

Answer: We had the variance derivation of PCA for $k = 1$ being seeking to maximise the variance of the projections, i.e. maximising $\sum_{i=1}^n (u^T x_i)^2$ (where we use the fact that the vectors x_i have been mean zeroed already to ensure that the mean of $u^T x_i$ is 0, so we are indeed capturing the sample variance). So then we're equivalently maximising $u^T (\sum_{i=1}^n x_i x_i^T) u$ which shows us that the optimal unit vector u is the eigenvector of the largest eigenvalue of the sample covariance matrix.

In the minimal projection error setup we want to instead minimise over unit vector u the value of $\sum_{i=1}^n \|x_i - f_u(x_i)\|^2$, however $f_u(x_i) = (u^T x_i)u$, so we're minimising over

$$\sum_{i=1}^n \|x_i - (u^T x_i)u\|^2 = \sum_{i=1}^n (x_i - (u^T x_i)u)^T (x_i - (u^T x_i)u) = \sum_{i=1}^n x_i^T x_i - (u^T x_i)^2$$

where the last line comes through since $u^T u = 1$. Hence equivalent to the previous variance maximisation.