GIRGs thesis - Benjamin Dayan - Marc & Ulysse

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February 24, 2023

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1 Erdos Renyi Random Graphs

$G_{n,p}$ has uniform iid edge probability of p.

Expected Degree is $(n-1)p$, and $\deg(v) \sim \text{binomial}(n-1, p)$.

If we fix $p = \mu/n$ s.t. $E[\deg(v)] \overset{n\to\infty}{\to} \mu$, then $\deg(v) \overset{n\to\infty}{\sim} \text{Poisson}(\mu)$

1.1 Galton-Watson Branching process

GWBP with offspring dist. $\mathcal{D} \in \mathbb{N}_0$ generates a random rooted tree T where every vertex independently has $\mathcal{D}$ children, e.g. $\mathcal{D} = \text{Poisson}(\mu)$.

Theorem (2.3 - connected component structure like GWBP trees)**.** This is a weird statement. We show it precisely by showing that $P(|C_u| = s) \overset{n\to\infty}{\to} P(|T| = s)$, for any node $u \in V$. This we do by coupling the connected component produced (as a BFS) with the GWBP (only concentrating on outgoing edges (to non-visited nodes)).

Further with any connected component (e.g. of 5 nodes) produced as such, it has $\to 0$ probability of containing any additional cyclic edges (ignored in the GWBP). This corroborates e.g. $E[\#\text{triangles}] = \Theta(n^3p^3)$ is constant.

Ok