Answer:

$$\begin{aligned}
\theta_{MAP} &= \mathsf{argmin}_\theta - \log p(y|x,\theta) + \lambda||\theta||_2^2 \\
&= \mathsf{argmin}_\theta - \log\left(\exp(-\frac{(y-X\theta)^T\Sigma(y-X\theta)}{2})\right) + \lambda||\theta||_2^2 \\
&= \mathsf{argmin}_\theta \frac{\sigma^2}{2}||y-X\theta||_2^2 + \lambda||\theta||_2^2
\end{aligned}$$

We take ∇_θ of the RHS and set to zero to find the minimal θ,

$$\begin{aligned}
\frac{\sigma^2}{2}(-2X^Ty + 2X^TX\theta) + 2\lambda\theta = 0 \\
(\sigma^2X^TX + 2\lambda I)\theta = \sigma^2X^Ty \\
\theta = (\sigma^2X^TX + 2\lambda I)^{-1}\sigma^2X^Ty
\end{aligned}$$