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A new interpretation of Shen Kuo's *Ying Biao Yi*

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Abstract This article analyzes the method of orienting a gnomon developed by the eleventh century Chinese scientist Shen Kuo and described in his *Ying Biao Yi*. I argue that Shen Kuo's criticism of the traditional orientation method was built on his belief that the earth is flat. The method Shen Kuo presented aims first to find the center of the earth, and only then to orient the gnomon to the cardinal directions. In addition, Shen Kuo developed two new techniques for improving observation with a gnomon: the first method sets the gnomon in a closed chamber with only a small slit for the entrance of the noon sunlight, thereby reducing ambient light and making it easier to see the gnomon's shadow. Shen's other innovation is to use a second gnomon together with the first. This can greatly weaken the shadow's penumbra.

1 Introduction

Shen Kuo 沈括 is one of the most famous scientists of the Song Dynasty and among the most important figures in the history of science in China. The astronomical treatise of the Songshi 宋史 dynastic history calls Shen Kuo "knowledgeable

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and good at the literary arts. He mastered astronomy, local chronicles, musical harmonics, the calendar, medicine and divination, and left works in all these fields.” (括博學善文, 于天文、方志、律曆、音樂、醫藥、蔔算, 無所不通, 皆有所論著,). Sivin (1975) describes Shen Kuo as “the most exceptional of the polymathic statesmen who flourished in the eleventh century.”

Shen Kuo was born in 1031 at Hangzhou in southern China. He became an important figure in both politics and science several decades later. His work in the field of astronomy was especially remarkable. In 1073, the second year he was appointed as the supervisor of the Astronomy Bureau, Shen Kuo presented three treatises to the emperor: *Hun Yi Yi* 渾儀議, *Fu Lou Yi* 浮漏議, and *Ying Biao Yi* 景表議. These three treatises, together called *Shen Kuo San Yi* 沈括三議 (literally “Three arguments from Shen Kuo”) are recorded in the *Song shi* 宋史. In them, Shen Kuo suggested to the emperor that he could better the work of the Astronomy Bureau by improving astronomical instruments for, remarked Shen Kuo, “the people who work in the Astronomy Bureau are no better than those mediocre people at the marketplace; they are basically ignorant of celestial configurations, astronomical charts and instruments” (日官皆市井庸販，法象圖器，大抵漫不知). Shen Kuo believed that careful observation was one of the key ways to improve the Astronomy Bureau’s work, for which better astronomical instruments were critically important. The emperor approved his suggestions, which did lead to improved instrumentation (Chen 2003, pp. 472–474).

The three articles of the *Shen Kuo San Yi* discussed the deficiency of the armillary sphere, clepsydra, and gnomon and suggested significant ways to improve them. Previous research, e.g., by Chen (2003, pp. 472–474) has established that Shen Kuo made great improvements to the armillary sphere in his *Hun Yi Yi* and to the clepsydra in the *Fu Lou Yi*. However, research on the *Ying Biao Yi*, which deals with the gnomon, has focused only upon the steps of the method used in measuring the length of its shadow (Chen 2003, pp. 472–474), paying less attention to the concepts that underpin Shen Kuo’s method. Agreement has not even been reached on the specific steps to be followed in making observations with the gnomonas set out in the *Ying Biao Yi*. This article focuses on Shen Kuo’s reason for criticizing the traditional orientation method and on his cosmological model. The effect of his method for measuring the length of the gnomon’s shadow will also be discussed.

2 Text and translation of Shen Kuo’s *Ying Biao Yi*

步景之法，惟定南北為難。古法置槷爲規，識日出之景與日入之景。晝參諸日中之景，夜考之極星。極星不當天中，而候景之法取晨夕景之最長者規之，兩表相去中折以參驗，最短之景爲日中。然測景之地，百里之間，地之高下東西不能無偏，其間又有邑屋山林之蔽，倘在人目之外，則與濁氛相雜，莫能知其所蔽，而濁氛又系其日之明晦風雨，人間烟氣塵坌變作不常。

臣在本局候景，入濁出濁之節，日日不同，此又不足以考見出沒之實，則晨夕景之短長未能得其極數。

參考舊聞，別立新術。候景之表三，其崇八尺，博三寸三分，殺一以爲厚者。圭首剡其南使偏銳。其趺方厚各二尺，環趺刻渠受水以爲准。以銅為之。表四方志墨以爲中，刻之，綴四繩，垂以銅丸，各當一方之墨。先約定四方，以三表南北相重，令趺相切，表別相去二尺，各使端直。四繩皆附墨，三表相去左右上下以度量之，令相重如一。自日初出，則量西景三表相去之度，又量三表之端景之所至，各別記之。至日欲入，候東景亦如之。長短同，相去之疏密又同，則以東西景端隨表景規之，半折以求最短之景。五者皆合，則半折最短之景爲北，表南墨之下爲南，東西景端爲東西。五候一有不合，未足以爲正。

既得四方，則惟設一表，方首，表下爲石席，以水準之，植表于席之南端。席廣三尺，長如九服冬至之景，自表趺刻以爲分，分積爲寸，寸積爲尺。爲密室以棲表，當極爲霤，以下午景，使當表端。副表并趺崇四寸，趺博二寸，厚五分，方首，剡其南，以銅爲之。凡景表景薄不可辨，即以小表副之，則景墨而易度。

The most difficult thing is to determine south–north direction in measuring the gnomon shadow (at noon). The ancient method is to erect a pole at the center of a circle and mark the gnomon shadow (falling on the circle) at sunrise and sunset. The result is verified by the gnomon shadow at noon in daytime, and checked with the Pole Star at night. The Pole Star is not located right on the pole of the heaven. Whereas in another so-called shadow-measuring method, the longest shadows in the morning and evening are first marked on the circle, and then the middle way on the segment joining the two marks is to be found. When you get the shortest shadow (on this point) it is mid-day. However, (for) hundreds of Li surrounding the site of measurement, the elevation and east–west orientation could not be without unevenness. In between, there must also be towns, houses, hills, and woods blocking (the rays from the sun), which are not able to be known when they are located outside of the range of vision and mixed with the turbid fog. Still worse, the turbid fog depends on whether the day is clear, dull, windy, or rainy, while smoke and dust from the human world rise, fall, and change unpredictably. I, as your servant, have made the gnomon observation at the Bureau (of Astronomy) and found the time differs every day for the sun to rise from and set into the vagueness (on the horizon). This makes it impossible to find the true sunrise and sunset, and thus you cannot get the extremes of the shadow-lengths in the morning and evening.

Referring to old information, I have invented a new method. Vertically set three copper gnomons of 8 Chi high, 3.3 Cun wide, and 2.2 Cun thick. Cut off the south part of the top of each gnomon to get a wedge-shaped tip. The base of each gnomon is a cube with each side of 2 Chi. Around the top surface of each base, a groove is chiseled to contain water for horizon keeping. Both gnomons and bases are made of copper. Mark the middle line with ink on each of the four vertical surfaces of the gnomons.

Carve holes on the lines and hang from them four plumb lines with copper balls, each parallel to the ink lines on corresponding direction. Roughly determine the four directions of the east, west, south, and north. Then put the three gnomons in a row along the south–north direction and let their bases touch each other, making a distance of 2 Chi between two neighboring gnomon. Set every gnomon vertical and make the four plumb lines parallel to the corresponding ink lines. Measure the distances between every part of the three gnomons, making sure that they are strictly in a row. When the sun rises, measure the distances between the shadows of the three westward gnomons, as well as the length of the shadows. Record the results. When the sun sets, measure the shadows of the three eastward gnomons in the same way. When the shadows have the same lengths and are with equal intervals, join the ends of the western and eastern shadows of each gnomon, and find the mid-way point for the shortest shadow. When the five data conform to each other, the shortest shadow pointing to the mid-way is in the direction of north, while the ink lines on the south surface of the gnomons points down to south. The eastern and western ends of the shadows are pointing to east and west. Any one of the five data being inconsonant, all of them cannot be used for the determination of the cardinal directions.

After knowing the four (cardinal) directions, one can set up a single gnomon (of 8 Chi high), with a squared top. A flat stone bed is laid out at the foot of the gnomon and set horizontal with water, while the gnomon is erected at the south end of the bed. The bed is 3 Chi wide, as long as the length of the noon shadow on winter solstice of the place. Beginning from the foot of the gnomon, carve the scale division Fen (on the surface of the bed). Accumulating Fen makes up Cun and accumulating Cun makes up Chi. Build a closed chamber to house the gnomon, and open a slot on the roof right above the top of the gnomon to let the sun light in for noon shadows. The minor gnomon is 4 Cun high with the base. The base is 2 Cun wide and 5 Fen thick, with a squared top. Cut off the south part of the top to make it into a wedged-shaped tip. The minor gnomon is also made of copper. When the shadow of the major gnomon is too vague to be recognized, one can use the minor gnomon to make the shadow darker so as to measure it easily.

3 Shen Kuo's criticism of the traditional orientation method

One of the fundamental tools of observational astronomy in ancient China was the gnomon. Extensive discussions of the gnomon appear in early Chinese sources such as the *Kao Gong Ji* 考工記, written at the end of *Chunqiu* 春秋 period (770–403BC), and the *Zhou Bi Suan Jing* 周髀算經, written during the Han 漢 dynasties (205 BC–AD 220) (Cullen 1996). Later, Chinese astronomers devoted significant efforts to improving methods of measuring the length of the shadow cast by a gnomon (Pan 2005).

At the beginning of his *Ying Biao Yi*, Shen Kuo remarked that the most difficult step in setting up a gnomon is to determine the cardinal directions which the shadow cast by the gnomon will be measured against. Shen Kuo described the method of orientation used in ancient China as follows:

> The ancient method is to erect a pole at the center of a circle and mark the gnomon shadow (falling on the circle) at sunrise and sunset. The result is verified by the gnomon shadow at noon in daytime, and checked with the Pole Star at night. The Pole Star is not located right on the pole of the heaven. Whereas in another so-called shadow-measuring method, the longest shadows in the morning and evening are first marked on the circle, and then the middle way on the segment joining the two marks is to be found. When you get the shortest shadow (on this point) it is mid-day.

Shen Kuo here refers to two traditional methods of measuring the length of the gnomon's shadow. The first method is recorded in the *Kao Gong Ji*:

> 匠人建國，水地，以懸置槷，（以懸）眡以景。為規，識日出之景與日入之景。晝參諸日中之景，夜考之極星，以正朝夕。

> In order to know the direction when building a city, the constructor should do as follows: take advantage of water to make sure one small area is flat, and then set a stick vertical on the ground. After that, the observer should draw a circle on the ground. When the sun rises and sets, the two intersections of the shadow of the stick and the circle should be marked. The line joining the two intersections points east and west. Orientation can be certain after referring to the stick's shadow at midday and the North Star at night.

In this method the east–west line is determined by connecting the positions of a fixed length of the shadow cast by the gnomon at sunrise and sunset. The second method referred by Shen Kuo uses the length of the gnomon shadow to orientate through measuring the length of the gnomon shadow. This time the endpoints of the shadow cast by the gnomon at sunrise and sunset are measured. Joining the middle of the line that connects them to the tip of the mid-day shadow (when the shadow is shortest) provides a north–south line. This second method is also related to the procedure described in the *Kao Gong Ji*. Zheng Xuan 鄭玄 (AD 127–200), an influential commentator and scholar at the end of Eastern Han, further explained the *Kao Gong Ji* method, noting that the semi-diameter of the circle should be equal to the length of the gnomon's shadow when the sun rises and sets. Many later astronomers uncritically followed Zheng Xuan's method in measuring the length of the gnomon shadow. Shen Kuo's criticism of the traditional method was aimed directly at Zheng Xuan's explanation.

Continuing his discussion, Shen Kuo remarked:

> However, (for) hundreds of Li surrounding the site of measurement, the elevation and east–west orientation could not be without unevenness. In between, there must also be towns, houses, hills and woods blocking (the rays from the sun), which are not able to be known when they are located outside of the range of vision and mixed with the turbid fog. Still worse, the turbid fog depends on whether the day is clear, dull, windy or rainy, while smoke and dust from the human world rise, fall and change unpredictably.

The difficulties Shen Kuo identified in the ancient measuring method can be divided into two categories: first, the effect of natural circumstance, e.g., hills, woods, or others things blocking the view of the horizon, and second, turbid air. When the sun is rising or setting, the sun's light is passing through the hazy air near to the ground. Shen Kuo gave an example from his own experience to illustrate the problem: as supervisor of the Astronomy Bureau, when he used the gnomon in making astronomical observations, he found that the extent of turbid air differs every day. As a result, his observations were notably affected since it was hard to determine the longest shadow of the gnomon in the morning or evening and as a consequence difficult to accurately establish the cardinal directions. Shen Kuo here repeatedly emphasized that the weak point of the existing method of using the gnomon is the effect of natural circumstance and turbid air, both of which are hard to control.

Shen Kuo began his discussion with the statement that "(for) hundreds of Li surrounding the site of measurement, the elevation and east–west orientation could not be without unevenness." Due to the curvature of the earth, the shadow cast by a gnomon will never reach the length of 100 Li (about 50 km). Assuming that the earth is spherical all places on the its surface have their own north–south meridian, and there is no need to know the east–west direction when using the gnomon. So why did Shen Kuo stresses the importance of the east–west direction and give the figure of 100 Li? I suggest Shen Kuo was here working with a cosmological model in which the earth is flat. If the earth is flat, it is possible that the shadow cast by a gnomon can reach 100 Li or even further. Furthermore, the earth will have a center point and a north–south direction that is unique to that point (Guan 2005). An observer cannot get the true south–north direction if the measuring site is not on the meridian line which passes through the center of the earth.

In China, opinion on where the center of the earth is located changed during of the Tang 唐 dynasty (seventh to tenth centuries AD) and the Song 宋 dynasty (10th to 13th centuries AD). Before the end of the Tang dynasty, it was generally held that the center of the earth was at Yangcheng 陽城. However, in AD724 a nation-wide astronomical geodetic survey undertaken by the Buddhist monk Yi Xing— 一行 showed that Yangcheng was not at the center of the earth. The *Xin Wu Dai Shi* 新五代史 (New History of the Five Dynasties), compiled in the eleventh century, describes Yi Xing's work as follows:

> 古者植圭于陽城，以其近洛故也。蓋尚慊其中，乃在洛之偏東。開元十二年，遣使天下候影，南距林邑，北距橫野，中得浚儀之岳台，應南北弦，居地之中。大周建國，定都于汴，樹圭置箭，測岳台晷漏，以爲中數。晷漏正，則日之所至，氣之所應，得之矣。

> The ancient people believe the center of the earth is at Yangcheng, that is because it is near to Luoyang 洛陽. In Kai Yuan 開元, 12th year (AD 724), the emperor sent people to the whole country to make observations. According to the south place Linyi 林邑, and the north place Hengye 橫野, the observers found that the center place which corresponds to the south and north direction should be Yuetai 岳台(a place in Kaifeng 開封). The great Zhou 周 established the country and placed the capital at Bian 汴 (another name of Kaifeng). The emperor set up the gnomon and clepsydra at Kaifeng. The result of measurement showed that it is according to the data in the center of the earth. This means Yuetai is the right place in correspondence with the sun and the Qi 氣 (a day marking one of the 24 divisions

of the solar year in the traditional Chinese calendar) which is the right place for the center of the earth.

Subsequent authors therefore placed the center of the earth at Kaifeng 開封 (Guan 2000). The flat-earth cosmology was one of several competing cosmologies in China and was still widely accepted as late as the Song dynasty. I will return to the question of whether Shen accepted this cosmological model in the final part of this article.

4 Shen Kuo's orientation method

Following his criticism of the traditional method of setting up the gnomon, Shen Kuo presented his own method, known as the "three gnomon method." The steps of Shen Kuo's "three gnomon method" have been clearly explained by Chen Meidong as follows:

> The steps are listed as follows: First vertically set three copper gnomons on flat ground. The gnomons are all 8 Chi 尺 high, 3.3 Cun 寸 wide and 2.97 Cun 寸 deep. Then, cut the south side of each gnomon wedge shaped and curve a sink on the base of the gnomon which is a 2 Chi cube to make the gnomon level by means of observing the water level. Mark the middle of each face of the base, and then hang a copper ball straight to each sign. Put the three gnomons in a row in the south–north direction and let their bases touch each other, with a distance of 2 Chi between each gnomon. Measure the distance between each part of the three gnomons to make sure the three gnomons are strictly in a row. Finally, measure and record the length of the shadow of each gnomon and the angle between each two gnomons when the sun rises and sets, giving a total of five data. When all the five data coincide, we can then get to know the south–north direction using the second traditional measuring method. (Chen 2003, pp. 472–474)

Shen Kuo's three gnomon method is shown in Fig. 1. Figure 1a–c represent three gnomons, d–f represent the gnomon shadow when the sun rises, and g–i represent the gnomon shadows when the sun sets. When the length of d–f equals the length of g–i, the angle between de and ef equals the angle between gh and hi, and we can then use the second traditional measuring method to get the south–north direction.

Shen Kou claimed that his three gnomon method is a more effective method when compared to the traditional *Kao Gong Ji* method. As we have seen, Shen Kuo argued gnomon shadow observations are effected by the location of the gnomon and by turbid air obscuring the sun's rays when the sun is close to the horizon. As Shen Kuo remarked, the effects of turbid air cannot be reduced whatever method is used. Similarly, it is impossible to eliminate the presence of hills, cities, etc., blocking the sun's rays close to the horizon. Thus, it seems likely that Shen Kuo's "three gnomon method" can only have been formulated to resolve the problem of whether the observation site is at the center of the earth.

Shen Kuo stated that once all the five data (the length of the gnomon shadows and the angles between them) coincide it is possible to use the second traditional method to obtain the north–south direction. Figure 2 illustrates Shen Kuo's argument. In Fig. 2, d represents the newly risen sun, e the setting sun, and a–c the three gnomons; the

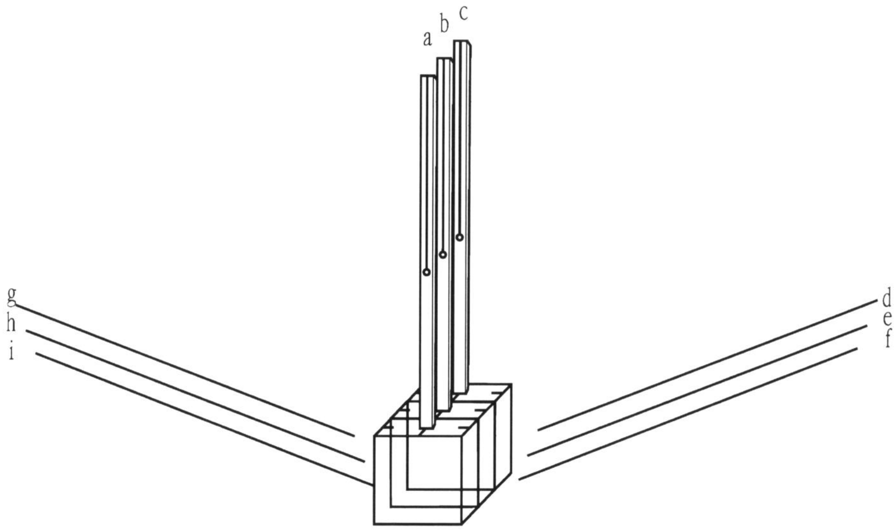

Fig. 1 The three gnomon method

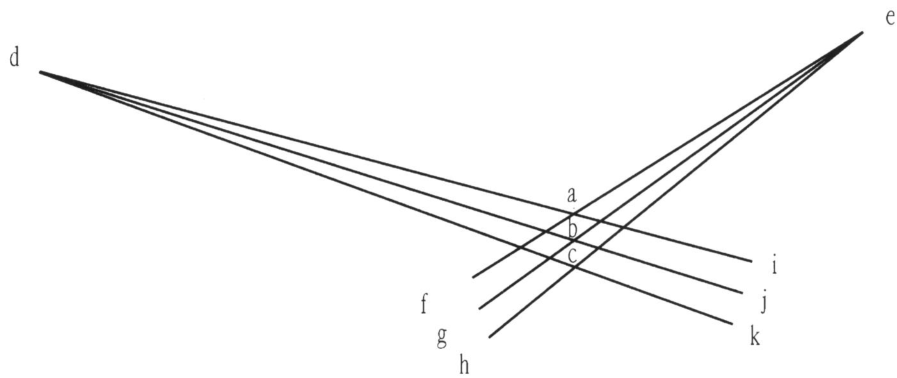

Fig. 2 The three gnomon method assuming the flat-earth cosmology. In the illustration the gnomons are not at the center of the earth and so the shadow lengths at sunrise and sunset differ

lengths of gnomon shadows in the morning are ai, bj, and ck, and the lengths in the evening are af, bg, and ch. According to the flat-earth cosmology, the sun rises and sets at opposite points to the center of the earth (d and e in the diagram). This means that the distances between these two points and the center of the earth are equal. Therefore, when the observation site is not at the center of the earth, the distance between the sun and the observation site must be different at sunrise and sunset. Since the height of the gnomon does not change, the length of the gnomon shadow will therefore be different in at sunrise and sunset. Thus, in the figure, ai, bj, and ck are not equal to af, bg, and ch. Similarly, the angles between the tips of the shadows in the morning must be different to those in the evening. However, if the measurement is carried out at the center of the earth the distance between the sun and the gnomons is the same at sunrise and sunset, and so the lengths of the three gnomon shadows must be equal and similarly the distances between the tips of the gnomon shadows must also be equal. As a result, working in the flat-earth cosmology, whether the observation site is at the center of the earth or not is the key factor in whether the five data coincide.

Shen Kuo's three gnomon method was a development of an earlier five gnomon method used to find the center of the earth proposed by Zu Geng 祖暅 during the Southern Dynasties 南朝 (420–589 AD). This method is also mathematically consistent with a flat-earth cosmology and was commented on with approval by later authors (Guan 2000). Zu Geng's method is described in the *Sui Shu* 隋書 dynastic history as follows:

> 先驗昏旦，定刻漏，分辰次。乃立儀錶于准平之地，名曰南表。漏刻上水，居日之中，更立一表于南表影末，名曰中表。夜依中表，以望北極樞而立北表，令參相直。三表皆以懸准定，乃觀。三表直者，其立表之地，即當子午之正。三表曲者，地偏僻。每觀中表，以知所偏：中表在西，則立表處在地中之西，當更向東求地中；若中表在東，則立表處在地中之東也，當更向西求地中。取三表直者，爲地中之正。又以春秋二分之日，且始出東方半體，乃立表于中表之東，名曰東表，令東表與日及中表參相直。是日之夕，日入西方半體，又立表于中表之西，名曰西表。亦從中表西望西表及日，參相直。乃觀三表直者，即地南北之中也。若中表差近南，則所測之地在卯酉之南；中表差在北，則所測之地在卯酉之北。進退南北，求三表直正東西者，則其地處中，居卯酉之正也

First check the time of sunrise and sunset, set up the water clock, define the time of Shi Chen 時辰 (a time period which equals to two hours). Set a gnomon in a flat area, call it the south gnomon. Then run the water clock, set another gnomon called the middle gnomon at the end of the shadow of the south gnomon at noon. Set up the north gnomon at midnight, make the top of it in a line with the top of the middle gnomon and the North Star. Then begin observation after knowing the three gnomons are all vertical. If the three gnomons are in a line, the place should be at the right south–north direction; if the gnomons are not in a line, the place is not right and the observer can know where the right place should be referring to the position of the middle gnomon. If it is in the west to the line, it means the measuring site is in the west to the center of earth, the observer should move to the east to find the center of earth, if it is in the east, it means the measuring site is in the east to the center of earth, the observer should move to the west to find it. When the three gnomons are straightly in a line, the measuring site should be at the middle of the earth on east–west direction. In the equinoxes, when half the sun has risen in the morning, set the east gnomon in the east of the middle gnomon which is in a line with the sun and the middle gnomon. When half the sun has set in the evening of that day, set the west gnomon in the west of the middle gnomon which is in a line with the sun and the middle gnomon. The observer should refer to the position of the middle, the east and the west gnomons to know the position of the measuring site. If the middle gnomon is in the south, it means the measure site is in the south to the center of earth, the observer should move to the north to find the center of earth, if it is in the north, the observer should move

> to the south to find the center of earth. When the three gnomons are straightly in a line, the measuring site should be at the middle of the earth on south–north direction. When the east–west direction and the south–north direction are all right, the measuring site should be the center of earth.

Rather than using the gnomon at sunrise and sunset, this method requires the use of a water clock to determine the moment of midnight, bringing a further possible source of error into the determination of the cardinal directions. This problem is eliminated in Shen Kuo's three gnomon method which uses the shadow cast by the gnomon at sunrise and sunset directly.

5 Shen Kuo's shadow measurement method

Once he has explained how to set up the gnomon, Shen Kuo presents a new technique for measuring the length of the shadow cast by the gnomon:

> After knowing the four (cardinal) directions, one can set up a single gnomon (of 8 Chi high), with a squared top. A flat stone bed is laid out at the foot of the gnomon and set horizontal with water, while the gnomon is erected at the south end of the bed. The bed is 3 Chi wide, long as the length of the noon shadow on winter solstice of the place. Beginning from the foot of the gnomon, carve the scale division Fen (on the surface of the bed). Accumulating Fen makes up Cun and accumulating Cun makes up Chi. Build a closed chamber to house the gnomon, and open a slot on the roof right above the top of the gnomon to let the sun light in for noon shadows. The minor gnomon is 4 Cun high with the base. The base is 2 Cun wide and 5 Fen thick, with a squared top. Cut off the south part of the top to make it into a wedged-shaped tip. The minor gnomon is also made of copper. When the shadow of the major gnomon is too vague to be recognized, one can use the minor gnomon to make the shadow darker so as to measure it easily.

Shen Kuo highlights two innovations in observational technique in this passage: first, the observation is to be conducted in a closed chamber with only a small slit to allow the sun's rays at noon into the room, and, secondly, a second gnomon is to be used together with the main gnomon in making the observation. Compared to the traditional method of observing in an open area, observing in a closed chamber reduces the scattering of sunlight, thereby increasing the contrast between the shadow and the surrounding environment. Two suggestions have recently been put forward as to how the second gnomon was used together with the first gnomon to improve the observations. Li (1983) has argued that "one can aim at the sun with the naked eye, making the head of the second gnomon, the head of the (first) gnomon and the center of the sun in a line." Li's explanation is problematical, however, as the second gnomon is only 4 Cun high (about 13 cm and only one 20th the height of the first gnomon) and so it will be quite difficult to observe from the head of the second gnomon to the head of the first gnomon. A more likely explanation has been proposed by Chen (2003, pp. 472–474)

> Although the sun is quite weak, observers can still see a beam shoot in the closed room from the split on the roof. Then move the second gnomon to make the head

of it just contact the beam to make the shadow clearer. The purpose of doing so is to diminish the phenomenon of penumbra on the accuracy of the measurement. We know that the sun is round, which leads to a penumbral zone at the end of the shadow and makes it hard to see the exact position of the shadow tips. The higher the gnomon is, the more seriously this phenomenon appears. Since the second table is only 4 Cun high which means the phenomenon of penumbra is much weaker, moving the second gnomon to make the head of it just contact the beam can greatly make the shadow clearer. I have done some imitation experiments which can also prove this.

6 Conclusion

In his *History of Ancient Chinese Astronomical Instruments* (2005, pp. 51–52), Pan Nai stated that there are six factors which can affect the accuracy of gnomon shadow measurement: the accuracy of the time of winter solstice and summer solstice; the accuracy of the north–south direction; whether the gnomon is vertical to the ground; whether the base of gnomon is level or not; the impact of penumbral zone on determining the length of the shadow; and the traditional method of measurement (referring to the traditional measurement of the length of gnomon shadow on the solstice). In these six factors, the accuracy of determining the time of winter solstice and summer solstice and the measurement of the length of gnomon shadow at the solstice had been significantly improved by the Song Dynasty. The problem of whether the gnomon is vertical to the ground could be resolved by hanging a vertical string down the gnomon. Similarly, the problem of whether the base of gnomon is level or not could be resolved using a surface of water. Thus, by the Song dynasty, the major problems in making observations using a gnomon were determining an accurate north–south direction and the impact of the penumbra in reading the length of the shadow. Shen Kuo's suggestions for improvements in the method of using the gnomon attempted to solve these two problems. First, his three gnomon method aimed to allow the north–south direction to be determined more accurately. Secondly, by placing the gnomon in a covered room and using a second subsidiary gnomon to help define the edge of the shadow, Shen Kuo made a significant improvement to the accuracy with which the length of the shadow could be measured.

As I have argued, Shen Kuo's three gnomon method for determining the north–south direction was based upon a flat-earth cosmology. In his article on the armiliary sphere, the *Hun Yi Yi*, Shen Kuo stated that:

> 臣觀古之候天者，自安南都護府至浚儀太岳台才六千里，而北極之差凡十五度，稍北不已，庸詎知極星之不直人上也？臣嘗讀黃帝《素書》："立于午而面子，立于子而面午，至于自卯而望酉，自酉而望卯，皆曰北面。立于卯而負酉，立于酉而負卯，至于自午而望南，自子而望北，則皆曰南面。"臣始不諭其理，逮今思之，乃常以天中爲北也。常以天中為北，則蓋以極星常居天中也。《素問》尤為善言天者。

> When I examined the astronomical observation in ancient times, I found that although there is only 6000 thousand Li between Annan 安南 and Junyi 浚儀, the difference of degrees of the North Star is 15 degrees. Su Wen is good at talking about the sky. If we go straight to the north a little further, would not the North Star be straight over our head? In Huang Di's 黃帝*Su Shu* 素書 it stated, when you stand at the south pole you will face north, when you stand at the north pole you will face south, when you stand at the east to see the west or stand at west to see the east, you will still face north. When you stand at the east with your back to the west, when you stand at the west with your back to the east, when you stand at the south pole to see south, when you stand at the north pole to see north, you will all face south. I did not get a clear idea of this argument until recent times. This condition happens because we believe the center of sky is north, that is because the North Star is always at the center of sky. Su Wen is good at talking about the sky.

Li (1995) has argued that here Shen Kuo expressed his belief in the cosmological theory of a canopy heaven with a round earth (but *not* the spherical heaven model that was used by several other astronomers of this time). However, on closer inspection of Shen Kuo's text, we can see that he is trying to interpret the words of Su Wen: "I did not get a clear idea of this argument until recent times" indicates that Shen Kuo himself had not thought this before. Rather than presenting his own view, Shen Kuo was trying to explain why Su Wen believed in a center of the sky, which is also the center of a circular earth in the theory of canopy heavens. We cannot use this paragraph as evidence that Shen Kuo himself believed that the earth is round. In his *Ying Biao Yi*, Shen Kuo clearly worked with a flat-earth cosmology. It is possible, however, that Shen Kuo at other times assumed other cosmological models, in particular the spherical heaven model that was adopted by several other Song dynasty astronomers.

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