Report - Delta-Vega hedging project

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Summary of the project

From the data provided (stock history, column B) in the file *Dataset*, we want to create the portfolio history of replication of a european call with maturity on **31/12/2018**, strike $\mathbf{K = 6}$. To do that, we are going to use a synthetic replication portfolio formed from a stock, cash and another K6.5 european call option (to neutralize the effect of σ on the price of the option).

Overview of the data

```
##          Date Prix.action Prix.option.strike.6 Prix.option.strike.6.5
## 1  02/10/2017       6.742                   NA                     NA
## 2  03/10/2017       6.955                   NA                     NA
## 3  04/10/2017       6.910              1.02096                0.59093
## 4  05/10/2017       7.006              1.09932                0.65885
## 5  06/10/2017       7.000              1.09306                0.65440
## 6  09/10/2017       6.950              1.04813                0.61591
## 7  10/10/2017       6.856              0.96879                0.54811
## 8  11/10/2017       6.795              0.91992                0.51253
## 9  12/10/2017       6.744              0.88119                0.49517
## 10 13/10/2017       6.561              0.74526                0.41163
```

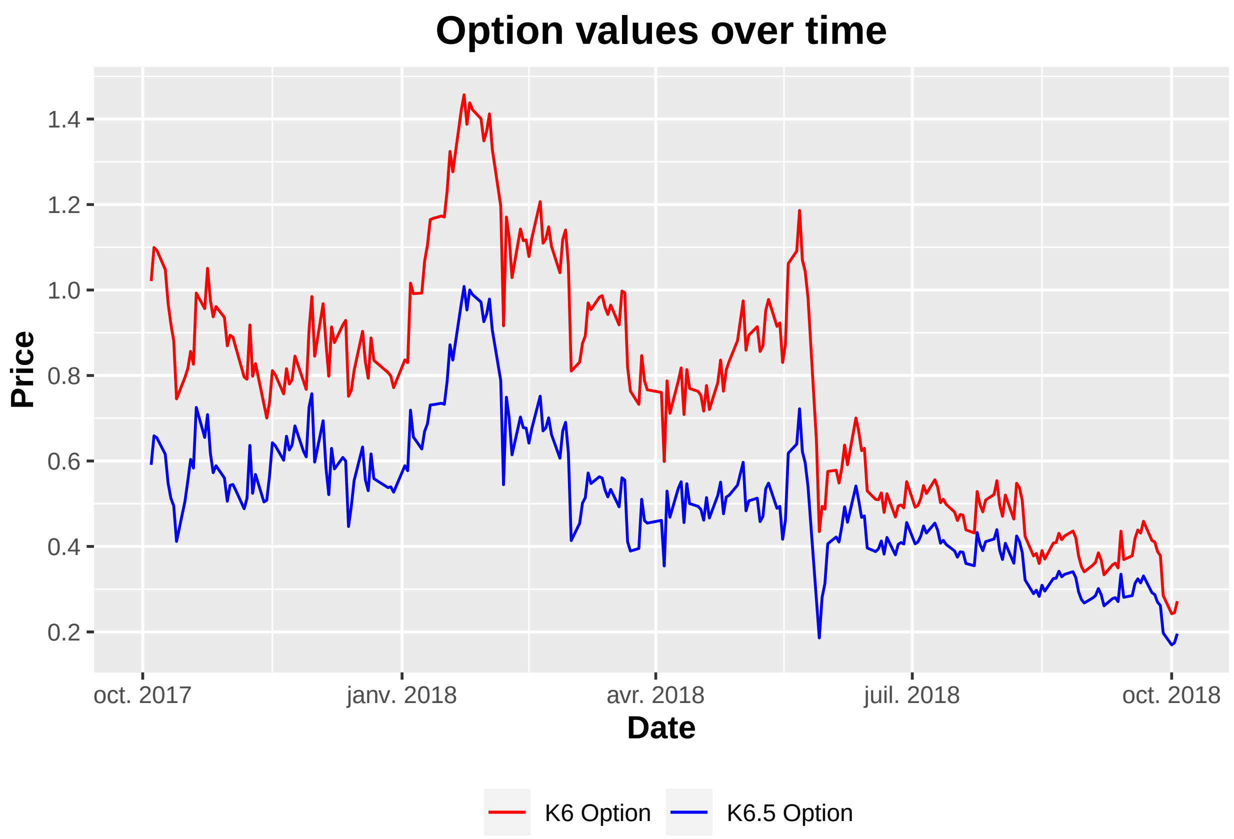
I. Introduction

First, let's compute now the vector of maturity thanks to the following formula $T_i = \frac{(T_{mat} - t_i)}{365}$.

Overview of the vector of maturities

```
##  [1] 1.246575 1.243836 1.241096 1.238356 1.235616 1.227397 1.224658 1.221918
##  [9] 1.219178 1.216438
```

Let's visualize the value of the two options over time.



II. Delta-Vega hedged portfolio with non constant volatility

In this part, we are going to build a synthetic delta-Vega hedged portfolio **formed by a stock, cash and a K6.5 call option**. The value of the portfolio **must be equal to the value of the option** to be replicated.

Overview of the principles of **delta-vega hedging** :

Option 1	S	Option 2	Liquidité
δ	1	δ'	0
ν	0	$\nu' \frac{\partial \sigma'}{\partial \sigma}$	0
1	q	q'	ℓ

We follow the same reasoning as for the previous project to find the following system :

$$\begin{cases} q_t + q'_t \delta'_t = \delta_t & (1) \\ q'_t = \frac{\nu_t}{\nu'_t \frac{\partial \sigma'}{\partial \sigma}} & (2) \\ q_t S_t + l_t + q'_t C'_t = C_t & (3) \end{cases}$$

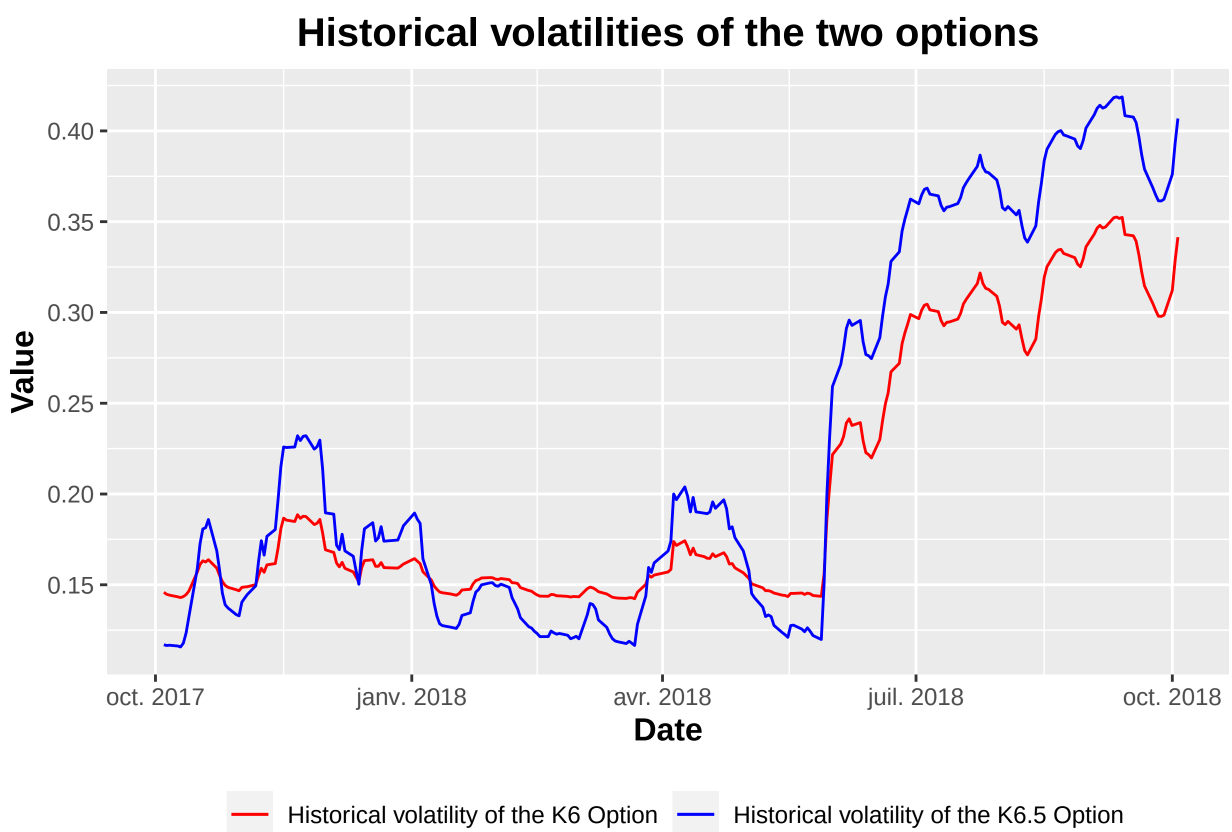
$$\begin{cases} q_t = \delta_t - \delta'_t \frac{\nu_t}{\nu'_t \frac{\partial \sigma'}{\partial \sigma}} & (1) \\ q'_t = \frac{\nu_t}{\nu'_t \frac{\partial \sigma'}{\partial \sigma}} & (2) \\ l_t = C_t - q'_t C'_t - q_t S_t & (3) \end{cases}$$

On the array above, we have different parameters. First, $\delta_t = N\big(d_1(t)\big)$ with $d_1(t) = \frac{1}{\sigma\sqrt{T_t}}\Big[ln\big(\frac{S_t}{K}\big) + \big(r + \frac{1}{2}\sigma^2\big)T_t\Big]$, S_t the value of the stock and T_t the vector of maturities. Furthermore, we have $\nu = \frac{\partial C}{\partial \sigma} = S\sqrt{T}N'(d_1)$, and $\frac{\partial \sigma'}{\partial \sigma} = \frac{\sigma'_{t+1} - \sigma'_t}{\sigma_{t+1} - \sigma_t}$. Here, σ_t and σ'_t represent the volatilities of the K6 and K6.5 options.

Besides, **we have another set of three parameters** : q, l and q' corresponding to the quantities of stock, cash and K6.5 option to be held in our portfolio. We can deduce the **general formula** to obtain the value of the portfolio over time :

$V_t = V_{t-1} + q_{t-1}(S_t - S_{t-1}) + q'_{t-1}(C'_t - C'_{t-1})$

In a first while, let's compute the **implied volatilities** of the K6 and K6.5 options over time using optimization algoritms (Newton-Raphson & bisection method) and get an overview of these



Now, let's compute the ν_t of the two options (thanks to the formula mentioned above) and get an overview of these

```
## [1]       NA       NA 1.955273 1.806168 1.805329 1.854272 1.978616 2.063676
## [9] 2.136711 2.361668
```

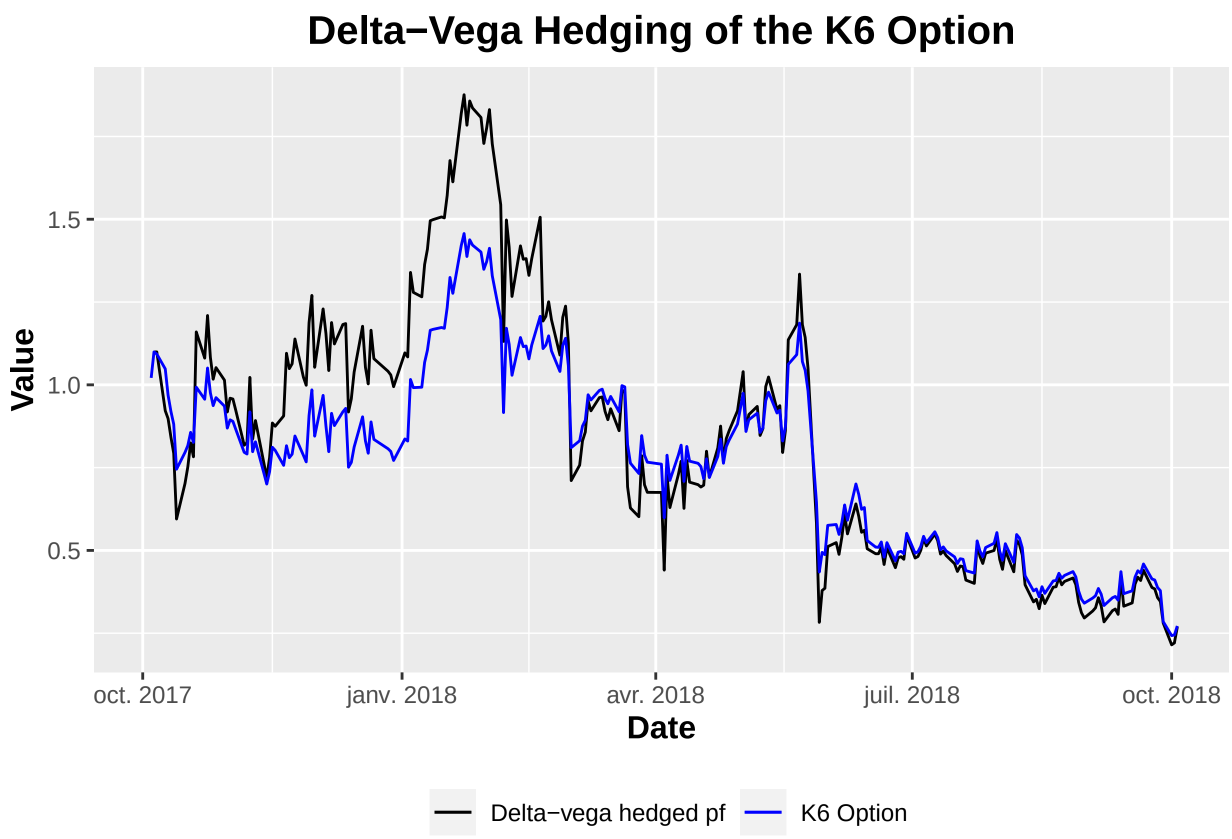
```
## [1]       NA       NA 2.662209 2.529913 2.535572 2.590180 2.697073 2.759635
## [9] 2.805467 2.859897
```

Then, we have to compute the ratio of volatility increments $\frac{\partial \sigma'}{\partial \sigma} = \frac{\sigma'_{t+1} - \sigma'_t}{\sigma_{t+1} - \sigma_t}$ .As a reminder, we have to consider the following formula : $\frac{\partial C'}{\partial \sigma} = \frac{\partial C'}{\partial \sigma'} \frac{\partial \sigma'}{\partial \sigma}$

Now, let's compute the value of q, l and q' (reminder: the first three values can't be computed)

```
##     q_option_2     l_cash    q_stock
## 1           NA         NA         NA
## 2           NA         NA         NA
## 3           NA         NA         NA
## 4    1.9572303   3.988951 -0.5965104
## 5   -2.8270452 -17.612368  2.9364923
## 6    1.6263586   2.333805 -0.3291174
## 7    0.5683886  -2.309322  0.4326973
## 8    0.1961939  -3.781565  0.6771052
## 9    0.1493102  -3.894161  0.6971259
## 10   0.1804891  -3.507017  0.6367905
```

Finally, we use the previous parameters and the iteration process mentioned above in order to build the history of the portfolio of replication :



Interpretation : As we can see on the chart above, the two times series are almost equal over time, especially from March, 2018. We can deduce that the hedge method is really efficient. To improve the quality of the replication, we could add an other call option to the portfolio to build a delta-gamma-vega hedged portfolio.