

Answer: semi supervised EM algorithm for Gaussian mixture model is straightforward. We have log likelihood $l_{\text{semisup}}(\theta) = l_{\text{unsup}}(\theta) + \alpha l_{\text{sup}}(\theta)$ which we wish to maximise, where here $\theta = \phi, \mu, \Sigma$. for the E-step we compute $Q_i(z_i) = p(z_i|x_i; \phi, \mu, \Sigma)$ for the unsupervised points $x_i \in \{x_1, \dots, x_n\}$. We parametrise the distribution $Q_i(z_i) \in \mathbb{R}^k$ for k choices of Gaussian distribution by writing

$$\begin{aligned}
 Q_i(z_i)_j &= w_j^{(i)} = p(z_i = j|x_i; \phi, \mu, \Sigma) \\
 &= p(x|z) \frac{p(z)}{p(x)} \\
 &= p(x_i|z_i = j) \frac{p(z_i = j)}{p(x_i)} \\
 &= N(\mu_j, \Sigma_j) \frac{\phi_j}{\sum_{l=1}^k p(x_i|z_i = l)p(z_i = l)} \\
 &= \frac{N(x_i; \mu_j, \Sigma_j)\phi_j}{\sum_{l=1}^k N(x_i; \mu_l, \Sigma_l)\phi_l}
 \end{aligned}$$