Report - Delta hedging project

Benjamin DAVILA

10/12/2020

Summary of the project

From the data provided (stock history, column B) in the file *Dataset*, we want to create the portfolio history replication of a European call with maturity on **31/12/2018**, strike $\mathbf{K = 6}$.

Overview of the data

```
##          Date Prix.action Prix.option.strike.6 Prix.option.strike.6.5
## 1  02/10/2017       6.742                   NA                     NA
## 2  03/10/2017       6.955                   NA                     NA
## 3  04/10/2017       6.910              1.02096                0.59093
## 4  05/10/2017       7.006              1.09932                0.65885
## 5  06/10/2017       7.000              1.09306                0.65440
## 6  09/10/2017       6.950              1.04813                0.61591
## 7  10/10/2017       6.856              0.96879                0.54811
## 8  11/10/2017       6.795              0.91992                0.51253
## 9  12/10/2017       6.744              0.88119                0.49517
## 10 13/10/2017       6.561              0.74526                0.41163
```

I. Introduction

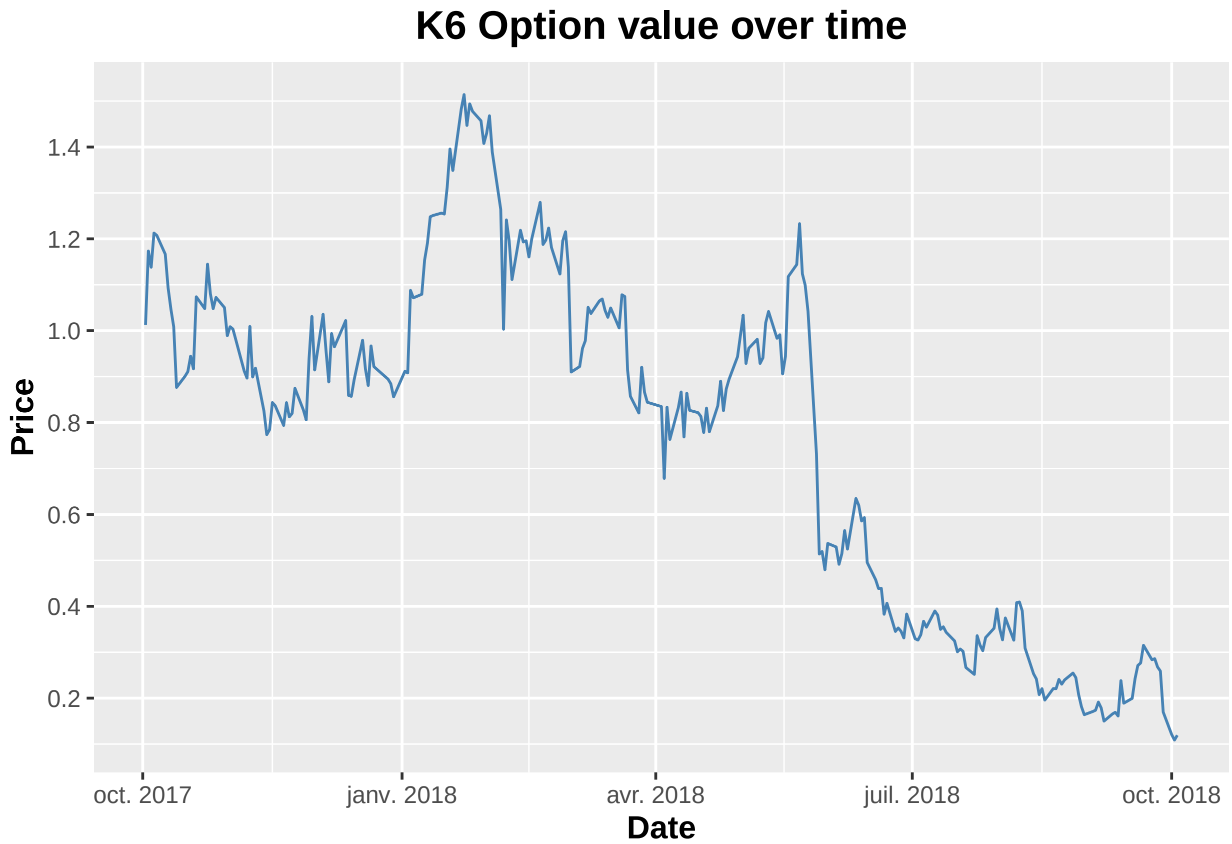
Let's assume here that $\sigma_{cte} = 20\%$. Let's compute now the vector of maturity thanks to the following formula : $T_i = \frac{(T_{mat} - t_i)}{365}$.

Overview of the vector of maturities

```
##  [1] 1.246575 1.243836 1.241096 1.238356 1.235616 1.227397 1.224658 1.221918
##  [9] 1.219178 1.216438
```

Then, we are going to use the **Black-Scholes formula** to price the corresponding european option call Black-Scholes formula : $C(S,t) = S_t N(d_2) - e^{-r(T-t)} K N(d_2)$ with $r = 0$, $\sigma = 20\%$ and $K = 6$

Overview of the call value over time



II. Delta-hedged portfolio with constant volatility

In this part, we are going to build a synthetic delta-hedged portfolio **formed by cash and underlying** associated with the option. The value of the portfolio **must be equal to the value of the option** to be replicated. The iteration process of replication is sum up below. Here, $\delta_t = N\big(d_1(t)\big)$ with $d_1(t) = \frac{1}{\sigma\sqrt{T_t}}\Big[ln\big(\frac{S_t}{K}\big) + \big(r + \frac{1}{2}\sigma^2\big)T_t\Big]$, S_t the value of the stock and T_t the vector of maturities

t	Option	Réplication	Poche action	Poche euros
0	C_0	$V_0 = C_0$	$\delta_0 S_0$	$C_0 - \delta_0 S_0$
1	C_1	$V_1 = V_0 + \delta_0 (S_1 - S_0)$	$\delta_0 S_1$ $\delta_1 S_1$	$C_0 - \delta_0 S_0$ $V_1 - \delta_1 S_1$
2	C_2	$V_2 = V_1 + \delta_1 (S_2 - S_1)$	$\delta_1 S_2$ $\delta_2 S_2$	$V_1 - \delta_1 S_1$ $V_2 - \delta_2 S_2$

We can deduce the general formula to obtain the value of the portfolio over time :

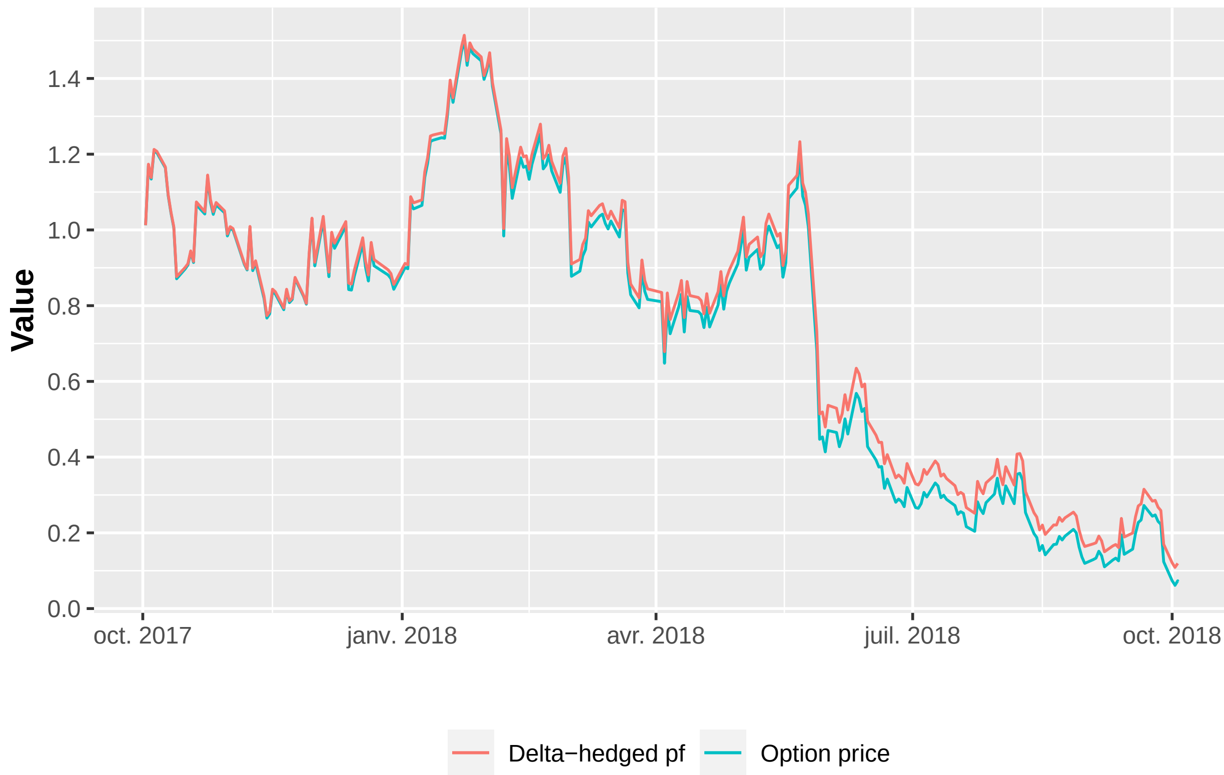
$V_t = V_{t-1} + \delta_{t-1}(S_t - S_{t-1})$

Now, let's compute the value of δ_t over time and give an overview of it

```
##  [1] 0.7368957 0.7804467 0.7719176 0.7903820 0.7894580 0.7805751 0.7621479
##  [8] 0.7496394 0.7388317 0.6968998
```

Here we are, we are going to use the previous iteration method mentioned above to build the portfolio history replication of the K6 option

Delta Hedging of the K6 Option with constant volatility



Interpretation : As we can see on the chart above, the two times series are almost equal over time. We can deduce that the hedge method is really efficient with a constant volatility.