University of Colorado Boulder

ASEN 3128 - Aircraft Dynamics

Assignment 6 - Afternoon Section

Assignment 6

Author:
Benjiman Smith
105979656

Professor:
D Lawrence

27 February, 2020

The objective of this assignment was to utilize equations and knowledge derived in class to design a wing-tail setup based off of given paramters derrived in the lab document.

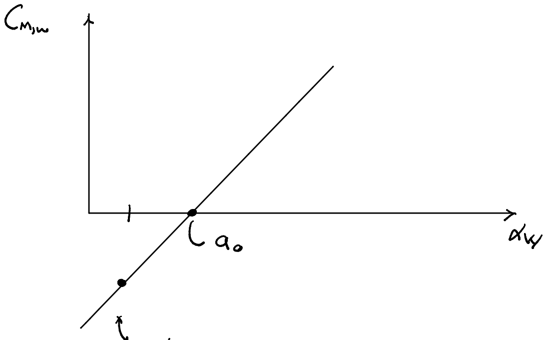
I. Nomenclature

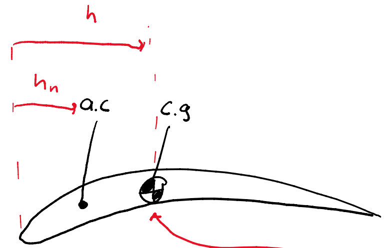
a_t	=	Lift slope of the tail[$1/deg$]
a_w	=	Lift slope of the wing [$1/deg$]
b_w	=	span of the wing [m]
c_w	=	chord of the wing [m]
C_{lw}	=	Coefficient of lift of the wing [$unitless$]
$C_{m_{AC,w}}$	=	Coefficient of the moment of the aerodynamic center of the wing [$unitless$]
C_{mw}	=	Coefficient of the moment of the wing [$unitless$]
h	=	location of the center of gravity measured aft of the leading edge of the wing [m]
h_n	=	location of the neutral point of the whole aircraft measured aft of the leading edge of the wing[m]
h_{nw}	=	location of the neutral point of the wing measured aft of the leading edge of the wing[m]
K	=	Static Margin [$unitless$]
ℓ_t	=	length of the tail [m]
L	=	Lift [N]
P	=	Power [kg/ms]
Re	=	Reynold's Number [$unitless$]
S_t	=	Planform area of the tail [m^2]
S_w	=	Planform area of the wing [m^2]
W	=	Weight [N]
$\frac{\partial C_{mw}}{\partial \alpha}$	=	pitch stiffness [m/deg]
ρ	=	air density [kg/m^3]
μ	=	viscosity [kg/ms]

Assignment #6

Friday, February 28, 2020 10:36 AM

1) Show that a positively cambered wing cannot be statically stable as a flying wing (i.e. without empennage) for any c.g. location.



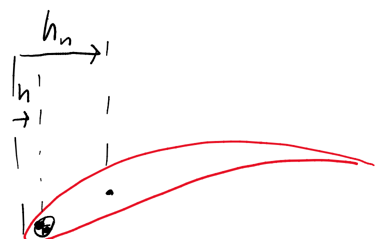


$$C_{M_w} = C_{M_{ac_w}} + \underbrace{C_L(h-h_n)}_{>0}$$

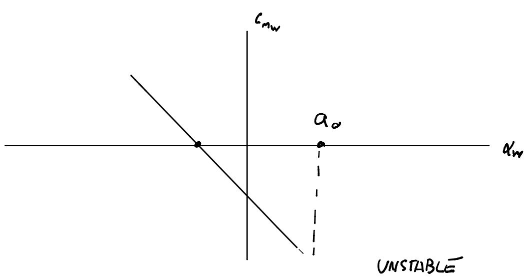
AS SEEN FROM THE PLOT, THE $\frac{\partial C_m}{\partial \alpha}$ IS POSITIVE, MEANING A POSITIVE INCREASE IN A.o.A WILL HAVE A RESULTING POSITIVE MOMENT. THIS SHOWS THAT THE CAMBERED WING IN THIS CONFIGURATION ISN'T STATICALLY STABLE, & WILL NOT FLY AS ALL THREE NECESSARY FLIGHT PARAMETERS AREN'T MET: LIFT ✓, BALANCE ✓, STABILITY X $\Rightarrow$ FLIGHT X

LIFT $\rightarrow$ LARGE a_0; BALANCE $\rightarrow$ $C_{m_w}=0$ @ TRIM; STABILITY $\rightarrow$ $\frac{\partial C_{m_w}}{\partial \alpha_w} < 0$ @ TRIM

& CONDITION #2: C.G AHEAD OF NEUTRAL PT.



$$C_{M_w} = C_{M_{ac_w}} + \underbrace{C_L(h-h_n)}_{<0}$$



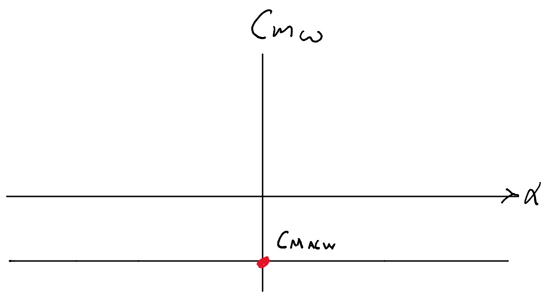
AS SEEN FROM THE PLOT, $\frac{\partial C_{m_w}}{\partial \alpha}$ IS NEGATIVE, & $C_{m_w} = 0$ AT TRIM

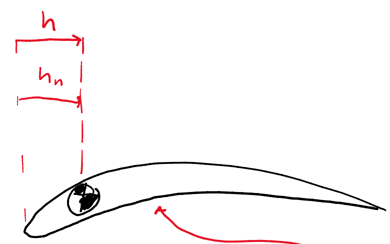
BUT THE a_0 IS NEGATIVE $\Rightarrow$ AIRFOIL ISN'T PRODUCING ENOUGH LIFT

$\therefore$ WHEN THE C.G IS BEHIND THE NEUTRAL PT., THE CAMBERED AIRFOIL IS STATICALLY STABLE, AS $\frac{\partial C_{m,w}}{\partial \alpha}$ IS NEGATIVE, BUT CANNOT FLY AS ALL THREE NECESSARY FLIGHT PARAMETERS AREN'T MET: LIFT X, BALANCE ✓, STABILITY ✓ $\Rightarrow$ FLIGHT X

LIFT $\rightarrow$ LARGE a_0; BALANCE $\rightarrow$ $C_{m_w}=0$ @ TRIM; STABILITY $\rightarrow$ $\frac{\partial C_{m_w}}{\partial \alpha_w} < 0$ @ TRIM

§ CONDITION #3: CG AT NEUTRAL PT





$$C_{M_w} = \underline{C_{M_{ACW}}} + C_L\underbrace{(h-h_n)}_{=0}$$

AS SEEN FROM THE PLOT, $\frac{\partial C_{Mw}}{\partial \alpha}$ IS ZERO, & C_{mw} = IS A CONSTANT NEGATIVE VALUE.

THE a_o DOESN'T EXIST => AIRFOIL ISN'T PRODUCING ENOUGH LIFT

∴ WHEN THE C.G IS AT THE NEUTRAL PT., THE CAMBERED AIRFOIL ISN'T STATICALLY STABLE, AND CANNOT FLY AS ALL THREE NECESSARY FLIGHT PARAMETERS AREN'T MET: LIFT, BALANCE, STABILITY => FLIGHT

X X X X

LIFT → LARGE a_o

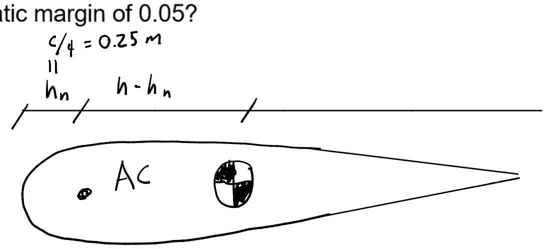
BALANCE → $C_{m_w} = 0$ @ TRIM

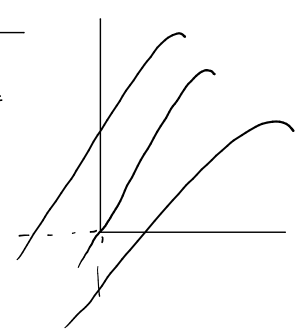
STABILITY → $\frac{\partial C_{m_w}}{\partial \alpha_w} < 0$ @ TRIM

---

2) Consider a rectangular flying wing with a $C_{m_{acw}}$ of 0.02 about the quarter-chord, a lift curve slope of 0.1/deg, a C_{L_w} at zero angle of attack of 0, a span of 10m, and a chord of 1.0 [m].

   a. Where should the c.g. be placed relative to the leading edge to produce a static margin of 0.05?





GIVENS: $C_{m_{acw}} = 0.02$ ABT QUARTER CHORD

$a_o = 0.1$ /deg $\qquad c = 1$ m

$C_{l @ \alpha = 0} = 0 \qquad b = 10$ m

C.G IS AHEAD OF VEHICLE n.p

STATIC MARGIN ≡ $\underbrace{h_n - h = K_n}$ EQ'N 2.3,6

$\Rightarrow 0.05 = 0.25 - h \Rightarrow \boxed{h = 0.20 \text{ m AFT}}$

b. What is the resulting pitch stiffness?

$$\frac{\partial C_m}{\partial \alpha} = 0 + \underbrace{a_w}(h - h_{n_w}) = C_{m_\alpha}$$

$-C_{m_\alpha}$ = PITCH STIFFNESS

$$\Rightarrow -C_{m_\alpha} = -a_w(h - h_{nw})$$

$$\Rightarrow -C_{m_\alpha} = -0.1\ /\text{deg}\,(0.2\text{m} - 0.25\text{m}) = \boxed{0.005\ \text{m}/\text{deg} = -C_{m_\alpha}}$$

c. What is the trimmed angle of attack?

$$C_m = C_{m_{ac_w}} + \underset{\substack{\Uparrow \\ a_w \alpha_w}}{C_{L_w}}(h - h_{n_w}), \quad C_m = 0 \text{ FOR TRIMMED FLIGHT}$$

$$\Rightarrow 0 = C_{m_{ac_w}} + C_{L_w}(h - h_{n_w})$$

$$\Rightarrow C_{L_w} = -\frac{C_{m_{AC_w}}}{(h - h_{n_w})} \Rightarrow a_w \alpha_w = \frac{-C_{m_{AC_w}}}{(h - h_{nw})}$$

$$\Rightarrow \alpha_w = -\frac{C_{m_{AC,w}}}{a_w(h - h_{n_w})} = \frac{-0.02}{(0.1\ /\text{deg})(0.20\text{m} - 0.25\text{m})}$$

$$\Rightarrow \boxed{\alpha_w = 4^\circ}$$

d. If the vehicle weighs 100 kg, what is the airspeed needed to fly at sea level? At Boulder?

IN BOULDER: $0.960\ \text{Kg}/\text{m}^3 = \rho$

ASSUMING THE AIRCRAFT IS TRIMMED, $L = W = (100\ \text{Kg})(9.81\ \text{m}/\text{s}^2) = 981\ \text{N}$

$C_L = a_0 \alpha = (0.1\ /\text{deg})(4^\circ) = 0.4$

$$L = \frac{1}{2}\rho V^2 c S C_L = V = \sqrt{\left(\frac{2L}{c\rho S C_L}\right)} \Rightarrow V = \sqrt{\left(\frac{2(981\text{N})}{(0.960\ \text{Kg}/\text{m}^3)(1\text{m})(10\text{m}^2)(0.4)}\right)} = \boxed{22.60392665\ \text{m}/\text{s} = V}$$

e. What is the chord Reynolds number for sea level flight? Boulder flight?

$\mu = 1.789\text{E}-5\ \text{Kg}/\text{ms}$

$$R_c = \frac{\rho V c}{\mu}$$

@ BOULDER: $\rho = 0.960\ \text{Kg}/\text{m}^3$

$$\Rightarrow Re = \frac{(0.960\ kg/m^3)(22.6039265\ m/s)(1.0\ m)}{1.789E{-5}\ kg/ms} = \boxed{1212955.251 = Re_{BOULDER}}$$

§ SEA LEVEL: $\rho = 1.225\ kg/m^3$

$\mu = 1.789E{-5}\ kg/ms$

° VELOCITY MUST BE CALCULATED FOR S.L FLIGHT

ASSUMING THE AIRCRAFT IS TRIMMED, $L = W = (100\ kg)(9.81\ m/s^2) = 981\ N$

$C_L = a_G\alpha = (0.1\ /deg)(4°) = 0.4$

$$L = \frac{1}{2}\rho V^2 c S C_L = V = \sqrt{\left(\frac{2L}{c\rho S C_L}\right)} \Rightarrow V = \sqrt{\left(\frac{2(981\ N)}{(1.225\ kg/m^3)(1m)(10m^2)(0.4)}\right)} = 20.01020148\ m/s = V$$

$$\Rightarrow Re_{S.L} = \frac{\rho_{S.L} V_{S.L} c}{\mu} = \frac{(1.225\ kg/m^3)(20.01020148\ m/s)(1.0\ m)}{1.789E{-5}\ kg/ms} = \boxed{1370178.693 = Re_{S.L}}$$

f. If L/D at the trimmed angle of attack is 10, how much power is required to fly?

§ POWER REQUIRED

$$T\cdot V = D\cdot V = \frac{L}{L/D} V = \boxed{\frac{WV}{L/D} = P}$$

USUALLY >> 1

§ BOULDER

$$P = \frac{(981\ N)(22.60392665\ m/s)}{10} = \boxed{2217.445204\ \frac{Nm}{s} = P_{BOULDER}}$$

§ SEA LEVEL

$$P = \frac{(981\ N)(20.01020148\ m/s)}{10} = \boxed{1963.000765\ \frac{Nm}{s} = P_{S.L}}$$

g. What is the pitch stiffness about the trimmed angle of attack in units of Nm/deg?

§ SEA LEVEL

$$\text{PITCH STIFFNESS} = -\underbrace{C_{m_\alpha}}_{m/deg} \times \underbrace{\frac{1}{2}\rho V^2 S\bar{c}}_{N} = (-0.005\ m/deg)\left(\frac{1}{2}(1.225\ \frac{kg}{m^3})(20.01020148\ m/s)^2(10m^2)(1m)\right) = -12.2625\ \frac{Nm}{DEG}$$

§ BOULDER

$$\text{PITCH STIFFNESS} = -\underbrace{C_{m_\alpha}}_{m/deg} \times \underbrace{\frac{1}{2}\rho V^2 S\bar{c}}_{N} = (-0.005\ m/deg)\left(\frac{1}{2}(0.960\ \frac{kg}{m^3})(22.60392665\ m/s)^2(10m^2)(1m)\right) = -12.2625\ \frac{Nm}{DEG}$$

SHOULD BE THE SAME

h. What is the relationship between pitch stiffness (in Nm/deg) and angle of attack for variation in c.g. location? Can c.g. movement be an effective control for angle of attack?

The relationship between pitch stiffness and angle of attack for varying C.G location is as follows.

$$\frac{\partial C_m}{\partial \alpha} = 0 + \underbrace{a_w}_{C_L/\alpha}(h - h_{n_w})\left(\frac{1}{2}\rho V^2 S c\right) = C_{m_\alpha}\left(\frac{1}{2}\rho V^2 S c\right)$$

$$\text{PITCH STIFFNESS} = -C_{m_\alpha} \Rightarrow \boxed{-C_{m_\alpha} = \frac{C_L}{\alpha}(h_{n,w} - h)}$$

THIS SHOWS THAT PITCH STIFFNESS & AoA ARE INVERSELY PROPORTIONAL, & AS THE LOCATION OF THE C.G CHANGES (h), THE AoA FOR TRIM WILL NEED TO CHANGE IN ORDER TO KEEP THE PITCH STIFFNESS CONSTANT; A NECESSARY CONSTRAINT FOR TRIM.

AS THE C.G MOVES FARTHER AFT FROM THE NEUTRAL POINT, A.O.A WILL NEED TO INCREASE TO KEEP $-C_{m\alpha}$ CONSTANT.

AS THE C.G MOVES FOREWARD, A.o.A WILL NEED TO DECREASE TO KEEP $-C_{m\alpha}$ CONST.

---

i. Design a tail for this wing that provides the same static margin as before, but with the c.g. at the quarter-chord of the wing. Assume the same lift curve slope for the tail airfoil as for the wing.

GIVENS: $K = 0.05$

$h = 1/4 \text{ m}$

$a_0 = 0.1 \; C_L/\text{DEG}$

$$C_{m\alpha} = a_w(h - h_n) + \underbrace{a_w(h_n - h_{n_w}) - V_H a_t}_{=0,\ \text{BY DEF OF } h_n} \quad \text{— SIMILAR TO EQN 2.3.3}$$

$$a_w(h_n - h_{n_w}) - a_t V_H = 0$$

RECALL: $h_n - h = K$

$$\Rightarrow \cancel{a_w}(h_n - h_{nw}) = \cancel{a_t} V_H \quad , \; a_w \;\&\; a_t \text{ ARE THE SAME}$$

$$\Rightarrow h_n = V_H + h_{nw}$$

$$\Rightarrow \Rightarrow \underbrace{h_n - h}_{K} = V_H + \underbrace{h_{nw} - h}_{0} \quad (h_{nw} = 0.25\text{m},\ h = 0.25\text{m})$$

$$\Rightarrow K = V_H = \frac{S_T l_T}{\bar{c} S} \Rightarrow S_T l_T = K(\bar{c} S) = (0.05)(1\text{m})(10\text{m}^2)$$

$$\boxed{\Rightarrow S_T l_T = 0.5\text{m}^3 \text{ IS THE TAIL REQUIREMENTS TO MEET THE STANDARDS DESCRIBED IN PART i).}}$$

---

j. Would it difficult to triple the pitch stiffness with a modification of this tail design?

RECALL: $$C_{m\alpha} = \underbrace{a_w(h - h_n)}_{K} + \underbrace{a_w(h_n - h_{n_w}) - V_H a_t}_{\text{ZERO, AS } h_n \text{ IS THE NEUTRAL PT. OF THE WHOLE BODY}}$$

WHERE $-C_{m\alpha}$ IS PITCH STIFFNESS

$\underbrace{\qquad\qquad}$ $\underbrace{\qquad\qquad}$

ZERO, AS h_n IS THE NEUTRAL PT. OF THE WHOLE BODY

WHERE $-C_{m_\alpha}$ IS PITCH STIFFNESS

$$\Rightarrow \quad -C_{m_\alpha} = a_w(\overbrace{h_n - h}^{K}) \quad \Rightarrow \quad -C_{m_\alpha} = a_w(V_H)$$

DERRIVED IN PT i

$$\Rightarrow \text{ TRIPLING PITCH STIFFNESS: } -3C_{m_\alpha} = a_w\left[\frac{3 l_T S_T}{\bar{c} S}\right]$$

$\therefore$ $\boxed{\text{IF } l_T S_T \text{ IS TRIPLED, PITCH STIFFNESS WILL BE TRIPLED.}}$