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GaleShapelyDrill4

	Day one	Day two	Day three	Day four	Day five
A	**1**, 3	**1**	1, **2**	**2**	**2**
B	**2**	2, **3**	**3**	**1**, 3	**1**
C					**3**

1. *In any execution of the algorithm, if a woman receives a proposal on dayi , then she receives some proposal on every subsequent day until the algorithm terminates.*

When a woman receives a proposal, she is the man's number one choice that has not yet rejected him. The man will continue to propose to her until she receives a different proposal that she accepts. In that case the cycle continues with this new man. Therefore, as soon as she receives a single proposal, she will continue to receive either that same proposal or a better one until the end of the algorithm.

2. *In any execution of the algorithm, if a woman receives no proposal on dayi , then she receives no proposal on any previous dayj , $1 \leq j < i$*

As proven by statement one, once a woman receives a proposal, she will continue to receive a proposal until the end of the algorithm. Therefore, the only way for a woman to not receive a proposal on a given day is if she was never previously proposed to.

3. *In any execution of the algorithm, there is at least one woman who only receives a single proposal.*

We know that Gale Shapely gives us a stable solution to the matching problem and the program terminates (Leff, slide 36). The algorithm terminates once every woman accepts a proposal, and a woman must accept her best proposal on any given day. The only reason why the algorithm does not terminate on any given day is because there is at least one woman that has not yet received a proposal. If she has not received a proposal today, she has never received one in the past (Statement two). The algorithm terminates when the last woman who has not yet receives a proposal receives her first proposal. On that day she has a proposal that she must accept, and all other women also have a proposal since once a woman is proposed to, she is never left without a proposal (Statement one). At the end of the execution of the algorithm at least one woman receives her first and only proposal which she must accept and cause the algorithm to be terminated.