Answer: Noting that $0 = \frac{\partial}{\partial \eta} \int p(y;\eta) dy = \int \frac{\partial}{\partial \eta} p(y;\eta) dy = \int p(y;\eta) [y-a'(\eta)] dy$ we get that $E(Y|\eta) = \int p(y;\eta) y dy = \int p(y;\eta) a'(\eta) dy = a'(\eta)$