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Emergence of classical trajectories in quantum systems: the cloud chamber problem in the analysis of Mott (1929)

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Abstract We analyze the paper "The wave mechanics of α-ray tracks" Mott (Proc R Soc Lond A 126:79–84, 1929), published in 1929 by N. F. Mott. In particular, we discuss the theoretical context in which the paper appeared and give a detailed account of the approach used by the author and the main result attained. Moreover, we comment on the relevance of the work not only as far as foundations of Quantum Mechanics are concerned but also as the earliest pioneering contribution in decoherence theory.

1 Introduction

N. F. Mott[1] proposed a theoretical explanation of the α-particles tracks observed in a Wilson cloud chamber in the paper "The wave mechanics of α-ray tracks"

[1] Sir Nevill Francis Mott (30 September 1905 to 8 August 1996) was an English physicist. He won the Nobel Prize for Physics in 1977 for his work on the electronic structure of magnetic and disordered systems. The award was shared with P. W. Anderson and J. H. Van Vleck (for further details see e.g. B. Pippard, Biographical Memoirs of Fellows of the Royal Society, **44**, 314–328, 1998).

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(Mott 1929), published in 1929. The work appeared in the early stage of the debate about the interpretation of Quantum Mechanics. It was mainly motivated by the attempt to clarify the meaning of the wave-particle duality and the crucial role of the measurement process. In this context the paper can be surely regarded as one of the seminal contributions by the quantum theory founders.

In addition to its foundational relevance, we also consider Mott's paper extremely important as the first pioneering example of the relatively recent research approach known as decoherence theory (Giulini et al. 1996). The aim of that theory is to explain the emergence of a quantum particle classical behavior as a result of its interaction with the environment. In fact, it is known that for an isolated quantum particle an approximate classical evolution can be derived in the limit $\hbar \to 0$ only for very special classes of initial states, like WKB or coherent states (see e.g. Robert 1998). However, in many important physical situations, quantum particles exhibit a quasi classical evolution for initial states drastically different from the ones in those classes. Decoherence is precisely the dynamical mechanism, entirely predictable on the basis of quantum evolution laws, responsible for the appearance of a particle classical behavior as a consequence of the interaction with the environment. In concrete situations the approach consists in providing models of a non trivial quantum environment which interacts with the particle going through it.

Our aim is to show that Mott's approach stands entirely inside this line of thought. Indeed, the author analyses the α-particle dynamics assuming a spherical wave as initial state. He attempts to explain the observed straight tracks left by the α-particle as a consequence of its interaction with the atoms of the vapor using a typical decoherence theory attitude. As a matter of fact, Mott's paper is less known than it deserves. In the following we describe the context in which the work appeared, detail the contents of the article and comment on its relevance both in the foundations of Quantum Theory and in the framework of decoherence theory. The paper is organized as follows.

In Sect. 2 we briefly describe the experimental and theoretical background in which Mott's paper appeared. In particular, we briefly recall how a Wilson chamber works and we summarize the fundamental ideas leading to the formulation of Quantum Mechanics according to the so-called Copenhagen or standard interpretation.

In Sect. 3 we give an account of the first theoretical arguments proposed by Born (1927), Heisenberg (1929–30) and Darwin (1929) to explain the tracks observed in a cloud chamber in the framework of quantum theory.

In Sect. 4 we present the introductory considerations appearing in Mott's paper, describe the three-particle model presented by the author and give a precise formulation of the main result.

In Sect. 5 we describe in some detail Mott's arguments, essentially based on perturbation theory and stationary phase methods, leading to the result.

In Sect. 6 we conclude with some critical remarks on relevance and modernity of Mott's paper, some limitations of his approach and, finally, possible re-examinations and improvements of the result.

2 The Wilson chamber and quantum theory

Roughly speaking, a tracking chamber is a device where atoms or sub-atomic particles are detected and their very short dynamical life is recorded. The prototype of such a device has been the cloud chamber, constructed by Wilson in the years 1911–12 (see e.g. Leone and Robotti 2004 for a description of the original apparatus). The importance of the cloud chamber was immediately realized and the device was extensively used to investigate the properties of many different atomic and sub-atomic particles. In particular, it was used to observe and characterize the "ionizing radiation" emitted by radioactive sources, which was precisely the case examined in Mott's paper. During the last years of the 19-th century Wilson was experimentally analyzing the conditions of fog formation in air saturated with water vapor. He examined the role of electric charges as condensation nuclei for the excess of vapor. In the first years of the 20-th century "*...ideas on the corpuscular nature of alpha- and beta-rays had become much more definite, and I had in view the possibility that the track of an ionizing particle might be made visible and photographed by condensing water on the ions which it liberated*" (from Wilson's Nobel lecture, Stockholm, December 1927). In fact, he made available an experimental apparatus operating schematically as follows. The air saturated with water vapor, contained in a chamber, is submitted to a very fast expansion lowering its temperature and driving the mixture towards a supersaturated state. The α-particle, released by a radioactive source placed in the chamber, interacts with the atoms of the gas inducing ionization. The ionized atoms trigger the formation of small drops of water near their positions. The sequence of such drops is the visible track that one can directly observe in the chamber. Wilson had to solve the complex experimental problem of instantaneous and synchronized illumination and photography of the sequence of drops. The track is regarded as the experimental manifestation of the "trajectory" of the α-particle and, as a matter of fact, is accurately described as the trajectory of a classical particle (relativistic or non relativistic according to the initial particle velocity) in a classical electromagnetic field. In particular, one observes straight lines whenever no electromagnetic field is present.

The advent of Quantum Mechanics in the years 1925–27, together with its standard interpretation, made the effectiveness of the classical description of the tracks rather problematic (see e.g. Falkenburg 1996; Carazza and Kragh 2000). In order to outline the context at the time the cloud chamber problem was first approached we briefly summarize here some crucial steps which led to the final formulation of the theory (for a detailed analysis we refer to Jammer 1989; Cushing 1994; Stepansky 1997).

As it is well known, the decisive contributions to the elaboration of the theory came from Heisenberg (1925), together with Born et al. (1925) in 1925, and from Schrödinger (1978) in 1926. The two approaches, known as matrix mechanics and wave mechanics respectively, were based on radically different physical descriptions of atomic phenomena.

The former approach is characterized by the explicit rejection of the classical notions of position and velocity in the description of atomic phenomena, in favor of a new kinematics based on observable quantities, like frequencies and amplitudes of the emitted or absorbed radiation in quantum jumps. As a consequence, Matrix Mechanics appears as an abstract algebraic formalism where any visualized or intuitive description of a microscopic object is abandoned.

On the contrary, according to Schrödinger, such intuitive space-time description is still possible if one admits that a microscopic object is described by a wave instead of a point particle. On the basis of the analogy between optics and mechanics, he was able to derive the evolution equation for the wave $\psi(x, t)$. Moreover, he also proposed a first physical interpretation of $e|\psi(x, t)|^2$ as the density charge at the point x and time t of the microscopic object with total charge e. The mathematical equivalence of the two theories was soon proved by Schrödinger himself leaving open the problem to provide the correct physical interpretation of the formal structure. The Schrödinger proposal encountered severe difficulties due to the fact that, in general, the solutions of the evolution equation inevitably spread in space as time grows.

The finally accepted interpretation was given by Born (1926) in 1926. According to his proposal, $|\psi(x, t)|^2$ is the probability density to find the object in x at time t. After Born, Quantum Mechanics was accepted more generally as a theory which can only provide probabilistic predictions (of the position or of any other observable relative to a microscopic object).

A further important step in the direction of a deeper understanding of the formalism was made in 1927 by Heisenberg. In his paper on the uncertainty principle Heisenberg (1927) he was able to prove the uncertainty relations for position and momentum as a consequence of the non commutativity of the corresponding operators q and p. The physical implications of the uncertainty relations were illustrated through the famous thought experiment with a gamma-ray microscope to determine the exact position of an electron. The conclusion was that a more accurate determination of the electron's position implies a less accurate determination of the momentum and vice versa. According to Heisenberg, the operational impossibility to determine both position and momentum of a particle was the origin of the statistical nature of Quantum theory, making, at the same time, the classical notions of position, momentum and trajectory inadequate to describe the microscopic world.

The work of Heisenberg on uncertainty relations stimulated Bohr to clarify his point of view on the interpretation of the theory. The main purpose of Bohr was to harmonize wave and matrix mechanics in a unified and coherent description of atomic phenomena. The occasion was the lecture delivered at the congress in Como, on September 1927 (Bohr 1928), and also the discussions in the subsequent Solvay Conference in Brussels, on October 1927 (Bacciagaluppi and Valentini 2009). Bohr's approach soon became the core of the so-called Copenhagen or standard interpretation of quantum mechanics. The main features of such interpretation scheme can be roughly summarized as follows: (i) completeness, (ii) wave-particle duality, (iii) lack of causal and space-time description, (iv) crucial role of the observation.

(i) Completeness means that the wave function ψ describes the actual state of an individual system and the probabilistic meaning of $|\psi|^2$ has an ontological character, i.e. it does not refer to our ignorance of hidden parameters.
(ii) Wave-particle duality should be understood in the sense that a microscopic system cannot be described in terms of only one of the two classical concepts of wave or particle. The system rather behaves either as a wave or as a particle, depending

on the context or, more precisely, on the experimental apparatus used to observe the system. Therefore wave and particle behavior are two mutually exclusive and complementary aspects of the microscopic object essence.

(iii) According to Bohr, Heisenberg uncertainty relations show the impossibility of having a causal description (based on the precise determination of the momentum) and a space-time description (based on the precise determination of the position) simultaneously. In this sense classical concepts like causality and space-time descriptions must also be considered as two mutually exclusive and complementary aspects of the description of a quantum system.

(iv) The last important point is the role of the observation. A measurement apparatus must be considered as a classical object, characterized by a precise determination of its classical properties. The measurement process consists in the interaction between the microscopic system and the classical apparatus. The result of a measurement is the determination of one of the possible complementary properties of the quantum system. The crucial point is that the measured property of the system cannot be thought as pre-existing to the measurement process. The property is produced only as the result of the interaction with the classical apparatus. More precisely, the emerging property is determined by the whole experimental context, i.e. the specific preparation of the system and the characteristics of the apparatus. This, in particular, means that the state of the system immediately before the measurement (when the system does not possess the given property) is different from the state immediately after the measurement (when the system possesses the property). The abrupt change of the state of the system determined by the measurement process is called wave packet reduction.

It is worth mentioning that the above conception of the measurement process is the most delicate aspect of the Copenhagen interpretation and it has raised a long debate that still continues. Here we only briefly mention some problematic points.

First, it is not explained why all or part of the experimental device, despite being made of atoms, must behave as a classical object with well-defined classical properties.

Moreover it is not clear where the borderline between the measurement apparatus (characterized by a classical behavior) and the system (characterized by a quantum behavior) should be put. The problem is usually solved pragmatically in each specific situation but, at a conceptual level, the ambiguity remains.

Finally, one has to renounce to describe the interaction of the system with the apparatus using the Schrödinger equation. This fact was formalized in 1932 by von Neumann (1955) who postulated two different kinds of evolution for the system: a genuine quantum evolution governed by the Schrödinger equation when the system is not measured and a different (stochastic and non linear) evolution producing the wave packet reduction when the system is measured.

We only want to mention that many attempts have been made to clarify these conceptual problems within the framework of the standard interpretation but a reasonable and universally accepted solution has not been found.

From the point of view of our analysis it is interesting to observe that the problematic aspects of the system-apparatus interaction are immediately evident when one approaches the description of a quantum particle in a cloud chamber. In fact, for a quantum mechanical description of the process one must take into account that the initial state of the particle emitted by the source does not have the form of a semi-classical wave packet but rather a highly correlated continuous superposition of states with well localized position and momentum. In particular, according to the first theoretical analysis of the radioactive decay given by Gamow (1928), the emitted α-particle must be described by a wave function having the form of a spherical wave, with center in the radioactive nucleus and isotropically propagating in space. Therefore, the non trivial problem arises as to how such a superposition state can produce the observed classical trajectories. As we shall see in the next sections, in such explanatory scheme a crucial role is played by the act of 'observation", where the system to be measured can be the α-particle (in particular its position) as well as the atoms of the vapor (in particular their excitation).

3 Early contributions

In this section we shall briefly summarize the first theoretical attempts to explain the apparent contradiction between the highly correlated superposition state of the α-particle and the observed tracks in the chamber. In particular, we shall consider the contributions of Born (1927), Heisenberg (1929–30) and Darwin (1929).

3.1 Born (1927)

The theoretical explanation of the observed tracks in a cloud chamber was already approached by Born (for the first time to our knowledge) in 1927 during the general discussion at the Solvay conference (Bacciagaluppi and Valentini 2009). In his words: "*Mr. Einstein has considered the following problem: a radioactive sample emits α -particles in all directions; these are made visible by the method of the Wilson cloud chamber. Now, if one associates a spherical wave with each emission process, how can one understand that the track of each α -particle appears as a (very nearly) straight line? In other words: how can the corpuscular character of the phenomenon be reconciled here with the representation by waves?*"

According to Born, the answer can be given resorting to the notion of "*reduction of the probability packet*" discussed by Heisenberg (1927). According to this notion, the observation of the electron position by light of wavelength λ would produce the reduction of the probability packet for the position of the electron to a region of linear size λ. In the subsequent evolution the packet spreads in space until a new observation reduces it again to a packet of width λ. Then, in Heisenberg's words: "*Every determination of position reduces therefore the wave packet back to its original size λ*". This mechanism of reduction would be responsible for the appearance of a (nearly) classical trajectory of the electron.

The same idea is used by Born in the case of the cloud chamber. Here the observation by means of light must be replaced by ionization of the atoms of the vapor in the chamber: "*As soon as such ionization is shown by the appearance of cloud droplets, in order to describe what happens afterwards one must reduce the wave packet in the immediate vicinity of the drops. One thus obtains a wave packet in the* *form of a ray, which corresponds to the corpuscular character of the phenomenon*". It is worth to emphasize that, according to this reasoning, the whole process is described in terms of the interaction of a quantum system (the α-particle) with a classical measurement apparatus (the atoms of the vapor). Such interaction, which is not described by Schrödinger equation, produces "reduction" of the spherical wave to a wave packet with definite position and momentum.

Following a suggestion by Pauli, Born continues discussing the possibility to describe both the α-particle and the atoms of the vapor as constituents of a unique quantum system, whose wave function depends on the coordinates of all the particles of the system. In particular, Born proposes, as an example, a simplified one dimensional model consisting of the α-particle and only two atoms. The α-particle is initially in a superposition state of two wave packets with opposite momentum and position close to the origin. The two cases where atoms are placed on the same side or on opposite sides with respect to the origin are considered. Born discusses, at a purely qualitatively level, the probability that during the time evolution of the whole system the two atoms will be hit in the two different cases. We simply give the conclusions reached by Born without going into details (for a quantitative analysis of the same model, see e.g. Dell'Antonio et al. 2008).

The first statement he makes, without invoking the reduction of the wave packet, is that the α-particle has negligible probability to hit both atoms unless they are on the same side with respect to the origin.

On the other hand, Born concludes his argument having an explicit recourse to the reduction postulate, saying: "*To the reduction of the wave packets corresponds the choice of one of the two directions of propagations*", and the choice is made as soon as one observes the excitation of an atom, as a consequence of the collision. Only starting from such an observation the evolution of the α-particle can be described as a real classical trajectory.

In conclusion, it seems that according to Born a more detailed description of the α-particle in a cloud chamber taking into account the presence of the environment is in fact possible "*but this does not lead us further as regards the fundamental questions*".

3.2 Heisenberg (1929–30)

As already mentioned, Born's analysis was explicitly inspired by the considerations of Heisenberg (1927) on the applicability of the notion of classical trajectory of an electron. It is remarkable that Heisenberg himself explicitly reconsidered the cloud chamber problem in his lectures at the University of Chicago in 1929, published in Heisenberg (1927), following the same line of thought of Born but with considerably more details. His approach, described in chapter 5 of the book, can be considered an exhaustive qualitative investigation of the problem according to the standard interpretation of Quantum Mechanics. This analysis had a deep influence on the physics community for many years.

Heisenberg's first (rather extreme) remark is that the whole experimental situation could be satisfactorily described using only classical mechanics, but it might also be of interest to discuss the problem from the point of view of quantum theory.

He stresses that, approaching a quantum theoretical description, one is immediately faced with the problem of separating the quantum system from the apparatus. In the case of the cloud chamber, one has two different reasonable choices: (a) the quantum system consists of the α-particle alone (and then the molecules of the vapor play the role of the measurement apparatus); (b) the quantum system consists of the α-particle and the molecules of the vapor. It should be emphasized that the physical descriptions of the experimental situation obtained following the two different choices are explicitly considered equivalent. Heisenberg's line of reasoning in examining the two cases proceeds as follows:

In case (a) the single molecule of the vapor measures the position of the α-particle. Assume that the molecule (supposed at rest) occupies a volume Δq around the point q_1 and t_1 is the collision time between α-particle and molecule. As result of the measurement process, the state of the α-particle is suddenly reduced and therefore at time t_1 it has position q_1 with spread Δq. On the other hand one knows the position q_0 of the α-particle at time t_0, when it leaves the radioactive source. Since no external force is present, one infers that the momentum of the α-particle at time t_1 is $p_1 = M(q_1 - q_0)/(t_1 - t_0)$, where M denotes the mass. It is then possible to conclude that for $t > t_1$ the α-particle is described by the free evolution of a wave packet starting from q_1, with initial spread Δq and momentum along the straight line γ joining q_0 and q_1. Therefore, the center of the wave packet moves along the same line γ. During the time evolution the wave packet inevitably spreads out. On the other hand the α-particle collides with other molecules placed along γ and after each collision the same measurement process of the position takes place. In this way the spreading is repeatedly reduced and the wave packet remains focused around the straight line γ, corresponding to the observed "trajectory" of the α-particle.

Let us consider case (b), where the molecules of the vapor are considered part of the quantum system. Heisenberg starts with an interesting claim that would have probably deserved further analysis: in case (b) the procedure to account for the observed trajectories is more complicated but, on the other hand, it allows to hide the role of the reduction of the wave packet.

Then he goes on to describe a simplified model made of the α-particle plus only two molecules. The molecules are non interacting, their centers of mass are fixed in the positions a_I, a_{II} and the internal coordinates are denoted by q_I, q_{II}. It is assumed that the Hamiltonians of the two molecules have a complete set of eigenfunctions $\varphi_{n_I}(q_I)$, $\varphi_{n_{II}}(q_{II})$, corresponding to a discrete set of eigenvalues labeled by integers n_I, n_{II}. The initial state of the system is chosen in the form of a product of the ground states of the molecules (labeled by n_I^0, n_{II}^0) times a plane wave with momentum p for the α-particle.

The interesting object to compute is the probability that both molecules are excited and the result of the computation is that such a probability is significantly different from zero only if the momentum p is parallel to the line joining a_I and a_{II}. Since the passage of the α-particle is indirectly observed through the excitations of the molecules, the result explains why one can only see straight trajectories in a cloud chamber.

The solution of the Schrödinger equation for the three-particle system is approached treating the interaction between the α-particle and the molecules as small perturbation of the free dynamics and assuming that the momentum p is large with respect to the spacing of energy levels of the molecules. The perturbative computation is not developed in all details by Heisenberg. Here we only summarize the main steps of his procedure in order to clarify the line of reasoning.

The Schrödinger equation is solved by iteration and at the first order the wave function can be written in the form

$$\psi^{(1)} = e^{-i\frac{t}{\hbar}E_0} \sum_{n_I, n_{II}} w^{(1)}_{n_I n_{II}}(x)\varphi_{n_I}(q_I)\varphi_{n_{II}}(q_{II}) \tag{3.1}$$

where E_0 denotes the total energy of the system, x the position coordinate of the α-particle and the coefficients $w^{(1)}_{n_I n_{II}}$ satisfy an equation which is easily derived from the original Schrödinger equation. From Born's rule it follows that $|w^{(1)}_{n_I n_{II}}(x)|^2$ represents the probability density (at first order) to find the α-particle in x when the molecules are in the states labeled by n_I, n_{II}.

From the equation for $w^{(1)}_{n_I n_{II}}$ it is immediately evident the first result: the probability that both molecules are excited is zero at first order.

The second result claimed by Heisenberg is definitively less evident and nevertheless it is stated without detailed proof. It says that $w^{(1)}_{n_I n^0_{II}}$ is significantly different from zero only in a strip parallel to p located behind the molecule I, whose thickness (close to the molecule) is of the same order of the dimension of the molecule. The same kind of result is obviously true for $w^{(1)}_{n^0_I n_{II}}$.

Then he continues with the analysis of the wave function at the second order

$$\psi^{(2)} = e^{-i\frac{t}{\hbar}E_0} \sum_{n_I, n_{II}} w^{(2)}_{n_I n_{II}}(x)\varphi_{n_I}(q_I)\varphi_{n_{II}}(q_{II}) \tag{3.2}$$

Writing the equation for $w^{(2)}_{n_I n_{II}}$ and exploiting the results obtained at the first order, he derives the desired final result. The probability density at second order $|w^{(2)}_{n_I n_{II}}(x)|^2$, with $n_I \neq n^0_I$ and $n_{II} \neq n^0_{II}$, is significantly different from zero only if the following two situations occur: the molecule II is in the strip of $w^{(1)}_{n_I n^0_{II}}$ or the molecule I is in the strip of $w^{(1)}_{n^0_I n_{II}}$.

The procedure can be iterated with an arbitrary number of molecules and therefore the linearity of the trajectories is proved.

At the end of the analysis, Heisenberg makes a second claim on the problem of the wave packet reduction. In particular, he explains that in case (b) the reduction takes place when one arranges a measurement process "to observe" the excitation of the molecules. This simply means that the unavoidable line of separation between the system and the apparatus has been moved to include the molecules in the system. In this sense one should probably understand the previous claim that the reduction in case (b) is hidden.

In summary, Heisenberg's analysis of the cloud chamber, like Born's analysis, insists to consider the treatments in case (a) and (b) as conceptually equivalent. This belief is founded on the fact that in any case the recourse to the reduction of the wave packet cannot be avoided.

3.3 Darwin (1929)

A further relevant contribution to the explanation of the trajectories in a cloud chamber was given by Darwin (1929) in 1929. In this paper one does not find any analysis of specific models. Nevertheless there is a detailed discussion of the problem and it is clearly stated a possible strategy for an approach entirely based on the use of Schrödinger equation.

Darwin approaches a collision problem in the framework of wave mechanics with the aim to "*take a problem which would be regarded at first sight as irreconcilable with a pure wave theory, but thoroughly typical of the behavior of particles, and show how in fact the correct result arises naturally from the consideration of waves alone*". He emphasizes that in order to obtain the correct predictions on the behavior of a given system $\mathcal{S}$ one has to take into account its interaction with (part of) the environment $\mathcal{E}$. Therefore the wave function ψ is not a wave in ordinary three dimensional space but rather it is a function of the coordinates of $\mathcal{S}$ and of $\mathcal{E}$. Only when such ψ has been computed, the probabilistic predictions on $\mathcal{S}$ are obtained by taking an average over all possible final configurations of $\mathcal{E}$. Such procedure, even if "*discouragingly complicated*", can account for the particle-like behavior working only on ψ and without invoking any act of observation. It is worth noticing that the program outlined by Darwin essentially coincides with the basic strategy of modern decoherence theory.

After these considerations, Darwin discusses a concrete example where the intuitive particle behavior can be derived from the analysis of the wave function. That part of the analysis is not directly connected with the cloud chamber and therefore it is not relevant for our purposes. However, in the final part of the paper, one finds some other interesting considerations.

He examines the case of the ray tracks of α-particles in a cloud chamber, "*one of the most striking manifestations of particle characters*", in connection with Gamow's theory of radioactive decay (Gamow 1928), distinguishing two different points of view.

According to the first one "*we must regard Gamow's calculations as determining only the probability of disintegration, and that when this has taken place, we start the next stage by assigning a definite direction for the motion of the α -particle; after which we reconvert it into a wave, but now on a narrow front, so as to find its subsequent history*".

As an alternative to this point of view, he first notices that α-rays can in principle exhibit diffraction and therefore it is reasonable to assign a real existence to the spherical wave outside the nucleus. Then he discusses a possible wave description of the experiment. The wave function ψ is a function of the coordinates of the α-particle and of the coordinates of the atoms in the chamber and, before the first collision, it is a product of the spherical wave for the α-particle times a set of stationary (in general ground) states for the atoms. "*But the first collision changes this product into a function in which the two types of coordinates are inextricably mixed, and every subsequent collision makes it worse*". Such complicated function contains a phase factor and "*without in the least seeing the details, it looks quite natural to expect that this phase factor will have some special character, such as vanishing, when the various co-ordinates satisfy a condition of collinearity*".

It is interesting to notice that Darwin clearly identifies stationary phase arguments as the crucial technical tools required to predict the particle-like behavior.

Then he continues: "*so without pretending to have mastered the details, we can understand how it is possible that the ψ function, so to speak, not to know in what direction the track is to be, but yet to insist that it should be a straight line. The decision as to actual track can be postponed until the wave reaches the uncovered part, where the observations are made*". This approach seems to have a general validity. In Darwin's view the wave-particle duality proposed by Bohr can be avoided. The whole quantum theory can be based on the wave function ψ, considered as the central object from which all the particle or wave properties can be accurately described, at least until a real measurement is performed. In his words "*it thus seems legitimate to suppose that it is always admissible to postpone the stage, at which we are forced to think of particles, right up to the point at which they are actually observed*".

4 Mott's paper

The program enunciated by Darwin was concretely realized by Mott in his seminal paper of 1929. In the introduction Mott recognizes to have been inspired by Darwin's paper in his attempt to explain the typical particle-like properties of an α-particle in a cloud chamber using only wave mechanics. He admits that such a point of view seems at first sight counterintuitive, since "*it is a little difficult to picture how it is that an outgoing spherical wave can produce a straight track; we think intuitively that it should ionise atoms at random throughout space*". Like Heisenberg, Mott points out that the crucial point is to establish the borderline between the system under consideration and the measuring device. He recalls the two possible approaches: in the first the α-particle is the quantum system under consideration (and the gas of the chamber is part of the measuring device) while in the second approach the quantum system consists of the α-particle and of the atoms of the gas.

Mott proceeds toward a detailed analysis of the problem following closely the latter approach. He claims that the intuitive difficulty mentioned above can be overcome since it arises from our erroneous "*tendency to picture the wave as existing in ordinary three dimensional space, whereas we are really dealing with wave functions in multispace formed by the co-ordinates both of the α -particle and of every atom in the Wilson chamber*".

The model considered by Mott consists of the α-particle, initially described by a spherical wave centered at the origin, and the electrons in two hydrogen atoms. The nuclei of the atoms are supposed at rest in the fixed positions $\mathbf{a_1}, \mathbf{a_2}$, with $|a_1| < |a_2|$. It is assumed that the α-particle does not interact with the nuclei and that the interaction between the two electrons is negligible. The interaction between the α-particle and the electrons is assumed to be weak. The model is essentially the same considered by Heisenberg in case (b). Nevertheless, as we shall see, Mott's analysis is definitely more detailed and therefore the outcome appears more transparent and convincing.

The main result of the paper can be summarized in the following statement: under suitable assumptions (which will be specified later) the two hydrogen atoms cannot both be excited (or ionized) unless $\mathbf{a_1}, \mathbf{a_2}$ and the origin lie on the same straight line.

We shall describe how Mott derives the result trying to follow the original notation and line of reasoning. Main objects of the investigation are periodic solutions $F(\boldsymbol{R}, \boldsymbol{r_1}, \boldsymbol{r_2})e^{iEt/\hbar}$ of the Schrödinger equation for the three particle system, where $\boldsymbol{R}, \boldsymbol{r_1}, \boldsymbol{r_2}$ denote the coordinates of the α-particle and of the two hydrogen atom electrons respectively. The function F (depending parametrically on E) is solution of the stationary Schrödinger equation

$$-\frac{\hbar^2}{2M}\Delta_R F + \left(-\frac{\hbar^2}{2m}\Delta_{r_1} - \frac{e^2}{|\boldsymbol{r_1} - \boldsymbol{a_1}|}\right) F + \left(-\frac{\hbar^2}{2m}\Delta_{r_2} - \frac{e^2}{|\boldsymbol{r_2} - \boldsymbol{a_2}|}\right) F \\ - \left(\frac{2e^2}{|\boldsymbol{R} - \boldsymbol{r_1}|} + \frac{2e^2}{|\boldsymbol{R} - \boldsymbol{r_2}|}\right) F = EF \tag{4.1}$$

where Δ_x is the Laplacian with respect to the coordinate x, M is the mass of the α-particle, m is the mass of the electron, $-e$ is the charge of the electron so that $2e$ is the charge of the α-particle.

The solution of Eq. (4.1) can be conveniently expanded in series of eigenfunctions of the two electrons in the hydrogen atoms. More precisely, let ψ_j be the j-th eigenfunction of an hydrogen atom centered in the origin, with ψ_0 denoting the ground state. Then the corresponding eigenfunctions of the atoms in $\mathbf{a_1}, \mathbf{a_2}$ are

$$\Psi_j^I(\boldsymbol{r_1}) = \psi_j(\boldsymbol{r_1} - \boldsymbol{a_1}), \quad \Psi_j^{II}(\boldsymbol{r_2}) = \psi_j(\boldsymbol{r_2} - \boldsymbol{a_2}) \tag{4.2}$$

We notice that here it is tacitly assumed that the index j can be an integer or a real positive number (and correspondingly ψ_j can be a proper eigenfunction or a generalized eigenfunction).

Exploiting completeness of the system of eigenfunctions, we have the following representation for F

$$F(\boldsymbol{R}, \boldsymbol{r_1}, \boldsymbol{r_2}) = \sum_{j_1, j_2} f_{j_1 j_2}(\boldsymbol{R})\Psi_{j_1}^I(\boldsymbol{r_1})\Psi_{j_2}^{II}(\boldsymbol{r_2}) \tag{4.3}$$

The Fourier coefficients $f_{j_1 j_2}(\mathbf{R})$ of the expansion have a direct physical interpretation. Indeed, exploiting Born's rule, the probability to find the first atom in the state labeled by j_1 and the second atom in the state labeled by j_2 is

$$\int d\boldsymbol{R}|f_{j_1 j_2}(\boldsymbol{R})|^2 \tag{4.4}$$

One can pictorially say that the "wave function" of the α-particle is $f_{00}(\mathbf{R})$ if both atoms remain in the ground state, $f_{j_1 0}(\mathbf{R})$, $j_1 \neq 0$, if the first atom is in the j_1-th excited (or ionized) state and the second in the ground state, $f_{j_1 j_2}(\mathbf{R})$, $j_1, j_2 \neq 0$, if both atoms are excited (or ionized).

The analysis will show that $f_{00}(\mathbf{R})$ is a (slightly deformed) spherical wave and $f_{j_1 0}(\mathbf{R})$, $j_1 \neq 0$, is a wave packet emerging from $\mathbf{a_1}$ with a momentum along the line $\overline{O\mathbf{a_1}}$. This means that the second atom can be excited by such wave packet only if $\mathbf{a_2}$ lies on the line $\overline{O\mathbf{a_1}}$. Thus the desired result will follow, i.e. $f_{j_1 j_2}(\mathbf{R})$, $j_1, j_2 \neq 0$, is approximately zero unless the condition of co-linearity is satisfied.

5 Derivation of the result

Exploiting the assumed weakness of the interaction between α-particle and electrons, the computation is carried out using second order perturbation theory. If we write

$$F = F^{(0)} + F^{(1)} + F^{(2)} + \cdots \tag{5.1}$$

then by successive approximation one has for $n \geq 1$

$$\begin{aligned}&-\frac{\hbar^2}{2M}\Delta_R F^{(n)} + \left(-\frac{\hbar^2}{2m}\Delta_{r_1} - \frac{e^2}{|\mathbf{r}_1 - \mathbf{a}_1|}\right)F^{(n)} + \left(-\frac{\hbar^2}{2m}\Delta_{r_2} - \frac{e^2}{|\mathbf{r}_2 - \mathbf{a}_2|}\right)F^{(n)} - EF^{(n)}\\ &= \left(\frac{2e^2}{|\mathbf{R} - \mathbf{r}_1|} + \frac{2e^2}{|\mathbf{R} - \mathbf{r}_2|}\right)F^{(n-1)}\end{aligned} \tag{5.2}$$

For $n = 0$ one has to solve the unperturbed equation:

$$\begin{aligned}&-\frac{\hbar^2}{2M}\Delta_R F^{(0)} + \left(-\frac{\hbar^2}{2m}\Delta_{r_1} - \frac{e^2}{|\mathbf{r}_1 - \mathbf{a}_1|}\right)F^{(0)}\\ &+\left(-\frac{\hbar^2}{2m}\Delta_{r_2} - \frac{e^2}{|\mathbf{r}_2 - \mathbf{a}_2|}\right)F^{(0)} - EF^{(0)} = 0\end{aligned} \tag{5.3}$$

As solution of (5.3) one chooses a diverging spherical wave times the ground state of the two atoms

$$\begin{aligned}F^{(0)}(\boldsymbol{R}, \boldsymbol{r}_1, \boldsymbol{r}_2) &= f_{00}^{(0)}(\boldsymbol{R})\Psi_0^I(\boldsymbol{r}_1)\Psi_0^{II}(\boldsymbol{r}_2),\\ f_{00}^{(0)}(\boldsymbol{R}) &= \frac{e^{ik|\boldsymbol{R}|}}{|\boldsymbol{R}|},\\ k &= \frac{\sqrt{2M(E-2E_0)}}{\hbar}\end{aligned} \tag{5.4}$$

where E_j denotes the j-th eigenvalue of the hydrogen atom and $f_{00}^{(0)}$ is the outgoing solution of the Helmotz equation. Notice that the use of stationary Schrödinger equation forces Mott to choose a solution not in L^2 which, strictly speaking, is not legitimate. In particular, the probabilistic interpretation (4.4) fails for $f_{00}^{(0)}$. For the first order term $F^{(1)}$ we write

$$F^{(1)}(\boldsymbol{R}, \boldsymbol{r}_1, \boldsymbol{r}_2) = \sum_{i_1, i_2} f_{i_1 i_2}^{(1)}(\boldsymbol{R})\Psi_{i_1}^I(\boldsymbol{r}_1)\Psi_{i_2}^{II}(\boldsymbol{r}_2) \tag{5.5}$$

Substituting (5.5) into (5.2) for $n = 1$ one has

$$\sum_{i_1,i_2} \left(-\frac{\hbar^2}{2M}\Delta_R + E_{i_1} + E_{i_2} - E \right) f^{(1)}_{i_1 i_2}(\boldsymbol{R})\Psi^I_{i_1}(\boldsymbol{r_1})\Psi^{II}_{i_2}(\boldsymbol{r_2})$$
$$= \left(\frac{2e^2}{|\mathbf{R}-\mathbf{r_1}|} + \frac{2e^2}{|\mathbf{R}-\mathbf{r_2}|} \right) f^{(0)}_{00}(\boldsymbol{R})\Psi^I_0(\boldsymbol{r_1})\Psi^{II}_0(\boldsymbol{r_2}) \tag{5.6}$$

Multiplying the above equation by $\Psi^I_{j_1}(\boldsymbol{r_1})\Psi^{II}_{j_2}(\boldsymbol{r_2})$ and integrating over the coordinates of the electrons one obtains the equation for $f^{(1)}_{j_1 j_2}(\boldsymbol{R})$

$$\left(-\frac{\hbar^2}{2M}\Delta_R + E_{j_1} + E_{j_2} - E \right) f^{(1)}_{j_1 j_2}(\boldsymbol{R})$$
$$= f^{(0)}_{00}(\boldsymbol{R}) \left(\delta_{j_2 0} \int d\boldsymbol{r}_1 \frac{2e^2}{|\mathbf{R}-\mathbf{r_1}|} \Psi^I_0(\boldsymbol{r_1})\Psi^I_{j_1}(\boldsymbol{r_1}) \right.$$
$$\left. + \delta_{j_1 0} \int d\boldsymbol{r}_2 \frac{2e^2}{|\mathbf{R}-\mathbf{r_2}|} \Psi^{II}_0(\boldsymbol{r_2})\Psi^{II}_{j_2}(\boldsymbol{r_2}) \right) \tag{5.7}$$

Equation (5.7) can be rewritten in the more compact form

$$\left(\frac{\hbar^2}{2M}\Delta_R + E - E_{j_1} - E_{j_2} \right) f^{(1)}_{j_1 j_2}(\boldsymbol{R}) = K_{j_1 j_2}(\boldsymbol{R}) \tag{5.8}$$

where

$$K_{j_1 j_2}(\boldsymbol{R}) = f^{(0)}_{00}(\boldsymbol{R}) \Big(\delta_{0 j_2} V_{j_1 0}(\boldsymbol{R} - \boldsymbol{a}_1) + \delta_{j_1 0} V_{0 j_2}(\boldsymbol{R} - \boldsymbol{a}_2) \Big) \tag{5.9}$$

$$V_{ij}(\boldsymbol{x}) = -\int d\boldsymbol{y} \frac{2e^2}{|\boldsymbol{x}-\boldsymbol{y}|} \psi_i(\boldsymbol{y})\psi_j(\boldsymbol{y}) \tag{5.10}$$

The computation of the first order term $F^{(1)}$ is now reduced to the solution of the non-homogeneous Helmotz Eq. (5.8). Mott refers to the treatise by Courant and Hilbert, Methoden der Mathematischen Physik, chap. 5, par. 10, to assert that the most general solution is

$$f^{(1)}_{j_1 j_2}(\boldsymbol{R}) = G_{j_1 j_2}(\boldsymbol{R}) + \frac{M}{2\pi\hbar^2} \int d\boldsymbol{R}' K_{j_1 j_2}(\boldsymbol{R}') \frac{e^{\pm ik'|\boldsymbol{R}-\boldsymbol{R}'|}}{|\boldsymbol{R}-\boldsymbol{R}'|},$$
$$k' = \frac{\sqrt{2M(E - E_{j_1} - E_{j_2})}}{\hbar} \tag{5.11}$$

where $G_{j_1 j_2}$ is an arbitrary solution of the homogeneous equation $(\Delta + k'^2)G_{j_1 j_2} = 0$. In our case the phase in the exponential must be taken with the sign + since we are interested in waves diverging from $\boldsymbol{a}_1$ or $\boldsymbol{a}_2$. Moreover Mott argues that $G_{j_1 j_2}$ "*represents* *streams of particles fired at already excited atom*" while, as initial condition, we have both atoms in their ground state. Therefore one must require

$$G_{j_1 j_2} = 0 \tag{5.12}$$

Furthermore, from (5.9) one sees that $K_{j_1 j_2}(\boldsymbol{R}) = 0$ if both j_1 and j_2 are different from zero and therefore, by (5.11), one also has $f^{(1)}_{j_1 j_2} = 0$ if both j_1 and j_2 are different from zero.

From these preliminary considerations a first conclusion can be drawn: at first order in perturbation theory the probability that both atoms are excited is always zero.

The result is not surprising since, as Mott remarks, in perturbation theory the probability that one atom is excited is a first order quantity while the probability that both atoms are excited is a second order quantity. This explains why the second order term $F^{(2)}$ is required in order to obtain an estimate of a double excitation occurrence.

The further crucial step is to give an approximate expression for $f^{(1)}_{j_1 0}$ and $f^{(1)}_{0 j_2}$. From (5.11), (5.12) and (5.9), for $f^{(1)}_{j_1 0}$ one has

$$f^{(1)}_{j_1 0}(\boldsymbol{R}) = \frac{M}{2\pi\hbar^2}\int d\boldsymbol{y}\, f^{(0)}_{00}(\boldsymbol{y}+\boldsymbol{a}_1) V_{j_1 0}(\boldsymbol{y}) \frac{e^{ik'|\boldsymbol{R}-\boldsymbol{a}_1-\boldsymbol{y}|}}{|\boldsymbol{R}-\boldsymbol{a}_1-\boldsymbol{y}|}, \qquad j_1 \neq 0 \tag{5.13}$$

and analogously for $f^{(1)}_{0 j_2}$. In order to find the required approximate expression Mott introduces the following assumptions:

(a) the "observation point" $\boldsymbol{R}$ is far away from the origin and the atom, i.e. $|\boldsymbol{a_1}| \ll |\boldsymbol{R}|$;
(b) the collisions for the α-particle are almost elastic, i.e. $k - k' \ll k$;
(c) the α-particle has a high momentum k, i.e. k^{-1} is much less than the effective linear dimension of the atom.

Exploiting assumption (a) one obtains the asymptotic formula:

$$f^{(1)}_{j_1 0}(\boldsymbol{R}) \simeq \frac{e^{ik'|\boldsymbol{R}-\boldsymbol{a}_1|}}{|\boldsymbol{R}-\boldsymbol{a}_1|}\frac{M}{2\pi\hbar^2}\int d\boldsymbol{y}\, f^{(0)}_{00}(\boldsymbol{y}+\boldsymbol{a}_1) V_{j_1 0}(\boldsymbol{y}) e^{-ik'\boldsymbol{u}_1(\boldsymbol{R})\cdot\boldsymbol{y}} \tag{5.14}$$

where

$$\boldsymbol{u}_1(\boldsymbol{R}) = \frac{\boldsymbol{R}-\boldsymbol{a}_1}{|\boldsymbol{R}-\boldsymbol{a}_1|} \tag{5.15}$$

Using the explicit expression of $f^{(0)}_{00}$ [see (5.4)] and assumption (b) one can write:

$$f^{(1)}_{j_1 0}(\boldsymbol{R}) \simeq \frac{e^{ik'|\boldsymbol{R}-\boldsymbol{a}_1|}}{|\boldsymbol{R}-\boldsymbol{a}_1|}\mathcal{I}(\boldsymbol{u}_1(\boldsymbol{R})) \tag{5.16}$$

$$\mathcal{I}(\boldsymbol{u}_1(\boldsymbol{R})) = \frac{M}{2\pi\hbar^2}\int d\boldsymbol{y}\frac{V_{j_1 0}(\boldsymbol{y})}{|\boldsymbol{y}+\boldsymbol{a}_1|} e^{ik(|\boldsymbol{y}+\boldsymbol{a}_1| - \boldsymbol{u}_1(\boldsymbol{R})\cdot\boldsymbol{y})} \tag{5.17}$$

One sees that $f^{(1)}_{j_1 0}(\boldsymbol{R})$ has the form of a wave diverging from $\boldsymbol{a}_1$, whose amplitude $\mathcal{I}$ is given by the integral in (5.17) and it is explicitly dependent on the direction $\boldsymbol{u}_1(\boldsymbol{R})$. The crucial point is now to evaluate such amplitude.

By assumptions (c), the integral in (5.17) is a highly oscillatory integral and then stationary phase arguments can be used. The leading term of the asymptotic expansion for $k \to \infty$ is determined by the value of the integrand at the critical points of the phase, i.e. for points $\boldsymbol{y}$ such that

$$\nabla_{\boldsymbol{y}}\Big(|\boldsymbol{y}+\boldsymbol{a}_1| - \boldsymbol{u}_1(\boldsymbol{R})\cdot\boldsymbol{y}\Big) = \frac{\boldsymbol{y}+\boldsymbol{a}_1}{|\boldsymbol{y}+\boldsymbol{a}_1|} - \boldsymbol{u}_1(\boldsymbol{R}) = 0 \tag{5.18}$$

On the other hand, the integrand in (5.17) is very small except in a neighborhood of $\boldsymbol{y}=0$. Therefore one obtains the condition

$$\boldsymbol{u}_1(\boldsymbol{R}) \simeq \frac{\boldsymbol{a}_1}{|\boldsymbol{a}_1|} \tag{5.19}$$

Using condition (5.19) in (5.15) one has that the amplitude $\mathcal{I}$ is significantly different from zero only for those $\boldsymbol{R}$ such that $\boldsymbol{R}-\boldsymbol{a}_1$ is (almost) parallel to $\boldsymbol{a}_1$, i.e. the observation point $\boldsymbol{R}$ must be (almost) aligned with the first atom and the origin.

From the above argument one concludes that $f^{(1)}_{j_1 0}(\boldsymbol{R})$ is approximately given by a wave diverging from $\boldsymbol{a}_1$ with an amplitude very small except for $\boldsymbol{R}$ satisfying (5.19), i.e. except in a small cone with vertex in $\boldsymbol{a}_1$ and pointing away from the origin. An analogous analysis is valid for $f^{(1)}_{0 j_2}(\boldsymbol{R})$ and therefore the computation of the first order term $F^{(1)}$ is completed.

The next step is to consider the second order term $F^{(2)}$. Proceeding as above, one has:

$$F^{(2)}(\boldsymbol{R},\boldsymbol{r}_1,\boldsymbol{r}_2) = \sum_{i_1,i_2} f^{(2)}_{i_1 i_2}(\boldsymbol{R})\Psi^I_{i_1}(\boldsymbol{r}_1)\Psi^{II}_{i_2}(\boldsymbol{r}_2) \tag{5.20}$$

and

$$\begin{aligned}
&\left(-\frac{\hbar^2}{2M}\Delta_R + E_{j_1} + E_{j_2} - E\right) f^{(2)}_{j_1 j_2}(\boldsymbol{R}) \\
&= \int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \left(\frac{2e^2}{|\boldsymbol{R}-\boldsymbol{r}_1|} + \frac{2e^2}{|\boldsymbol{R}-\boldsymbol{r}_2|}\right) F^{(1)}(\boldsymbol{R},\boldsymbol{r}_1,\boldsymbol{r}_2)\Psi^I_{j_1}(\boldsymbol{r}_1)\Psi^{II}_{j_2}(\boldsymbol{r}_2) \\
&= \delta_{0 j_2}\sum_{i_1} f^{(1)}_{i_1 0}(\boldsymbol{R})V_{j_1 i_1}(\boldsymbol{R}-\boldsymbol{a}_1) + f^{(1)}_{j_1 0}(\boldsymbol{R})V_{0 j_2}(\boldsymbol{R}-\boldsymbol{a}_2) \\
&\quad +\delta_{j_1 0}\sum_{i_2} f^{(1)}_{0 i_2}(\boldsymbol{R})V_{j_2 i_2}(\boldsymbol{R}-\boldsymbol{a}_2) + f^{(1)}_{0 j_2}(\boldsymbol{R})V_{j_1 0}(\boldsymbol{R}-\boldsymbol{a}_1)
\end{aligned} \tag{5.21}$$

In the case $j_1, j_2 \neq 0$ one finds

$$\begin{aligned}
&\left(-\frac{\hbar^2}{2M}\Delta_R + E_{j_1} + E_{j_2} - E\right) f^{(2)}_{j_1 j_2}(\boldsymbol{R}) \\
&\quad = f^{(1)}_{j_1 0}(\boldsymbol{R})V_{0 j_2}(\boldsymbol{R}-\boldsymbol{a}_2) + f^{(1)}_{0 j_2}(\boldsymbol{R})V_{j_1 0}(\boldsymbol{R}-\boldsymbol{a}_1)
\end{aligned} \tag{5.22}$$

We recall that $|\boldsymbol{a}_1| < |\boldsymbol{a}_2|$, $V_{j_1 0}(\boldsymbol{R} - \boldsymbol{a}_1)$ is negligible except for $\boldsymbol{R} \simeq \boldsymbol{a}_1$ and $f^{(1)}_{0 j_2}(\boldsymbol{R})$ is negligible except for $\boldsymbol{R}$ in a small cone with vertex in $\boldsymbol{a}_2$, pointing away from the origin. This means that the last term in the r.h.s. of (5.22) is very small. The same kind of argument shows that the first term in the r.h.s. of (5.22) is negligible except when the second atom is (approximately) aligned with the first atom and the origin. Thus the main result of the paper follows: the probability that both atoms are excited

$$\int d\boldsymbol{R}|f^{(2)}_{j_1 j_2}(\boldsymbol{R})|^2, \quad j_1, j_2 \neq 0 \tag{5.23}$$

is (approximately) zero unless $\boldsymbol{a}_1$, $\boldsymbol{a}_2$ and the origin lie (approximately) on the same straight line. If one agrees that the (amplified) effect of the excitations of the atoms is the true observed phenomenon in a cloud chamber then the result can be rephrased saying that one can only observe straight tracks. In this sense Mott provides an explanation of the straight tracks observed in the chamber entirely based on the Schrödinger equation.

6 Concluding remarks

We want to conclude with a few comments on the different theoretical approaches to the cloud chamber problem discussed in the previous sections.

As was pointed out before, according to Born and Heisenberg, it is definitely equivalent to consider the atom of the vapor as a (classical) measurement device (case a) or as a part of the quantum system to be described by Schrödinger dynamics (case b). Such a position is meant to guarantee the consistency of the standard interpretation of quantum theory. In particular, the authors are interested in stressing the unavoidable role of wave packet reduction as the crucial rule ensuring the right correspondence between theory and observed physical world. From a purely foundational point of view their reasoning aims to make the axiomatic scheme work in any chosen way to address the cloud chamber dynamical problem and, as such, it has been adopted and shared by the majority of the physics community for a long time.

However, from the concrete point of view of the physical description of quantum systems, the equivalence of the two approaches (a) and (b) seems difficult to maintain. In fact, one may find hard to accept the claim that an atom of the vapor is a classical measurement device of the position of the α-particle. After all, the atom is a microscopic system on the same ground of the α-particle and there is no a priori reason to regard it as a classical system. Agreeing with this point of view, one should concede that approach (b) is surely more natural. In particular, it has the important advantage to allow a quantitative analysis taking into account explicitly the physical parameters characterizing system and environment. It is only on the basis of such a quantitative investigation that it is possible to clarify the conditions under which the interaction with the environment produces the appearance of classical trajectories.

We want to stress this crucial point: the classical behavior of the α-particle is far from being universal. A completely dissimilar behavior has to be expected if the values of the physical parameters are different. On the other hand, consider Heisenberg's argument about "the atom as a measurement apparatus", within the framework we called approach (a). The resulting reduction of the wave packet should drive the α

particle in a state where the position is close to the ionized atom and the momentum has a direction deduced by the successive positions around the source (at time zero) and around the ionized atom (after the flight time to the first ionized atom). The argument seems independent of the interaction strength, while quantum analysis clarifies that, as expected, only an almost zero-angle scattering process (very low energy exchange) can guarantee that particle momentum will approximately lie in a small cone with axis on the line connecting source and ionized atom.

A further point to be emphasized concerns the relevance of the result obtained in the approach b). It is shown that, under the right conditions and with high probability, one can only observe straight tracks. This is a highly non trivial result obtained exploiting Schrödinger dynamics without having any recourse to the wave packet reduction rule. The reduction should only be invoked at the stage of the "observation of the actual track" described by the α-particle. It is remarkable that, as we have seen in Sect. 3, this aspect was understood by Darwin even before the quantitative analysis was performed by Mott.

On the basis of the above considerations, it seems to us that, from the physical point of view, the approach followed by Mott is more significant and challenging. In some sense, one can say that it establishes the validity conditions of the effective dynamical behavior of the system which is only postulated in case (a) without real physical motivations. It is interesting to notice that awareness of this fact is clearly expressed by Darwin while it remains only implicit in Mott, who prudently avoids expressing a preference.

From an historical point of view, it is interesting to understand the reasons why the line of research initiated by Mott was not further developed and remained almost neglected for many years. One reason could be the influence and the authority of the position expressed by Born and Heisenberg. The consequence has been to discourage the new approach to the problem with the motivation that it was ineffectively more complicated without giving real advantages from the conceptual point of view.

The whole problem of classical behavior in quantum systems has been rediscovered, starting around the eighties of last century, when remarkable experimental progress has made a detailed exploration of the classical/quantum border possible. This progress has motivated the development of decoherence theory, based on the construction of theoretical models of "system plus environment" where the dynamics of the emergence of classical behavior in a quantum system could be analyzed and quantified. Therefore, any investigation based on the wave packet reduction is of no help and one is forced to consider the problem entirely in the context of Schrödinger dynamics.

As examples of successful applications of decoherence theory we mention the reduction of interference effects for the state of a heavy particle due to scattering by light particles (Joos and Zeh 1985; Hornberger and Sipe 2003; Adami et al. 2006) or the molecular localization for pyramidal molecules, like NH_3, due to the dipole-dipole interaction among the molecules (Claverie and Jona-Lasinio 1986; Grecchi et al. 2002).

It is worth underlining that the pioneering work of Mott is entirely within the same line of thought. Moreover, we emphasize that the relevance of such a research is more related to the concrete and applied analysis of quantum systems rather than to foundational aspects.

We want to conclude commenting on some limitations present in Mott's analysis and on possible developments of his work. The first point concerns the use of the stationary Schrödinger equation which prevents a clear description of the time evolution of the three-particle system. In particular, it is missing a physically meaningful definition of the initial state and an explicit time-dependent analysis of the successive interactions of the α-particle with the two atoms. A second aspect is related to the proof techniques used by Mott which exploit perturbation theory and stationary phase arguments. There is a clear lack of control of the conditions in which these methods are applicable and there is no estimate of error arising from the various approximations used by the author (for a recent attempt to revisit Mott's model we refer to Dell'Antonio et al. 2010).

We finally notice that a further development of Mott's work is lacking in the literature. In particular, the model should be generalized considering a more generic initial state for the particle, the presence of external force fields and an environment made by an arbitrary number of model atoms. In this way one could analyze the possible emergence of a classical trajectory of the quantum particle in a more realistic model. In our opinion, such analysis would surely be of great interest both from the theoretical and from the applicative point of view.

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