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Investigations of the coordinates in Ptolemy's *Geographike Hyphegesis* Book 8

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Abstract In Book 8 of his *Geographike Hyphegesis* Ptolemy gives coordinates for ca. 360 so-called noteworthy cities. These coordinates are the time difference to *Alexandria*, the length of the longest day, and partly the ecliptic distance from the summer solstice. The supposable original conversions between the coordinates in Book 8 and the geographical coordinates in the location catalogue of Books 2–7 including the underlying parameters and tabulations are here reconstructed. The results document the differences between the Ω - and Ξ -recension. The known difference in the longitude of *Alexandria* underlying the conversion of the longitudes is examined more closely. For the ecliptic distances from the summer solstice of the Ω -recension, it is revealed that they were originally computed by means of a so far undiscovered approximate, linear conversion. Further it is shown that the lengths of the longest day could be based on a linear interpolation of the data in the *Mathematike Syntaxis* 2.6.

Keywords Ptolemaios · *Geographike Hyphegesis* Book 8 ·
Conversion of coordinates · Longitude · Latitude · Length of the longest day ·
Ecliptic distance

1 Introduction

Ptolemy's *Geographike Hyphegesis* (GH/Geography) compiled in about 150 AD contains a location catalogue in Books 2–7 with about 6,300 ancient localities and their positions expressed by geographical longitude Λ and latitude Φ . Three hundred and

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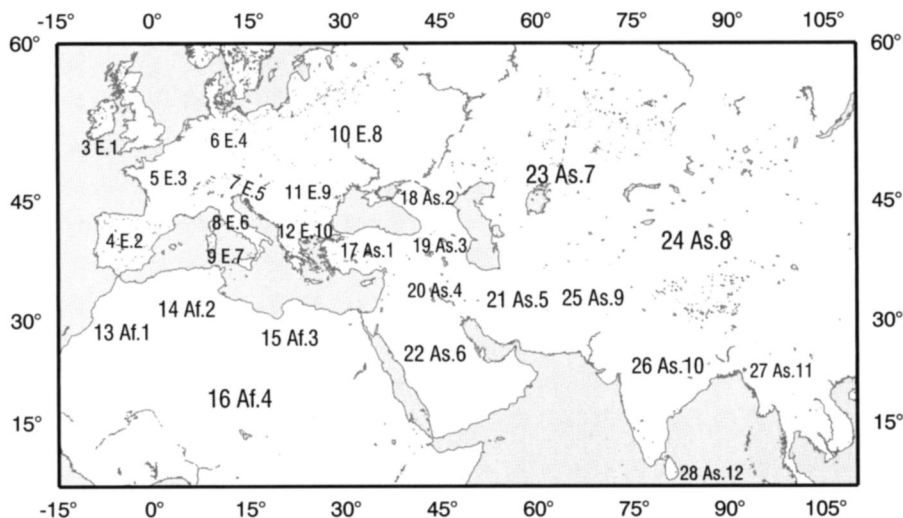


Fig. 1 Overview of the chapters in Book 8 (E. Europe, Af. Africa, As. Asia)

sixty of these localities are additionally listed with coordinates in Book 8 of the Geography. In contrast to the location catalogue, the positions in Book 8 are expressed by means of

- the time difference A (in hours) from the location to *Alexandria*,
- the length of the longest day M (in hours) at the location,
- the ecliptic distance S (in arc degree) of the sun from the summer solstice at the time when the sun reaches the zenith (for locations between the tropics).

The localities in Book 8 are selected cities, the so-called *poleis episemoi* (“noteworthy cities”). They are arranged in 26 chapters; each chapter is related to one of the maps Ptolemy divided the *Oikoumene* into (the inhabited world known to the Greeks and Romans). Figure 1 shows a general view of the sites of the single maps (see also Stückelberger and Graßhoff 2006).

Since the positions of the *poleis episemoi* are given in the location catalogue and in Book 8, a comparison of the positions of both sources is possible after a conversion of coordinates. The *poleis episemoi* also appear in another of Ptolemy’s works, the *Procheiroi Kanones* (Handy Tables; hereinafter denoted Canon). Unlike Book 8, the coordinates are given there in form of geographical coordinates as in the location catalogue of the Geography.

A conversion of the A - and M -data in Book 8 into geographical coordinates can be found in Cuntz (1923) and in Stückelberger and Mittenhuber (2009). The latter contains an edition of the Canon with a compilation of the corresponding geographical coordinates of the GH 2–7 and the converted values $\Lambda(A)$ and $\Phi(M)$ on the basis of Book 8. Cuntz examines four maps of Europe (GH 8.5–8). He supposes that A and M in Book 8 originate from the geographical coordinates in Books 2–7. By his assumptions about the precision of A and M as well as about tabulations underlying A and M , he makes an attempt to explain the coordinate values in Book 8 (see Sects.

2 and 3). Honigmann (1929, p. 64 ff.) assumes that the data of the *poleis episemoi* in Book 8 as well as in Books 2–7 were derived from works by a predecessor of Ptolemy.

In the following, the conversions of coordinates between the location catalogue and Book 8 are investigated. The underlying calculation methods and their parameters are worked out as well as the underlying tabulations. Deviations between the coordinates in Book 8 and in the location catalogue are explained as comprehensively as possible by the resolution (precision) of the coordinates. As source for the data of the Geography, the edition of Stückelberger and Graßhoff (2006) was used (for changes on the data see Appendix). It contains the coordinates of the two main recensions of the Geography, the Ω - and Ξ -recension. The latter one has been handed down only by the X-manuscript; its location catalogue ends in Book 5, Chapter 13, so that a direct comparison with the data in Book 8 is not possible. Both Ω and X are considered in the present work. Stückelberger and Graßhoff (2006, p. 44) state that many M -values of X seem to be based on the coordinates of the Canon. The relation between the coordinates in Book 8 and the Canon is only broached here. The coordinates used are based on the edition of the Canon by Stückelberger and Mittenhuber (2009).

2 Time difference to *Alexandria*

The conversion between the time difference to *Alexandria* A (in hours) and the longitude Λ (in degree) is

$$\Lambda = A \cdot 15^\circ/\text{h} + \Lambda_A \quad (1)$$

with the longitude of *Alexandria* Λ_A (in degree). Ptolemy gives two different values for Λ_A in the Geography, in the location catalogue $60^\circ 30' =: \bar{\Lambda}_A$ (4.5.9) and in Book 8 $4\text{ h} =: \tilde{\Lambda}_A$ (8.15.10; the distance between *Alexandria* and the zero meridian); in GH 7.5.14, he gives both values. The difference between both parameters is $\Delta\Lambda_A := \bar{\Lambda}_A - \tilde{\Lambda}_A = 60^\circ 30' - 60^\circ = 30' = 2\text{ min}$.

Cuntz (1923, p. 101 ff.) investigates the coordinates of the 3. to 6. map in GH 8.5–8 (Europe, 42 localities). He states that both $60^\circ 30'$ and $60^\circ 00'$ underlie the time differences A_i ($i = 1, 2, \dots$, number of coordinates n) and that it is not possible to determine the Λ_A the coordinates are based on in all cases. His explanation for different Λ_A is that Ptolemy upgraded the value $60^\circ 30'$ to $60^\circ 00'$ and added the updated A_i in a further column besides the values underlying $60^\circ 30'$; later copyists chose one of the values. Stückelberger and Graßhoff (2006, p. 44) mention that 60° primarily underlies the data of Ω as well as $60^\circ 30'$ the data of X.

2.1 Conversion of coordinates and accuracy

Ptolemy explicitly declares in GH 8.2.1 that the time differences A_i in Book 8 are approximate values; thus, equality after a conversion between the data of Books 2–7 and Book 8 is not to be expected. For the following conversions of coordinates, the

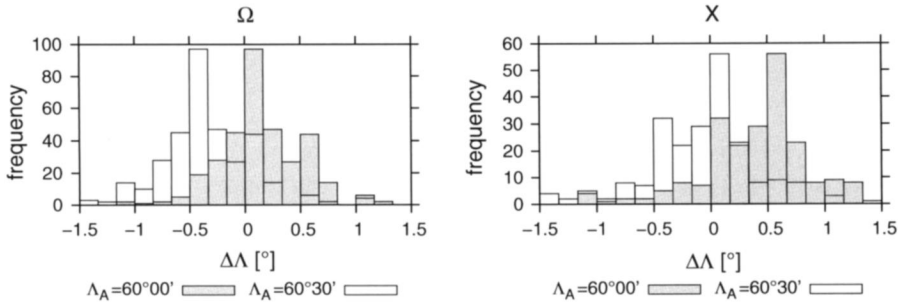


Fig. 2 Frequency of differences $\Delta\Lambda_i = \Lambda_i - \Lambda(A_i)$ between the longitudes Λ_i of the location catalogue and the longitudes $\Lambda(A_i)$ computed from the time differences A_i to *Alexandria* of Book 8 by means of Eq. (1) for the Ω -recension (*left*) and the X-manuscript (*right*); Λ_A is the longitude of *Alexandria* underlying the conversion

difference Δy between the conversion $y(x)$ of a coordinate x and the given coordinate value y is defined as

$$\Delta y = y - y(x). \tag{2}$$

Then Δy is the error of y with respect to $y(x)$.

Both $\bar{\Lambda}_A$ and $\tilde{\Lambda}_A$ were used here for the conversion of coordinates. For the conversion $A \mapsto \Lambda$ by Eq. (1), Fig. 2 shows the frequencies of differences $\Delta\Lambda_i = \Lambda_i - \Lambda(A_i)$ lying within specified intervals (interval width $10'$). For Ω the conversion with $\tilde{\Lambda}_A$ gives a maximal frequency near zero and with $\bar{\Lambda}_A$ near $-30'$. That is the negative systematic difference $\Delta\Lambda_A$ and indicates that the majority of the coordinates were originally converted with $\tilde{\Lambda}_A = 60^\circ$. In X, the conditions are converse so that $\bar{\Lambda}_A = 60^\circ 30'$ must be the main parameter of the original conversion. The frequency distribution is bimodal; the distance of the lower second peak from the main peak is $30'$. Hence, the second peak is probably caused by A_i which are based on the corresponding other Λ_A .

The size of the differences $\Delta\Lambda_i$ can mainly be explained by a original conversion $\Lambda \mapsto A$ in conjunction with a rounding of A . Apart from a few exceptions, the largest denominator of the fractions of the A_i is 30 ($1/30 \text{ h} = 2 \text{ min}$). Cuntz (1923, p. 108) assumes an original precision of $1/12 \text{ h} = 5 \text{ min} (= 1^\circ 15')$.¹ In general, if a number is rounded with a precision (resolution) $a = 1/d$, the expected maximal rounding error is

$$e_{\max} = \pm \frac{a}{2} = \pm \frac{1}{2d}. \tag{3}$$

For $a = 5 \text{ min}$, e_{\max} is $2.5 \text{ min} \approx 2/3^\circ$. This agrees with that part of the tail of the frequency distribution which is not extended by the systematic difference $\Delta\Lambda_A$ (see Fig. 2).

Cuntz (1923, p. 108) gives a tabulation of A - and Λ -values with a step size of $\Delta A = 5 \text{ min}$ ($\Delta\Lambda = 1 1/4^\circ$) by which A -values may have been determined. Cuntz

¹ Honigmann (1929), who discusses Cuntz' work, does not dissent.

supposes that in the case of a Λ_i which does not equal a tabulated Λ the nearest tabulated Λ was chosen. In the end Cuntz cannot explain several A_i by means of the tabulation. The use of a tabulation described by Cuntz was applied here to the entire A -data of Book 8. A tabulation was generated for $\bar{\Lambda}_A$ and $\tilde{\Lambda}_A$. In the case of Ω 61 % and in the case of X 51 % of the A -values determined by one of both tabulations equal the A_i given in Book 8. Furthermore a tabulation with $\Delta A = 2.5$ min was tested; the result are 61 and 55 %. That does not argue for the use of a tabulation. Cuntz gives the example of *Lugdunum* (8.5.5) with $\Lambda = 23^\circ 15'$ and $A = -2^h 30^m$. The nearest tabulated Λ is 23° so that A becomes $-2^h 30^m$. However, this might also be the result of Eq. (1) and a rounding: $A(\Lambda) = -2^h 29^m$.

2.2 Determination of Λ_A and the resolution

Considering the single chapters of Book 8, there are chapters whose longitudes are converted with one of both Λ_A mainly; in other cases, the underlying Λ_A cannot be recognized reliably. The question of the Λ_A underlying a single A led to an attempt to determine Λ_A for each locality by means of the following method. Λ_i is converted with $\bar{\Lambda}_A$ to $\bar{A}(\Lambda_i)$ and with $\tilde{\Lambda}_A$ to $\tilde{A}(\Lambda_i)$ by Eq. (1). \bar{A}_i and \tilde{A}_i are rounded with different precisions a to determine the apparent resolution of the given A_i . In general terms, the two coordinates x and y of different types are given as well as the converted coordinate $y(x)$. Furthermore, a main resolution a_s is set. The procedure applied is then

Procedure ROUND:

- (1) Round $y(x)$ to the precision a_s ; if $\Delta y = y - y(x) = 0$ end.
- (2) For each of the precisions $a < a_s$ downwards: round $y(x)$; if $\Delta y = 0$ end.
- (3) For each of the precisions $a > a_s$ upwards: round $y(x)$; if $\Delta y = 0$ end.

If Δy becomes 0, the apparent resolution of y is found; if Δy does not become 0 until the end, an erroneous y or x can be assumed.

Procedure ROUND was applied to the \bar{A}_i and \tilde{A}_i . The results of a comparison with the given A_i were valued as follows:

- (1) rounded $\bar{A}_i = A_i$ or/and rounded $\tilde{A}_i = A_i$:
 - (a) only rounded $\bar{A}_i = A_i$: A_i is apparently based on $\bar{\Lambda}_A$;
 - (b) only rounded $\tilde{A}_i = A_i$: A_i is apparently based on $\tilde{\Lambda}_A$;
 - (c) rounded $\bar{A}_i = A_i$ and rounded $\tilde{A}_i = A_i$: $\Delta \bar{A}_i = A_i - \bar{A}_i$, $\Delta \tilde{A}_i = A_i - \tilde{A}_i$
 - (i) $|\Delta \bar{A}_i| < |\Delta \tilde{A}_i|$: A_i is possibly based on $\bar{\Lambda}_A$;
 - (ii) $|\Delta \bar{A}_i| > |\Delta \tilde{A}_i|$: A_i is possibly based on $\tilde{\Lambda}_A$;
 - (iii) $|\Delta \bar{A}_i| = |\Delta \tilde{A}_i|$: no decision possible;
- (2) rounded $\bar{A}_i \neq A_i$ and rounded $\tilde{A}_i \neq A_i$: no decision possible, maybe a corrupted A_i -value.

The possible resolutions for Procedure ROUND were chosen according to the denominators of the fractions which occur in Book 8. The resolutions used for Ω are (in h):

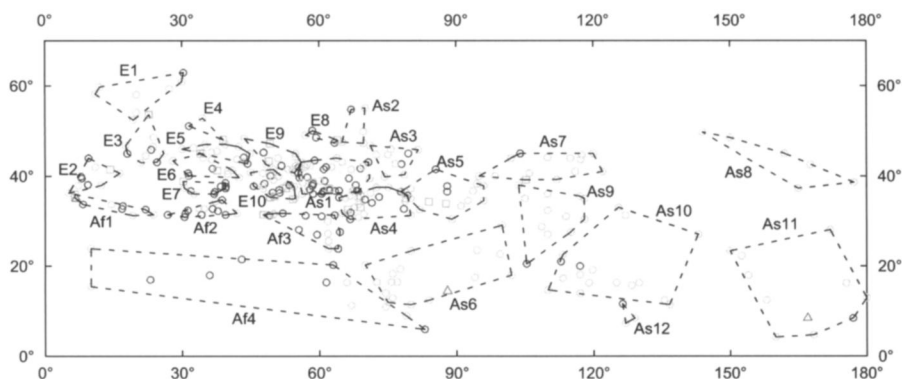


Fig. 3 Classification of the localities in Book 8 of the Ω -recension according to the longitude Λ_A of Alexandria which underlies a conversion of the longitudes Λ_i of the location catalogue into the time differences A_i to Alexandria of Book 8; blue/circle $\bar{\Lambda}_A = 60^\circ 30'$, yellow/circle $\tilde{\Lambda}_A = 60^\circ 00'$, green/square $\bar{\Lambda}_A$ or $\tilde{\Lambda}_A$, red/triangle A does not fit with Λ . (Color figure online)

$1/360, 1/60, 1/30, 1/24, 1/15, 1/12, 1/10, 1/8, 1/7, 1/6$, and $1/5$.² For X $1/60$ and $1/7$ were not used, and $1/20$ and $1/9$ were added. The main resolution was set to $a_s = 1/12$ h (following Cuntz 1923) and for a further run to $a_s = 1/24$ h. The results for the portions of apparent resolutions are:

- $a_s = 1/12$ h:
 - Ω : $a = a_s$: 61 %, $a < a_s$: 34 %, $a > a_s$: 4 %, no fitting rounding: 1 %;
 - X : $a = a_s$: 51 %, $a < a_s$: 36 %, $a > a_s$: 9 %, no fitting rounding: 4 %;
- $a_s = 1/24$ h:
 - Ω : $a = a_s$: 61 %, $a < a_s$: 24 %, $a > a_s$: 14 %, no fitting rounding: 1 %;
 - X : $a = a_s$: 55 %, $a < a_s$: 23 %, $a > a_s$: 19 %, no fitting rounding: 4 %.

The result indicates that not only $a_s = 1/24$ h is the main resolution, but $a_s = 1/12$ h also comes into consideration. The result of Procedure ROUND applied with $a_s = 1/24$ h shows that ca. 80 % of the localities have only small absolute differences $< 0.5 \cdot 1/24$ h = $1/48$ h = 1.25 min (cf. Eq. (3)) between A_i and $A(\Lambda_i)$ converted by Eq. (1). Further, almost all A_i can be explained by conversion based on either $\bar{\Lambda}_A$ or $\tilde{\Lambda}_A$ and a rounding. Hence, the vast majority of the differences between the given A_i and the converted $A(\Lambda_i)$ are explicable by the precision of the A_i . Consequently, the location catalogue and Book 8 show a high level of interdependence.

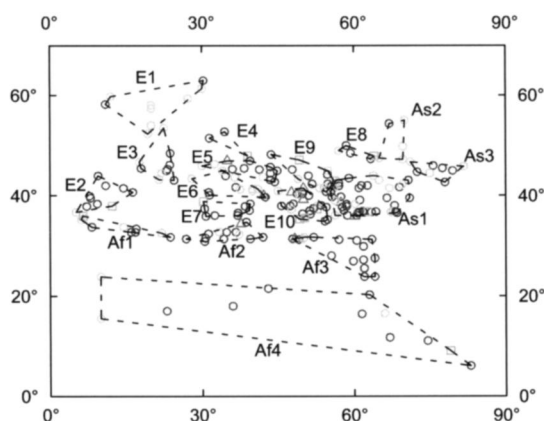
The determined frequencies of $\bar{\Lambda}_A$ and $\tilde{\Lambda}_A$ are

- Ω : $\bar{\Lambda}_A$: 107 (31 %), $\tilde{\Lambda}_A$: 209 (61 %), $\bar{\Lambda}_A$ or $\tilde{\Lambda}_A$: 25 (7 %), no fitting rounding: 3 (1 %);
- X : $\bar{\Lambda}_A$: 137 (62 %), $\tilde{\Lambda}_A$: 63 (29 %), $\bar{\Lambda}_A$ or $\tilde{\Lambda}_A$: 11 (5 %), no fitting rounding: 9 (4 %).

This again reveals that $\bar{\Lambda}_A$ dominates in X and $\tilde{\Lambda}_A$ in Ω (with a ratio of ca. 2:1).

² The denominator 360 does not really occur but was introduced because of the occurrence of the sum $1/8 + 1/90 = 49/360$ (see Stükelberger and Graßhoff 2006, p. 883, n. 65).

Fig. 4 Classification of the localities in Book 8 of the X-manuscript according to the longitude Λ_A of *Alexandria* which underlies a conversion of the longitudes Λ_i of the location catalogue into the time differences A_i to *Alexandria* of Book 8; blue/circle $\bar{\Lambda}_A = 60^\circ 30'$, yellow/circle $\bar{\Lambda}_A = 60^\circ 00'$, green/square $\bar{\Lambda}_A$ or $\tilde{\Lambda}_A$, red/triangle A does not fit with Λ . (Color figure online)



Figures 3 and 4 show the result of the classification for each locality in the Ptolemaic coordinate system ($\bar{\Lambda}_A$: blue/circle, $\tilde{\Lambda}_A$: yellow/circle, $\bar{\Lambda}_A$ or $\tilde{\Lambda}_A$: green/square, no fitting rounding: red/triangle). In Ω , the dominant $\bar{\Lambda}_A$ occurs nearly exclusively in the east, and in the west also $\bar{\Lambda}_A$ occurs often. There are in Ω 13 chapters in which $\bar{\Lambda}_A$ dominates: Europe 1, 3, 5, 6, 9, Asia 3, 5–11; in 2 chapters $\tilde{\Lambda}_A$ dominates: Europe 7, Africa 2. In X $\bar{\Lambda}_A$ dominates in 8 chapters: Europe 2, 5, 7, 10, Africa 2–4, Asia 1; $\tilde{\Lambda}_A$ dominates in 2 chapters: Europe 1, Asia 2.

2.3 Comparison of Ω and X

Finally, the A_i of Ω and X were compared. Three hundred and forty three values are equal and 143 values have differences, thereof 16 localities also have differences in Λ in the location catalogue so that $143 - 16 = 127$ A_i remain with actual differences between Ω and X (37%). Besides the expected systematic difference of $\Delta\Lambda_A = 30' = 2$ min also other differences occur comparably often: 2.5 min $= 1/24$ h, 3 min $= 1/20$ h, 4 min $= 1/15$ h, and 5 min $= 1/12$ h. This is explicable by the precision of the A_i . As an example *Clunia* (8.4.5) is considered: In Ω A is -3^h15^m , that is $\Lambda = 11^\circ00'$ converted with $\bar{\Lambda}_A = 60^\circ$ to $A = -3^h16^m$ and rounded to the nearest $1/12$ h. In X A is -3^h20^m , that is Λ converted with $\tilde{\Lambda}_A = 60^\circ30'$ to $A = -3^h18^m$ and rounded to the nearest $1/12$ h. The systematic difference of $\Delta\Lambda_A = 2$ min becomes 5 min here.

Figure 5 shows the localities without (green/circle) and with differences in A between Ω and X (Ptolemaic system). Those with differences are divided into localities with difference in Λ (yellow/triangle), without difference in Λ (red/triangle), and the remaining localities (magenta/square) which have no Λ in the location catalogue of X. Localities with differences disperse across the whole area; however, they often occur in local groups. In the west, there is a larger amount of differences.

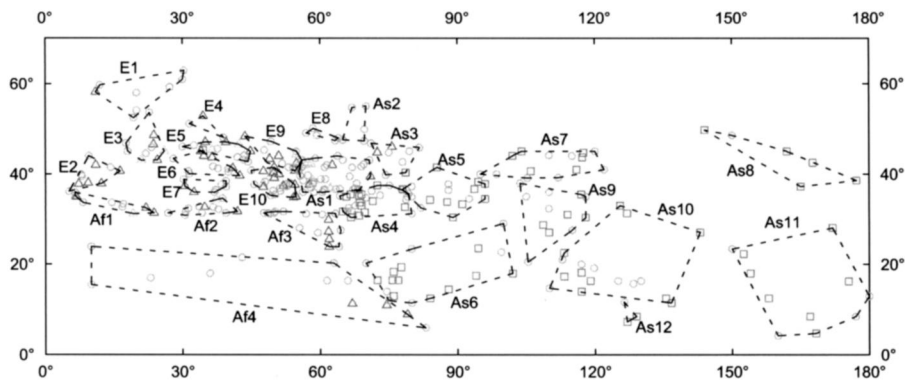


Fig. 5 Differences in the time differences A_i to Alexandria between Book 8 of the Ω -recension and the X-manuscript; green/circle no difference in A , other difference in A , red/triangle no difference in the longitude Λ of the location catalogue, yellow/triangle difference in Λ , magenta/square no comparison of Λ . (Color figure online)

3 Length of the longest day

The length of the longest day M increases from 12 h at the equator ($\Phi = 0$) to 24 h for $\Phi = 90^\circ - \varepsilon$. Ptolemy derives the computation of Φ from M and vice versa in his *Mathematike Syntaxis* (MS) 2.3. The modern formulation of that relation between M and Φ is

$$\cos\left(\frac{1}{2}M \text{ 15}^\circ/\text{h}\right) = -\tan \Phi \tan \varepsilon \tag{4}$$

(M in hours) with the obliquity of the ecliptic ε .³ In MS 2.6 Ptolemy gives a compilation of parallels with specific M including their latitudes (see Sect. 3.5).

Ptolemy does not explicitly mention the ε underlying the conversion between Φ and M in the Geography. In GH 7.6.7 he gives the ratio $\varepsilon : \pi/2 \approx 4/15$, that is the roughly rounded value $90^\circ \cdot 4/15 = 24^\circ =: \varepsilon_r$ for ε . In MS 1.12 Ptolemy states that the arc between the solstice points (i.e. 2ε) is approximately $(11/83) \cdot 360^\circ$. That is $47^\circ 42' 40''$ so that ε becomes $23^\circ 51' 20'' =: \varepsilon_m$ (Manitius 1912, p. 44). Other appearances of ε_m are MS 1.14 and 2.4, where Ptolemy uses this value for exemplary computations, as well as the “Table of Inclination” in MS 1.15, where ε_m occurs as the size of the arc of meridian with an ecliptic longitude of 90° (see Sect. 4). The correct value of Ptolemy’s time is $23^\circ 41' =: \varepsilon_c$ so that ε_m has an error of about $11'$.

Rawlins (1985) gives two groups of cities of Book 8, on the one hand 10 cities, whose M and Φ could be related by Eq. (4) through $23^\circ 50' \approx \varepsilon_m$, on the other hand 7 cities, whose M and Φ could be related through $23^\circ 55' =: \varepsilon_h$. Rawlins attributes the latter value to Hipparchus (see also Rawlins 1982) so that the original conversion

³ Trigonometric problems could be solved accurately in antiquity by using tabulations of chords as it can be found in MS 1.11. On ancient trigonometry used by Ptolemy and described in his MS see Neugebauer (1975, p. 21 ff.).

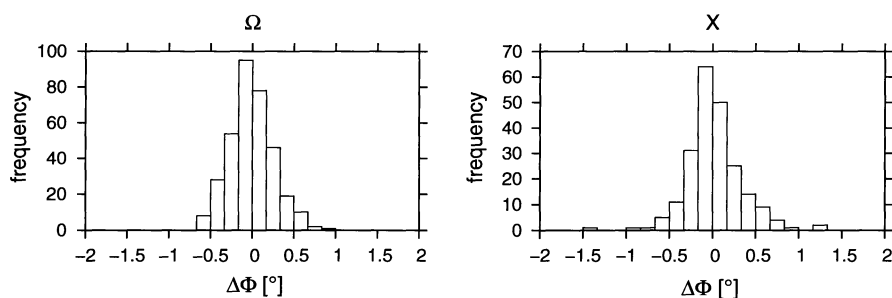


Fig. 6 Frequency of differences $\Delta\Phi_i = \Phi_i - \Phi(M_i)$ between the latitudes Φ_i of the location catalogue and the latitudes $\Phi(M_i)$ computed from the lengths of the longest day M_i of Book 8 by Eq. (4) for the Ω -recension (left) and the X-manuscript (right)

would have been done with that value before Ptolemy. A further hint to $\varepsilon_r = 24^\circ$ used in Ptolemy's data was detected by Neugebauer (1975, pp. 236, 245 ff.), who found ε_r underlying the data in the tables for the first and last visibility of the planets in MS 13.10. The question of the ε underlying the M_i in Book 8 is considered in the following.

3.1 Conversion of coordinates and accuracy

A conversion between Φ and M was carried out for both directions $M \mapsto \Phi$ and $\Phi \mapsto M$ by means of Eq. (4), and the differences $\Delta\Phi_i = \Phi_i - \Phi(M_i)$ and $\Delta M_i = M_i - M(\Phi_i)$ to the given values Φ_i and M_i were computed. For $M \mapsto \Phi$ with $\varepsilon = \varepsilon_m$ Fig. 6 shows the frequencies of differences $\Delta\Phi_i$ lying within specified intervals (interval width $10'$). The distribution has a bell-shaped form with its maximum at ≈ 0 . The majority of the absolute values of the differences is not larger than $1/2^\circ$.

As a measure of the consistency between given coordinates y_i and converted coordinates $y(x_i)$ the median absolute deviation

$$\text{MAD} = \text{med}(|y_i - y(x_i)|) \quad (5)$$

($i = 1, \dots, n$) and the mean deviation

$$\text{MD} = \frac{1}{n} \sum_{i=1}^n |y_i - y(x_i)| \quad (6)$$

are used here (Hampel 1974; Sachs 1992, p. 337). Unlike the MD, the MAD is resistant against outliers (i.e. abnormally large differences $y_i - y(x_i)$ here).

For both directions of conversion four conversions were carried out with ε_m , ε_h , ε_r , as well as experimentally with ε_c . Table 1 shows the MAD, MD, and the number of coordinates n . Because of its resistant property the MAD is lower than the MD. In most cases value ε_m leads to the lowest measures of precision; only in the case of $M \mapsto \Phi$ the MAD based on ε_h is $1'$ (X) and $2'$ (Ω) smaller than the MAD based on

Table 1 MAD (Eq. (5)) and MD (Eq. (6)) after conversions of coordinates of the Ω -recension, the X-manuscript, and the Canon; $M \mapsto \Phi$: deviations $\Phi_i - \Phi(M_i)$ between the latitudes $\Phi(M_i)$, computed from the lengths of the longest day M_i of Book 8, and the Φ_i of Books 2–7/the Canon; $\Phi \mapsto M$: deviations $M_i - M(\Phi_i)$ between the $M(\Phi_i)$, computed from the Φ_i of Books 2–7/the Canon, and the M_i of Book 8; $S \mapsto \Phi$ and $\Phi \mapsto S$: as $M \mapsto \Phi$ and $\Phi \mapsto M$ but for the distances S_i of the sun from the summer solstice of Book 8; n is the number of coordinates; the values of the obliquity of the ecliptic underlying the conversions are: $\varepsilon_m = 23^\circ 51' 20''$, $\varepsilon_h = 23^\circ 55'$, $\varepsilon_r = 24^\circ$, $\varepsilon_c = 23^\circ 41'$

Conversion		Ω B.2–7/B.8		Canon/ Ω B.8		X B.2–5/B.8		Canon/X B.8	
ε		MAD	MD	MAD	MD	MAD	MD	MAD	MD
n		344	344	346	346	220	220	354	354
$M \mapsto \Phi$	ε_m	9.7'	13.0'	10.7'	17.5'	9.6'	13.6'	10.7'	19.8'
	ε_h	7.9'	13.6'	11.6'	18.1'	8.5'	14.4'	10.7'	20.3'
	ε_r	11.4'	15.9'	12.8'	20.1'	12.0'	17.3'	11.8'	22.2'
	ε_c	14.3'	18.4'	15.2'	22.3'	15.4'	18.6'	15.8'	25.1'
	ε_m	0.9 ^m	1.3 ^m	1.0 ^m	1.7 ^m	1.0 ^m	1.5 ^m	1.1 ^m	1.9 ^m
$\Phi \mapsto M$	ε_h	0.9 ^m	1.4 ^m	1.2 ^m	1.8 ^m	1.1 ^m	1.6 ^m	1.1 ^m	2.0 ^m
	ε_r	1.2 ^m	1.6 ^m	1.5 ^m	2.1 ^m	1.5 ^m	2.0 ^m	1.4 ^m	2.2 ^m
	ε_c	1.5 ^m	1.9 ^m	1.6 ^m	2.3 ^m	1.8 ^m	2.1 ^m	1.7 ^m	2.5 ^m
n		58	58	59	59	13	13	58	58
$S \mapsto \Phi$	ε_m	18.7'	22.0'	19.3'	27.3'	0.5'	3.5'	1.1'	8.6'
	ε_h	21.2'	23.1'	21.7'	28.0'	3.2'	5.4'	3.1'	10.2'
	ε_r	22.4'	24.8'	22.4'	29.4'	6.9'	8.1'	6.6'	12.6'
	ε_c	13.1'	21.0'	15.0'	27.3'	7.5'	9.5'	7.5'	12.9'
	ε_m	72.4'	104.6'	86.2'	117.5'	1.5'	31.2'	3.5'	50.0'
$\Phi \mapsto S$	ε_h	81.5'	117.4'	82.7'	124.1'	11.6'	57.1'	11.3'	64.3'
	ε_r	78.6'	131.7'	79.6'	138.6'	23.5'	82.2'	21.8'	80.4'
n		55	55	54	54	11	11	54	54
	ε_c	57.4'	90.1'	72.7'	110.3'	23.0'	32.2'	22.2'	57.1'

ε_m . Regarding $\Phi \mapsto M$ as the main direction, the results does not argue against ε_m underlying the data.

A conversion of coordinates and a comparison with given coordinate values was also carried out for the M_i of Book 8 and the latitudes of the Canon. For $\varepsilon = \varepsilon_m$ the resulting MAD slightly increases in comparison to the location catalogue by 1' or 0.1 min, respectively, and the MD increases noticeably by 6' or 0.4 min, respectively (see Table 1). Consequently, the close relation between the M -data of X and the latitudes of the Canon stated by Stückelberger and Graßhoff (2006, p. 44) cannot be supported by means of the present computations.

The size of the differences $\Delta\Phi_i = \Phi_i - \Phi(M_i)$ can be explained by a rough resolution of the M -data and a disadvantageous propagation of their errors (assuming that the original direction of conversion was $\Phi \mapsto M$). Cuntz (1923, p. 104) assumes that the original resolution was $1/12$ h. The largest denominators occurring in Ω among the fractions of the M_i are 150 (*Palura* (8.26.6)) and 60 (e.g. *Ptolemais Herme* (8.15.13), *Omanon* (8.22.12), *Saue* (8.22.15)); however, they are exceptional values. In X the largest denominator is 30. In both Ω and X the denominator 12 occurs often so that the main resolution of M -values seems to be $1/12$ h = 5 min. From fractions like $n/6 = 2n/12$ and $n/4 = 3n/12$ the precision is not known for sure, rougher resolutions than $1/12$ h are also possible. For $30^\circ \leq \Phi \leq 40^\circ$ Fig. 7 exemplarily shows the data points (Φ_i, M_i) of Ω and X as well as the curve of the trigonometric conversion

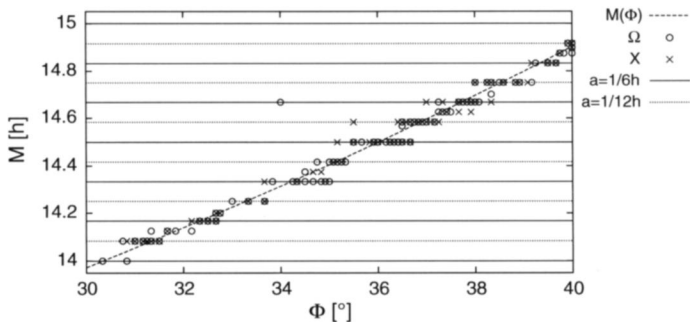


Fig. 7 Plot of the lengths of the longest day M_i of Book 8 against the latitudes Φ_i of the location catalogue for the Ω -recension and the X-manuscript, trigonometric conversion function $M(\Phi)$ (derived from Eq. (4)); the horizontal lines show the position of M -values with resolutions of $a = 1/6h$ and $a = 1/12h$

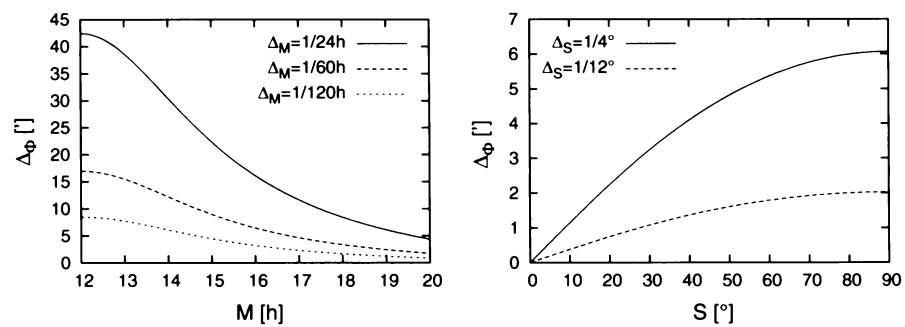


Fig. 8 Propagation of errors for the computation of the latitude Φ from the length of the longest day M (left) and the distance S from the summer solstice (right) for different errors Δ_M and Δ_S

function $M(\Phi)$ (derived from Eq. (4), $\varepsilon = \varepsilon_m$). The data points are arranged in horizontal groups, within which the M -values of different Φ_i are equal. Most of these groups have a distance of $1/12h$ so that their resolution is apparently $1/12h$ (dotted lines). Also M_i with other resolutions become apparent. The plot shows, too, that equal Φ_i were converted to different M_i . That can be explained by a rougher precision, i.e. $a = 1/6h$. Regarding only the lines of $a = 1/6h$ (solid), no Φ_i with different M_i occur.

The error of an M affects the corresponding latitude $\Phi(M)$ according to the value of that latitude. In general, the estimated effect of an error Δ_x in x on the value of the function $y(x)$ is

$$\Delta_y = \frac{dy}{dx} \Delta_x \tag{7}$$

with the derivative dy/dx of $y(x)$ with respect to x . Applied to Eq. (4) the variation Δ_Φ of Φ according to the error Δ_M of M can be determined.

Figure 8 shows the propagation of errors for the range of M in Book 8 ($12h \leq M \leq 20h$) and for the resolutions $a = 1/60h$, $1/30h$, and $1/12h$; the variations were set to $\Delta_M = a/2$ (Eq. (3)). Most of the localities in Book 8 have an $M < 17h$. For them a resolution of $1/12h$ can cause a significant loss of accuracy, the variation Δ_Φ

of Φ is $> 10'$. Δ_Φ increases for $M \rightarrow 12\text{h}$ to $2/3^\circ$. Accordingly, an ancient conversion $\Phi \mapsto M$ with a precision of $1/12\text{h}$ (or more imprecise) caused a significant loss of accuracy in the latitude expressed by M . Even exceptionally high precisions as $1/60\text{h}$ could have a noticeable effect as the example of *Ptolemais Herme* shows: $M = 13^{43}/60\text{h}$, $\Delta_M = 0.5 \cdot 1/60\text{h}$, $\Delta_\Phi = 7'$. Therefore, a large denominator in the fraction of M does not generally mean that the value is given with an unrealistically high precision.

3.2 Distribution of the coordinate differences

For the coordinate differences $\Delta M_i = M_i - M(\Phi_i)$ after the conversion $\Phi \mapsto M$ with $\varepsilon = \varepsilon_m$ it was tested by means of the χ^2 -test of goodness of fit (e.g. Sachs 1992, p. 420 ff.) whether they are normally distributed. The test statistic is in general

$$\chi^2 = \sum_{i=1}^k (n_i - np_i)^2 / (np_i) \quad (8)$$

with the class number k , the observed frequencies n_i of class i , the hypothetical probabilities p_i , and $n = \sum_{i=1}^k n_i$. Null hypothesis of the test is the normal distribution assumption. Under this hypothesis χ^2 follows a χ^2 -distribution with $f = k - 1 - u$ degrees of freedom, where u is the number of the unknown parameters. The two parameters of the normal distribution, the location parameter μ and the scale parameter σ , were estimated by the arithmetical mean of the ΔM_i and by the root mean square deviation (with respect to the mean). A few outlying, very large differences, which are not characteristic for the distribution form, were excluded. According to the assumed crudest resolution $a = 1/4\text{h}$ of the M_i (see Sect. 3.4) the limit for $|\Delta M_i|$ was set to $a/2 = 7.5\text{min}$ (Eq. (3)). The level of significance α used is 5%. The results are:

- Ω : $n = 341$, $k = 10$, $f = 7$, $\mu = 0.06\text{min}$, $\sigma = 1.58\text{min}$, $\chi^2 = 26.9$;
- X : $n = 216$, $k = 10$, $f = 7$, $\mu = -0.17\text{min}$, $\sigma = 1.83\text{min}$, $\chi^2 = 27.5$.

The quantile is in both cases $q_{1-\alpha, f} = 14.1$ so that the normal distribution assumption has to be rejected.

The χ^2 -test was repeated for the Laplace distribution. Its two parameters μ and σ were estimated by the median and by the MD (Eq. (6), but here with respect to the computed median). The results are:

- Ω : $n = 341$, $k = 10$, $f = 7$, $\mu = 0.06\text{min}$, $\sigma = 1.16\text{min}$, $\chi^2 = 12.4$;
- X : $n = 216$, $k = 10$, $f = 7$, $\mu = 0.03\text{min}$, $\sigma = 1.35\text{min}$, $\chi^2 = 7.6$.

For Ω and X χ^2 goes below the quantile $q_{1-\alpha, f} = 14.1$. Accordingly it can be assumed that the differences ΔM_i follow (at least approximately) a Laplace distribution.

3.3 On the original direction of conversion

Ptolemy states in GH 8.2.1 that the M -data are computed from latitudes. However, it is known from a few localities that Ptolemy adopted their M -values from Hipparchus

Table 2 Examples of the conversion of the latitude Φ of the location catalogue into the length of the longest day $M(\Phi)$ by Eq. (4) with $\varepsilon = \varepsilon_m$; M is the coordinate value of Book 8

Locality	Φ	M		$M(\Phi)$
	($^\circ$)	(h)	(h, min)	(h, min)
<i>Arbela</i> (8.21.3)	37 1/4	14 5/8	14, 17.5	14, 17.2
<i>Athenai</i> (8.12.18)	37 1/4	14 5/8	14, 17.5	14, 17.2
<i>Karchedon</i> (8.14.5)	32 2/3	14 1/5	14, 12.0	14, 11.8
<i>Lysimachia</i> (8.11.7)	41 1/2	15 1/12	15, 05.0	15, 04.3
<i>Nikaia</i> (8.17.7)	41 1 1/12	15 1/8	15, 07.5	15, 07.1
<i>Rhodos</i> (8.17.21)	36	14 1/2	14, 30.0	14, 29.9
<i>Tarentum</i> (8.8.4)	40	14 1 1/12	14, 55.0	14, 54.2

(Berggren and Jones 2000, p. 29). Hipparchus attributed them to parallels with particular lengths of the longest day (*Meroë* (8.16.9), *Ptolemais Theron* (8.16.10), *Sone* (8.15.15), *Rhodos* (8.17.21), and *Massalia* (8.5.7)). From these localities can be expected that their latitudes in the location catalogue are derived from M in Book 8. Rawlins (1985, 2008) mentions the possibility of a conversion $M \mapsto \Phi$ for major cities; in Rawlins (1985) examples are given. Exemplarily, the localities shall be considered which Rawlins relates to ε_h . The contrary conversion $\Phi \mapsto M$ was done by means of Eq. (4) with ε_m ; the result is shown in Table 2. As can be seen, each computed $M(\Phi)$ agrees satisfactorily with the value in Book 8. Hence, the coordinates of these localities also support the direction of conversion $\Phi \mapsto M$.

To provide evidence for the original direction of conversion it was examined how satisfactorily the coordinate differences $\Delta\Phi_i = \Phi_i - \Phi(M_i)$ and $\Delta M_i = M_i - M(\Phi_i)$ after both conversions $M \mapsto \Phi$ and $\Phi \mapsto M$ can be explained by the precision of the coordinates. Unfortunately, the precision of Φ and M is not constant and not known in each case; however, a maximal resolution can be given for the Φ -values and a main resolution for the M -values. Thereby limits $L_{\Delta\Phi}$ and $L_{\Delta M}$ for the differences $\Delta\Phi_i$ and ΔM_i can be given so that the numbers of differences was determined, which go below these limits.

For the Φ -values of the *poleis episemoi* a maximal resolution of $1/2^\circ$ can be assumed, definitely the actual resolution is often more precise (e.g. $1/12^\circ$ or $1/6^\circ$ (see Marx 2011)). According to Eq. (3) the limit $L_{\Delta\Phi}$ is then $0.5 \cdot 1/2^\circ = 1/4^\circ$. For the M -values a main resolution of $1/12$ h (or more imprecise; see Sect. 3.1) can be assumed so that $L_{\Delta M} = 0.5 \cdot 1/12$ h = $1/24$ h = 2.5 min was used as a limit.

For both conversions $M \mapsto \Phi$ and $\Phi \mapsto M$ the parameters ε_m and ε_h were used. The result is:

- ε_m : Ω : 69 % $|\Delta\Phi| \leq L_{\Delta\Phi}$, 88 % $|\Delta M| \leq L_{\Delta M}$; X: 66 % $|\Delta\Phi| \leq L_{\Delta\Phi}$, 82 % $|\Delta M| \leq L_{\Delta M}$;
- ε_h : Ω : 68 % $|\Delta\Phi| \leq L_{\Delta\Phi}$, 87 % $|\Delta M| \leq L_{\Delta M}$; X: 66 % $|\Delta\Phi| \leq L_{\Delta\Phi}$, 81 % $|\Delta M| \leq L_{\Delta M}$.

The test conditions for the conversions $M \mapsto \Phi$ and $\Phi \mapsto M$ were not the same, for $\Delta\Phi$ the limit was advantageously large and for ΔM small. Nevertheless, the amount of ΔM_i going below their limit is noticeably larger than in the case of $\Delta\Phi_i$. Thus,

the results argue for a main direction of conversion of $\Phi \mapsto M$. The resulting M_i of this conversion were round values, whose inaccuracies affect the $\Delta\Phi_i$ computed here after the back conversion $M \mapsto \Phi$.

3.4 The obliquity of the ecliptic

The question of which value ε underlies the M -data was investigated more in detail chapter by chapter, assuming that the majority of the localities of a chapter were converted with the same value of ε . An approach to determine the underlying ε is an adjustment of ε , which also leads to information about the accuracy of the result. Solving Eq. (4) for M gives the observation equation of the adjustment model

$$M_i + v_i = f(\varepsilon) = 2 \frac{1}{15^\circ} \arccos(-\tan \Phi_i \tan \varepsilon), \quad i = 1, \dots, n \quad (9)$$

with the observation M_i and its correction v_i . The solution of this non-linear adjustment problem can be obtained iteratively by means of a linearisation (see e.g. Jäger et al. 2005, p. 116). In Sect. 3.2 the χ^2 -test of goodness of fit showed that the differences ΔM_i with respect to a conversion with ε_m follow rather a Laplace distribution than a normal distribution. It shall be assumed here that the Laplace distribution can also be expected for errors of M_i whose conversion is based on $\varepsilon \neq \varepsilon_m$. The maximum likelihood estimation for the Laplace distribution is the L_1 -norm estimation, which is applied here (see e.g. Jäger et al. 2005, p. 117 ff.).

To estimate the accuracy of the adjusted parameter ε , the term $q = (\mathbf{A}^\top \mathbf{A})^{-1}$ is needed, whose one-column design matrix \mathbf{A} of the linearised adjustment problem contains the derivatives $df_i(\varepsilon)/d\varepsilon$ of f_i with respect to ε . Then the $(1 - \alpha)100\%$ confidence interval for ε can be estimated according to Dielman and Pfaffenberger (1982, 1990) by

$$\varepsilon \pm \Delta\varepsilon = \varepsilon \pm z_{\alpha/2} \lambda \sqrt{q} \quad (10)$$

with the quantile $z_{\alpha/2}$ of the standard normal distribution as well as with

$$\lambda = \frac{\hat{v}_{m+\mu} - \hat{v}_{m-\nu}}{4\nu/n}. \quad (11)$$

Value m is the index of the median of the computed corrections v_i in the ordered sequence \hat{v}_i . Values $\hat{v}_{m+\mu}$ and $\hat{v}_{m-\nu}$ are the corrections having a distance ν from \hat{v}_m in the ordered sequence.

The investigations in Sects. 3.1 and 3.3 show that there are inaccurate M_i with larger errors, possibly partly corrupt.⁴ The L_1 -norm estimation of ε and the estimation (10) of $\Delta\varepsilon$ have the advantage to be resistant against outlying observations.

⁴ Examples for corrupt or roughly given M -values are *Nisaia* (8.23.6) and *Arbis* (8.25.14) in Ω (Stückelberger and Mittenhuber 2009, pp. 205, 209)

Due to their small number of observations (≤ 3), the chapters Europe 4 and Asia 12 were excluded from the parameter estimation. λ was computed using all v_i of the remaining chapters. ν must be small but larger than the half of the number u of adjustments of ε ; it was set to $11 + u/2 = 18$ for X and to $18 + u/2 = 30$ for Ω ⁵. For the confidence interval of ε , the probability 95 % was chosen. In summary, the results are

- Ω : $n = 338$, $u = 24$, $\lambda = 0.7'$, smallest ε : $23^\circ 46' \pm 8'$ in Europe 10, largest ε : $24^\circ 07' \pm 30'$ in Asia 11;
- X: $n = 217$, $u = 16$, $\lambda = 1.0'$, smallest ε : $23^\circ 30' \pm 44'$ in Africa 4 (anomalously inaccurate) and $23^\circ 45' \pm 11'$ in Europe 10, largest ε : $23^\circ 56' \pm 11'$ in Europe 9.

With the exception of Asia 11 in Ω and Africa 4 in X, the adjusted values of ε deviate from ε_m in the order of some arc minutes ($\leq 10'$). Only in the case of Asia 3 in Ω , ε_m lies outside of the estimated 95 %-confidence interval ($\varepsilon = 24^\circ 02' \pm 9'$). Apart from this exception, the adjustment does not result in significant deviations from ε_m , and accordingly the results do not argue against ε_m underlying the M -values in general.

Moreover, ε_m need not be rejected as long as the differences $\Delta M_i = M_i - M(\Phi_i)$ based on ε_m can be explained by rounding. Exemplarily Asia 3 in Ω with the significant deviation from ε_m is considered. Each of its ten M_i can be explained by a kind of rounding as it can be found among the M_i of Book 8 (see Sect. 3.5). There are eight M_i explicable by a rounding to the nearest $1/12$ h. Further, in the case of *Thospia*, the precision could be $1/24$ h ($M(\Phi = 39^\circ 50') = 14^h 53.2^m \approx 14^{21/24} \text{ h} = 14^{7/8} \text{ h} = M$) and in the case of *Albana* $1/6$ h ($M(\Phi = 45^\circ 50') = 15^h 36.7^m \approx 15^{4/6} \text{ h} = 14^{2/3} \text{ h} = M$). Apart from that, M of *Artemita* is not explicable by a conversion with ε_r (which is to be expected in view of the adjustment) in conjunction with any kind of rounding ($M(\Phi = 40^\circ 20') = 14^h 57.7^m$, but $M = 14^h 55.0^m$).

Finally, all M_i were used for an L_1 -norm-adjustment of one parameter ε . For Ω is obtained $\varepsilon = 23^\circ 52' \pm 2'$ and for X $\varepsilon = 23^\circ 52' \pm 3'$. Among the different values for ε introduced here, only ε_m lies within both 95 % confidence intervals.

3.5 Conversion by means of linear interpolation

It is much more probable that a conversion of coordinates was originally done by means of a tabulation and not by means of a single, trigonometric computation from Eq. (4) for each locality. Cuntz (1923, p. 101 ff.) assumes an underlying tabulation (M_j , $\Phi(M_j)$) with a step size of $1/12$ h = 5 min. Using this tabulation, Cuntz cannot explain each M -value in the examined chapters (8.3–8). An example is *Gesoriacum* (8.5.6) with $\Phi = 53^\circ 30'$ and $M = 16^h 50^m$. The nearest Φ in Cuntz's tabulation is $53^\circ 39'$ which leads to $M = 16^h 55^m$. Thus, Cuntz supposes that $\Phi = 53^\circ 30'$ is not the original value underlying M .

⁵ In an L_1 -norm estimation of u unknown parameters, (at least) u corrections become zero. The choice $\nu' = 11 = \nu - u/2$ for X is reconcilable with the values used in Dielman and Pfaffenberger (1990). According to the ratio $(n_\Omega - u_\Omega)/(n_X - u_X) = 1.6$, ν was set to $\nu = 1.6\nu' + u/2 = 18 + 12 = 30$ for Ω .

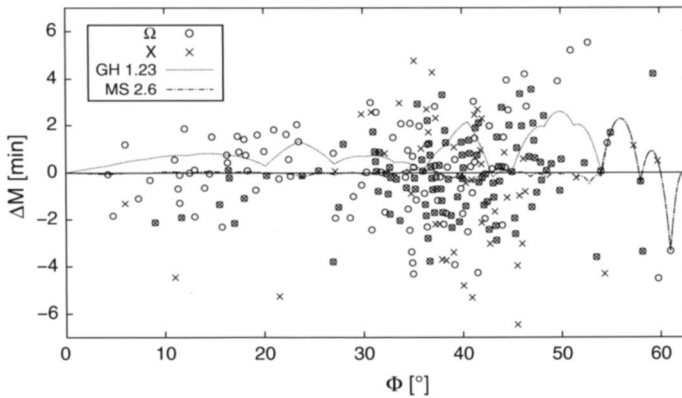


Fig. 9 Differences $\Delta M = M - M(\Phi)$ between the length of the longest day $M(\Phi)$, trigonometrically computed from the latitude Φ (Eq. (4)), and the M -data of Book 8 (Ω -recension and X-manuscript) as well as M resulting from an linear interpolation of the M -data of GH 1.23 and MS 2.6

Another approach to explain the M -data in Book 8 is taking recourse to known Φ - and M -data given by Ptolemy. There are two datasets with latitudes and corresponding M -data in the Geography and the MS, these are the compilations of parallels in GH 1.23 and MS 2.6. It is conceivable that such data were used in conjunction with an interpolation, a method applied by Ptolemy (Pedersen 2011, p. 84). Considering the mentioned example of *Gesoriacum*, $M(\Phi)$ computed from Eq. (4) is $16^{\text{h}}54^{\text{m}}$. The same result is obtained by means of the compilation of the parallels in GH 1.23 and a linear interpolation. The slope a of the 17th and 18th parallels in GH 1.23 is

$$\begin{aligned} a &= (M_{j+1} - M_j) / (\Phi_{j+1} - \Phi_j) \\ &= (17^{\text{h}}00^{\text{m}} - 16^{\text{h}}30^{\text{m}}) / (54^{\circ}00' - 51^{\circ}30') = 0.2 \text{ h}^{\circ}. \end{aligned} \quad (12)$$

The approximated M of *Gesoriacum* is then

$$M = 16^{\text{h}}30^{\text{m}} + a \cdot (\Phi - 51^{\circ}30') = 16^{\text{h}}54^{\text{m}}. \quad (13)$$

This value rounded to the nearest $1/6$ is $16^{\text{h}}50^{\text{m}}$, the value in the location catalogue. As the plot in Fig. 7 shows, a precision of $1/6$ h can be assumed for part of the M -data (see Sect. 3.1).

The accuracy of a linear interpolation of the data in GH 1.23 and MS 2.6 was determined by computing the difference $\Delta \tilde{M}(\Phi) = \tilde{M}(\Phi) - M(\Phi)$ between the interpolated value $\tilde{M}(\Phi)$ and the trigonometric computed value $M(\Phi)$ (Eq. (4)). Figure 9 shows $\Delta \tilde{M}(\Phi)$ and additionally the differences $\Delta M_i = M_i - M(\Phi_i)$ between the M_i of Book 8 and the $M(\Phi_i)$ computed from the Φ_i of the location catalogue (a few data points with $|\Delta M| > 7$ min are not plotted). For $\Phi > 54^{\circ}$, the curves of the

Table 3 Estimated percentages of different resolutions a of the lengths of the longest day M of Book 8 (Ω -recension and X-manuscript) in the case that they originate from a trigonometric conversion (Eq. (4)) or from a linear interpolation of the M -data in MS 2.6 and GH 1.23; last row: M does not fit with Φ

	Ω			X		
	Trig.	Interp. MS	Interp. GH	Trig.	Interp. MS	Interp. GH
$a < 1/12\text{h}$	12	12	11	3	3	2
$a = 1/12\text{h}$	72	72	68	71	69	65
$a > 1/12\text{h}$	12	12	18	21	22	24
$M \not\approx M(\Phi)$	4	3	4	5	6	9

interpolated GH- and MS-data are equal⁶. For $\Phi < 54^\circ$, the interpolation of the MS-data is very accurate ($|\Delta\tilde{M}| < 0.5\text{ min}$); for $\Phi \geq 54^\circ$, the majority of the differences ΔM_i coincide with the curve $\Delta\tilde{M}(\Phi)$. This indicates that the M_i of Book 8 are based on an interpolation of the data in MS 2.6. The interpolation of the GH-data produces errors up to 3 min. Its curve $\Delta\tilde{M}(\Phi)$ does not coincide with the distribution of the differences ΔM_i in general. However, some data points lie next to the curve, and three data points coincide with the curve so that an interpolation of the data in GH 1.23 can be assumed at least in these cases (Ω : *Harmuza* (8.22.21), *Aspithra* (8.27.11); Ω and X: *Ambrodax* (8.21.17)).

Finally, the trigonometric computation ($\varepsilon = \varepsilon_m$) and the linear interpolation were compared with regard to the amount of M_i explicable by their precision. For this purpose, Procedure ROUND was applied (see Sect. 2). As it was shown in Sect. 3.1, different resolutions come into consideration. The possible resolutions of M were chosen according to the denominators of the fractions occurring in Book 8. In addition to the standard resolution $a_s = 1/12\text{h}$ there are in Ω the more precise a (in h) $1/15$, $1/20$, $1/24$, $1/30$, $1/60$, and $1/150$ as well as the less precise $1/10$, $1/8$, $1/6$, and $1/4$. The more precise a in X are (in h) $1/15$ and $1/30$ as well as the less precise $1/8$, $1/6$, and $1/4$. The result is given in Table 3. The more precise the resulting resolutions are, the more probable the conversion can be considered. The trigonometric conversion and the interpolation of the MS-data have nearly the same result; the interpolation of the data in GH 1.23 has a worse result than the other two methods. Thus, a conversion by means of the data in MS 2.6 is very probable. The amount of M_i being explainable with a resolution of $1/12\text{h}$ is 72 % for Ω and 69 % for X.

The results in Table 3 show that in the case of Ω 84 % and in the case of X 72 % of the localities have only small absolute differences $< 0.5 \cdot 1/12\text{h} = 1/24\text{h} = 2.5\text{ min}$ (cf. Eq. (3)) between M_i and $M(\Phi_i)$ converted by means of the data of MS 2.6. Almost all M_i (ca. 95 %) can be explained by a conversion (data of MS 2.6 or Eq. (4)) and rounding. Hence, the majority of the differences between Book 8 and the location catalogue are explicable by the precision of the M -data. Accordingly, both datasets show a high level of interdependence.

⁶ There are more parallels in MS 2.6 than in GH 1.23 from 45° latitude, but from 54° to 63° , there are some interpolated values in MS 2.6 so that a linear interpolation of the data in MS 2.6 and in GH 1.23 leads to the same result for latitudes $> 54^\circ$.

There is a lower percentage of M_i (3 % in Ω , 6 % in X) differences ΔM_i of which are inexplicable by one of the types of rounding used. These as well as lower precise explicable M -values occur in nearly all chapters of Book 8. Possible reasons for M_i and $M(\Phi_i)$ not fitting are

- (1) M is rounded incorrectly⁷ or is slightly erroneous due to a propagation of rounding errors in intermediate results; for example possibly *Asturica August* (8.4.5) with $M = 15^h25^m$ (in Ω and X) and $M(\Phi) = 15^h22.2^m \approx 15^h20^m$;
- (2) M is wrong due to a corruption; for example possibly *Palmyra* (8.20.10) in Ω with $M = 14^h40^m$ and $M(\Phi) = 14^h19^m \approx 14^h20^m$;
- (3) Φ is wrong due to a corruption; for example *Osika* (8.19.9) in X with $\Phi = 45^\circ30'$, $M = 15^h27.5^m$, $M(\Phi) = 15^h34^m$ (in contrast Ω : $\Phi = 44^\circ45'$ and Canon: $\Phi = 44^\circ30'$).

It was tested for the remaining inexplicable M_i , whether a conversion with ε_h or ε_r brings an explanation of their fractions. For this purpose Procedure ROUND was repeated. As a result, a few M_i can be explained by a rounding, but there still remain inexplicable values. However, it is not necessary to assume another value than ε_m ; each of the problematic M_i can be explained by Error 1: *Asturica August*: see above; *Luppia* (8.6.3): $M(\Phi) = 16^h44^m$ became 16^h50^m instead of 16^h40^m (resolution $\frac{1}{6}h$); *Ravenna* (8.8.5): as *Asturica August*; *Capua* (8.8.6): $M(\Phi) = 15^h02^m$ became 15^h05^m instead of 15^h00^m ; *Mallos* (8.17.44): $M(\Phi) = 14^h32.8^m$ became 14^h34^m instead of 14^h32^m (resolution $\frac{1}{30}h$); *Medaba* (8.20.20): $M(\Phi) = 14^h02^m$ became 14^h05^m instead of 14^h00^m .

4 Distance from the summer solstice

At places located between the tropics the sun is at the zenith two (for $|\Phi| < \varepsilon$) or one (for $|\Phi| = \varepsilon$) times within the year. For this date, the distance S of the sun from the summer solstice on the ecliptic is given in Book 8. The quantity $l = 90^\circ - S$ is the ecliptic longitude, reckoned from the vernal equinox. There are about 60 localities in Book 8 with given S . In MS 1.14, Ptolemy explains the computation of the distance of the sun from the equator (declination) by means of l ⁸. When the sun is at the zenith, its declination equals the latitude Φ of the locality. Thus, the calculational relations can be used to determine l or S from Φ . The relation between S and the latitude Φ is

$$\sin \Phi = \sin \varepsilon \cos S. \quad (14)$$

In MS 1.15, Ptolemy gives a list of tabulated $\Phi(l_j)$ for $l_j = 1^\circ, \dots, 90^\circ$ in steps of 1° ("Table of Inclination"). A further source of S -data is MS 2.6, where Ptolemy arranges data about specific parallels (see Sect. 3.5); however, it includes only 5 data pairs (Φ_j, S_j) . For the parallels with $\Phi < \varepsilon$ S is given there. In the following both

⁷ A known inconsistency of this kind, but in latitude, occurs in GH 1.23 in the latitude $\Phi = 43^\circ05'$ of the 13th parallel. Since M of this parallel is 15^h15^m , the correct value for Φ should be $43^\circ00' \approx \Phi(M) = 43^\circ01'$ which was given by Ptolemy in MS 2.6.

⁸ In MS 1.14, the ecliptic longitude l is reckoned from the autumnal equinox.

compilations of S -data in the MS are taken into consideration besides Eq. (14) as a possible basis of the S -data in Book 8 of the Geography.

4.1 Conversion of coordinates and accuracy

Firstly, the precision of the S_i is considered to get an idea, which size of coordinate differences are to be expected after a conversion between S and Φ . The largest denominator of the fractions of the S_i is 6 in Ω and 8 in X ; the main resolutions seem to be $1/3^\circ$ and $1/4^\circ$. Solving Eq. (14) for Φ , the variation Δ_Φ of Φ due to an error Δ_S in S can be estimated by Eq. (7). The computation of propagation of errors was done exemplarily for the resolutions $1/6^\circ$ and $1/2^\circ$. The variations were set to the expected maximal rounding errors (Eq. (3)): $\Delta_S = 0.5 \cdot 1/6^\circ = 1/12^\circ$ and $\Delta_S = 0.5 \cdot 1/2^\circ = 1/4^\circ$. Figure 8 shows the result. The effect of a variation in S on Φ is very low (in comparison to M and Φ). For an inaccurate resolution of $1/2^\circ$ the expected maximal error in Φ increases only to $6'$; it goes to zero for $S \rightarrow 0$ (i.e. $\Phi \rightarrow \varepsilon$).

The conversion between S and Φ was carried out for both directions $S \mapsto \Phi$ and $\Phi \mapsto S$ by means of Eq. (14) with the four different ε introduced in Sect. 3, and the differences $\Delta\Phi_i = \Phi_i - \Phi(S_i)$ and $\Delta S_i = S_i - S(\Phi_i)$ to the given values Φ_i and S_i were computed. Since X ends in GH 5.13, only 16 S -values can be considered for the comparison of Book 8 and the location catalogue (these are *Syene* (8.15.15) and Chapter 16 (Asia 4)). From the differences between the converted and the original coordinates, the MAD and the MD were computed by Eqs. (5) and (6); the results are given in Table 1.⁹

In the case of X , the $MAD = 2'$ for $\Phi \mapsto S(\varepsilon_m)$ is very small and goes below the expected result with regard to the assumed main precisions of $1/3^\circ$ and $1/4^\circ$. In contrast, the $MD = 31'$ is considerably larger. It is increased by four large differences.¹⁰ The MAD for $\varepsilon \neq \varepsilon_m$ is significantly larger than for ε_m ; that argues for ε_m underlying the S -data of X .

Stückelberger and Graßhoff (2006, p. 44) mention that the S -data of X are considerably more precise than the S -data of Ω . Using a conversion based on Eq. (14), this can be confirmed. In the case of Ω , the MAD for $\Phi \mapsto S(\varepsilon_m)$ is $72'$. There are many differences $\Delta S_i > 1^\circ$ which cannot be explained by the resolution of the S_i . A wrong value for ε underlying the reconversion can be excluded; the other (eligible) values do not lead to considerably smaller coordinate differences. Thus, either corrupt data must be assumed for Ω or that the conversion done here is wrong. Regarding the known sources of S -data, the tabulation in MS 1.15 has a step size of 1° for l , and Ptolemy's Handy Tables include a similar tabulation with a step size of 3° (Neugebauer 1975, p. 979). Thus, the usage of these tabulations (without interpolation) would result in S -values with fractions equal to zero. That does not hold for the data in Book 8. In

⁹ In the case of $\Phi \mapsto S(\varepsilon_c)$, the number n of the converted latitudes is smaller because values $\Phi > \varepsilon_c$ cannot be converted by Eq. (14). *Arbis* (8.25.14) is excluded from the computations because of its exceptionally wrong $S = 0^\circ$ in Ω .

¹⁰ These localities are *Syene* (8.15.15) and *Autolalai* (8.16.3) with the crude value $S = 0^\circ$ for $\Phi = 23^\circ 50'$ as well as *Dere* (8.16.12) and *Mosylon* (8.16.13).

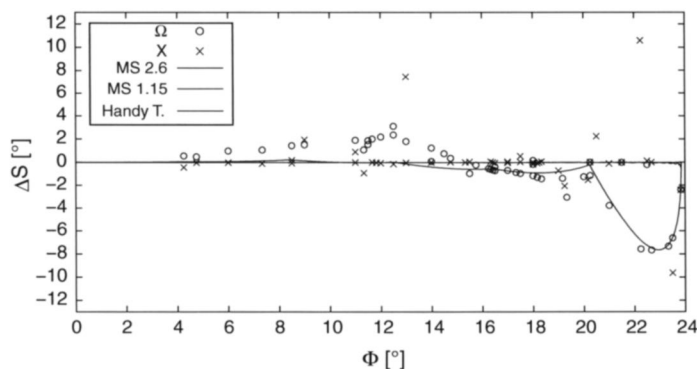


Fig. 10 Differences $\Delta S = S - S(\Phi)$ between the distances $S(\Phi)$ from the summer solstice, trigonometrically computed from the latitude Φ (Eq. (14)), and the S -data of Book 8 (Ω -recension and X-manuscript) as well as S resulting from an linear interpolation of the S -data of MS 1.15, MS 2.6, and Handy Tables ($\Delta S \approx 0$ for MS 1.15 and Handy Tables)

Fig. 10, the differences $\Delta S_i = S_i - S(\Phi_i)$ are plotted against the latitude.¹¹ The plot indicates a dependence between ΔS and Φ of Ω ; regionally ΔS increases or decreases with increasing Φ . This suggests that the S -data in Ω are not determined by means of the conversion (14).

Table 1 also gives the MAD and the MD for the differences $\Delta \Phi_i$ and ΔS_i computed from the Φ_i and converted $S(\Phi_i)$ of the Canon. In the case of Ω the results do not change significantly in comparison to the location catalogue of the Geography. In the case of X, the Canon enables a comparison for all S -data (in contrast to the incomplete location catalogue of X). Using the Canon, the small MAD = 3.5' for $\Phi \mapsto S(\epsilon_m)$ confirms the high accuracy of the S -data which has already been observed for the small sample size of the location catalogue. The large MD of 50' shows, as in case of the location catalogue, that there are also some latitudes which do not agree with the respective S .

Due to the incomplete location catalogue of X the missing latitudes were experimentally substituted by the values of Ω as well as of the Canon to compute the $S(\Phi_i)$. In the case of Ω , there are eight localities more than in the case of the Canon with large differences $|\Delta S_i| > 1^\circ$.¹² Figure 10 shows the ΔS_i based on the latitudes of the Canon. In the case of X there is no comparable dependence between ΔS and Φ as detected for Ω . Most of the differences are near zero. Consequently, the coordinate differences of X reveal a close connection between its S -data and the latitudes of the Canon.

Furthermore, Fig. 10 shows the curves of the differences $\Delta \tilde{S}(\Phi) = \tilde{S}(\Phi) - S(\Phi)$ between the trigonometrically computed $S(\Phi)$ (Eq. (14)) and the value $\tilde{S}(\Phi)$ determined by linear interpolations: (1) of the tabulation in MS 1.15, (2) of the tabulation in

¹¹ *Arbis* (8.25.14) with very large $|\Delta S|$ is excluded in the case of Ω .

¹² Localities with $|\Delta S| > 1^\circ$ occurring in Ω but not in the Canon are: *Kane* (8.22.9), *Gerra* (8.22.10), *Sapphara* (8.22.16), *Chaberis* (8.26.5), *Palura* (8.26.6), *Patala* (8.26.10), *Barbarei* (8.26.11), *Tosalei* (8.27.5), *Triglyphon* (8.27.7) and *Aspithra* (8.27.11).

the Handy Tables,¹³ and (3) of the data of the parallels in MS 2.6. The interpolations of the data in MS 1.15 and the Handy Tables lead to accurate results ($|\Delta\tilde{S}| < 2.3'$ and $|\Delta\tilde{S}| < 0.5^\circ$, respectively). Their error curves run close to zero so that they coincide with the majority of the differences ΔS_i occurring in X. Consequently, a conversion by means of one of both tabulations can be assumed for the predominant amount of accurate S_i in X. Because of the small number of data pairs (Φ_j, S_j) in MS 2.6, the interpolation of these data is significantly erroneous, especially for $\Phi > 20^\circ$. However, the curve $\Delta\tilde{S}$ fits the differences ΔS_i of some localities in Ω for $\Phi > 16^\circ$ well. This reveals the possibility of another type of interpolation underlying the data of Ω , see Sect. 4.2.

For the S -data of X and the auxiliary latitudes of the Canon, Procedure ROUND (see Sect. 2) was applied to explain as many S_i as possible by rounding. Because of the small MAD, the standard resolution was set to $a_s = 1/8^\circ = 7.5'$. The less precise resolutions are $a = 1/6^\circ$, $1/4^\circ$, $1/3^\circ$, $1/2^\circ$, and 1° . As a result 42 of 59 S_i can be explained by a rounding: $a = a_s$: 15, $a = 1/6^\circ$: 20, $a = 1/4^\circ$: 3, $a = 1/2^\circ$: 3, and $a = 1^\circ$: 1. Among the remaining 17 S_i there are 5 with the obviously approximative value $S = 0^\circ$.¹⁴ Furthermore, two S_i can be explained by a linear interpolation of the data in MS 2.6, the concerning localities are *Argyre* (8.27.10) and *Kattigara* (8.27.13). Their Φ is $8^\circ 30'$, the interpolated S is $68^\circ 49'$, that is $\approx 68\frac{3}{4}^\circ$, the S in Book 8. The remaining 10 problematic S_i probably have corruptions; for example, *Dere* (8.16.12) has $\Phi = 11^\circ 00'$ and $S = 62\frac{3}{4}^\circ$, but the converted value is $S(\Phi) = 61^\circ 51' \approx 61\frac{3}{4}^\circ$ so that an error of 1° in S is possible.

4.2 Linearly converted data in Ω

A few localities in Book 8 of Ω have accurate S -values with $|\Delta S| \leq 15'$, they are *Geira* (8.16.6), *Garama* (8.16.7), *Napata* (8.16.8), *Muza* (8.22.6), *Karminna* (8.22.23), and *Barbarei* (8.26.11). The localities *Syene* (8.15.15), *Berenike* (8.15.19) and *Autolalai* (8.16.3) have $\Phi = 23^\circ 50'$ and the approximative $S = 0^\circ$. *Arbis* (8.25.14) has the erroneous $S = 0^\circ$. The S -values of most of the other localities can be explained by a conversion of Φ by means of a piecewise linear function. The data reveal three regions for which a linear conversion was done separately. These regions cannot be made out well in the plot of the differences ΔS_i in Fig. 10 but by means of a plot of S against Φ , see Fig. 11. Obviously, the limits between the three regions are $\Phi_{12} \approx 12^\circ 30'$ and $\Phi_{23} \approx 20^\circ 15'$; they correspond to a division of the quadrant of the ecliptic into three equal parts with the range of $\approx 30^\circ$. Hence, the piecewise linear function of conversion is

$$S'(\Phi) = \begin{cases} S'_1(\Phi) = a_1\Phi + b_1 & \text{for } 0^\circ \leq \Phi < \Phi_{12} \\ S'_2(\Phi) = a_2\Phi + b_2 & \text{for } \Phi_{12} \leq \Phi < \Phi_{23} \\ S'_3(\Phi) = a_3\Phi + b_3 & \text{for } \Phi_{23} \leq \Phi < \varepsilon. \end{cases} \quad (15)$$

¹³ The tabulation of the Handy Tables is represented here by the data of the table in MS 1.15. To obtain a step size of 3° for l each third row was used.

¹⁴ These localities are *Syene* (8.15.15), *Autolalai* (8.16.3), *Gerra* (8.22.10), *Harmuza* (8.22.21), and *Tosalei* (8.27.5).

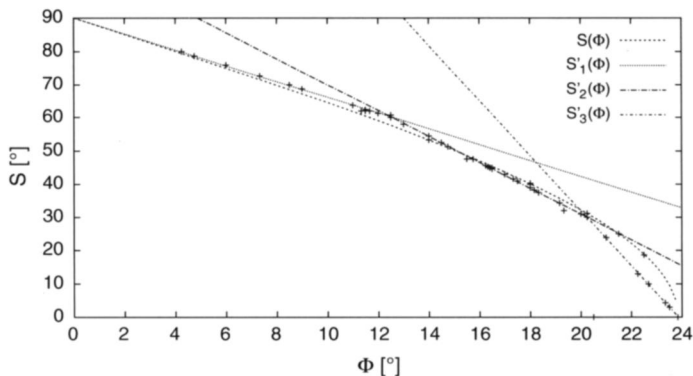


Fig. 11 Ω -recension: plot of the distances S_i from the summer solstice of Book 8 against the latitudes Φ_i of the location catalogue, trigonometric conversion function $S(\Phi)$ (derived from Eq. (14)), linear conversion functions S'_1 , S'_2 , and S'_3 (Eq. (15))

To define a linear functions two points are necessary. Two points (Φ, S) are known: $(0^\circ, 90^\circ)$ and $(\varepsilon, 0^\circ)$. Moreover, very probably *Badeo* (8.22.4) with $\Phi_B = 20^\circ 15'$ and $S_B = 30^\circ$ was used as end point of $S'_1(\Phi)$ and starting point of $S'_2(\Phi)$ as well as *Pityndra* (8.26.19) with $\Phi_P = 12^\circ 30'$ and $S_P = 60^\circ$ was used as end point of $S'_2(\Phi)$ and starting point of $S'_3(\Phi)$. S_B and S_P are inaccurate, but conspicuously round values, possibly intensionally approximative to simplify the computations.¹⁵ It is obtained with $(0^\circ, 90^\circ)$ and *Pityndra*:

$$a_1 = (S_P - 90^\circ)/(\Phi_P - 0^\circ) \approx -2.4 \quad (16)$$

$$b_1 = 90^\circ - a_1 \cdot 0^\circ = 90^\circ. \quad (17)$$

It is obtained with *Badeo* and *Pityndra*:

$$a_2 = (S_B - S_P)/(\Phi_B - \Phi_P) \approx -3.9 \quad (18)$$

$$b_2 = S_B - a_2 \cdot \Phi_B = 108^\circ 58' \approx 109^\circ. \quad (19)$$

It is obtained with *Badeo* and $(\varepsilon, 0^\circ)$:

$$a_3 = (0^\circ - S_B)/(\varepsilon - \Phi_B) \approx -8.37 \sim -8.3 \quad (20)$$

$$b_3 = 0^\circ - a_3 \cdot \varepsilon = 197^\circ 49' \approx 198^\circ, \quad (21)$$

where $\varepsilon = 23^\circ 50'$ is used, the latitude of localities with $S = 0^\circ$ in Book 8 (see above).

$S'(\Phi)$ is plotted in Fig. 11 together with the given data points (Φ_i, S_i) and the trigonometric conversion function $S(\Phi)$ based on Eq. (14). S' fits the inaccurate data points very well. Especially S'_3 is a bad approximation for the trigonometric

¹⁵ An accurate S for Φ_B exists in Ω . *Napata* (8.16.8) has $\Phi = 20^\circ 15'$ and $S = 31^\circ 10'$, the value computed from Eq. (14) is $S(\Phi) = 31^\circ 09'$.

Table 4 Ω -recension: linear conversion of the latitude Φ of the location catalogue into the distance $S'(\Phi)$ from the summer solstice by means of the piecewise functions $S'_1(\Phi)$, $S'_2(\Phi)$, and $S'_3(\Phi)$ (Eq. (15)) and remaining difference $\Delta S' = S - S'(\Phi)$ with respect to S of Book 8

Locality	Φ ($^{\circ}$, $'$)	S ($^{\circ}$, $'$)	S' ($^{\circ}$, $'$)	$\Delta S'$ ($'$)
$S'_1(\Phi)$				
<i>Adulis</i> (16.11)	11,20	62,00	62,48	−48
<i>Dere</i> (16.12)	11,00	63,45	63,36	9
<i>Mosylon</i> (16.13)	9,00	68,45	68,24	21
<i>Aromata</i> (16.14)	6,00	76,00	75,36	24
<i>Okelis</i> (22.7)	12,00	61,15	61,12	3
<i>Arabia</i> (22.8)	11,30	62,20	62,24	−4
<i>Palura</i> (26.6)	11,30	62,00	62,24	−24
<i>Takola</i> (27.3)	4,15	80,00	79,48	12
<i>Zabai</i> (27.4)	4,45	78,40	78,36	4
<i>Argyre</i> (27.10)	8,30	70,00	69,36	24
<i>Kattigara</i> (27.13)	8,30	70,00	69,36	24
<i>Nagadiba</i> (28.3)	8,30	70,00	69,36	24
<i>Talakori</i> (28.4)	11,40	62,00	62,00	0
<i>Maagrammon</i> (28.5)	7,20	72,40	72,24	16
$S'_2(\Phi)$				
<i>Iarzeitha</i> (16.4)	15,30	47,40	48,33	−53
<i>Thamondokana</i> (16.5)	17,00	43,00	42,42	18
<i>Mero</i> (16.9)	16,25	45,00	44,58	2
<i>Ptolemaïs Theron</i> (16.10)	16,25	45,00	44,58	2
<i>Town of Pudnos</i> (22.5)	16,30	44,40	44,39	1
<i>Kane</i> (22.9)	12,30	60,45	60,15	30
<i>Mara</i> (22.11)	18,20	37,30	37,30	0
<i>Omanon</i> (22.12)	19,20	32,00	33,36	−96
<i>Menambis</i> (22.13)	16,30	45,00	44,39	21
<i>Sabbatha</i> (22.14)	16,30	45,00	44,39	21
<i>Sabe</i> (22.15)	13,00	58,00	58,18	−18
<i>Sapphara</i> (22.16)	14,30	52,30	52,27	3
<i>Isle of Sarapias</i> (22.18)	17,30	41,00	40,45	15
<i>Simylla</i> (26.3)	14,45	51,20	51,29	−8
<i>Muziris</i> (26.4)	14,00	54,30	54,24	6
<i>Chaberis</i> (26.5)	15,45	47,36	47,35	1
<i>Barygaza</i> (26.12)	17,20	41,40	41,24	16
<i>Ozene</i> (26.13)	20,00	31,00	31,00	0
<i>Baithana</i> (26.14)	18,10	38,15	38,09	6
<i>Hippokura</i> (26.15)	19,10	34,20	34,15	5
<i>Karura</i> (26.16)	16,20	45,20	45,18	2
<i>Modura</i> (26.17)	16,20	45,20	45,18	2
<i>Orthura</i> (26.18)	16,20	45,20	45,18	2
<i>Triglyphon</i> (27.7)	18,00	39,00	38,48	12
<i>Marëura</i> (27.8)	12,30	60,00	60,15	−15
<i>Aspithra</i> (27.11)	16,15	45,40	45,38	2
<i>Sinai</i> (27.12)	13,00	58,00	58,18	−18
$S'_3(\Phi)$				
<i>Gerra</i> (22.10)	23,20	4,20	4,20	0
<i>Harmuza</i> (22.21)	23,30	3,00	2,57	3
<i>Samydake</i> (22.22)	22,40	10,00	9,52	8
<i>Patala</i> (26.10)	21,00	23,50	23,42	8
<i>Tosalei</i> (27.5)	23,20	4,20	4,20	0
<i>Tugma</i> (27.6)	22,15	13,00	13,19	−19

function $S(\Phi)$ based on Eq. (14). In Table 4 S_i , the computed $S'(\Phi_i)$ as well as the remaining difference $\Delta S'_i = S_i - S'(\Phi_i)$ are given for the 47 localities which have not been discussed so far. The differences $\Delta S'_i$ are explicable in almost every case by a reasonable rounding to the nearest 1° , $1/2^\circ$, $1/3^\circ$, or $1/4^\circ$. There remain 10 localities whose $S'(\Phi)$ cannot be rounded to the given S in this way. The reason could be peculiar roundings or scribal errors, also in an originally trigonometric converted S . For example, a linear conversion and a confusion of $1/3 = \gamma$ and $2/3 = \gamma o$ can be assumed for *Barygaza* ($S'(\Phi) = 41^\circ 24' \approx 41\ 1/3^\circ \neq 41\ 2/3^\circ = S$) as well as for *Maagrammon*. A trigonometric conversion and a scribal error can be assumed for *Iazeitha*, which has $S = 47^\circ 40'$ in Ω and the accurate value $48^\circ 40'$ in X .

Appendix: Changes to and errors in the data source

The following changes of coordinates values underlie the present investigations. Changes of known inconsistent values are: Φ of *Autolalai* (4.6.24) in X was changed from $27^\circ 20'$ to $23^\circ 50'$ (as Ω and Canon). *Argyre* (8.27.10) and *Kattigara* (8.27.13) were positioned on the northern hemisphere instead of the southern one. (The positive latitudes equal the more plausible values in the Canon. On the negative latitudes of Chinese places see also Rawlins 2008, p. 47.) Further changes are: *Palmyra* (8.20.10): $M = 14^h 40^m$ (also Nobbe 1843–1845) to $M = 14^h 20^m$ (Rawlins 2009) and *Apameia* (8.20.9): $A = 1/8\ h$ to $A = 5/8\ h$ (Nobbe 1843–1845; Rawlins 2009).

As a support for further investigations of the coordinates of Book 8, the errors shall be given here, which were found in the data base of Stückelberger and Graßhoff (2006). Data which are correct in the printout but erroneous in the database are given in Table 5.

Table 5 Errors in the data base of Stückelberger and Graßhoff (2006)

ID	Name	Ω/X	Error	Correction
8.3.7.1	<i>Eboracum</i>	X	Link to 2.3.16	2.3.17
8.5.7.3	<i>Narbo</i>	X	$M = 16^h 15^m$	$15^h 15^m$
8.5.7.4	<i>Vienna</i>	Ω	$A = 2^h 30^m$	$2^h 15^m$
8.5.7.4	<i>Vienna</i>	X	$A = 2^h 15^m$	$2^h 30^m$
8.5.7.4	<i>Vienna</i>	X	$M = 17^h 15^m$	$15^h 30^m$
8.8.6.1	<i>Aquileia</i>	Ω, X	$M = 15^h 05^m$	$15^h 30^m$
8.13.8.1	<i>Iol Caesarea</i>	X	$M = 14^h 20^m$	$14^h 15^m$
8.17.32.1	<i>Perge/Aksu</i>	X	$A = 0^h 30^m$	$0^h 05^m$
8.17.46.1	<i>Adana</i>	Ω	Link to 5.8.8	5.8.7
8.23.12.1	<i>Maruka</i>	Ω	$A = 3\ 5/6\ h$	$3\ 5/6\ h$
8.25.7.1	<i>Ortospana</i>	X	$M = 15^h 20^m$	$14^h 20^m$
8.27.7.1	<i>Trilingon</i>	Ω	$A = 7^h 16^m$	$6^h 16^m$

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