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On dating Hero of Alexandria

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Abstract The dating of Hero of Alexandria has been linked with the lunar eclipse of March 13, AD 62, since Otto Neugebauer discovered that this eclipse is the only one that can fit the one described in Hero's *Dioptra* 35. Although only a number of scholars claim that Hero himself observed the eclipse, almost all of them take Neugebauer's identification for granted. We use statistical and linguistic methods to criticize this assumption: all indices we have found point to the fact that the eclipse was merely invented as an example and, for that reason, that it cannot be used to determine Hero's life span.

1 Introduction

The dating of Hero of Alexandria has received the attention of modern scholarship since Neugebauer (1938) claimed that, on the one hand, the lunar eclipse (henceforth «eclipse») of March 13, AD 62, is that described in *Dioptra* 35 and, on the other hand, that the eclipse could have been observed by the Alexandrian scientist.[1] This has been generally accepted (Drachman 1950, 1972; Raïos 2000; Keyser 1988). However, recent works have cast some doubts on this conclusion: Souffrin (2000) resorted to probabilistic arguments, Sidoli (2005) proposed a new interpretation of the text of

[1] The direct observation of the eclipse by Hero has only been asserted explicitly by Drachmann (1972, n. 4).

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Dioptra 35 (Schöne 1903), pointing out that other eclipses fit the data of the eclipse there described (for a summary of these positions see Acerbi 2007). Recently, Sidoli corrected his last conclusion, recognizing that the only eclipse partially consistent with the data of *Dioptra* 35, in the five centuries between second century BC and third century AD, is the one identified by Neugebauer (Sidoli 2011). Contrary to Drachman's claim, Sidoli only regards this date as a *terminus post quem* for the dating of Hero.

The present discussion on the dating of Hero is based on factual and interpretation-laden data. To the first category belong the astronomical data concerning ancient eclipses, available in tables set out in a variety of forms (such as the NASA website);[2] to the second category belong the text of *Dioptra* 35 and the probabilistic treatment of the eclipses occurring in the period of time in which Hero is supposed to have lived. The relationships between these factors have formed the crux of a debate that, in our view, has been developed without a clear distinction between facts and assumptions. Thus, the hypotheses of the arguments set forth by the scholars involved in the debate have sometimes been defined without precision; in some cases, they are even marred by circular reasoning.

With regard to the first two factors, although a number of ambiguities and obscurities in the text of *Dioptra* 35 persist, it seems clear that the only eclipse *that actually happened* and that fits *relatively* well, both the data for Alexandria provided in the text of *Dioptra* 35 and the presumed life span of Heron, is that of March 13, AD 62. That said, no-one has discussed the margin of error implicit in the adverb «relatively». This, as we shall see, is crucial to a consistent probabilistic approach to the issue.

As for the probabilistic approach, Souffrin (2000) presented results based on the assumptions that the distribution of eclipses is uniform throughout the year, over long periods of time, and that the minimal period of time that certainly includes that of Hero's life is 500 years. However, the weakness of the probabilistic framework and errors in calculations make Souffrin's conclusion incorrect.[3]

2 Neugebauer (1938, p. 23, n. 39) used the Oppolzer and Ginzel Tables, whereas Souffrin (2000, p. 14) and Sidoli (2005, p. 251) consulted tables of ancient eclipses and performed additional calculations of their own.

3 In fact, Souffrin does not argue his probabilistic results: «un calcul simple indique alors que la *probabilité a priori* qu'une éclipse «convenable» ait eu lieu au moins une fois précisément un dixième jour avant le printemps dans un intervalle de 500 ans est de 0.75» (Souffrin 2000, p. 15). Souffrin does not show us how to do this «calcul simple», but perhaps it is $\frac{365}{500} = 0.73$. This result (and the calculation likely producing it) is not correct; the probability that *at least* one eclipse occurs on a particular day, if we consider a uniform distribution of eclipses, is $1 - \frac{CR_{364,500}}{CR_{365,500}}$, approximately equal to 0.58 ($CR_{a,b}$ means *combinations with repetition* of a elements grouped in sets of b elements. b can be greater than a because the operation allows repetition of elements). This calculation is not difficult, but cannot be described as «simple». Souffrin also performed a calculation taking into account the hour and not only the day of the eclipse. This too appears to be wrong: The correct calculation is $1 - \frac{CR_{4,379,500}}{CR_{4,380,500}} = 0.10248$ (where $4{,}380 = 365 \times 12$, and $4{,}379 = 365 \times 12 - 1$). We do not know the exact operations that Souffrin used as he only states that the result «est un peu supérieur[e] à 0.1»; since his previous result of 0.75 is incorrect, it is reasonable to think that this is incorrect too, and that it is close to the result of the exact calculation only by chance. In fact, Souffrin's second result could be based on a simple operation, *close* to his former one: $\frac{500}{365 \times 12}$, that is approximately equal to 0.11. The resemblance between this result and the exact result is due to the fact that the formulae are asymptotic.

In this paper, we shall review these factors and propose an analysis based mainly on philological arguments but also on statistical ones. The goal is to set up a framework for the interpretation of the text and a well-founded estimate of the probability that a random date and hour fits a certain eclipse that actually happened in Alexandria on that date and at that hour, within a selected period of time.

In Sect. 2, we discuss the meaning of some terms and phrases related to the expression of time in the Greek language and, more specifically, in Greek astronomy. In Sect. 3, we analyze the passage of the *Dioptra* in which the eclipse is described. After the analysis of the text, we add historiographical data in order to delimit the timeframe of Hero's life. Once the timeframe and the interpretative framework are fixed, in Sect. 4, we turn to probability. First, we define a simple probabilistic model derived from our interpretation of the text. Within this model, we calculate the probability that an eclipse happened under the conditions given by the text and in a period of time coinciding with the timeframe. To this end, we set up the complete table of all eclipses between the dates of the timeframe, classifying them according to the hour and the day of the month in which they occurred, regardless of the year.[4] In Sect. 5, we sum up the conclusions derived from this statistical analysis and discuss some methodological issues involved in the use of statistical techniques in the treatment of dating problems.

2 Preliminaries

In this section, we present some astronomical, mathematical, and linguistic facts related to the reckoning of time in Greek civilization that will allow us to back up some features of our subsequent argument.

2.1 The modern expression of ancient dates

The identification of dates in Greek texts can generate some confusion simply because there appears to be no complete agreement on how to express them, not even in modern literature.[5] Most texts use the dating system called *Anno Domini*, based on the (supposed) birth date of Christ: The number of the year before/after this date, always starting with year 1, is accompanied by the abbreviation BC or AD.[6] Two calendars refer to this dating system, the Gregorian calendar and the Julian calendar, though scholars usually do not expressly declare the calendar used. Most often, a date is simply, and tacitly, expressed in terms of its corresponding calendar (i.e., Julian, before October 4, 1542, and Gregorian, onward from the immediately subsequent day, which was October 15). Historians normally date events in this way.[7]

[4] We have used the complete tables of ancient eclipses provided by NASA Eclipse Web Site (2013).

[5] This is not a minor problem, since the error (that Sidoli himself acknowledged: Sidoli 2011, p. 7) in Sidoli (2005) is precisely due to a misinterpretation of the dating method used by the table of eclipses he himself had used.

[6] See Meeus (1998) and Mosshammer (2008, chapter 2) for part of the information contained in this section.

[7] In recent years, some scholars have advocated the use of the religiously neutral abbreviations BCE (for «Before Common Era») to replace BC, and CE (for «Common Era») to replace AD. See Mosshammer (2008, p. 34)

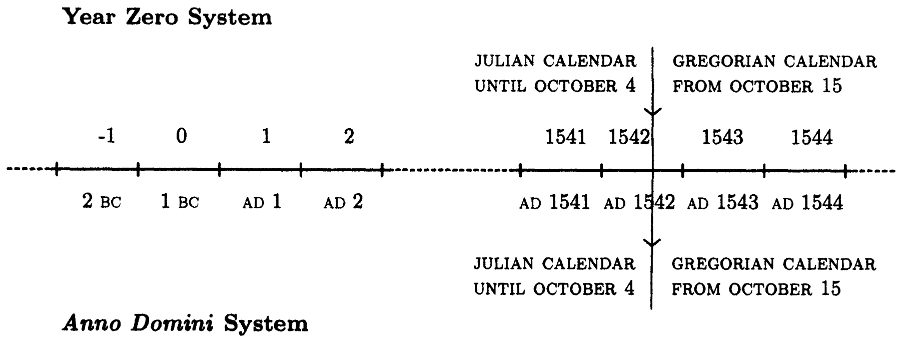

Fig. 1 Basic diagram of different dating systems

The *Anno Domini* system has one major problem: the absence of a so-called *Year Zero*. As already said, in fact, the year immediately after 1 BC is AD 1. The absence of '0' complicates matters, especially if computational tools are used. Therefore, in an astronomical context and sometimes among historians of astronomy, the astronomical notation or *Year Zero notation* is used, which identifies year 1 BC with year 0. As a consequence, all years preceding year 0 are decreased by one year with respect to the *Anno Domini* date, a − or + being added to all of them according to their preceding or following year 0, respectively (i.e., using the notation of integer numbers). In this way, year 2 BC becomes −1, 3 BC becomes −2, and so on (see Fig. 1).[8] We shall use both conventions, still giving prominence to the Year Zero notation. When using the *Anno Domini* convention, the Latin AD precedes the numeral while the English BC follows the numeral.[9]

2.2 Some aspects of the expression of time in the Greek world

The measurement of time and its linguistic expression appear to be self-evident data, but they are highly developed and codified conventions of the International System of Units subsumed in common speech. Of course, in ancient times this was not the case, and every civilization (and quite often regional cultures within a civilization) had its own perception and measurements of time and also some particular expressions for

[8] The famous eclipse of Arbela, for example, is placed by some texts in year 331 BC (e.g., Berggren and Jones 2000, p. 29), in other texts in −330 (e.g., Neugebauer 1975, p. 668, n. 30 and the NASA website). Steele (2000) even uses both notations: the Year Zero notation in the tables, the *Anno Domini* notation in the text. For a concise but complete discussion of dating conventions see http://eclipse.gsfc.nasa.gov/SEhelp/dates.html).

[9] See Mosshammer (2008, p. 34). There are other minor issues on dating, but they do not affect our analysis. One of them, however, must be briefly mentioned: Souffrin (2000, p. 14) assigns the date of March 14 and not March 13 (namely that provided by all other sources) to the eclipse of year 62. This is probably due to the fact that in 1979 the *Terrestrial Dynamical time* (TD) or, simply, *Terrestrial Time* (TT), was introduced. This parameter, which takes into account relativistic effects in the measurement of time, is used to calculate the *TD of Greatest Eclipse*, namely «the instant when the distance between the center of the moon and the axis or earth umbral shadow cone reaches a minimum». Therefore, the TD of Greatest Eclipse is expressed without reference to a place, which is the usual way in modern tables. The TD of Greatest Eclipse of that observed in Alexandria on March 13, 62, was March 14 at 00:53:05. As a consequence, the date of the eclipse without reference to a place is March 14 and not March 13.

them. In ancient Greek culture, terms or expressions such as «hour», «day», «after three days», «two days ago» and the like, could be polysemic, and it may even be that the modern meaning did not feature among those assumed. The issue becomes even more delicate when astronomical texts are at issue, in which these terms are crucial. In addition, the meaning of the above terms often has, in this context, two levels: the technical level and the level of common language.[10]

Let's start with hours. In astronomical treatises, we find two kinds of hours: *seasonal* or *unequal* or *local* hours, and *equinoctial* or *equal* hours.[11] An equinoctial hour (henceforth simply denoted «hour») almost matches an ordinary hour of ours: it is one of the 24 equal parts into which is divided 1 day. Seasonal hours are usually different for day and night. A nocturnal seasonal hour is one of the 12 portions of equal length in which a night is divided; a daylight seasonal hour has an equivalent definition. Obviously, daylight and nocturnal seasonal hours are almost identical around the equinoxes. On the other hand, the maximum difference between the daylight seasonal hour and the nocturnal seasonal hour occurs at solstices: At the winter solstice, daylight hours last about 48 min and nocturnal hours about 1 h and 12 min all around the Mediterranean sea. Equinoctial hours do not exactly match the hours nowadays in use. The current UTC system (Coordinated Universal Time), which defines time zones, does not use equinoctial hours.

So, when we need to indicate the moment of an eclipse, we must choose the time system (nocturnal hours[12] or UTC time) and be consistent with it, taking into account that the UTC time system is not equivalent to an *equinoctial hours* system.[13] Moreover, the «nocturnal hour» of a particular place should be deduced from the UTC time zone, the hour of the sunset in this place on that particular night and the duration of that night. The hour of the sunset must be exact and not related to the time zone.

The ways Greek culture used to refer to the time in which an event occurs is also a crucial issue, but it is sometimes overlooked. Two basic forms of reference can be singled out: We shall call them the *point way* and the *range way*. The *point way* places an event at a specific time point: «at midnight», for example. On the other hand, in Greek Astronomy, the use of the *range way* was very common: «in the third hour», for example. This expression means «between two and three»: in a more formalized way, the hour is taken to be an interval, in this case [2, 3]. Each way must

[10] Note that the meaning of a specific time expressions in Greek culture can be *unclear* because of our inaccurate understanding of its exact meaning, not because it is intrinsically obscure. We shall return to this issue presently.

[11] In the introduction to his translation of the *Almagest*, Toomer (1984, p. 23), defines and discusses these terms.

[12] Recall that a lunar eclipse can only happen during night.

[13] For example, Sidoli (2005, p. 28) says that the starting time of the eclipse of March 13, 62, in Alexandria was 22:50. Assuming that the UTC time is intended (though it is not specified), the eclipse must occur at 21:50 in Rome, the time in its time zone, 1 h earlier than in Alexandria, and not at 21:40 as Sidoli claims, because in the UTC system the exact longitude of a place does not matter, only its time zone does. Nocturnal hours could be used to take into account differences of time due to longitude, but the two systems must not be mixed.

use its own operation pattern; the point way uses classical arithmetic,[14] whereas the range way must use interval arithmetic, which is substantially different from classical arithmetic.[15]

Let's continue with days. There is, even today, a basic ambiguity: «day» could mean either the 24-h period[16] or the period of daylight. In addition, as Toomer points out (Toomer 1984, p. 23), another ambiguity arises: our 24-h day is a *mean solar day* «of uniform length ... <but> in antiquity, where the normal means of telling time was the sundial, <the ancient day was> usually reckoned by the *true solar day*, of varying length, the time taken by the sun to go from one meridian crossing to the next on a specific day».

Another problem, hardly ever considered, concerning a day considered as a *nychtemeron*, is its starting moment. According to Pliny the Elder:

> The actual period of a day has been differently kept by different people: the Babylonians count the period between two sunrises, the Athenians that between two sunsets, the Umbrians from midday to midday, the common people everywhere from dawn to dark, the Roman priests and the authorities who fixed the official day, and also the Egyptians and Hipparchus, the period from midnight to midnight.[17]

In the biblical context, two methods of day reckoning (from sunset to sunset, or from sunrise to sunrise) «coexisted harmoniously» (Bacchiocchi 1985). There has been some scholarly discussion about the beginning of the day in Greek culture (see Melville 1902; Brewster 1980), but most scholars refer to sunset.

It is worth noting that this issue had already been settled by ancient astronomers; for example, Ptolemy avoids any ambiguity in this regard by using «double dates» (Toomer 1984, p. 12):

> <Ptolemy> frequently characterizes the day of an observation by expressions like [...] 'Pachon 17/18' [...] In antiquity the 'civil epoch' of the day was either dawn (as in Egypt) or sunset (as in Babylon). In either system, an event which took place in the daylight would be on the same 'day' but one which took place in the night would be on 'day n' for those using dawn epoch and 'day n + 1' for those using sunset epoch. Hence ambiguity was possible. Ptolemy uses double dates (which are found *only* for night-time observations) to avoid this ambiguity.

[14] The duration of an eclipse starting at exactly 1:00 a.m. and ending at 3:30 a.m. is the difference of two numbers, $3{:}30 - 1{:}00 = 2{:}30$; so the eclipse lasted 2 h and a half.

[15] If an eclipse occurs in the third hour, i.e., in the interval [2, 3], and ends in the sixth hour, i.e., in the interval [5, 6], it has not lasted $6 - 3 = 3$ h, but $[5, 6] - [2, 3] = [2, 4]$, i.e., it has lasted «between 2 and 4 h»: the result of an operation on intervals is also an interval (see Cloud et al. 2009).

[16] Called by Ptolemy νυχϑήμερον, *night and day*. Toomer's translation does not use a specific term for it, and translates it as «day» (Toomer 1984, p. 23). We shall adopt the lexical calque *nychtemeron*.

[17] «ipsum diem alii aliter observauere: babylonii inter duos solis exortus, athenienses inter duos occasus, umbri a meridie ad meridiem, uulgus omne a luce ad tenebras, sacerdotes romani et qui diem finiere civilem, item aegyptii et hipparchus a media nocte in mediam.» *Historia Naturalis*, 2.79, ed. E. H. Warmington, transl. H. Rackham, HUP, 1938.

In short, there are two basic ambiguities in the ancient conception of a day: It can refer to daylight or to the *nychtemeron* and the beginning of the *nychtemeron* could be set at sunset or at sunrise. Context seldom clears up these ambiguities.

An issue that will prove crucial in our discussion is that of temporal phrases that include the term «day», ἡμέρα, accompanied by a numeral, i.e., time complements used to delimit a certain number of days. The meaning of these complements is not uniform, either in the genitive, in the dative or in the accusative (with or without preposition). Modern translations appear to be even less homogeneous than the original texts; it is easy to find contradictory translations, as we shall see presently.

In general, if we read in a Greek text that a certain phenomenon happens/will happen/has happened in a range of «n days» this could mean two things: The elapsed time is n days or the elapsed time is $n-1$ days (the latter reading is sometimes called «inclusive timing», since the starting day and the end day are taken into account). Thus, we read in Thucydides 4.90.3: ἡμέρᾳ δὲ ἀρξάμενοι τρίτῃ [lit. third] ὡς οἴκοθεν ὥρμησαν ταύτην τε εἰργάζοντο καὶ τὴν τετάρτην [lit. fourth] καὶ τῆς πέμπτης [lit. fifth] μέχρι ἀρίστου, «the work was begun on the *third* day after leaving home, and continued during the *fourth*, and till dinner-time on the *fifth*» (translation R. Crawley, 1903; emphasis ours, as always henceforth). On the other hand, in Athenaeus' *Deipnosophistae* 83ef, we read: εἶτ' ἄρδεται διὰ τετάρτης [lit. four] ἢ πέμπτης [lit. five] ἡμέρας, «it is then watered every *three* or *four* days» (translation Ch. B. Gulick, Loeb Classical Library, 1927). In general, it appears that the criteria for choosing one or another interpretation are unclear. We read in Aeschines, *Against Ctesifont* 77: ἑβδόμην [lit. seventh] δ' ἡμέραν τῆς θυγατρὸς αὐτῷ τετελευτηκυίας. The translation of Ch. D. Adams (HUP, 1919) reads «and though it was but the *seventh* day after the death of his daughter», while the French translation by V. Martin (Les Belles Letres, 1962) reads «*six* jours s'étaient écoulés depuis sa mort». Furthermore, Plutarch cites this passage in *Demosthenes* 22.2: ἑβδόμην ἡμέραν τῆς θυγατρὸς αὐτοῦ τεθνηκυίας, ὡς ὁ Αἰσχίνης φησί, and B. Perrin (Loeb, 1919) translates: «although his daughter had died only *six* days before, as Aeschines says». Another example, from *The Online Liddell–Scott–Jones Greek-English Lexicon* (LSJ). The entry ἡμέρα contains this example: «τρίτην [lit. third] ἡμέραν ἥχων *two* days after one's arrival, Th.8.23», but the translation of R. Crawley (J.M. Dent and Co., New York, 1903) reads: «on the *third* day after his arrival».

In short, although we have just presented a few examples, it is clear that the choice between the inclusive and the non-inclusive form is not a matter of course.[18] This

[18] This issue assumes quite bizarre connotations when considering paradigmatic relations between adjectives linked to time intervals. For example, the Great Panathenaea or the Olympica were called *penteteric* festivals, πεντετηρικαί [lit. period of 5 years], but were repeated, according to our way of counting, every 4 years (the same as our modern Olympic games). In the LSJ, πεντετηρικός [lit. period of 5 years] is defined as «happening every five years, quinquennial, cf. πενθετηρικός»; if we look up this last term, it is identified with πεντετηρίς [lit. period of 5 years], and the definition reads «festival celebrated every fifth year (inclusively)», whereas the entry πενταετηρικός [lit. period of 5 years] reads «falling every four (=five inclusive) years, quinquennial». One cannot but get confused by these definitions. The entry τετραετερικός [lit. period of 4 years] directs back to τετραετηρίς [lit. period of 4 years], and this entry reads «a quadrennial festival», or «period of four years», without indication of inclusivity. Still, another paradigmatically related adjective, τριετερίς [lit. period of 3 years] reads «triennial festival, i.e., celebrated every third year (inclusively), = in alternate years», and shortly afterward, «cycle or period of three (two) years», which is

feature has also produced discussions about the translation of certain expressions of this type in the New Testament (see for example, Clark 1978).

2.3 The treatment of eclipses in the *Almagest*

In this section, we shall analyze the expressions used by Ptolemy to denote the precise time in which an eclipse occur. We have analyzed 19 eclipses reported in the *Almagest* and also the famous eclipse reported in *Geography* 1.4.1, observed simultaneously in Carthage and Arbela a few days before the battle of Gaugamela. Four formulae are used to denote the time in which an eclipse occurs or in which a phase of an eclipse occurs; they are, in decreasing order of frequency: πρό + genitive (usually translated by Toomer as «at ... »), μετά + accusative (usually translated as «after ... »), genitive absolute (usually translated with a clause beginning with «when ... »), or only genitive (usually translated as «at ... »). In addition, all combinations of the two first formulae with the last two are also used. For example, in Toomer (1984, p. 191, H302.16-17)[19] we read that the eclipse starts μετὰ τὴν ἀνατολὴν μιᾶς ὥρας ἱκανῶς παρελθούσης, «well over an hour after moonrise», namely, a μετά+ accusative complement added to a genitive absolute. Among the expressions using only the genitive,[20] we have found no examples denoting a particular hour interval. They always refer to a point in time: «exactly at midnight» (p. 191), «at the middle of the sixth hour [of night]» (p. 206) and «at midnight» (p. 208). This fact confirms that the simple genitive can be used to indicate a point in time where an eclipse occurred, or a particular phase of it.

Another important fact is that in the *Almagest*, when the term «eclipse» is used without specifying its phase (even if this is not usual), it always refers to the midpoint of the eclipse. Moreover, in the examples we have found, the eclipse is total (Toomer 1984, p. 206 and 208). In Toomer (1984, p. 208, H332.7-11) we read:

Footnote 18 continued

obviously self-contradictory. There are no adjectives *διετηρικός and *διετερίς [lit. period of 2 years] in the LSJ. Another group of adjectives of time is even more telling in this regard and perhaps worse treated by the LSJ: those ending with −ετηρος, with the same meaning as the former group, «period of n years ». In the LSJ, there are adjectives of this kind for $n = 2, 3, 4, 5, 6, 9, 10, 100$, but the definitions are not always consistent with each other. For $n = 3, 5, 9$, the definition is «n years old ». For $n = 5$, there is another adjective with the same ending: πενθέτηρος, «in the fifth year (inclusively)». For $n = 10$, δεχαέτηρος, «ten-yearly: χρόνος, a space of 10 years ... more freq. as Subst., period of 10 years, a space of 10 years». For $n = 100$, ἑκατονταέτηρος, «of a hundred years». Probably, $n = 2$ shows most clearly the issue; in this case, the prefix is not a numeral, but a preposition indicating duality, ἀμφί, and the adjective is ἀμφιέτηρος, «celebrated in yearly festivals». In other words, in this case, we read 'two' literally, but we must interpret 'one' (inclusive interpretation), whereas for $n > 2$, we must conclude the contrary (except in one of the two words for $n = 5$). Therefore, either the system of these adjectives is not entirely consistent or it is not well understood—or both.

19 When preceded by the Greek text, the references to Toomer's translation of the *Almagest* include some numbers preceded by 'H', which is a reference to Heiberg's edition as is also partly (i.e., without the indication of lines) given in Toomer's book. Thus, H302.16-17 means 'Heiberg's edition, p. 302 lines 16–17' (all passages quoted happen to be contained in the first volume of this edition). This double reference makes also easier to find these passages in Toomer's translation.

20 This is important since it is the way in which the eclipse of *Dioptra* 35 is referred to.

τούτων δὲ τῶν ἐκλείψεων πρώτη ...γενομένη δὲ τῷ β′ ἔτει Μαρδοκεμπάδου κατ' Αἰγυπτίους Θὼθ ιη′ εἰς τὴν ιθ′ ἐν μὲν Βαβυλῶνι τοῦ μεσονυκτίου, «the first of these eclipses ...occurred in the second year of Mardokempad, Thoth[I] 18/19 in the Egyptian calendar [−719 Mar. 8/9], at midnight in Babylon». «At midnight», τοῦ μεσονυκτίου, is in the genitive, the eclipse was total and lasted 2 h, but the total phase only took 1 min, at 23:50, according to the NASA website; the term «eclipse», in this case, refers strictly to the midpoint of a total eclipse. On the other hand, Ptolemy (Toomer 1984, p. 206) also describes a total eclipse, lasting virtually 1 h with the central phase of the eclipse also lasting only an instant, and the expression that refers to this moment, ὥρας ς' μέσης, «at the middle of the sixth hour», is in the genitive. Therefore, both examples show, firstly, that the term «eclipse» may simply refer to the central phase of an eclipse, usually a total eclipse, without any explanatory complement, and, secondly, that the precise time in which the eclipse is total can be expressed in the genitive.[21]

Finally, we analyze the eclipse of September 20, 331 BC, described in Ptolemy's *Geography* 1.4.1.[22] To indicate the time of the eclipse, a genitive is used exactly as in the eclipse described in *Dioptra* 35: τὴν ἐν μὲν 'Αρβήλοις πέμπτης ὥρας φανεῖσαν, ἐν δὲ Καρχηδόνι δευτέρας, «the <eclipse> in Arbela takes place *in the fifth hour*, while in Carthage *in the second*». It was a total eclipse, and the central phase took place between 18:52 and 19:56 in Carthage (20:52 and 21:56 in Arbela). The eclipse had already begun at sunset and lasted until 21:03 in Carthage (19:46 and 23:03 in Arbela). On 20 September, sunset occurs at around 18:00 both in Carthage and in Arbela. The description of the eclipse in Carthage is well adapted to the period of total eclipse and the description in Arbela differs by about half an hour from the accurate data.[23] Therefore, the eclipse in Arbela also corroborates the hypothesis that the genitive can

[21] All of this can be explained if we consider the meaning of «eclipse» and its relation with the verb from which it derives: ἐκλείπω, «leave out», «abandon». An «eclipse» can be understood as the entire period in which the phenomenon occurs (range meaning) and also as its most remarkable feature (punctual meaning), namely the moment in which the moon is covered by the earth's shadow at a maximum degree—that is, when the moon is in the strictest sense of the verb «left out»; this very moment can be defined with the same word as the entire period: «eclipse». It is telling that sometimes, in the description of a lunar eclipse, when the earth's shadow covers the moon to the maximum extent (i.e., at the middle of the eclipse), the verb is used without any complement, and in this case indicates the central phase of the eclipse (Toomer 1984, p. 253, H419.16): ἐξέλειπεν ἡ σελήνη ἀπ' ἄρκτων τό ἥμισυ τῆς διαμέτρου, «the moon was eclipsed half its diameter from the north»; whereas when the beginning of the eclipse is referred to, the main verb used is «to begin» and not «to be eclipsed» (Toomer 1984, p. 284, H478.2): ἤρξατο ἐκλείπειν ἡ σελήνη, «the moon began to be eclipsed». To sum up, the verb «to be eclipsed» and the derived noun «eclipse» contain in their semantic field the complete duration of the event as well as the midpoint of it. It should be noted, however, that in astronomical texts the noun is usually accompanied by complements that specify the exact phase even if the phase is the central phase.

[22] Together with Hero's passage in *Dioptra* 35, they are the only recorded eclipses in antiquity observed in two places simultaneously.

[23] According to Berggren and Jones (2000, pp. 29–30), «Ptolemy's report is [...] in serious error for Arbela whether it refers to the middle or the beginning of the eclipse». This conclusion seems to be the *communis opinio*. But the fifth hour in Arbela is, approximately, the interval [22:00, 23:00], and the central phase of the eclipse took place in Arbela, as we have seen, in the interval [20:52, 21:56]. Therefore, the end of this phase of the eclipse almost coincides with the beginning of the fifth hour, which suggests that the error is not so *serious* if we compare it with the mean value of the error on eclipse timings made by the Greek astronomers listed in Ptolemy's *Almagest*: −0.38 (Steele 2000, p. 101) (i.e., −23 min).

be used in time expressions to indicate the central moment of the eclipse (perhaps only for total eclipses, as the examples of this kind in Ptolemy are total eclipses).[24] Of course, a more detailed study of the question is needed, but we can safely say that this interpretation is fully compatible with the way eclipses are recorded in Ptolemaic texts (we have found no passages supporting the contention that ancient Greek astronomers used the beginning of an eclipse as the default reference time).

3 The text of the eclipse and its interpretation

We use for *Dioptra* 35 the new edition in Acerbi and Vitrac (2014, pp. 103–106); the still standard edition of the complete treatise is (Schöne 1903, in particular p. 302):

> δεδόσθω δή, εἰ τύχοι, τὴν μεταξὺ Ἀλεξανδρείας καὶ Ῥώμης ὁδὸν ἐχμετρῆσαι τὴν ἐπ᾽ εὐθείας. τήν γε ἐπὶ κύκλου περιφερείας μεγίστου τοῦ ἐν τῇ γῇ προσομολογουμένου τοῦ ὅτι περίμετρος τῆς γῆς σταδίων ἐστὶ μ$^{\kappa\epsilon}$ καὶ ἔτι ͵β, ὡς ὁ μάλιστα τῶν ἄλλων ἀκριβέστερον πεπραγματευμένος Ἐρατοσθένμς δείκνυστιν ἐν <τῷ> ἐπιγραφομένῳ περὶ τῆς ἀναμετρήσεως τῆς γῆς. τετηρήσθω οὖν ἔν τε Ἀλεξανδρείᾳ καὶ Ῥώμῃ <ἡ> αὐτὴ ἔκλειψις τῆς σελὴνης· εἰ μὲν γὰρ ἐν ταῖς ἀναγραφείσαις εὑρίσκεται, ταύτῃ χρησόμεθα· εἰ δὲ οὔ, δυνατὸν ἔσται ἡμᾶς αὐτοὺς τηρήσαντας εἰπεῖν διὰ τὸ τὰς τῆς σελήνης ἐκλείψεις διὰ πενταμήνων καὶ ἑξαμήνων γίνεσθαι. ἔστω οὖν εὑρημένη ἐν τοῖς εἰρημένοις κλίμασιν αὕτη <ἡ> ἔχλειψις, ἐν Ἀλεξανδρείᾳ μὲν νυκτὸς ὥρας ε ἐν Ῥώμῃ δὲ ἡ αὐτὴ νυκτὸς ὥρας τρεῖς · δηλονότι τῇ αὐτῇ νυκτί. ἔστω δὲ καὶ ἡ νύξ, τουτέστιν ὁ ἡμερήσιος κύκλος καθ᾽ οὗ φέρεται ὁ ἥλιος ἐν τῇ εἰρημένῃ νυκτί, ἀπέχων ἀπὸ ἰσημερίας ἐαρινῆς ὡς ἐπὶ τροπὰς χειμερινὰς ἡμέρας ι·

We translate this passage:

> Then, let it be necessary to measure, say, the path between Alexandria and Rome along a line–or rather along a great-circle arc on the earth–if it has been

[24] A comment by Sidoli (2005, p. 258, n. 37) seems to contradict this conclusion: «Steele (2000, pp. 100–102) shows that early Greek astronomers generally recorded the times of eclipse observations by the beginning of the eclipse». We have not found anything that could justify this statement in the whole book of Steele. It appears to be a mistake, probably due to a misinterpretation of these words (Steele 2000, p. 101): «The eclipse timings made by the early Greek astronomers are listed in Table 3.4. Unlike the Babylonian observations, there appears to be a systematic error in the times of all of these observations. The mean value of this error is −0.38 h. The time of the start of the eclipse in 141 BC is almost an hour early. This is noticeably less accurate than the other records in this group which suggests that there may be some problem with the record. Britton has suggested that the time may relate to the middle of the eclipse rather than the beginning. This would reduce the error to −0.09; however, *there is no real justification for making this correction* and so it seems better simply to say that this record is somehow corrupt» (emphasis added). But it is the expression in Ptolemy's *Almagest* of the eclipse of 141 BC that leaves no doubt as to the exact time of the eclipse, not the usage of ancient astronomers: «the moon began to be eclipsed», we read in Toomer (1984, p. 284). It is worth noting that (Neugebauer 1975, p. 846, n. 12) suggests, from the eclipse reported in *Dioptra* 35, that «for the discussion of ancient eclipse records it is of interest to notice that there the "time" of the eclipse refers to the beginning» without adducing evidence other than the actual eclipse of 62. This is a circular argument.

agreed that the circumference of the earth is 252,000 stades–as Eratosthenes, having worked rather more accurately than others, showed in his book entitled *On the Measurement of the Earth*. Now, let <the> same lunar eclipse have been observed at Alexandria and Rome. If one is found in the records, we will use that, or, if not, it will be possible for us to state our own observations because lunar eclipses occur at 5 and 6 month intervals. Now, let an eclipse be found <in the records>—this one, in the stated regions: in Alexandria in the 5th hour of the night, and the same one in the 3rd hour in Rome— obviously the same night. And let the night–that is, the day circle with respect to which the sun moves on the said night–be 9 (or 10) days from the vernal equinox in the direction of the winter solstice.

Some textual aspects deserve special attention:

- The original Greek text omits the article in two key expressions «same eclipse» and «this eclipse», <ἡ> αὐτὴ ἔχλειψις and αὕτη <ἡ> ἔχλειψις. All editors have decided to restore them in both cases.[25] Although the restitution of the article is required by grammar in the first case,[26] it is surprising that in a document with so few article restitutions,[27] there are two such errors so close.[28] The first article should be restored, but even so the reference is to *an* indeterminate eclipse.[29]

[25] Note that, in both cases, the word that immediately precedes the feminine article ἡ ends with the Greek letter η (*êta*). It may be that a copyist transcribed only one of the letters η (this kind of quite common scribal error is called «haplography»). It is not easy to assess this possibility, but in the whole *Dioptra*, we have found 55 pairs of consecutive words such that the first of them ends with an *êta* and the second begins with the same letter; the feminine article ἡ features in 40 such pairs. On the other hand, only one restitution presents a situation as the one in our passage: it is εἴ γε ὅλη <ἡ> περίμετρός ἐστι μ^{κε} καὶ ͵β at Schöne (1903, 306.14). Therefore, it seems unlikely that the absence of both articles in our passage is the result of a mistake of copy.

[26] In Greek, when the meaning of αὐτός is «the same », as in this case, the term must be preceded by an article.

[27] The Teubner edition of *Dioptra* has only 20 article restitutions. As we have said, the first restitution is required but the second is not. A demonstrative + noun may not require an article when the noun with which οὗτος agrees stands as its Predicate or when the Predicate is not so distinctly separated from the Subject; the Greek αὕτη ἔχλετψτς, «this eclipse», is correct. In any rate, even in the presence of the article, the meaning of this noun phrase can still be indefinite (see note 29).

[28] In fact, there is also another article restitution a few words before (Schöne 1903, 302.16), the dative singular masculine, τῷ.

[29] The article is required by grammar prescriptions (so that the syntagma is determinate), but its meaning can still be definite or indefinite, because the semantic opposition definite/indefinite is there neutralized. The same applies to English language, because one must write «the same eclipse», with a determinate article, and not «*a same eclipse» which is an incorrect grammatical construction. Other languages, such as Catalan, allow using use both determinate and indeterminate articles in this context (indeterminate: «*un* mateix eclipse», determinate: «*el* mateix eclipse»). Therefore, in Greek (and in English), the semantic opposition definite/indefinite is neutralized, because the (determinate) article is mandatory, and the definite/indefinite meaning of the expression is dictated by the context. In the present instance, the facts that the passage is formulated in a mathematical style (see also *infra*) and that the next reference to «eclipse» does not need the article restitution, as we have seen in note 27 (i.e., it is an indefinite reference, because Greek has no indefinite articles), suggest that the syntagma «the same eclipse» must be read as indefinite. In mathematical texts, neutralization is very common. For example, we read in *El*.1.30 αἱ τῇ αὐτῇ εὐθείᾳ παράλληλοι καὶ ἀλλήλαις εἰσὶ παράλληλοι, «straight lines parallel to *the same straight line* are parallel to each other» (in Catalan the article must be indefinite: «rectes paral·leles a

- The translation of αὕτη <ἡ> ἔκλειψις must be «this eclipse», and not «the same eclipse», as some scholar translates, because the pronoun is demonstrative. Moreover, as we have said, it is not necessary to restore the Greek article.
- In the passage in which specific details of the eclipse are given, there is an evident articulation of the verbs «observe», τηρέω, and «find», εὑρίσχω, that we have emphasized in our translation: «Now, let the same lunar eclipse *have been observed* at Alexandria and Rome. If one is *found* in the records, we will use that, or, if not, it will be possible for us to state our own *observations*[30] because lunar eclipses occur at 5 and 6 month intervals. Now, let *an eclipse be found <in the records>—this one, in the stated regions: in Alexandria …»*. The syntagma «in the records» has been added in order to further emphasize the articulation of the verbs «observe» and «find».[31] In addition, we have highlighted the absence of the article and recovered the meaning of the demonstrative.[32] All of this leads us to suspect that the author chose the option of a registered eclipse, although the expression falls in the hypothetical mode of expression, in agreement with mathematical style.[33]
- The passage translated in the previous item is in fact problematic: ἔστω οὖν εὑρημένη ἐν τοῖς εἰρημένοις κλίμασιν αὕτη <ἡ> ἔκλειψις, ἐν Ἀλεξ-ανδρείᾳ μὲν νυκτὸς ὥρας ε ἐν Ῥώμῃ δὲ ἡ αὐτὴ νυκτὸς ὥρας γ, δηλονό-τι τῇ αὐτῇ νυκτί. The critical apparatus Acerbi and Vitrac (2014, p. 106) shows that the text is corrupted here. The *crux* is in the description of the eclipse in Rome, which reads, literally (without accents and breaths), δε εν αυτης νυκτος ωρας τρεις, that the editors correct in this way: δὲ ἡ αὐτὴ νυκτὸς ὥρας γ.[34] A drastic correction, because an article is again restituted and the expression ἡ αὐτή, «the same», which originally (and without article) appears to refer to «night», becomes

Footnote 29 continued

una mateixa recta són paral·leles entre elles»). There is no doubt that the noun phrase «the same line» is indefinite even though a definite article is mandatory in Greek and English (but not in Catalan). The first article αἱ, «the» (feminine and plural), is also mandatory in Greek (for the noun is qualified by a complement in form of a complex syntagma), but not in English « straight lines parallel to the same straight line», because the meaning of the expression is indefinite. Here, we have in Greek, again, the neutralization of the opposition definite/indefinite.

30 The Greek text does not use the noun «observations» but a participle τηρήσαντας qualifying the subject of the substantive clause, «<it will be possible for us,> when observing».

31 The sentence «to find an eclipse in the stated regions» sounds a bit strange in Greek, and it is much more reasonable to repeat the complement of the same verb where the verb last appeared, as Greek language frequently avoids repeating verb complements when the verb is repeated. Therefore, we must translate «to find <in the records> an eclipse in the stated region»). The expression without the repeated complements sounds so strange, even in translation, that Souffrin changed the verb («to find») to «to observe» (Souffrin 2000, p. 13): «Admettons que cette éclipse *est observée* dans les régions mentionnes». A. Rome makes the same slip: «supposons que la même éclipse *ait été observée* dans les deux régions […]» (Rome 1923, p. 237).

32 In the translation of Sidoli (2011) the demonstrative could be confused with the adjective αὐτός, «the same».

33 This was already pointed out in Acerbi (2007).

34 Acerbi and Vitrac (2014, p. 106) proposes δὲ τῆς αὐτῆς νυκτὸς ὥρας γ, «and in the same night in the 3rd hour», deleting the subsequent sintagma δηλονότι τῇ αὐτῇ νυκτί, «obviously the same night».

a complement of «eclipse». However, the transmitted text is obviously corrupt and should be amended, but we must also be aware of the very fact of the corruption.
- As we have seen in Sect. 2.2, a period of «n days » can be understood, although not always, as a period of «$n - 1$ days» because, in the Greek cultural context, the first day of a period can also be counted.

We can now turn to the interpretation of the passage. We are told that the eclipse is seen in Alexandria and in Rome, 9 or 10 days before the vernal equinox, at the third hour of night in Rome and at the fifth hour in Alexandria. The first problem, as noted by many scholars, seems to be that «the time difference that Hero states between Alexandria and Rome is 2:00 h, whereas they are actually only about 1:10 h apart, so that it is clear that at least one of these data sets does not correspond to an accurately observed eclipse» (Sidoli 2011, p. 56). But these data sets are not as straightforwardly inconsistent as it may seem. It should be pointed out, first, that Hero never says that the difference between Alexandria and Rome is 2 h: He only sets up the hours of the eclipse, which, following the interval arithmetic we have introduced in Sect. 2.2, must lead to the conclusion that the time difference between the two cities is $[4, 5]-[2, 3] = [1, 3]$, i.e., between 1 and 3 nocturnal hours, which is a raw approximation but not incorrect (the exact difference is 1:10 nocturnal hours only if we consider the difference in longitude and assuming that nocturnal hours are equivalent to 1 h on that particular night).

We can see this in a different way. First of all, note that the local times of the sunset in Alexandria and Rome usually do not match, and the difference seems always to be of about 7 min in mid-March.[35] So, the first nocturnal hour begins at different hours in each of these places. With this time datum, it is possible to imagine eclipses occurring in the fifth hour in Alexandria and in the third in Rome: If we assume that the night starts at 18:07 in Alexandria and at 18:14 in Rome,[36] and calculate the exact nocturnal hours,[37] an eclipse observed between 22:09 and 22:17 in Alexandria would be observed between 21:09 and 21:17 in Rome. In Alexandria, this is the fifth nocturnal hour, whereas in Rome it is the third nocturnal hour. Therefore, the eclipse could fit the data of *Dioptra* 35.[38] However, the eclipse of 62, that was *actually* observed in Alexandria and Rome, was observed at 22:39 in Alexandria and does not fit this interval.[39]

[35] For example, sunset in Alexandria on March 13, 2013 was at 18:07 and in Rome at 18:14 (see http://www.timeanddate.com). We do not know if these times change throughout history—probably they do, but the difference will not change, or the change is very small, because the difference in longitude does not change, and this is the most important factor in this difference.

[36] As said above, we have taken the 2013 times, because we do not know the exact hour of sunset in the year 62.

[37] On March 13, the nocturnal hour is almost 1 h long: in Alexandria, 1:00:30, and in Rome, 1:00:55.

[38] It is worth noting that this range of 87 min (10 if 1 takes into account only the difference in longitude between Alexandria and Rome) is 13 % of a whole hour. In other words, 13 % of the fifth nocturnal hour in Alexandria matches 13 % of the third nocturnal hour in Rome.

[39] Nor do the hours set up by other researchers. Neugebauer and Sidoli give 22:50 for the beginning of the eclipse in Alexandria, Souffrin calculates it at 22:34, and the NASA tables we use set it at 22:39. There are no great differences, but none of these could fit the description of the eclipse in *Dioptra* 35. However, the NASA website offers an estimate of the error (http://eclipse.gsfc.nasa.gov/SEcat5/uncertainty.html): «Morrison and Stephenson (2004) propose a simple parabolic relation to estimate the standard error (σ) which is valid over the period 1000 BCE to 1200 CE: $\sigma = 0.8 * t^2$ seconds, where: $t = \frac{\text{year}-1820}{100}$». If

A second problem is that «Hero does not state whether the time he gives in *Dioptra* 35 is the beginning or the middle of the eclipse» (Sidoli 2011, p. 57). The genitive used is not very accurate, but from what we have shown in Sect. 2.2, the fifth hour in Alexandria should contain the middle of the eclipse and there is some likelihood that the eclipse itself was total. There is another possible meaning of the genitive, which is more restrictive: that the eclipse only occurs in this hour.[40]

Another issue is the source of the eclipse record: actual observation, tables, or an invention (or even a mix of them). The formulation of the text can help us to find out the source. The passage is sharply set in a mathematical style: It is introduced by two conditional sentences in the present tense, with apodosis in the future tense (the second of which also uses a modal expression of possibility). The second sentence introduces a fairly crude remark: Eclipses occur in periods of 5 or 6 months. Thus, according to the text, in the absence of a suitable eclipse in the tables, it is necessary to wait for an actual eclipse before making the calculations; this seems a rather weird requirement. Our translation is interpretation-laden, as it assumes that the author chooses to *find the eclipse <in records>*. But the expression which states the finding of the eclipse is set in the hypothetical mode: the periphrastic verbal form, ἔστω εὑρημένη, is equivalent to the imperative passive perfect typical of the mathematical language of proofs. In short, the text in which the eclipse is described reveals the usual generalizing intent of mathematical texts, and this is even highlighted, as we have seen above, by the indefinite character of some crucial syntagmata containing the term «eclipse». This fact shows, then, that the option of an eclipse actually recorded in a table or an almanac is quite unlikely. By converse, an invented eclipse is more likely.

In summary, the eclipse data are vague, because we know for sure neither the day (this depends on the date of the vernal equinox and on the number of days we should subtract, 9 or 10, from it, and even on the interpretation of the term «day»), nor the phase in the moment in which it was registered (although the expression employed could denote a total eclipse in its full phase), nor the duration (perhaps an eclipse lasting a single nocturnal hour?). If this is combined with the fact that the formulation is in the hypothetical mode, we can conclude that the details of the eclipse seem not to matter to the man who redacted the text; the eclipse description appears to be a mere formality and it is probably nothing more than an example. The possibility of an eclipse observed by Hero himself must be ruled out as well.[41] In any case, it is

Footnote 39 continued
we substitute year = 62, the error is 247.25 s, just over 4 min. Therefore, a range for the beginning of the eclipse with an almost absolute confidence is [22:35, 22:43]. The lower limit is greater by 18 min than the third nocturnal hour in Rome. In any case, our ignorance of the exact time of sunset in Alexandria and Rome on that day introduces uncertainties that cannot be estimated.

[40] In fact, the optimal eclipse to calculate the distance between two cities is one that lasts only one nocturnal hour in both cities, because the observers of the eclipse need not describe any phase of it (beginning, end, etc.) to accurately delimit the exact moment of the eclipse. We might think that the number of lunar eclipses that lasts only one nocturnal hour is very low, but this is not true: Whatever period of years is chosen, the proportion of eclipses that lasts a single nocturnal hour is always close to 18 %.

[41] For the interpretation of the eclipse, we have not used at all the analemmatic construction and its explanation, set forth in *Dioptra* 35. Sidoli (2005, 2011) and Acerbi and Vitrac (2014, pp. 110–115) discussed such diagrams and text. The diagram is based essentially on a semicircle representing hours, divided into 12 equal parts. Each such part represents a nocturnal hour. We did not use it in our

impossible to discard the possibility that the eclipse was a mixture of some or all of these elements (observation, recording and invention).

3.1 Eclipses that actually happened and that fit our interpretation

Our database of eclipses contains all lunar eclipses that actually happened in Alexandria, after −331 and before +350, namely after the foundation of Alexandria and before the upper bound of Hero's life span (see just below), and that fits our interpretation of Heron *Dioptra* 35 *reasonably* well. There is no eclipse in this database that fits our interpretation of the eclipse exactly.[42] If we do not take into account the hour in Rome and we count any eclipse, total or partial, in which the beginning matches the fifth hour in Alexandria,[43] only the eclipse of March 13, 62, fits the description.[44]

It is difficult to evaluate the suitability of this eclipse, almost unanimously accepted from Neugebauer on, as the eclipse that marks the *terminus post quem* of Hero's life, or even the *acme* of Hero's activity.[45] To better evaluate the pertinency of this eclipse, we turn to summarize some historiographic assessments of the best timeframe containing Hero's life span.

This is a scholarly debate which involves tricky textual issues and requires a mastery of the work of Hero and of the authors who quote him (or seem to quote him), or who are quoted by him (or seem to be quoted by him). A fairly accepted reference timeframe was established by Drachman, between −150 and +250. This timeframe has been accepted virtually without discussion, probably because determining a timeframe

Footnote 41 continued
interpretation because, besides being a very simplified geometric representation of the situation, it contains a notable inconsistency: The arc of circumference that represents night, called ΘΚΛ, is divided into 12 parts; ΘΜ represents five of these 12 parts, because «the eclipse was observed in Alexandria in (or from) the fifth hour; therefore, <point> M is in the same position as where the sun was when the eclipse occurred», ἐπειδήπερ πέμπτης ὥρας ἡ ἔκλειψις ἐτηρήθη ἐν Ἀλεξανδρείᾳ· ἔσται ἄρα τὸ Μ ὁμοταγὲς τῷ πρὸς ὃ ἦν ὁ ἥλιος τῆς ἐκλείψεως γενομένης. But it is clear that point M marks the beginning of the sixth hour (or the end of the fifth hour) and not the beginning of the fifth hour! To put it in modern terms, rather than choosing a corresponding *range* of the fifth hour, the author has preferred to simplify matters by choosing the first exact *point* in time after the fifth hour, corresponding to the very beginning of the sixth hour. In short, neither the analemmatic diagram nor the comments on it could be used in the interpretation of the text of the eclipse, because they amount to nothing more than a graphic simplification.

42 There is only one eclipse after −2000 and before 350 in the NASA database fitting quite well our text of *Dioptra* 35: the eclipse of March 13, −459. It was almost complete (magnitude 0.931), its middle point occurred at 22:16 in *Alexandria* (i.e., in the spot where Alexandria would be built) and at 21:16 in Rome, i.e., at the fifth hour in *Alexandria* and at the third in Rome.

43 As far as we know and we have said in Sect. 3, there is no reason to undermine these facts, but most scholars seem to pass by them. In the next section, we will assess statistically the consequences of this interpretation, even if we have not found any indication to support it.

44 Removing these two restrictions of our interpretation (eclipse in Rome, reference to the central hour of the eclipse), there are other eclipses in the entire NASA database before 350 that fit as well as the one of 62 of even better our text of *Dioptra* 35 (March 11, −1631 and March 13 −1520, −999, −980).

45 There are exceptions to this unanimity: A. Rome, the first scholar who studied the passage exhaustively, concluded that «le chapitre 35 de la *Dioptra* ne se rapporte pas à une éclipse réellement observée,» because the data associated with the eclipse in Rome and in Alexandria «sont tellement contradictoires qu'il est impossible qu'ils soient obtenus par l'observation» (Rome 1923, p. 248 and p. 247, respectively).

became a secondary issue since Neugebauer had attached an exact date very close to Hero's life, namely 62. Of course, if exact dating becomes something to prove rather than a hypothesis, the discussion concerning the optimal timeframe of Hero's life comes again to the fore.

Acerbi and Vitrac (2014, pp. 15–22) reviews all historiographical data on Hero's date, and therefore, we refer to it for any further explanation on the data that we offer only schematically. We have chosen two intervals. The maximal interval includes only two historiographical constraints: Hero is later than (or perhaps contemporary with) Apollonius (whose *Cutting off of an Area* he mentions at *Metrica* 3.10, 13, 15) and earlier than (or contemporary with) Pappus.[46] The first maximal interval is therefore $[-200, +350]$. A second interval, minimal but quite plausible, is $[-50, 200]$. We need, now, to assess, in these intervals, the likelihood of an eclipse chosen by chance.

4 Statistical considerations

In this section, we first set up a simple probabilistic framework for lunar eclipses within a determined range of time. Essentially, we actually work out the complete table of ancient lunar eclipses in Alexandria that occurred in a range of time that includes the conjectural life span of Hero. We can easily derive from this table the number of yearly nocturnal hours[47] with eclipses in this range. This calculation allows us to know exactly the probability that, in choosing a random nocturnal hour, in this hour, day and month there was an eclipse (from now on, the meaning of the term 'probability' will be exactly this). This probability can allow us to assess *somehow* the likelihood of the case that the eclipse in *Dioptra* 35 was chosen by chance. In making these calculations, we introduce a number of parameters, and accordingly, we obtain a variety of probabilities, depending on the choice of the parameters.[48]

4.1 Table building and parameters

To build the table, we collect all lunar umbral eclipses (total or partial) in Alexandria between the given dates[49] following these rules:

- Only month, day, and hour information of the Julian calendar is used. Data concerning the year are not taken into account.
- The hour is transformed into a nocturnal hour, identified in the table by the appropriate Roman numeral. For example, if on a particular day sunset occurs at 17:35

[46] The lower limit of the interval of Hero's life span cannot go beyond −286 (birth of Archimedes; Hero mentions him by name), the upper limit beyond, approximately, 320 (the date at which we can locate the *acme* Pappus, the first author quoting Hero in *Coll.* 8). Raïos (2000), Drachman (1972) reduced this interval from different considerations.

[47] We consider a year of 365 days and 1/4, i.e., $365.25 \times 12 = 4{,}383$ nocturnal hours.

[48] Although Souffrin and Neugebauer use probabilities, they do not make their hypotheses explicit. Neugebauer only observes that in a period of five centuries, only one eclipse fits *closely enough* the one reported by Hero; Souffrin also introduces the probability that an eclipse had occurred on a given day of the year in the same period of five centuries, but does not explain, as we have seen in note 3 above, what this means or what exactly we can deduce from this probability.

[49] The complete table of eclipses can be found in this file: http://www.webcitation.org/6WbQxULEf.

and sunrise of the next day occurs at 7:10 (so that night has lasted 13 h and 35 min), then each nocturnal hour will have been almost 68 min long, and nocturnal hour I starts at 17:35 and ends at 18:43, approximately. To calculate the nocturnal hours, we have used records of sunrise and sunset in Alexandria throughout 2012.[50] We call the triplet [nocturnal hour, day, month] a *triad*.
- Given one eclipse in the NASA tables, the cell of the triad in which the eclipse has occurred is increased by 1. Thus, cells marked with 0 show that in these triads there is no eclipse; if the content of a cell is equal or greater than 1, this means that in this triad there was at least one eclipse (if it is greater than 1, the eclipses obviously have happened in different years).

These rules derive from a *key criterion* for associating a triad to an eclipse that actually happened: The triad must have a nonzero intersection with the period from the beginning to the end of the eclipse.[51] This criterion is based on our interpretation of the text (see Sect. 3) relaxing the restrictions about the exact hour of the midpoint of the eclipse: The wording of the eclipse is so vague that we should accept all hours in which the eclipse was happening.

This interpretation is, perhaps, too restrictive, especially on one point: the day of the eclipse. The text of *Dioptra* 35 does not give indications as to whether we should count 9 or 10 days before the vernal equinox, as already discussed in Sect. 2.2. Nor are we told on which day the vernal equinox took place, nor are we presented with any method to calculate it.[52] We cannot know precisely when the author started his counting of days, since it could start the night of the eclipse or the subsequent morning, or even the subsequent night (as we have seen in Sect. 2.2, it is not evident whether one should count the days from sunset or from sunrise, or whether the term refers to daylight day or to *nychtemeron*). In sum, and beyond any semantic subtleties of the Greek language, the exact day of the eclipse cannot be inferred from the text and therefore an error margin should be accepted: The minimum of latitude is 1 day (before or after

[50] After almost 2000 years, some of these data may have slightly changed but the error is small. Moreover, this error is probably much less than the error in the calculation of time in antiquity and, therefore, can be considered negligible. The data are drawn from http://www.timeanddate.com/worldclock/astronomy.html?n=426&month=12&year=2012&obj=sun&afl=-11&day=1.

[51] For example, the triad [V, 13, March], i.e., the nocturnal time interval [4, 5] of March 13 (starting at 22.139 and ending at 23.144 in decimal format and UTC at Alexandria) is associated to the eclipse of March 13, 62 (whose beginning is at 22.65 and the end at 1.67 the next day, in decimal format) because the interval of the eclipse and the triad have a non-zero intersection.

[52] We read in (Sidoli 2011, p. 57): «If the time of the equinox was observed using an instrument like the equinox rings that Hipparchus and Ptolemy, quoting and discussing him, say were set up in Alexandria then it will not have been observed until the sunrise of 23 March. Another possibility is that the date of the equinox was taken from a calendrical type device, like a parapegma. In either case, the lunar eclipse could have been recorded as 10 days prior to the equinox and we are left with only some uncertainties concerning the stated times of the observations.» However, Sidoli does not seem to note that 10 days could well be interpreted as 9 days. His intention is to make the date and time of the actual equinox (18:33 in year 62 as stated in http://www.timeanddate.com/calendar/seasons.html?year=50&n=426) to agree with the data of the text: *10 days before the vernal equinox must be 13 March*. What is plainly true, however, is that the exact time that the author of the text (or his source) considered to be the vernal equinox is totally unknown. In fact, the vernal equinox occurs in a movable date over long periods of time, phenomenon due mainly to the *precession of equinoxes* and already known to Hipparchus, about 150 BC (see Neugebauer 1975, pp. 292ss.).

the eclipse).[53] Thus, the calculations must be repeated with the following *extension of the key criterion*: The triad must fit some instant between the beginning and the end of an eclipse that actually happened, or of an eclipse that actually happened exactly 1 day before or exactly 1 day after. Our first parameter is, therefore, whether to perform calculations according to the key criterion (KC) or to the extended key criterion (EKC).

Neugebauer and other scholars based their conviction that the reported eclipse is that of year 62 on the fact that the eclipse of 62 is unique over a long period during antiquity. Accordingly, we can impose this restriction: In the triad's hour, the eclipse must be unique among all registered eclipses. The parameter *uniqueness* covers this situation.

Another important parameter must be defined before starting calculations: The length of the time interval, measured in years, in which Hero's life span is entirely included. We shall call any such interval the *scenario*. The eclipses that actually happened taken into account in our calculations are exactly those that happened within the selected scenario. As said in Sect. 3.1, this scenario can only be set up with historiographical criteria and we have chosen only two of them: a maximal scenario, [−200, +350], and a minimal, but reliable, scenario, [−50, +200]. Of course, statistics cannot do anything to decide between them; on the contrary, the results obtained depend on the scenario adopted.

In selecting the scenario, we have introduced a simplification, since in general the relevant intervals for observed eclipses and for recorded eclipses are different. The former only can coincide with Hero's presumed life span, while in the second case, the lower bound of the interval can be further lowered. In the first case, a range too wide seems unrealistic, whereas a range too close to Hero's *acme* in his lower limit would rule out past eclipses that Hero could have had at hand. As these situations are too complex, another parameter is needed. We shall focus only on two basic *frameworks*, namely the intervals within which the observed eclipses are selected, for each scenario:

1. The framework exactly matches the scenario.
2. The framework extends the lower limit of the scenario, including the possibility that the eclipse could be drawn from an almanac.

We have established, therefore, four parameters: *key criterion/extended key criterion*, *uniqueness*, *scenario*, and *framework*. In the next section, we present the result tables for any combination of them, grouped by framework and scenario, and discuss them.

4.1.1 Results and discussion

In the first framework, we consider only the interval of Hero's life span. With scenario 1, [−200, 350] (see Table 1), there is an eclipse in 30.8 % of all nocturnal hours. Thus, if a triad is randomly chosen in this scenario, the probability that this triad fits, at least, one eclipse is almost 0.31. If the eclipse must be unique (almost 80 % of the triads with

[53] As a matter of fact, this suggestion seems to be accepted by all scholars, though not explicitly. So, Sidoli finds eclipses that «were 9–12 days before the equinox for this period»; an eclipse in this range would immediately become an acceptable candidate precisely because the exact day of the eclipse cannot be accurately determined.

Table 1 Table of eclipses occurring between −200 and 350

KC	Total	∅	With eclipse					
			Total	1	2	3		
#h		4,383	3,035	1,348				
				1,077	249	22		
p	1	0.692	0.308					
				0.246	0.057	0.005		
ple				0.799	0.185	0.016		
EKC	Total	∅	With eclipse					
			Total	1	2	3	4	5
#h	4,383	1,378	3,005					
				1,585	979	376	59	6
p	1	0.314	0.686					
				0.362	0.223	0.086	0.013	0.001
ple				0.527	0.326	0.125	0.020	0.002

Without error in the day (KC) or with an error margin of 1 day (EKC). Nocturnal hours (#h) without eclipse (∅) or with some eclipse. Probability of each set (p), and conditional probability (ple) that only one, or only two, or only three ... eclipses coincided with this triad, knowing that an eclipse has actually happened

eclipses contain only one eclipse), the probability that the triad fits the *only* eclipse in this hour is almost 0.25. If we perform calculations according to the *extended key criterion*, the probability that a given triad fits an eclipse grows substantially and is more than twice the probability that no eclipse occurs. Moreover, the probability that it fits only one eclipse is high, 0.362, greater than the probability that no eclipse occurs.

In scenario 2 (see Table 2), there is an eclipse in 15.3 % of all nocturnal hours. Thus, if a triad is randomly chosen in this scenario, the probability that this triad fits, at least, one eclipse is 0.153. If the eclipse must be unique (more than the 85 % of the triads with eclipses contain only one eclipse), the probability that the triad fits the *only* eclipse in this hour is almost 0.131. With the *extended key criterion*, the probability that no eclipse occurs (0.632) is almost twice the probability that there is an eclipse (0.368), and is nearly three times the probability that it fits only one eclipse (0.24).

In the second framework, we extend the lower limit of the scenarios. In the case of scenario 1, we choose year −330, whereas in scenario 2, year −200.[54] Thus, scenario 1 is now calculated on the interval $[-330, 350]$, while scenario 2 on the interval $[-200, 200]$ (see Table 3 for scenario 1, and Table 4 for scenario 2).

With scenario 1, $[-330, 350]$, there is an eclipse in 35.7 % of all nocturnal hours; therefore, the probability that a triad randomly chosen fits, at least, one eclipse is 0.357. If the eclipse must be unique (76 % of the triads with eclipses contain only one eclipse), the probability that the triad fits the *only* eclipse in this hour is 0.271. If we

[54] In the first case, we extend the lower limit to the foundation of Alexandria. In scenario 2, we extended the lower limit by 150 years to make it coincide, roughly, with the birth of Hipparchus. In any case, it is very difficult to give precise margins without knowing what and how ancient almanacs were actually used.

Table 2 Table of eclipses occurring between −50 and 200

KC	Total	Ø	With eclipse					
			Total	1	2	3	4	5
#h	4,383	3,712	671					
				573	96	2		
p	1	0.847	0.153					
				0.131	0.022	0.000		
ple				0.854	0.143	0.003		
EKC	Total	Ø	With eclipse					
			Total	1	2	3	4	5
#h	4,383	2,770	1,613					
				1,053	424	129	6	1
p	1	0.632	0.368					
				0.240	0.097	0.029	0.001	0.000
ple				0.653	0.263	0.080	0.004	0.001

Without error in the day (KC) or with an error margin of 1 day (EKC). Nocturnal hours (#h) without eclipse (Ø) or with some eclipse. Probability of each set (p), and conditional probability (ple) that only one, or only two, or only three ... eclipses coincided with this triad, knowing that an eclipse has actually happened

perform calculations according to the *extended key criterion*, the probability that a given triad fits an eclipse grows substantially, almost three times the probability that no eclipse occurs, 0.743. Moreover, the probability that it fits only one eclipse is high, 0.331, greater than the probability that no eclipse occurs (0.292) (Table 3).

In scenario 2 (see Table 4), if a triad is randomly chosen, the probability that this triad fits at least one eclipse is 0.238. If the eclipse must be unique (82 % of the triads with eclipses contain only one eclipse), the probability that the triad fits the *only* eclipse in this hour is almost 0.2. With the *extended key criterion*, the probability that no eclipse occurs (0.439) is lower than the probability that there is an eclipse (0.561), and is greater than the probability that it fits of only one eclipse (0.341), but there are no big differences.

Finally, Table 5 shows the 16 different results of the probability that an ancient eclipse is going on at a random nocturnal hour, covering a large spectrum from 0.131 (scenario/framework [−50, 200], normal *key criterion* and *unique eclipse*) to 0.743 (scenario/framework [−330, 200], *extended key criterion* and non *unique eclipse*).

4.1.2 Some statistical simplifications and issues

In processing our statistical data, we have tacitly adopted some simplifications. First, the calculations assume that each hour has the same probability of being chosen. Since nocturnal hours do not have the same duration (i.e., their probability distribution is not uniform), a slight deviation of the results (less than 4 or 5 %) may occur. Second, in leap years there is an extra day, February 29 according to our conventions, on

Table 3 Table of eclipses occurring between −330 and 350

KC	Total	Ø	With eclipse					
			Total	1	2	3	4	5
#h	4,383	2,820	1,563					
				1,188	315	52	7	1
p	1	0.643	0.357					
				0.271	0.072	0.012	0.002	0.001
ple				0.760	0.202	0.033	0.004	0.001
EKC	Total	Ø	With eclipse					
			Total	1	2	3	4	5
#h	4,383	1,125	3,258					
				1,452	1,103	498	152	43
p	1	0.292	0.743					
				0.331	0.252	0.114	0.035	0.010
ple				0.446	0.339	0.153	0.047	0.013

Without error in the day (KC) or with an error margin of 1 day (EKC). Nocturnal hours (#h) without eclipse (Ø) or with some eclipse. Probability of each set (p), and conditional probability (ple) that only one, or only two, or only three ... eclipses coincided with this triad, knowing that an eclipse has actually happened

Table 4 Table of eclipses occurring between −200 and 200

KC	Total	Ø	With eclipse					
			Total	1	2	3	4	5
#h	4,383	3,340	1,043					
				859	179	5		
p	1	0.762	0.238					
				0.196	0.041	0.001		
ple				0.824	0.172	0.005		
EKC	Total	Ø	With eclipse					
			Total	1	2	3	4	5
#h	4,383	1,922	2,461					
				1495	702	247	15	2
p	1	0.439	0.561					
				0.341	0.160	0.056	0.003	0.000
ple				0.607	0.285	0.100	0.006	0.001

Without error in the day (KC) or with an error margin of 1 day (EKC). Nocturnal hours (#h) without eclipse (Ø) or with some eclipse. Probability of each set (p), and conditional probability (ple) that only one, or only two, or only three ... eclipses coincided with this triad, knowing that an eclipse has actually happened

Table 5 Sum up of the probabilities that a triad randomly chosen fits an actually happened eclipse, depending on four parameters

Scenario	Framework	EKC		KC	
		All ec.	Unique ec.	All ec.	Unique ec.
[−200, 350]	[−330, 350]	0.743	0.331	0.357	0.271
	[−200, 350]	0.686	0.362	0.308	0.246
[−50, 200]	[−200, 200]	0.561	0.341	0.238	0.196
	[−50, 200]	0.368	0.24	0.153	0.131

which actual eclipses might have occurred. We have made minor corrections to the tables to include these eclipses, and this has obviously affected, although minimally, the results. Finally, an issue: The modern calculation of ancient eclipses and vernal equinoxes differs depending on our modern source (NASA, Oppolzer tables, ...), and usually, there is no information about the error margin of these calculations.[55]

5 Conclusions

A passage in Hero of Alexandria's *Dioptra* 35 gives some data of an eclipse allegedly observed in Alexandria and in Rome that has traditionally been associated with the eclipse that actually happened in Alexandria on March 13, AD 62. A careful analysis of this text shows that the description of the eclipse is vague and its formulation nicely fits the hypothetical mode of expression, very much in the demonstrative style of Greek mathematics. Our interpretation of these data does not match any eclipse in the largest interval that covers Hero's life span, from the foundation of Alexandria (−330) to the age of Pappus (350). All of this leads us to think that the eclipse was invented, or, at least, that the author of the text did not bother to chose a real one. If, going against the wording of the text, we insist on supposing that he did, we are led to suppose that the reported eclipse actually happened, it was quite likely extracted from an almanac and not observed by the author. Even the possibility that the author reworked the data of a recorded eclipse cannot be ruled out.[56] Finally, there are no arguments supporting the thesis that the eclipse, if it actually happened, was observed by Hero.

If we relax the interpretation of the text (we accept that the nocturnal hour described in the text could be any one throughout the eclipse and we rule out the description of the eclipse in Rome), the only eclipse, in the largest interval that covers Hero's life span, that matches this relaxed interpretation is the one of March 13, AD 62. Even if we accept this relaxed interpretation, it does not rule out that the eclipse was chosen by chance, and not observed or taken from a record. The only way to shed some light

[55] We assume that the error margin is not too big, but we cannot assure what is the precise meaning of *not too big*.

[56] Hypothesis launched in Acerbi and Vitrac (2014, p. 111(*xv*)) to justify some geometric operations performed in *Dioptra* 35: «Ici, on mesure tout l'intérêt d'avoir placer l'éclipse dans la troisième heure à Rome : si on la plaçait dans la quatriéme heure, il faudrait faire la trisection de l'arc ΥΣ (ou de l'angle au centre correspondant) ; en toute généralité, l'opération n'est pas si simple».

on the issue is to use the probabilistic approach. It is for that reason that we created the complete table of ancient lunar eclipses in Alexandria in a range of time that covers Hero's life span so that it includes all the nocturnal hours with eclipses. With this table, we have worked out the probability that a nocturnal hour randomly chosen coincides with an ancient eclipse in this range, using four adjustable parameters: the *scenario*, that is, a timeframe containing Hero's life span; the *key criteria* (the eclipse time must fit an assigned *triad* [hour, day, month] or, less strictly, the eclipse time must fit the month and the hour, but an error margin of 1 day is allowed on the day); the *uniqueness* of the eclipse; the *framework*, that is, the lower limit of the scenario. The results form a probability range that is very wide, from 0.131 to 0.743. If we had to identify one combination of parameters as the most plausible, we would pick up scenario 2 from framework 2, with a margin of error of 1 day and without uniqueness.[57] In this case, the probability is 0.561, that is to say that, even in this relaxed interpretation of the text, it is not at all unreasonable to think that the eclipse was chosen by chance.

In summary, all indices we found point to the fact that the eclipse was invented as an example and, for that reason, that it cannot be used as a marker for Hero's life span.

The methodology we have set up is based on the linguistic data of Hero's text and the statistical data of ancient eclipses. It is not a confirmative methodology, such as some researchers seem to adhere to (underlining some facts and undermining some others): We do not wish to refute or confirm any particular hypothesis, we only present all the facts that are known and we assess these facts, in some cases associating some probabilities to a fact, and in some cases acknowledging our ignorance of some aspect of the issue.

Other methodological points concerning the whole issue can be summarized as follows:

- Any statistical analysis should clearly distinguish factual and textual data, that is, ancient eclipses that actually happened from reported eclipses (and a margin of error should also be taken into account in the calculations). Mixing these two elements may give rise to circular arguments.
- The building of the table of eclipses has a number of adjustable parameters, and the results happen to differ appreciably depending on the values adopted for them. Such a decision obviously precedes the calculation of probability.
- The size of the time intervals associated to the scenario and to the framework induces an evident distortion of the results: As the size gets greater, the probability gets closer to 1, and as the size decreases, the probability gets closer to 0. In any case, if these interval is too large or too narrow, the model seems quite useless—and, moreover, we have no idea of the *right* size that the intervals must have to obtain maximal information: If they are too large, almost all nocturnal hours contain an eclipse that actually happened and this very fact offers no additional information

[57] See Sect. 3.1. It is in fact reasonable to extend the lower limit of the scenario to consider eclipses recorded up to −200. Moreover, there is no plausible argument that justifies assuming uniqueness (apart from the circular argument that the eclipse of year 62 is unique); on the contrary, there are many triads with a unique eclipse and, in fact, it is merely a *prejudice* that it is more likely that the eclipse is real if the eclipse fits a unique eclipse in tables. Finally, the error margin of 1 day is needed because the text does not specify the actual day of the eclipse—in fact it specifies no day at all—and contains textual and factual ambiguities that make an accurate calculation of the day impossible.

about the authenticity of an eclipse; if they are too narrow, few nocturnal hours contain an eclipse that actually happened, but we could, in fact, have the exact timespan of Hero in this narrow interval.
- The interpretation of the results of the probabilistic model must be accurate and careful. In our case, we have picked the interval $[-50, 200]$ with framework 2, and the probability associated to it is 0.561. This probability does not mean that the random choice of the data of the eclipse is more likely than the deliberate choice of a specific eclipse that actually happened (observed or recorded). This probability only translates a plain fact: The number of nocturnal hours with an eclipse in the conditions made explicit by the chosen parameters is the $56.1\,\%$. This is the only information we can be assured of. This information does, however, refute the assertion, implicit in some scholars' work, that the eclipse of *Dioptra* 35 is highly unlikely to have been invented by the author; this last statement would need some indication other than the researcher's intuition.

In short, the incompleteness and vagueness of the information concerning the phenomenon described in *Dioptra* 35 cannot be overcome by any probabilistic processing of the data concerning ancient eclipses, but this kind of processing may help the historical and linguistic analysis, especially to sideline, or even discard, unfounded claims.

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