

Answer:

$$J(\theta) = -\frac{1}{n} \sum_{i=1}^n \left(y^{(i)} \log(h_{\theta}(x^{(i)})) + (1 - y^{(i)}) \log(1 - h_{\theta}(x^{(i)})) \right),$$

So then

$$\begin{aligned} \nabla J(\theta) &= -\frac{1}{n} \sum_{i=1}^n \left(y^{(i)} \frac{1}{h_{\theta}(x^{(i)})} h(1-h)x^{(i)} - (1 - y^{(i)}) \frac{1}{1 - h_{\theta}(x^{(i)})} h(1-h)x^{(i)} \right) \\ &= -\frac{1}{n} \sum_{i=1}^n \left(x^{(i)} \left[(1-h)y^{(i)} - (1 - y^{(i)})h \right] \right) \\ &= -\frac{1}{n} \sum_{i=1}^n \left(x^{(i)} \left[(1-h)y^{(i)} - (1 - y^{(i)})h \right] \right) \\ &= -\frac{1}{n} \sum_{i=1}^n \left(x^{(i)} \left[y^{(i)} - h \right] \right) \end{aligned}$$

And then

$$\nabla \nabla J(\theta) = -\frac{1}{n} \sum_{i=1}^n (-xx^T h(1-h)) = H$$

So

$$z^T H z = \frac{1}{n} \sum_{i=1}^n \left(z x^{(i)} (x^{(i)})^T z^T h_{\theta}(x^{(i)}) (1 - h_{\theta}(x^{(i)})) \right) \geq 0$$

as $h, (1-h) \in (0, 1)$