**Answer:**

$$\begin{aligned}
\theta_{MAP} &= \text{argmax}_\theta \ \log p(y|x,\theta) + \log p(\theta) \quad \text{using fact that LR model has } p(\theta|x) = p(\theta) \\
&= \text{argmax}_\theta \ \log \left( \exp(-\frac{(y - X\theta)^T \Sigma (y - X\theta)}{2}) \right) + \log(\prod_i \frac{1}{2b} \exp\left(\frac{-|\theta_i|}{b}\right)) \\
&= \text{argmin}_\theta \ \frac{\sigma^{-2}}{2} ||y - X\theta||_2^2 + \frac{1}{b} ||\theta||_1
\end{aligned}$$

Hence finding $\theta_{MAP}$ is equivalent to solving the linear regression problem with $L_1$ regularization, i.e. minimising the loss $J(\theta) = ||y - X\theta||_2^2 + \gamma ||\theta||_1$, where here $\gamma = \frac{2\sigma^2}{b}$