] 1978, pp. 79–92).

[19] Burton (1945, pp. 360–361). Galileo did not introduce a mirror into the operation of his military compass. He was, however, well familiar with the optical features and capacities of various arrangements of mirrors, see Valleriani (2010, pp. 60–66) and Dupré (2005).

on the other, the heights of the observer's eye and that of the object. Thus, angles and magnitudes of visible objects could be manipulated via the interface of a flat mirror by changing its location.

3 Della Porta's *De refractione optices parte* (1593): quantitative analyses of reflection and refraction

3.1 Theory

In Book II of *De refractione*, Della Porta discusses reflection and refraction of solar rays in spherical interfaces.[20] He stated in the Preface that "... refraction through a sphere is reciprocal to, and in agreement with, reflection from a concave mirror...."[21] Here, we have a powerful claim concerning the relation between refraction and reflection. The reciprocal relation between refraction and reflection played a fundamental role in Della Porta's theory of refraction, and he proceeded—as we will see—to illustrate the theorems experimentally. He was not content with a purely theoretical, that is, geometrical, analysis of refraction and augmented the analysis with optical illustrations.

In this analysis, Della Porta builds on Euclid's *Elements*, Bk. IV, in which the geometrical properties of polygons inscribed in a circle are discussed. Della Porta begins the discussion: Let DABC (Fig. 3a) be a concave spherical mirror to which a ray of the Sun AB, parallel to the diameter CED, incident at B, and arc BC equals to one sixth of the circle circumference. The connected lines BC, CE, and EB form a triangle BCE in which angles CEB, EBC, and BCE are equal and the alternate angles CEB and ABE are also equal. Since the angle of incidence is equal to the angle of reflection, the incident angle ABE is equal to the angle of reflection CBE, and line BC is the path of the reflected ray. Therefore, in a spherical concave mirror, when a ray of the Sun, AB strikes along the chord of a hexagon, it is reflected toward the diameter DC at its lower terminus to point C.[22]

Della Porta continues: In a spherical concave mirror (Fig. 3b), when a ray of the Sun strikes along the chord of an octagon, the angle of incidence ABD is equal to the angle of reflection DBE, and the reflected ray BE intersects the diameter of the mirror's sphere perpendicularly; in a spherical concave mirror (Fig. 3c), when a ray of the Sun strikes along the chord of a dodecagon, the angle of incidence ABD is equal to the angle of reflection DBE, and the reflected ray BE intersects the diameter of the mirror at point E.[23]

On the basis of these relations, Della Porta inferred the following propositions: the angle of reflection, made by a hexagon (Fig. 3a, six-sided polygon) is 60°; the angle

[20] Note that the cone of rays issued from each point source of the Sun (i.e., an infinite object) will become parallel at the point of incident. On focal points and planes, and the location, size, and orientation of an image produced by a lens, see Hecht (1990, pp. 139–145) and Smith, A. Mark (2010b, 1: xciii–xcvii).

[21] Della Porta (1593, p. 35): "Sed quia pilae refractio reciprocationem, & conuenientiam quandam cum concaui speculi reflexione habet, ideo de eius reflexione nonnihil attingemus."

[22] Della Porta (1593, p. 36).

[23] Della Porta (1593, p. 37).

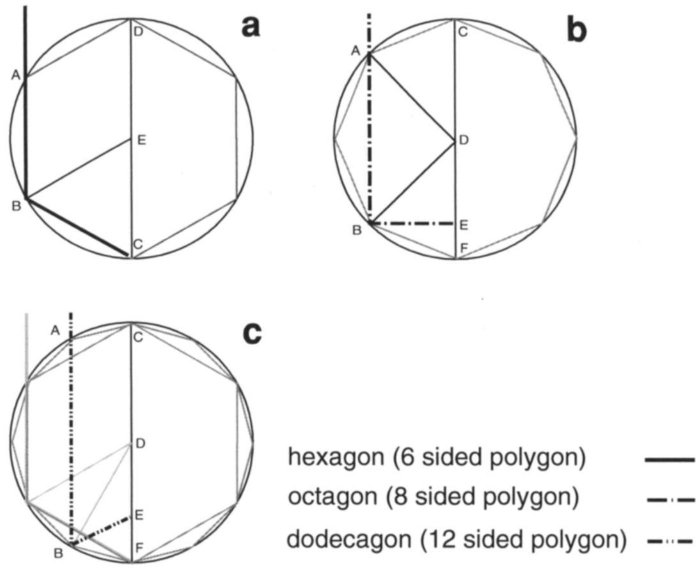


Fig. 3 Reflection in hexagon, octagon, and dodecagon

of reflection made by an octagon (Fig. 3b, eight-sided polygon) is 45°; the angle of reflection made by a dodecagon (Fig. 3c, 12-sided polygon) is 30°; the angle of reflection made by hexdecagon (Fig. 4a, 16-sided polygon) is 22.5°; and so the angles of reflection are getting more acute as the rays strike the spherical concave mirror nearer to the diameter. Hence, the point of reflection for each ray, respectively, is located along the diameter CE from the terminus at E (Fig. 4a), up to a point placed at one quarter of the mirror's diameter, where the burning point of the mirror is located.[24]

Della Porta noted that by placing the burning point, that is, the focal point of the mirror, at one quarter of its diameter, he corrected Euclid's long standing error in *Catoptrica*, where the burning point of a concave spherical mirror was placed at the center of the mirror's diameter.[25]

Della Porta now traced the reflection when ray AB strikes above the chord of a hexagon (Fig. 4b) at a height equal to a chord of a pentagon; the angle of incidence ABD is equal to the angle of reflection DBF. The path of the reflected ray BF falls beyond the diameter CDE, further than E, to the point of reflection F outside the spherical concave mirror.[26]

Della Porta concluded the discussion of reflection with a general proposition for the relation between the point of incidence, namely, the point where the ray is reflected, and the point of reflection, the point where the reflected ray intersects with the optical axis of the system (in this case, a spherical concave mirror). Let EBAC (Fig. 5a) be

[24] Della Porta (1593, pp. 38–39).

[25] Della Porta (1589, p. 271; 1593, p. 39), Zamberti (1537, p. 515), and Dupré (2005, pp. 156–157). On the term, *focus*, see Goldstein and Hon (2005, pp. 92–93).

[26] Della Porta (1593, p. 40).

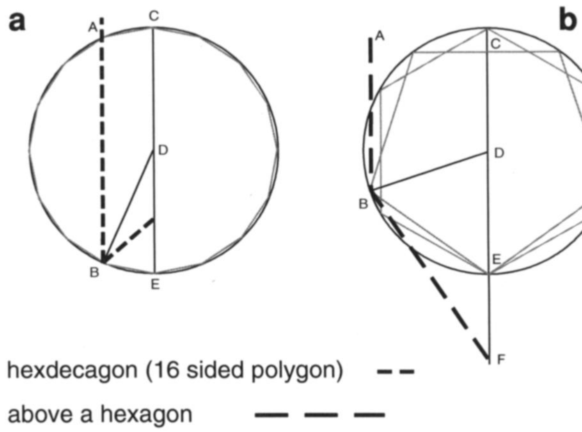


Fig. 4 Reflection in hexdecagon, and above hexagon

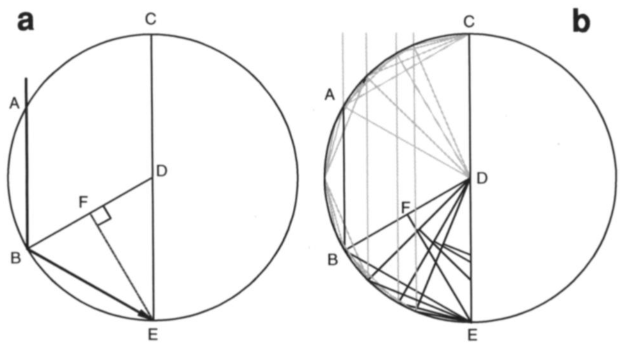


Fig. 5 The determination of the point of reflection

a spherical concave mirror, B a point of incidence on its circumference. To find the point of reflection on the diameter CE, draw line DB from the center of the circle, D, to the point of incidence, B, and mark point, F, at the middle of that line. From point F, draw a perpendicular toward DE. Point E, where the perpendicular FE intersects the diameter, CE, marks the place where the point of reflection is located. Because in triangles FBE and FDE the angles DFE and BFE are right angles, and lines BF and FD are equal, angles FDE and FBE are also equal. Lines DE and BE are equal and the point of reflection is located at the place where the two lines coincided, namely, E.[27]

Della Porta's geometrical construction is applicable for any given incident ray and shows how the optical path of the reflected ray could be traced (Fig. 5b): The intersection of a perpendicular, dropped from the mid-line connecting the point of incidence and the center of a spherical concave mirror, and the diameter of the mirror, always marks the point of reflection. In effect, applying an equilateral polygons inscribed by a circle, Della Porta established a method by which the relations among, (1) the height

[27] Della Porta (1593, pp. 40–41).

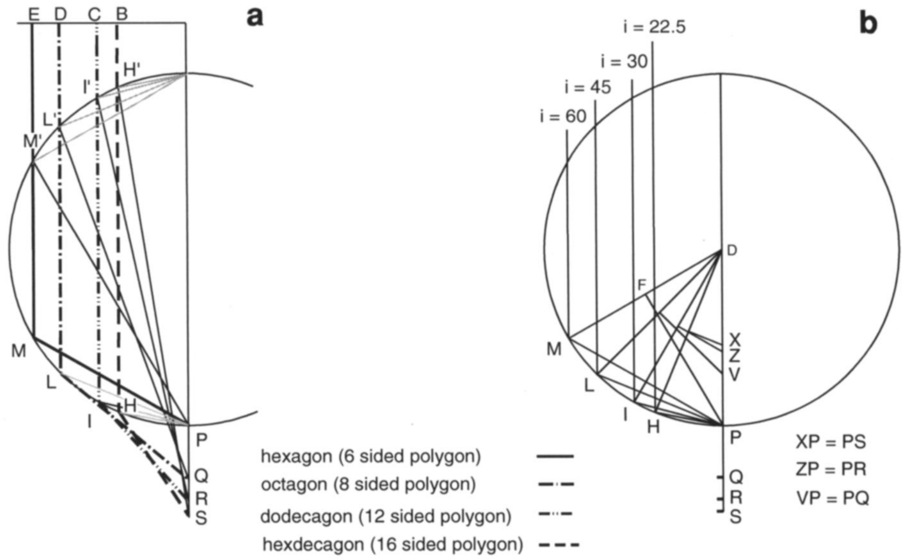


Fig. 6 Refraction in hexagon, octagon, dodecagon, and hexdecagon

of the incident ray denoted by its chord, (2) the angle of reflection, and (3) the radius of the mirror, could be calculated. Accordingly, the focal length of a concave spherical mirror could be determined in terms of the radius of the spherical mirror.

Thus far is Della Porta's analysis of reflection. Given the fundamental assumption of reciprocity, he now conducts the same analysis, applied to refraction. When ray BH (Fig. 6a) strikes at a height defined by a chord of a hexdecagon (16-sided polygon) the angle of reflection in a spherical concave mirror is 22.5°. The ray ascends toward the region of the semi-diameter to point X (Fig. 6b) placed between the terminus and a quarter of the radius of the sphere. Given that the path of the refracted ray and the distance of its point of refraction from the terminus P is reciprocally equal to the path and distance of the reflected ray, the refracted ray proceeds to an opposite direction along the optical axis below the sphere. Della Porta, therefore, marked the point of refraction at S, external to the sphere, where XP = PS.

For another example, ray CI (Fig. 6a) strikes at a height defined by a chord of a dodecagon (12-sided polygon) and the angle of reflection in a spherical concave mirror is 30°. In reflection, the ray ascends toward the region of the semi-diameter to point Z (Fig. 6b). Della Porta, therefore, marked the point of refraction at R, external to the sphere, where ZP = PR.[28] The same geometrical construction defines all the other points of refraction, Q and P. The path of the refracted rays is inscribed as a line connecting the points of incidence at the front surface of the sphere to the points of

[28] Della Porta (1593, pp. 42–43): "Radium CI, & erit punctus I, latus dodecagoni, scilicet latus GI, dimidium lateris exagoni, & refrangetur eius radius infra latus exagoni, scilicet extra diametrum in puncto R, nam quantum in concauo speculo supra diametri calcem [i.e., point P] feriebat in semidiametro, tantum hic infra extra diametrum desendet."

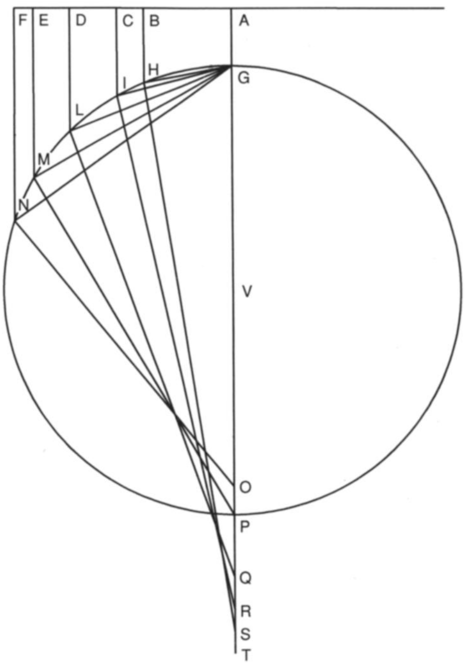


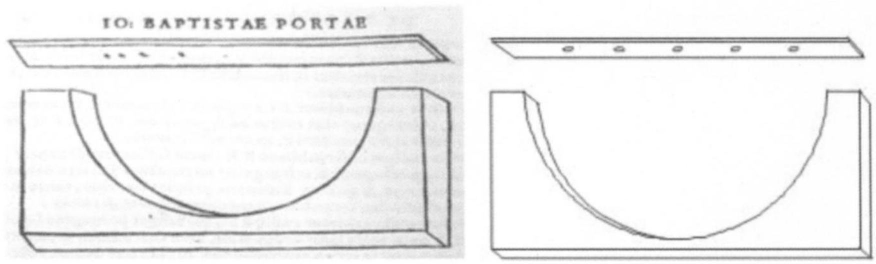
Fig. 7 Refraction through a glass sphere

refraction, that is (Fig. 6a), H′S for the ray incident at 22.5°, I′R for the ray incident at 30°, L′Q for the ray incident at 45°, and M′P for the ray incident at 60°.

Della Porta applies this geometrical construction to tracing the path of the rays of the Sun through a sphere (Fig. 7).[29] The ray AGVPT passes through the center of the sphere to infinity without being refracted, and its path marks the optical axis to serve as a reference line. Next, he traces the ray passing through E, striking at M. GM is a chord of a hexagon (six-sided polygon), thus the ray EM advances toward the terminus at P, forming line MP. Then, he traced ray CI, where point I is made by a chord of a dodecagon (12-sided polygon), which is refracted outside the diameter to point R. Then, he traced ray DL, where point L is made by a chord of an octagon (eight-sided polygon), which is refracted outside the diameter to point Q. Ray BH, where point H is made by a chord of a hexdecagon (16-sided polygon), is refracted outside of the diameter to point S. Ray FN, where point N is made by a chord of a pentagon, thus refracted inside the sphere to point O, above the terminus at point P. The geometrical construction which defines points of refraction is governed, according to Della Porta, by the following rule:

> [the path of the rays] is inverted to what is said for a spherical concave mirror. Where rays [of the Sun] ascend in a spherical concave mirror, here [in a glass sphere] they descend and vice versa. Only when the rays strike a point made by

29 Della Porta (1593, pp. 41–42).



Neapolitan foot is 26.37 cm; and a finger is 1.86 cm.

Fig. 8 A device for tracing refraction

> a chord of a hexagon, they ascend or descend [are reflected or refracted] toward the terminus, [thus meeting at P].[30]

For a given point of incidence, the distance between the terminus and the point where the reflected and refracted ray intersects the optical axis is the same, but the ray proceeds in opposite directions with respect to the optical interface. Once this relation between reflection and refraction is established, Della Porta could calculate the geometrical properties of an optical interface, whether it is a mirror or a sphere. The path of the rays is determined in terms of the height of the incident ray denoted by its chord, the angle of reflection/refraction, and the radius curvature of the interface. The result is the point at which the ray converges or diverges in relation to the optical axis of the interface.

3.2 Experimental illustrations

To illustrate his theoretical claims, Della Porta procured a transparent block made of well polished glass, a foot length, a foot width, and a finger breadth (Fig. 8). He cut out a slice of half a sphere of the surface of the block so it could be used as a refractive interface. On the top of the device, Della Porta placed an opaque plate with holes through which rays could pass and penetrate the device. Then, by tracing and inscribing on the side of the glass block the perceived path of the rays traversing the device he could actually see correlations between his geometrical construction and the actual path of the rays.[31]

To ease the adverse effects caused by the poor quality of the glass, Della Porta used spheres made of crystal glass (*pila crystallina*) which was considered much purer than the common glass used for lens making at the time. In Bk. II, Props. 22 and 23, he presented an analysis of the rays traversing through the spheres and added a figure to

[30] Della Porta (1593, p. 43): "Sed haec irregularitas hanc habet regulam, vt contraria sint ijs, quae de speculo sphaerico diximus concauo, nam vbi in concauo ascendebant, hic descendunt, & vbi ibi descendabant, hic ascendunt, solum in vtroq; latus exagoni metam habet, nam vtrunque diametric finem ferit."

[31] Della Porta (1593, pp. 43–45).

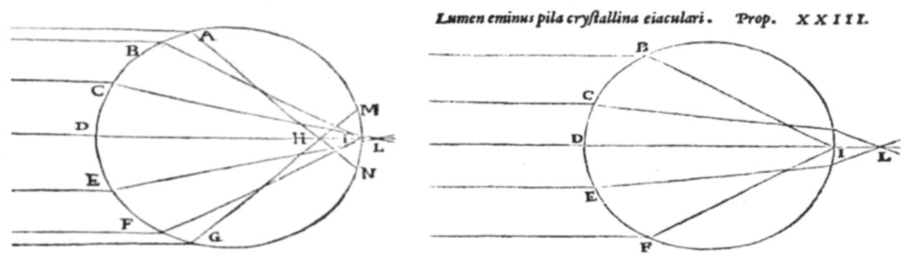


Fig. 9 Refraction through crystal spheres

clarify the argument (Fig. 9).[32] Modern optical simulations of the path of the rays BI and FI, incident at 60° into a glass sphere, suggest that the rays could only refract to point I, at the terminus of the sphere, that is, if Della Porta had used crystal glass of index of refraction of 1.732.[33]

When Della Porta made his experiments with spherical concave and convex interfaces, he also used elements made of common glass (*superficiem vitream conuexam*).[34] For comparison, the indices of refraction of the glass used by Galileo in his telescopes and spectacle lenses at the time varied between 1.512 and 1.546,[35] while the range of crystal (lead) glass's indices of refraction varied between 1.6 and 1.82.

To execute his measurements, Della Porta had to collimate the device. He lifted the board so that the ray could traverse the hole at E (Fig. 10). Della Porta used the same device for determining the place to which the rays of the Sun are refracting through spherical concave surface. In the same manner, he prepared a small glass board a foot length, half a foot width, and a finger breadth. Next, he placed on top of the glass a thin opaque board made of copper (or wood), equal in length and width to the original device, and in it he bored small, round holes to let the rays of the Sun travel through them. Then, he raised the device from the floor so the rays could pass through it, while the observer looks at them from the side.[36]

Della Porta used the device for tracing the path of solar rays refracting through a spherical concave interface, that is, a plano-concave interface (Fig. 11).[37] In his graphical report, Della Porta drew the path of the ray DI incident at 22.5°, ray CH incident at 30°, ray BG′ incident at 45°, and ray AF incident at 60°.[38]

Consider the path of ray AF incident at 60°, at the height of six-sided polygon: passing through the glass, along a straight line, the ray is refracted at F toward point

32 Della Porta (1593, pp. 62–63).

33 On the art of glass making and Della Porta's list of materials used for making various types of glass, see Neri ([1611] 2004, pp. 164–168, 297–307, 386–387, 391–392, 398) and Ilardi (1993, pp. 507–541).

34 Della Porta (1593, Bk. II, Prop. 5, p. 47).

35 Zik (1999, pp. 51–52, n. 70).

36 Della Porta (1593, pp. 43–45).

37 Della Porta (1593, Bk. II, Prop. 4, pp. 45–46): "Vera Loca determinare refractionum Solaris radii egredientis ex concaua spharali superficie."

38 In general, Della Porta's figures and discussion, especially in Bk. II, Props. 4 and 5, suffer from printing inaccuracies.

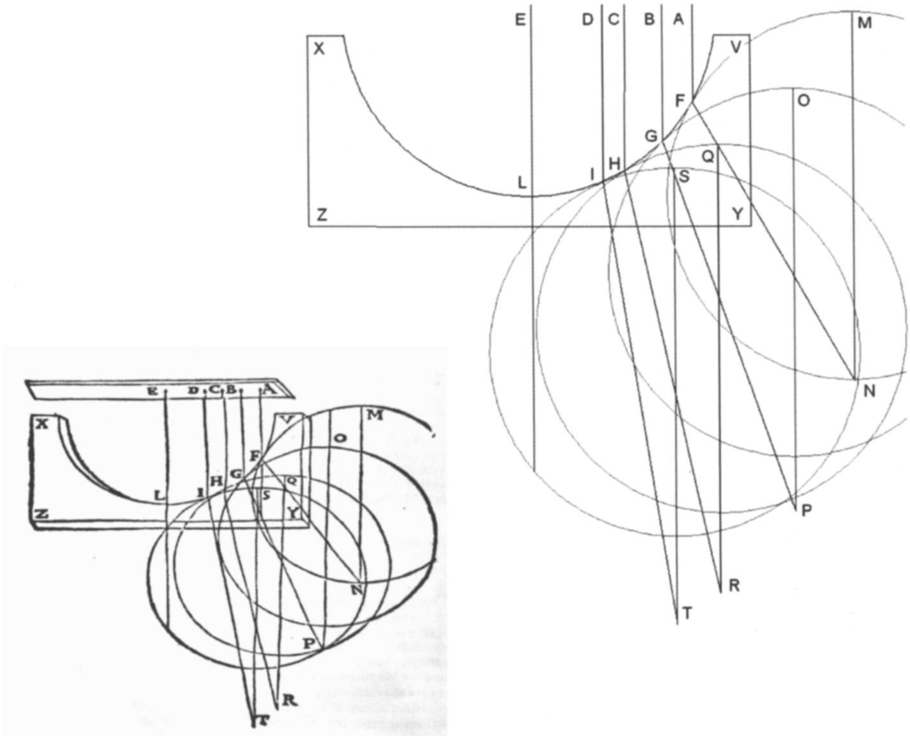


Fig. 10 Ray tracing through a spherical concave interface

T so angle TFS is equal to angle SFR. Ray BG′ is incident at 45°, at the height of eight-sided polygon. Passing through the glass, along a straight line, another ray is refracted at G′ toward point Q so angle QG′P is equal to angle PG′O. Ray CH incident at 30°, at the height of 12-sided polygon. Passing through the glass, along a straight line, the ray is refracted at H toward point N′ so angle N′HM is equal to angle MHL. Ray DI is incident at 22.5°, at the height of 16-sided polygon. Passing through the glass, along a straight line, the ray is refracted at point I toward point N so the angle NIK is equal to angle KIG. Della Porta noted that rays BG′ and AF will not refract toward points Q and T. We know that the reason for this is that rays BG′ and AF are incident at points G′ and F in an angle greater than the critical angle of 48.5°.

Points N and N′ (Fig. 11), which Della Porta marked in his observational report, can also be determined according to the geometrical approximation he had developed (i.e., the discussion related to Fig. 6). Being incident at 22.5° (Fig. 12a), at the height of a chord of 16-sided polygon, ray DI is refracted toward point N. X is the point to which ray IK is reflected by a spherical concave mirror QKR, intersecting the optical axis LR. Thus, point G is determined by the extension of the distance XR to the other side of the interface QKR, that is, XR = RG. Since a ray refracted through a concave interface will bend outside of the optical axis, the path of the refracted ray will follow line IN, forming equal angles NIK = KIG. The same is with ray CH incident at 30°

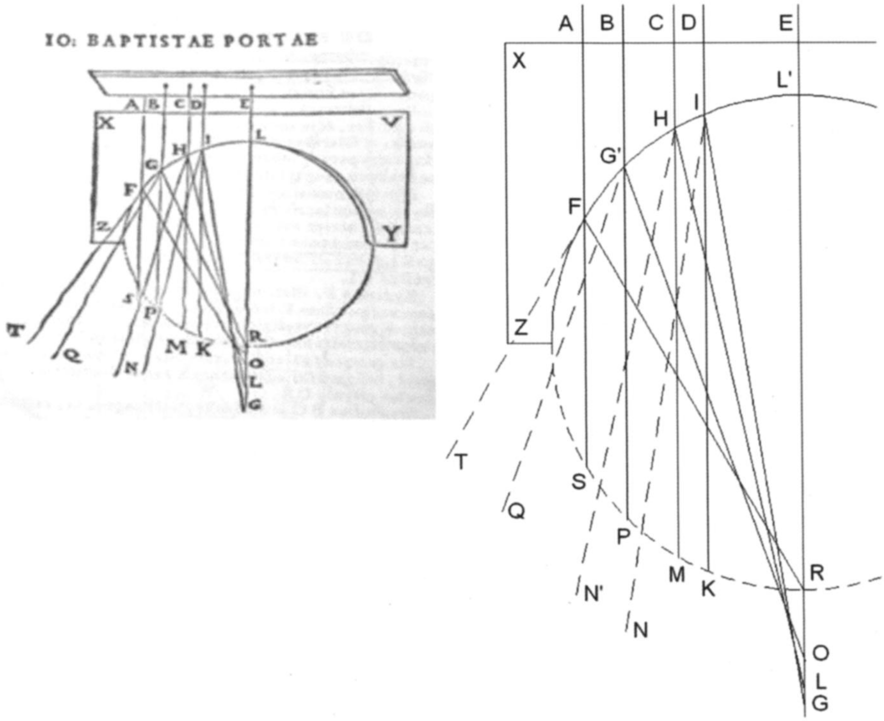


Fig. 11 Refraction through a plano-concave interface

(Fig. 12b), at the height of a chord of 12-sided polygon. Ray CH is refracted toward point N′. Point Z is the point to which ray HM is reflected by a spherical concave mirror QMR, intersecting the optical axis L′R. Thus, point L is determined by the extension of the distance ZR to the other side of the interface QMR, that is, ZR = RL. Since a ray refracted through a concave interface will bend outside of the optical axis, the path of the refracted ray will follow line HN′, forming equal angles MHL = MHN′. Note that the slope angle of the diverging ray IN (Fig. 12a) is the same as slope angle IG, but they proceed to different directions. This holds for the slope angles of ray CH (Fig. 12b), that is, the diverging slope angle HN′ and the converging slope angle HL. However, since the distance RG (Fig. 12a), is larger than the distance RL (Fig. 12b), the slope angle of ray CH (Fig. 12b) is less acute than the corresponding slope angle of ray DI (Fig. 12a).

Della Porta then accounted for the path of the solar rays refracting through a spherical convex interface, that is, a plano-convex interface (Fig. 13).[39] He drew the path of ray DI incident at 22.5°, ray CH incident at 30°, ray BG incident at 45°, and ray AF incident at 60° at the height of six-sided polygon. To trace the path of the converging rays, Della Porta applied the slope angles of the corresponding rays refracted by the

[39] Della Porta (1593, Bk, II, Prop. 5, p. 47): "Vera loca determinare Solaris radii refractionis egredientis ex conuexa spharali superficie."

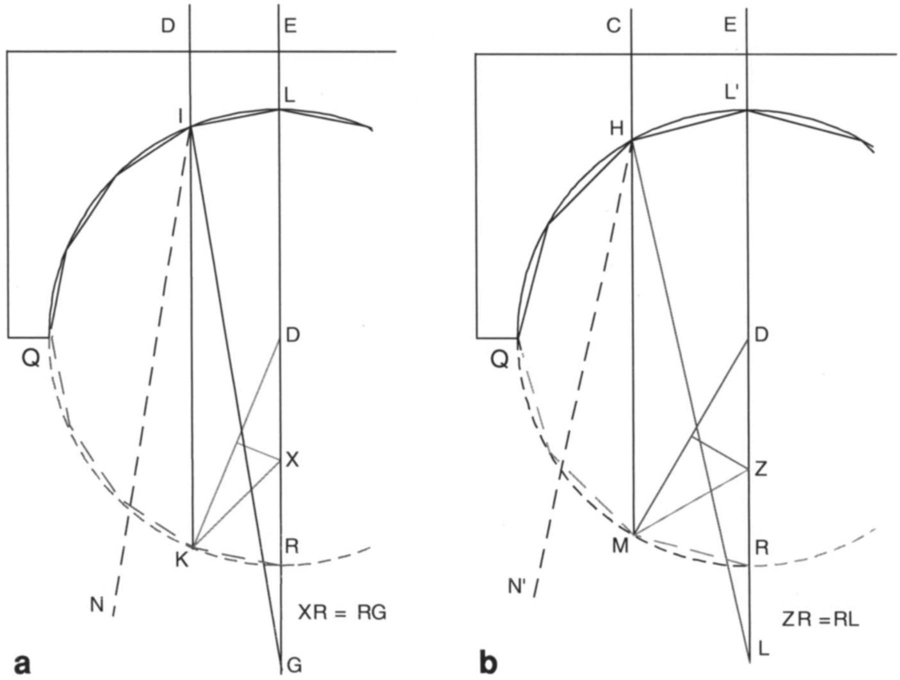


Fig. 12 Geometrical scheme of refraction through a plano-concave interface

concave surface. Passing through the glass, along a straight line, ray AF is refracted at F toward point Q so angle QFM is equal to angle MFR. In this case, the refracted ray will not advance toward point R because the angle of incidence is above the critical angle of 48.5°. Ray BG, incident at 45°, at the height of eight-sided polygon, passes through the glass along a straight line, and is refracted at G toward point S so angle SGN is equal to angle NGX.[40] Ray CH, incident at 30°, at the height of 12-sided polygon, passes through the glass along a straight line, and is refracted at H toward point T, so angle THO is equal to angle OHY. Ray DI, incident at 22.5°, at the height of 16-sided polygon, passes through the glass along a straight line, and is refracted at point I toward point V, so angle VIP is equal to angle PIZ.

3.3 Application

Della Porta's theory of refraction delineates a quantitative scheme for tracing rays incident at various angles to optical interfaces. In particular, rays with angles of incidence below the chord of 16-sided polygon (22.5°). Excluding considerations of image formation and visual perception, Della Porta developed a geometrical method by which one can define a point (*punctus refractionis*) where the refracted rays meet the optical

[40] Ray BG will not advance toward point X since the incident angle at point G is above the critical angle of 48.5°. Della Porta acknowledged this constraint (1593, Bk II, Prop. 4, p. 46).

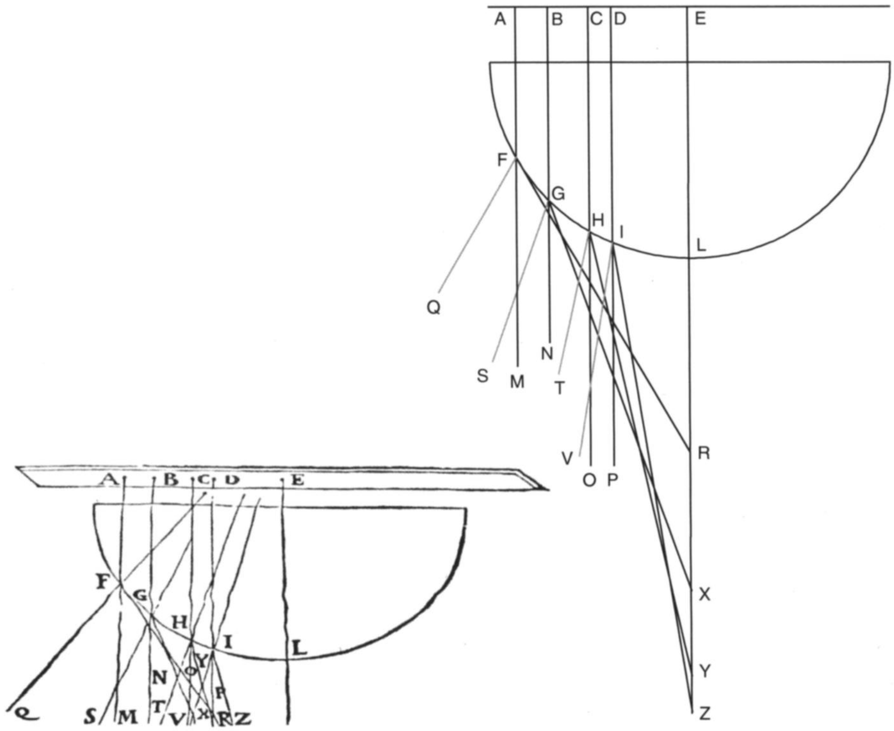


Fig. 13 Refraction through a plano-convex interface

axis of the interface. This was the only available quantitative theory of refraction at the time. It was a theory which skilled opticians, using a straightedge, compass, and table of chords,could easily apply in their practice.[41]

Della Porta's practice was based on a systematic tracing technique for illustrating the path of the refracted rays in terms of their directions and the angles by which they converge or diverge. Given reciprocity between reflection and refraction, Della Porta extended the focal length of the reflected rays to the other side of the interface, and marked the point of refraction (i.e., the burning point). Della Porta applied the same slope angles of the rays to concave and convex interfaces. For example, the diverging slope angle of ray DI (Fig. 12a), that is slope angles IN = IG, were applied to the corresponding ray DI (Fig. 13) incident at the convex interface and refracted along slope angle IZ (Fig. 13). From Della Porta's geometrical scheme one could infer the distances LG (Fig. 12a) and LZ (Fig. 13), in terms of the radius curvature of the interface. Accordingly, the distance LG (Fig. 12a), from the back surface of the plano-concave interface and the distance LZ (Fig. 13), from the back surface of the plano-convex interface, for angles incident below the chord of 16-sided polygon, would be about 2.4 times the radius curvature. This technique of ray tracing suggests an approximation

[41] For Della Porta's commentary on Chapter X of Ptolemy's *Almagest* and a copy of his table of chords, see Della Porta ([1605] 2000, pp. 108–128).

by which a ratio of about 2.4 exists between the radius curvature and the distance of the burning points of plano-concave and plano-convex interfaces.

Essentially, Della Porta's theory of refraction makes the following claims: For a given incident ray, (1) the larger the radius of an optical interface was, the further away the converging/diverging ray would intersect the optical axis from the optical interface; and (2) the larger the radius of an optical interface was, the smaller the angle of refraction would be. Accordingly, one could mark a point (*punctus refractionis*) along the optical axis, identified with what we define today as the focal point of an optical interface.

4 A proposed solution

We conjecture that Galileo ingeniously combined two distinct optical theories which together offer insight into the working of the telescope. We claim that Galileo could link Della Porta's quantitative theory of reflection and refraction with the Euclidean optical theory of surveying instruments. Galileo could consider magnification in surveying instruments, based on reflection, an analogue for telescopic magnification. He could rely on Della Porta's theory to secure, first, a faithful relation between reflection and refraction and, second, an algorithm for determining lens specifications for optical properties required for improving magnification. Galileo stated in *Sidereus nuncius* that he improved the telescope on the basis of his knowledge of refraction (*doctrinae de refractionibus*).[42] It stands to reason that Galileo was familiar with Della Porta's optical works since he had them in his private library.[43] Galileo could apply Della Porta's theory of refraction which develops the essential quantitative geometrical relations by which the properties of optical elements are determined:

1. The relation between the radius of curvature and the angle of reflection/refraction of the incident ray; and,
2. The relation between the radius of curvature and the point at which the reflected/refracted rays intersect with the optical axis.

These are elements of a theory by which specifications of lenses can be calculated and produced accordingly.[44]

Here is the plausibility argument we put forward: Galileo knew about the spyglass; an instrument made of a tube and two lenses of which the one facing the object is convex, and the other, closest to the eye, is concave. From his experience with surveying instruments (i.e., his military compass), Galileo knew how to manipulate images of remote objects with the goal of calculating the heights of the objects. Similarly, the

[42] Galileo ([1610] 1989, p. 37) and Favaro (1890–1909, 3: 60): "Per quae ad consimilis Organi inventionem devenirem, me totum converterem; quam paulo post, doctrinae de refractionibus innixus, assequutus sum."

[43] For a list of Galileo's books on astronomy and physics, astrology and philosophy of the occult, cosmography and geography, natural sciences, optics and catoptrics, mathematics, and mechanics, see Favaro ([1886] 1964, pp. 246–270, esp. 262–263).

[44] On the principles of the telescope and their mathematical formulation, see Smith, Warren (1990, pp. 235–239).

spyglass is an instrument that manipulates images of remote objects with the goal of seeing them closer. Could there be analogical reasoning that would help in clarifying the functioning of the spyglass, and turn it into a scientific instrument?

The rules of measurement by sighting, applied by Galileo for the operation of his military compass, correlate linear magnitudes with apparent angles. These rules offered Galileo a method for manipulating images of objects, and he could thus draw an analogy between the following two procedures. In the case of surveying instruments, one looks at an image formed with a flat mirror by reflection, while in the case of the spyglass, one looks through a lens at an image which is formed with another lens by refraction. Galileo could regard the procedure by which an observer OL (Fig. 14a), looks at the image, I′, of object AB reflected by the plane mirror located at E, as analogical to the procedure of looking through lens GM at the image, I, of object AB (Fig. 14b), formed by lens CD.

The apparent magnitude of reflected image I′ (Fig. 14a) of object AB, is determined by the visual angle OI′L subtended at the eye of observer OL. Now, shortening the distance of the mirror from object AB, that is, EZ, will result in increasing angle AI′B. Given that the angle of incidence and the angle of reflection are equal, the angle OI′L will increase too by the same amount. To maintain the line of sight between the observer's eye and the top of the image of object AB, the observer will have to come closer to the mirror, thus shortening KE. Greater visual angles, AI′B and OI′L, yield greater apparent magnitude of the image of object AB making it to be seen closer and larger. The relation between triangles ABI′ and I′OL (Fig. 14a) could be suggestive for the spyglass. Compare image I′ with an image I, formed by a lens CD (Fig. 14b) at a distance DE. Assuming Della Porta's quantitative analysis of refraction, as long as the angles of incidence are below 22.5°, one can proceed safely by replacing refraction with reflection.

In the setup of lenses CD and GM (Fig. 14b), the apparent size of image I of object AB is determined by the geometrical relations of triangle A′B′I. Image I is now considered an object at which the observer looks through lens GM, and thus a second triangle is formed, namely, IHQ. Note that these triangles are formed by each lens independently, according to its respective optical properties (i.e., convex lens converges the rays and form a united cone while concave lens diverges the rays and form an inverted cone).[45] The parameters of triangles A′B′I and IHQ could be therefore manipulated separately. Sharing the same optical axis and a mutual conjugate point at E, any change of the triangles' sides B′I and IQ will affect, respectively, the angles A′IB′ and HIQ. Analogical to the case of surveying instruments where enlarging angle OI′L subtends image I′ under a greater angle at the eye, in the case of the spyglass angle HIQ, enlarged by lens GM in relation to angle A′IB′, subtends image I under a greater angle at the eye.

Galileo could thus analyze the working of the spyglass in the following way. The observer looks through the spyglass at a distant object AB (Fig. 14b); its image, I, is formed by lens CD at distance DE. Looking at image I from a short distance, that

[45] In 1623, in *The Assayer*, Galileo states for the first time the different functioning of the lenses—the convex objective and the concave eyepiece: See Favaro (1890–1909, 6: 255) and Galileo ([1623] 1960, p. 209).

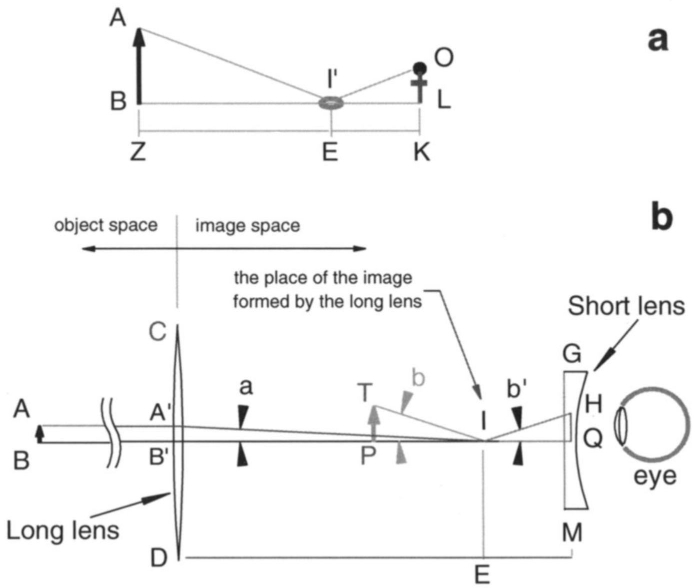


Fig. 14 The analogy: (**a**) Triangulation in surveying instruments, and (**b**) Triangulation in Galilean telescopic scheme

is, through a shorter lens GM, placed at distance ME from the image I, yields two different angles. Angle, *a* (A′IB′), produced by the longer lens CD, which is smaller than visual angle, *b*′ (HIQ), produced by the shorter lens GM. Presupposing Della Porta's claim regarding the reciprocal relation between the angle of reflection and that of refraction, image I of object AB, is presented to the eye at TP, under angle *b* which is equal to angle *b*′ subtended at the eye by lens GM.[46] Thus, the apparent size of a distant object, seen through a combination of two lenses of different "lengths," is enlarged, because angle *b*′, subtended at the eye by the shorter lens, is larger than angle *a* subtended by the object through the longer lens. In this way, the principle of magnification comes to fruition.

This analysis could facilitate the turning of the spyglass into a scientific instrument—the telescope. Galileo could account for the path of refracted rays, passing through a system of lenses, and thus comprehend how the telescope enlarges the angle by which an object is represented to the eye. What remained for him to do is to realize the analysis with lenses cut to specification—a technological accomplishment on its own right. The calculation could be carried out by Della Porta's approximation that a ratio of about 2.4 exists between the radius curvature and the distance of the burning point for plano-concave/convex interfaces.

[46] The height of TP (Fig. 14b) can be calculated for any given distance from M. This geometrical insight underpins the principle of the micrometer, the device later used by Galileo to measure apparent diameters and angular distances between the stars: see Drake and Kowal (1980).

4.1 A calculation

It is instructive to follow in detail the procedure of design and determination of the optical features and properties of a telescope like the one attributed to Galileo that magnifies 14 times. It is composed of a nearly plano-convex objective and a plano-concave eyepiece, and has an entrance pupil diameter of 26 mm. It is evident that the smallest radius curvature of the eyepiece of this telescope is about 50 mm. Therefore, our design will not include an eyepiece of radius curvature less than 50 mm.

For a given object at infinity (e.g., the Moon), which subtends an apparent angle of 30′,[47] we obtain the chord of this angle related to a radius of 60 units of a reference circle:[48]

$$\text{Ch } 0.5^\circ = 2R \cdot \sin {}^{\alpha}/_{2} = 120 \cdot \sin {}^{0.5}/_{2} = 120 \cdot 0.004363 = 0.523$$

To see the Moon enlarged 14 times one has to obtain a visual angle of:

$$0.5^\circ \cdot 14 = 7^\circ$$

The chord of this angle related to a radius of 60 units is obtained by:

$$\text{Ch } 7^\circ = 2R \cdot \sin {}^{\alpha}/_{2} = 120 \cdot \sin {}^{7}/_{2} = 120 \cdot 0.061 = 7.32$$

Consequently, magnification can be also expressed by the ratio of chords:

$$M = {}^{\text{ch}7}/_{\text{ch}0.5} = {}^{7.32}/_{0.523} = 13.996$$

Under these terms, the reciprocal sine of a visual angle subtended at the eye by a circle/diameter/object is inversely proportional to its distance. Accordingly, one can compute the distance (i.e., radius of an arc/circle) according to the sine of the subtended angle. Thus we obtain

Radius curvature of the long lens is, ${}^{1}/_{\sin 0.5:2} = 1/0.00436 = 229.36$ units, and
Radius curvature of the short lens is, ${}^{1}/_{\sin 7:2} = 1/0.061 = 16.38$ units.

Applying Della Porta's approximation, the focal point of a plano-convex/concave lens is located at a distance equal to 2.4 of its radius curvature. Consequently,

The focal point of the long lens is $229.36 \cdot 2.4 \approx 550$ units, and
The focal point of the short lens is $16.38 \cdot 2.4 = 39.3$ units.

Accordingly, the magnification of this design is 550:39.3 = 14.

Recall that we are limited to using an eyepiece of less than 50 units of radius curvature. Assuming radius curvature of 50 mm for the eyepiece, one has to obtain the

47 For a reference, a tower of 52 m height located at a distance of 6000 m from an observer, subtends an angle of ∼30′.

48 Ptolemy used radius of 60 unites for establishing his table of chords. On Ptolemy's computational techniques and calculation of chords, see Pedersen (1974, pp. 52–65).

ratio 50:16.38 = 3. The radius of curvature of the longer lens has to be enlarged by 3, thus $229.36 \cdot 3 \approx 688$ units. We present in the table the lens data of our design, made of common glass close to what Galileo used in his telescope.

LENS DATA: GAL X14

SURFACE	RADIUS CURVATURE	THICKNESS	RADIUS LENS	GLASS
Objective				
1	688[a]	2.5	25.000	BK7
2	0	1.2267 e + 03	25.000	AIR
Eyepiece				
3	0	3	13.000	BK7
4	50	10[b]	13.000	AIR

[a] The measure unit in the table is millimeter
[b] This is the eye-relief of the telescope

> The first column, SURFACE, denotes the surfaces of the optical system. Line 1 refers to the front surface of the objective, and line 2 refers to its back surface, while lines 3 and 4 refer to the front and back surfaces of the eyepiece, respectively. The second column, RADIUS CURVATURE, displays the radius of each surface of the lenses. The third column, THICKNESS, lines 1 and 3, display the central thickness of the lenses, and line 2 displays the distance between the lenses after optimization, while line 4 displays the distance of the eye from the last surface of the eyepiece. The columns, RADIUS LENS and GLASS, display the radius of the lenses and the material through which the rays travel, respectively. BK7 is a common material for optical components and windows made of borosilicate glass with the following specifications: refractive index, $n = 1.5168$, and dispersion value, $V = 64.17$

We obtained a Galilean telescope that magnify 13.76 times and its overall lens length is 1232.2 mm. As shown, our design and the ensuing computation rest only on optical magnification and the approximation derived from Della Porta's theory of refraction. Considerations related to image formation, visual perception, or derivations of the sine law of refraction were not relevant in this design process, nor an aperture stop of less than 26 mm is required for improving the resolving power of the telescope. All these elements, theoretical and practical, were available to Galileo. He could put them together and transform thereby the Dutch spyglass into a scientific instrument. He did, but he did not divulge his scheme.

5 Conclusion

The available theories of light and vision at the time Galileo obtained the Dutch invention, failed to provide the knowledge required for improving the performance of the telescope.[49] A change of theory was needed. We have argued that it came with the

[49] Scholars and mathematicians from the fifteenth and sixteenth centuries had not gone beyond the qualitative account of the sense of sight and visual perception; they did not seek quantitative analysis of reflection

quantitative calculations of refraction put forward by Della Porta in his *De refractione*. We have suggested a plausibility argument; Galileo could appeal to analogical reasoning that, formally, the geometry of magnification in surveying instruments is the same as that in the telescope. He could then use Della Porta's theory to render the transition from reflection to refraction reliable.

It takes a towering figure to appreciate what Galileo had accomplished. Kepler too was a great theoretician who made his own instruments.[50] It is not surprising that he was probably the only scientist at that time who, upon receiving a copy of *Sidereus nuncius* in April 1610, could understand precisely what Galileo had achieved:

> I am aware how great a difference there is between theoretical speculations and visual experience; between Ptolemy's discussion of the antipodes and Columbus' discovery of the new world, and likewise between the widely distributed tubes with two lenses and the apparatus with which you, Galileo, have pierced the heavens. But here I am trying to induce the skeptical to have faith in your instrument.[51]

To be sure, since this revolutionary time at the beginning of the seventeenth century, modern science has cultivated a good measure of faith in instruments, but we are still not entirely clear how Galileo transformed the spyglass into the astronomical telescope. This paper offers a new perspective on Galileo's dramatic accomplishment—turning a toy into a scientific instrument.

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Footnote 49 continued
and refraction in optical interfaces: see Smith, A. Mark (1998, pp. 40–44; 2001b, pp. 158–161, 2004; 2005, pp. 163–170; 2008, pp. xi–xii, xxxviii–xl; 2010a; 2010b, 1: xcvii–civ), Ilardi (2007, pp. 207–224), Malet (1990), Shapiro (1990, pp. 119–127, 165), Lindberg and Cantor (1985), and Lindberg (1972, pp. xx–xxi; 1976, pp. 178–190, 194).

50 Hon and Zik (2009). On Kepler and the telescope, see Zik (1999, pp. 38–39; 2003, pp. 486–490) and Malet (2003, pp. 107–136).

51 Kepler ([1610] 1965, p. 17).

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