Report - Delta hedging project

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Summary of the project

From the data provided (stock history, column B) in the file *Dataset*, we want to create the portfolio history of replication of a European call with maturity on **31/12/2018**, strike $\mathbf{K = 6}$.

Overview of the data

```
##          Date Prix.action Prix.option.strike.6 Prix.option.strike.6.5
## 1  02/10/2017       6.742                   NA                     NA
## 2  03/10/2017       6.955                   NA                     NA
## 3  04/10/2017       6.910              1.02096                0.59093
## 4  05/10/2017       7.006              1.09932                0.65885
## 5  06/10/2017       7.000              1.09306                0.65440
## 6  09/10/2017       6.950              1.04813                0.61591
## 7  10/10/2017       6.856              0.96879                0.54811
## 8  11/10/2017       6.795              0.91992                0.51253
## 9  12/10/2017       6.744              0.88119                0.49517
## 10 13/10/2017       6.561              0.74526                0.41163
```

I. Introduction

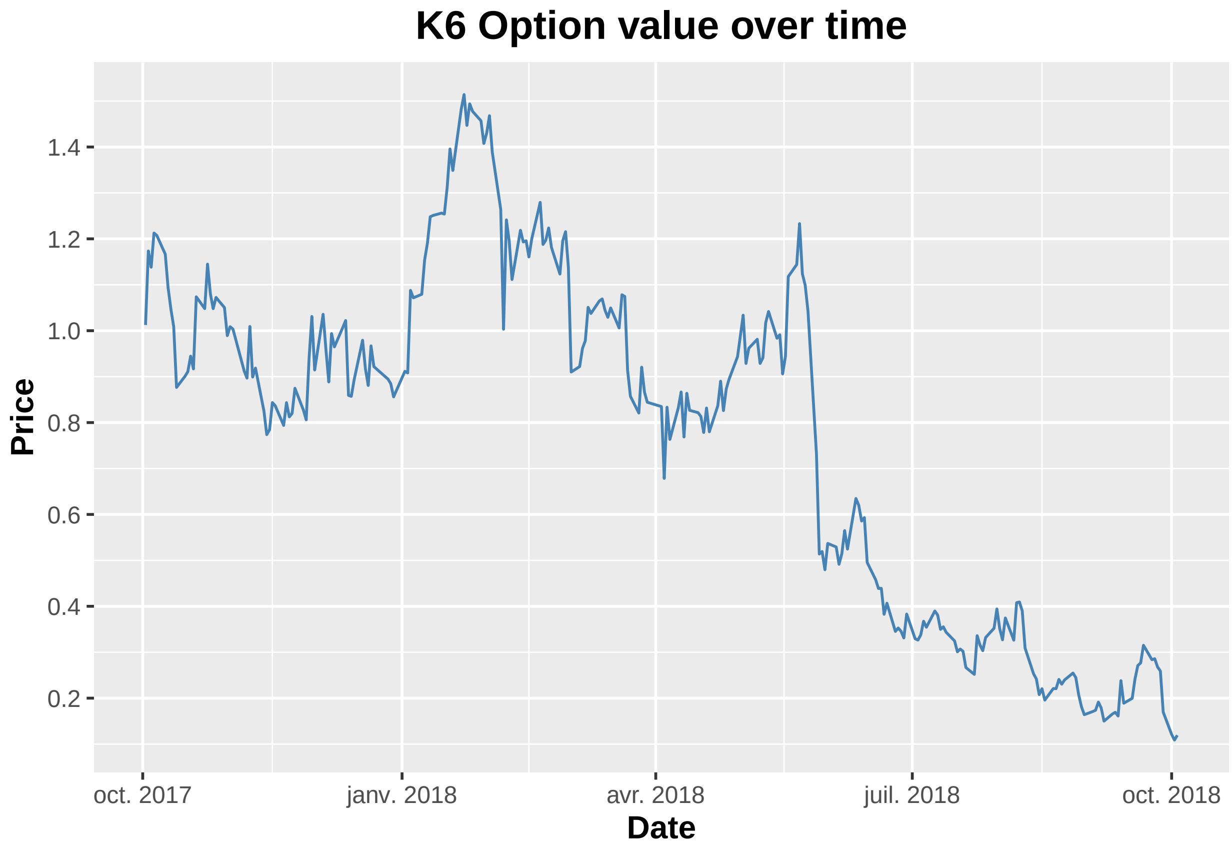
Let's assume here that $\sigma_{cte} = 20\%$. Let's compute now the vector of maturity thanks to the following formula $T_i = \frac{(T_{mat} - t_i)}{365}$.

Overview of the vector of maturities

```
##  [1] 1.246575 1.243836 1.241096 1.238356 1.235616 1.227397 1.224658 1.221918
##  [9] 1.219178 1.216438
```

Then, we are going to use the **Black-Scholes formula** to price the corresponding european option call Black-Scholes formula : $C(S,t) = S_t N(d_2) - e^{-r(T-t)} K N(d_2)$ with $r = 0$, $\sigma = 20\%$ and $K = 6$

Overview of the call value over time



II. Delta-hedged portfolio with constant volatility

In this part, we are going to build a synthetic delta-hedged portfolio **formed by cash and underlying** associated with the option. The value of the portfolio **must be equal to the value of the option** to be replicated. The iteration process of replication is sum up below. Here, $\delta_t = N\big(d_1(t)\big)$ with $d_1(t) = \frac{1}{\sigma\sqrt{T_t}}\Big[ln\big(\frac{S_t}{K}\big) + \big(r + \frac{1}{2}\sigma^2\big)T_t\Big]$, S_t the value of the stock and T_t the vector of maturities

t	Option	Réplication	Poche action	Poche euros
0	C_0	$V_0 = C_0$	$\delta_0 S_0$	$C_0 - \delta_0 S_0$
1	C_1	$V_1 = V_0 + \delta_0 (S_1 - S_0)$	$\delta_0 S_1$ $\delta_1 S_1$	$C_0 - \delta_0 S_0$ $V_1 - \delta_1 S_1$
2	C_2	$V_2 = V_1 + \delta_1 (S_2 - S_1)$	$\delta_1 S_2$ $\delta_2 S_2$	$V_1 - \delta_1 S_1$ $V_2 - \delta_2 S_2$

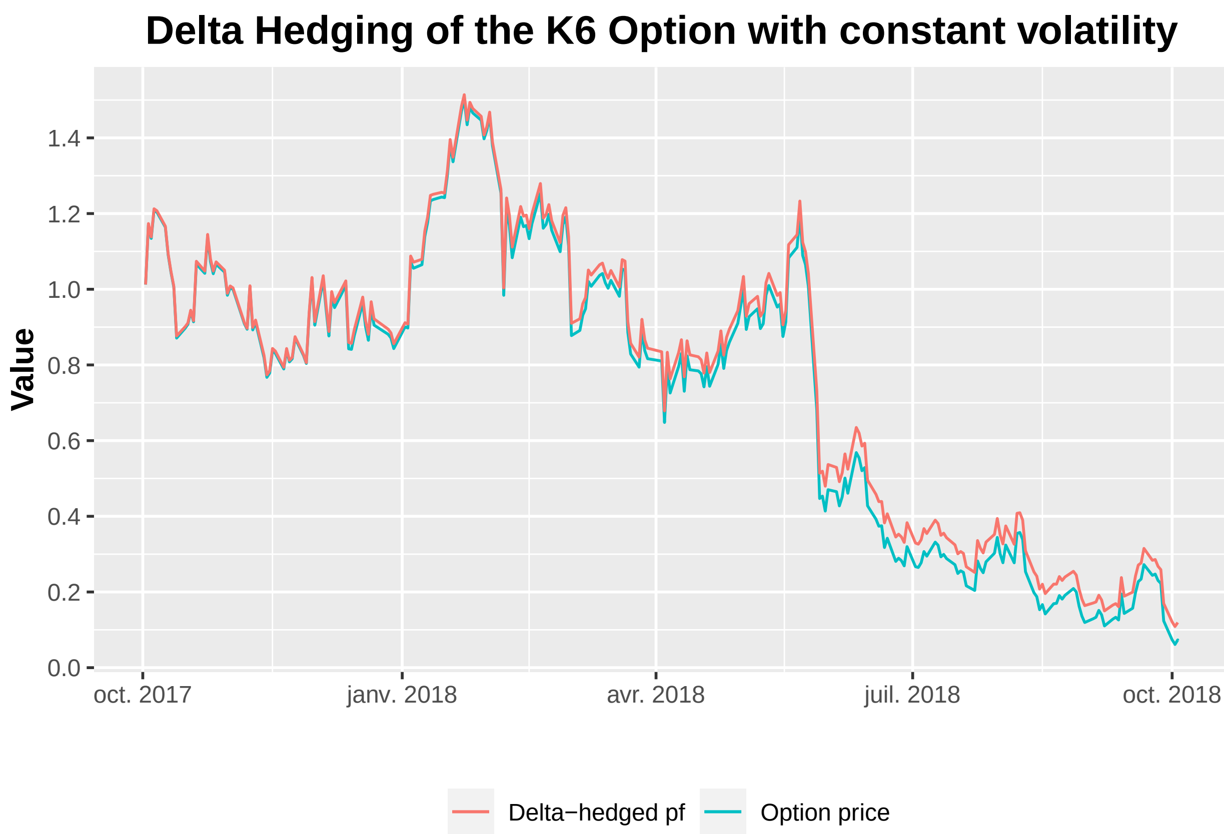
We can deduce the general formula to obtain the value of the portfolio over time :

$V_t = V_{t-1} + \delta_{t-1}(S_t - S_{t-1})$

Now, let's compute the value of δ_t over time and give an overview of it

```
##  [1] 0.7368957 0.7804467 0.7719176 0.7903820 0.7894580 0.7805751 0.7621479
##  [8] 0.7496394 0.7388317 0.6968998
```

Here we are, we are going to use the previous iteration method mentioned above to build the portfolio history of replication of the K6 option



Interpretation : As we can see on the chart above, the two times series are almost equal over time. We can deduce that the hedge method is really efficient with a constant volatility.