

Answer: The loss function, NLL, is $l(\theta) = -\log(p(y; \eta)) = -\log(b(y)) + a(\theta^T x) - y\theta^T x$. Then $\frac{\partial l(\theta)}{\partial \theta} = xa'(\theta^T x) - yx$ and $\frac{\partial^2 l(\theta)}{\partial \theta^2} = xx^T a''(\theta^T x) = xx^T \text{Var}(Y|\eta = \theta^T x)$. Hence the Hessian is positive semi-definite, as the variance is non-negative, by the fact that $z^T H z = (z^T x)^2 \text{Var}(Y|\eta = \theta^T x) \geq 0 \forall z$