Answer:

(a) $K_{ij} = K'_{ij} + K''_{ij}$ is still symmetric, and still positive definite via $a^T K a = a^T K' a + a^T K'' a$

(b) $K_i j = K'_i j - K''_i j$ is not necessarily symmetric semi-pos-def as e.g. $K'' = 2K'$ then $K = -K'$ is now symmetric semi-neg-def

(c) $K = aK'$ where $a > 0$ is clearly still a kernel

(d) $K = -aK'$ where $a > 0$ is clearly never a kernel

(e) $K(x, z) = K'(x, z)K''(x, z)$ i.e. $K_{ij} = K'_{ij} K''_{ij}$ (no summation) is a bit tricky. Consider that since K', K'' are both kernels, then e.g. $K'(x, z) = \alpha(x)^T \alpha(z)$ and $K''(x, z) = \beta(x)^T \beta(z)$ for some unknown dimensional maps α, β. Then writing α_i for $\alpha(x_i)$ for our arbitrary set of vectors under consideration $\{x_1, ..., x_n\}$ (that gives us the corresponding $n \times n$ matrix K_{ij}),

$$\begin{aligned}
a^T K a &= \sum_{i,j} a_i K_{ij} a_j = \sum_{i,j} a_i (\alpha_i^T \alpha_j \times \beta_i^T \beta_j) a_j \\
&= \sum_{i,j} a_i (\sum_k \alpha_{ik}^T \alpha_{jk})(\sum_l \beta_{il}\beta_{jl}) a_j \\
&= \sum_{k,l} \sum_i \left[a_i \alpha_{ik} \beta_{il} \left(\sum_j a_j \alpha_{jk} \beta_{jl} \right) \right] \\
&= \sum_{k,l} \left(\sum_i a_i \alpha_{ik} \beta_{il} \right)^2 \geq 0
\end{aligned}$$

(f) $K(x, z) = f(x)f(z)$ for a real valued func $f : \mathbb{R}^d \to \mathbb{R}$, then $a^T K a = a_i f_i f_j a_j = (\sum_i a_i f_i)^2 \geq 0$

(g) $K(x, z) = K'(\phi(x), \phi(z))$ for some func $\phi : \mathbb{R}^d \to \mathbb{R}^p$ then still as above, $a^T K a = a_i \phi_i \phi_j a_j \geq 0$

(h) $K(x, z) = p(K'(x, z))$ for some polynomial p with positive coefficients, then we can use the previous parts of the question to construct K and still be a valid kernel, e.g. $K', K'K', K'K'K', ...$ are all valid kernels from part (e), and we can add positive scaled kernels together and still have a kernel by other parts.