# Proofs0

Benjamin Katz

January 2022

## 1 Simple Example

Theorem: If x is an odd integer, x+1 is even.

- Let $x = 2a - 1$ where $a \in Z$
- Since $\frac{2a-1}{2} = a - \frac{1}{2}$, and $a - \frac{1}{2}$ is not an integer, x does not provide an integer value when being divided by 2 and is therefore odd
- $x + 1 = 2a$ and $\frac{2a}{2} = a$ where $a \in Z$
- Because $x + 1$ is divisable by 2 it is therefore an even number
- QED

## 2 Induction I

Theorem: For all natural numbers n, $3|(n^3 - n)$

- Base case: For $3|(k^3 - k)$ let k = 0, $3|(0^3 - 0)$ gives us $\frac{0}{3} = 0$ so P(0) is true
- Inductive Hypothesis: Lets assume $P(k)$ is true $3|(k^3 - k)$ for all values of k
- Inductive step: Lets prove that $P(k + 1)$ is true
  $((k + 1)^3 - (k + 1)) = k^3 + 3k^2 + 3k - k$
  A three is factorable from part of the equation
  $3k^2 + 3k = 3(k^2 + k)$ and therefore $3|(3k^2 + 3k)$
  $3|(k^3 - k)$ through the inductive hypothesis
- $P(k + 1)$ is true
- This completes the proof by induction
- QED

# 3 Induction II

Theorem: For all natural numbers $n > 1$, $n! < n^n$

- Base case: let n = 2, $2! < 2^2$ therefore $P(2)$ is true
- Inductive Hypothesis: Lets assume $P(k)$ is true where $P(k) = k! < k^k$
- Inductive Step: Lets prove that $P(k+1)$ is true
  $(k+1)! < (k+1)^{(k+1)}$
  $(k+1) * k! < (k+1)^{(k+1)}$
  Divide both sides by $(k+1)$
  $k! < (k+1)^k$
  if $k! < k^k$ is true due to the inductive hypothesis and $k^k < (k+1)^k$ is true
  then $k! < (k+1)^k$ is true due to the transitive property of inequalities
- We have therefore proven the inductive step and $P(k+1)$ is true
- This Completes the proof by induction
- QED