Answer: For events A, B, C we have

$$\begin{aligned} p(A|B) &= \frac{p(A,B)}{p(B)} = \frac{p(A,B,C)}{p(B,C)} \frac{p(B,C)}{p(B)} \frac{p(A,B)}{p(A,B,C)} \\ &= \frac{p(A|B,C)p(C|B)}{p(C|A,B)} \end{aligned}$$

So writing $t^{(i)} = 1$ as event A, $x^{(i)} = x$ as event B, and $y^{(i)} = 1$ as event C, we get

$$p(t=1|x) = \frac{p(t=1|x, y=1)p(y=1|x)}{p(y=1|x, t=1)} = \frac{p(y=1|x)}{\alpha}$$

Actually an alternative better way of showing the required relation is by conditioning $y^{(i)}, t^{(i)}|_{x^{(i)}=x}$, giving

$$\begin{aligned} p(y=1|x) &= \sum_t p(y=1|x, t=t)\; p(t=t|x) \\ &= p(y=1|x, t=1)\; p(t=1|x) + p(y=1|x, t=0)\; p(t=0|x) \\ &= \alpha\; p(t=1|x) \end{aligned}$$