Answer:

$$\begin{split} p(y=1\mid x;\phi,\mu_0,\mu_1,\Sigma) &= p(x|y=1,\phi,\mu_0,\mu_1,\Sigma) \frac{p(y=1)}{p(x|\phi,\mu_0,\mu_1,\Sigma)} \\ &= \frac{\phi f_{y=1}(x)}{\phi f_{y=1}(x) + (1-\phi) f_{y=0}(x)} \\ &= \frac{1}{1 + \frac{1-\phi}{\phi} \exp\left(-\frac{1}{2}(x-\mu_1)^T \Sigma^{-1}(x-\mu_1) + \frac{1}{2}(x-\mu_0)^T \Sigma^{-1}(x-\mu_0)\right)} \\ &= \frac{1}{1 + \frac{1-\phi}{\phi} \exp\left(-(\mu_0-\mu_1)^T \Sigma^{-1}x - \frac{1}{2}(\mu_1^T \Sigma^{-1}\mu_1 - \mu_0^T \Sigma^{-1}\mu_0)\right)} \\ &= \frac{1}{1 + \exp(-(\theta^T x + \theta_0))}, \end{split}$$