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I want to thank a few people.

Preface

This is an example of a thesis setup to use the reed thesis document class.

List of Abbreviations

QCD	Quantum chromodynamics
SCET	Soft collinear effective theory

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Abstract

The preface pretty much says it all.

Dedication

You can have a dedication here if you wish.

Introduction

0.1 The physics of elementary particles

Modern particle physics is, ultimately, the result of a confluence of an ancient problem and an ancient technique. The problem: how does nature work on the most fundamental level? The technique: to smash objects together and see what comes out.

Of course, thousands of years of scientific development have led us to a more nuanced and sophisticated understanding than, say, the early atomic theories of Democritus and Lucretius — though echoes of these theories remain. We now understand¹ that almost all visible matter and almost all observed forces are, at root, manifestations of interactions between 17 fundamental particles (though there may yet be more). We conceive of these particles as excitations in fields which permeate space-time — this is known as Quantum Field Theory.²

The goal of elementary particle physics³ is to understand these particles and fields at the deepest level.⁴ We would like to lay a complete framework, understanding their nature and their interactions to such an extent that one could, in principle, derive from first principles an explanation for any observed physical phenomenon. The field is a long way from that goal, but much progress has been made over the past centuries. Let us begin by summarizing what we know so far.

0.1.1 The Standard Model

The 17 known particles are organized in a framework called the Standard Model of particle physics [2, 3], displayed in schematic form in Fig. 1.

There are 12 particles, called **fermions**, which are the fundamental components of matter. These in turn are subdivided into **quarks** and **leptons**. Quarks, with names such as ‘up,’ ‘down,’ and ‘strange,’ combine in turn to form **hadrons** — among the hadrons are familiar composite particles like protons and neutrons.

Of the remaining 5 particles, called **bosons**, mediate interactions between par-

¹Or at least, believe to understand

²Space-time itself is a geometric entity — we perceive this geometry as the force of gravity, acting on both human and cosmic scales — described by the theory of General Relativity.

³Also known simply as particle physics or high-energy physics

⁴Some particle theorists are also trying to unite Quantum Field Theory with General Relativity. Whether they will ever succeed remains to be seen, but they do lots of pretty math

Standard Model of Elementary Particles

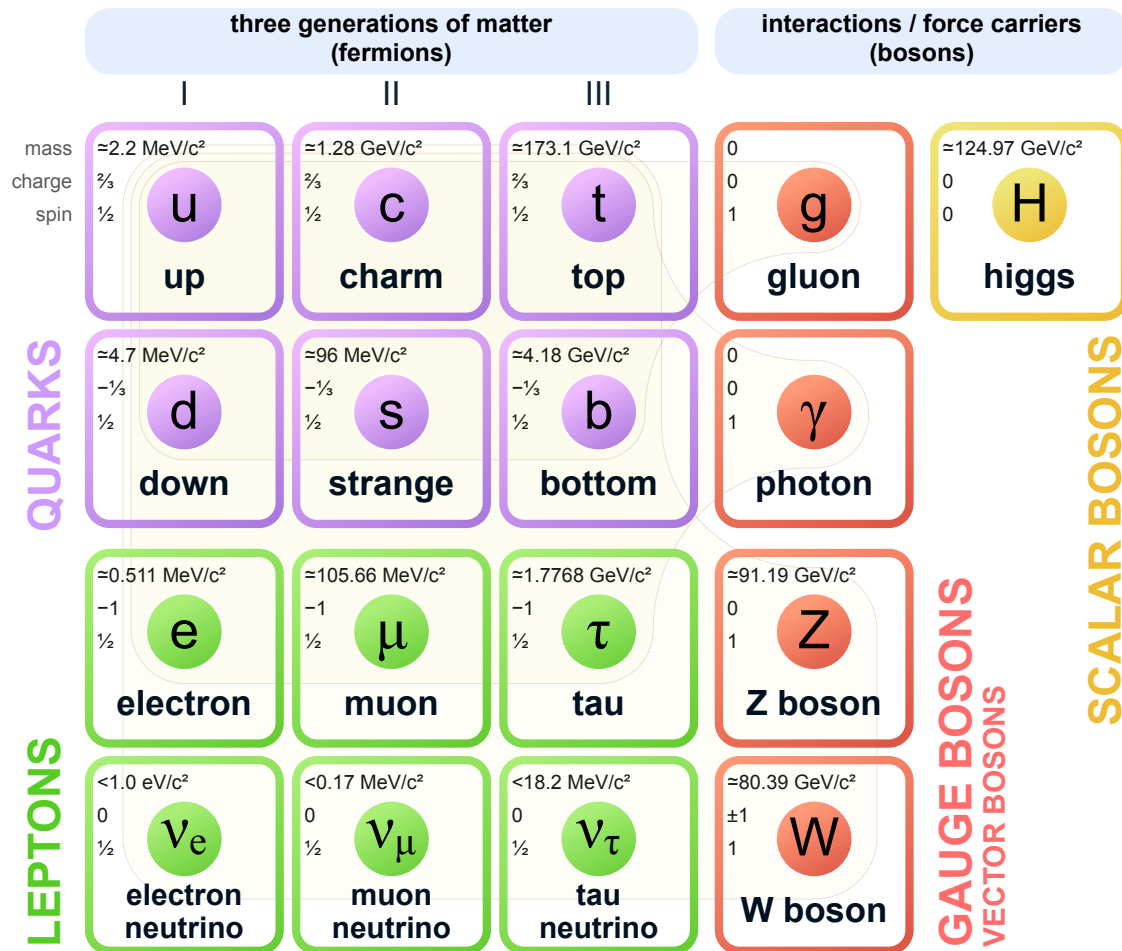


Figure 1: Diagram of the Standard Model of particle physics, as it currently stands.
From [1]

ticles. Four of these are responsible for three of the forces of nature: the **gluon** is responsible for the strong force; the **photon** is responsible for the electromagnetic force; and the **W and Z bosons** are responsible for the weak force. In terms of familiar phenomena, the strong force is what binds together the nuclei of atoms; the electromagnetic force is what makes possible most of chemistry, electronics, and modern technology; and the weak force is responsible for the decay of unstable nuclei and particles. The final boson, the **Higgs boson**, is responsible for giving mass to most of the fundamental particles, and in a technical sense is the keystone which holds the Standard Model together.

In the Standard Model, each fermion is beholden to some subset of these bosons. The neutrinos experience the weak force and so interact via the W and Z bosons. The electron, muon, and tau experience the weak force as well, but they can also interact electromagnetically through exchange of photons. The quarks experience all of the fundamental forces, and can interact via any of the force-carrying bosons. It is the interaction of quarks and gluons through the strong force which will occupy most of this thesis.

0.1.2 (In)completeness of the Standard Model

The Standard Model is a remarkable theory. It explains many observed phenomenon with unparalleled accuracy — in fact, the most accurate calculation in physics was performed in the context of quantum electrodynamics, the Standard Model explanation of electromagnetism [2] **[TODO: find and cite this]**. Experiment after experiment has confirmed predictions of the Standard Model, and in this sense it is a triumph of 20th-century physics.

In the present century, however, the strength of the theory is a significant problem in the field — for we know that the Standard Model is incomplete. While the theory is successful at describing small-scale physics, it is not nearly as successful on a cosmic scale: it has been observed that 84.4% of all matter in the universe is unknown to us and invisible except by gravitational signatures [4]. We would like to understand the composition of this so-called ‘dark matter.’ There are also hints that the Standard Model is incomplete which are visible in Earthly laboratories. Among these hints, one of the most famous is the existence of neutrino masses. While the Standard Model describes neutrinos as massless particles, the Super-Kamiokande experiment discovered evidence in 1998 that neutrinos have a (tiny) non-zero mass [5] — these observations have since been verified by numerous other experiments [4]. Finally, there are aesthetic considerations. The Standard Model, as it currently stands, has a multitude of parameters (e.g., the masses of different particles) that currently have no basis in the theory; they must be supplied externally. If possible, we would like to be able to predict these parameters eventually.⁵

Thus, the Standard Model is incomplete. What can we do about that? There are two primary approaches to finding a solution:

⁵One can hardly say they understand something if, when asked why a particular detail is just so, their response is no more sophisticated than “That’s just the way it is.”

1. **Searches for new particles:** one strategy is to design experiments that attempt to generate and detect new, previously unobserved particles. This is a well-worn technique in particle physics, responsible for many of the advances in the field over the last 20th century. Famous examples of new-particle discoveries include the J/ψ in 1974 [6, 7], the discovery of the top quark in 1995 [8, 9], and the discovery of the Higgs boson in 2012 [10, 11]. Historically, whenever new experimental frontiers have been explored, new discoveries have followed, and with them, new understanding. This is not, of course, a guarantee that such will continue in the future.⁶ Nevertheless, it is the underlying (if simplified) philosophy for many new particle searches at modern experiments.
2. **Precision measurements of Standard Model parameters:** another, complementary strategy is to measure parameters of the Standard Model very precisely, and compare these measurements to theoretical predictions. These parameters could be anything from particle masses, to the strength of coupling constants, to the probability of a particular event after a particular interaction. If a discrepancy emerges, then that is a clue about where to consider modifying the Standard Model. Note, however, that to put a precise experimental measurement into a proper context, it is necessary to have at hand a precise theory. If we could measure, say, the mass of the muon⁷ to 1 part in 10^{20} , it would do us no good if the theoretical prediction were only confident to 1 part in 10^5 . Theory and experiment must both be sufficiently advanced to take full advantage of a precise measurement.

This thesis is situated firmly in the latter camp. We will be interested in precision theoretical predictions of a particular observable, called the ‘groomed jet mass,’ measured in high-energy electron-positron collisions. More will be said on this topic in due time.

0.1.3 Particle physics experiments

0.2 Thesis goals

0.3 Technical and notational conventions

Before we embark, we must lay some mathematical ground rules. First, we hold Planck’s constant and the speed of light to be equal to unity: $\hbar = c = 1$. It turns out that non-unity values of these quantities are, for our purposes, redundant; when converting a given quantity back to SI units, the appropriate factors of c and \hbar can be intuited from context. The result is that all quantities will be measured in units of

⁶Past performance is not a guarantee of future returns.

⁷Compared to the state of the art, this would be an outrageously precise measurement. The mass of the muon is currently accepted to be $105.6583745(24)$ MeV [4], which is a precision of approximately 1 part in 10^8 .

energy. Physics where we will be working is at the GeV scale and higher. Therefore, to a high degree of accuracy, we will assume all particles to be massless.

Unless otherwise stated (and we *will* eventually state otherwise), we will work in 4 dimensions, comprising the usual three spatial dimensions and one temporal dimension. Vectors in 4 dimensions (called four-vectors) are denoted by a Greek-letter index and take the form

$$p^\mu = (p^0, p^1, p^2, p^3). \quad (1)$$

The 0-th component of a four-vector is its ‘time’ (or equivalent) component, and the others are the ‘spatial’ (or equivalent) components. Thus, for example, a four-vector representing position would be

$$x^\mu = (t, x, y, z), \quad (2)$$

while a four-momentum has the components

$$p^\mu = (E, p_x, p_y, p_z) \quad (3)$$

with energy taking the place of time. It is sometimes convenient to refer to lower-dimensional pieces of a four-vector (usually two or three of the spatial components). When doing so, we will denote the sub-vector using a bold-face letter:

$$p^\mu = (E, \mathbf{p}), \quad \mathbf{p} = (p_x, p_y, p_z). \quad (4)$$

As is standard in high-energy physics, we will neglect the effects of gravity and assume we are working in a flat space-time. When combining four-vectors, we will therefore use the ‘mostly minus’ metric⁸

$$\eta^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \quad (5)$$

We will also employ the Einstein summation notation, in which one sums over repeated indices in an expression (known as ‘contracting’ the index). Hence, for $p^\mu = (p^0, \mathbf{p})$ and $k_\mu = (k_0, \mathbf{k})$, we have

$$k_\mu p^\mu = k_0 p^0 + k_1 p^1 + k_2 p^2 + k_3 p^3. \quad (6)$$

With our choice of metric, there is little mechanical difference between a contravariant and a covariant four-vector; one picks up a formal minus sign in the spatial components, but that is all. We will, therefore, not distinguish between the two, and we will interchange upper and lower indices freely, bearing in mind that contracting an

⁸Also known as the ‘West Coast’ metric, among other names. The ‘East Coast’ metric takes the opposite sign convention. Our convention here is clearly the correct one, as it results in naturally positive masses.

index negates the spatial terms of the sum. Hence, for $p^\mu = (p^0, \mathbf{p})$ and $k^\mu = (k^0, \mathbf{k})$, we will write⁹

$$k^\mu p_\mu = k_\mu p^\mu = k^\mu p^\mu = k_\mu p_\mu = k^0 p^0 - \mathbf{k} \cdot \mathbf{p}. \quad (7)$$

The final term is the standard dot product between the three-vectors. This choice enables us to abuse notation in a convenient manner: we will often drop the Greek sub/superscript on four-vectors, and use the standard notation of linear algebra to indicate their contraction:

$$k \cdot p = k^0 p^0 - \mathbf{k} \cdot \mathbf{p}. \quad (8)$$

Let us end with a reminder about the connection between these four-vectors and the physical world. Suppose a particle has a momentum four-vector p^μ . Transforming our frame of reference to the particle's rest frame, we could write $p^\mu = (E, 0, 0, 0)$, where E is the particle's energy. But then, recalling the famous relation $E = mc^2 = m$ (since we set $c = 1$), we have

$$p^2 = p \cdot p = E^2 = m^2. \quad (9)$$

Thus, the square of a particle's four-momentum yields its squared mass. Recall now that we are assuming all particles to be massless; therefore, for any 'on-shell' particle (that is, a particle that could exist on its own and not just in some quantum fluctuation), we see that $p^2 = 0$, and also that $E^2 = \mathbf{p} \cdot \mathbf{p}$.¹⁰ This will greatly simplify our calculations later on.

⁹Sorry, Joel.

¹⁰This is not strictly an accurate proof of these properties, since massless particles move at the speed of light and one cannot boost into a light-like reference frame using Lorentz transformations. But the spirit of the argument is right, and the result is the same regardless.

Chapter 1

QCD, jet grooming, and SCET

First chapter here

1.1 Quantum Chromodynamics (QCD)

1.2 Jets

1.3 mMDT Grooming

1.4 Soft Collinear Effective Theory (SCET)

Chapter 2

Leading-order calculation

2.1 Setup

2.2 Dimensional regularization

2.3 Putting it all together

Chapter 3

Factorization formula

The first step on the path to an all-orders calculation is to derive a factorization formula for the heavy hemisphere mass cross section. The basic process for doing so is laid out in technical detail in Ref. [12], and an example of a similar flavor to our calculation is provided by Frye et al. in Ref. [13].¹ There are two primary steps in developing a factorization formula:

1. **Power counting:** this involves determining the possible radiative modes of an event and their dominant momentum scales. The term ‘power counting’ refers to the fact that for some momentum scale λ , different radiative modes have momenta that scale as different powers of λ .
2. **Factorization and refactorization:** Once the different radiative modes and energy scales are identified, we can use the framework of SCET to split the cross section into a convolution of terms describing different radiative modes. These terms themselves must then be split (refactored) into convolutions of terms, each of which depends, to leading order, only on a single energy scale.

3.1 Setup

Recall that the hemisphere mass is defined to be

$$\rho = \frac{1}{E_J^2} \sum_{i < j} 2p_i \cdot p_j \quad (3.1)$$

with E_J the jet energy and the sum ranging over all pairs of particles in the jet. Expanding out the dot product, we have

$$\rho = \frac{2}{E_J^2} \sum_{i < j} (E_i E_j - \mathbf{p}_i \cdot \mathbf{p}_j) = \frac{2}{E_J^2} \sum_{i < j} E_i E_j (1 - \cos \theta_{ij}) = \sum_{i < j} 2z_i z_j (1 - \cos \theta_{ij}). \quad (3.2)$$

¹Indeed, the calculation of Frye et al. is a more general factorization of mass-like variables in groomed jets. Setting $\alpha = 2, \beta = 0$ for their two-point energy correlation function $e_2^{(\alpha)}$ under soft drop grooming with angular exponent β yields the mMDT-groomed jet mass ρ . Their factorization is valid in the limit $\rho \ll z_{\text{cut}} \ll 1$, whereas we are interested in the limit $\rho \sim z_{\text{cut}} \ll 1$.

Here, z_i and z_j are the relative energy fractions of each particle and θ_{ij} is the angle between particles i and j .

Throughout the following discussion, with n^μ the jet direction and \bar{n}^μ the direction opposite the jet, we will describe momenta in light-cone coordinates

$$p^\mu = (p^-, p^+, p_\perp) \quad (3.3)$$

with

$$p^- = \bar{n} \cdot p \quad p^+ = n \cdot p \quad (3.4)$$

and p_\perp the components of momentum transverse to n . In these coordinates, the energy fraction with respect to total energy $E_J = Q$ is

$$z = \frac{p^+ + p^-}{2Q} \quad (3.5)$$

and, in the collinear limit, the angle to the jet axis is $\theta \approx p_\perp/p^0$ [13].

In an $e^+e^- \rightarrow \text{jets}$ event, there are two types of emission: resolved and unresolved. The essential difference is that a resolved emission is one which manifests itself as a jet at a particular scale of observation, while an unresolved emission does not. The presence of unresolved emissions can, however, perturb observable values of a resolved emission. **[TODO: check that this is a reasonable description]**

Suppose now that we have applied an mMDT groomer with energy fraction cutoff z_{cut} . Then every *resolved* emission must satisfy

$$z_i > z_{\text{cut}}, \quad (3.6)$$

while other emissions with $z_i < z_{\text{cut}}$ can only pass the groomer if they are at a sufficiently small angle to a resolved emission.

3.2 Power counting

3.2.1 Resolved soft emission

The primary emission contributing to the jet mass in the limit $\rho \sim z_{\text{cut}} \ll 1$ is a gluon emission z_i sensitive to both ρ and z_{cut} . In the presence of a hard quark (i.e., the jet) with $z_q \sim 1$, leading contributions to the jet mass of Eq. 3.2 is

$$\rho \approx \sum_i 2z_i(1 - \cos \theta_i) \quad (3.7)$$

where θ_i is the angle of emission i from the quark. Considering the case with only one such emission,² we have

$$\rho \approx z_i(1 - \cos \theta_i). \quad (3.8)$$

²This is the one-loop contribution **[is this accurate?]**.

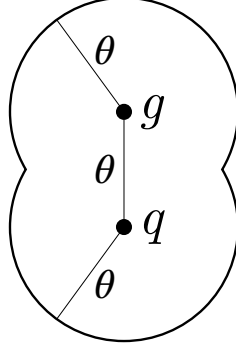


Figure 3.1: Head-on schematic of a quark jet q and a resolved gluon g . If the angle between the quark and the gluon is θ and the gluon (plus all lower-scale unresolved radiation) passes the groomer, then the mMDT groomer will accept all radiation within an angle θ of both the quark and the resolved gluon.

But since $z_i \sim z_{\text{cut}}$ and $\rho \sim z_{\text{cut}}$, this means that

$$\rho \approx \rho(1 - \cos \theta_i), \quad (3.9)$$

so

$$\cos \theta_i \ll 1. \quad (3.10)$$

This means that

$$\theta_i \sim \frac{\pi}{2}. \quad (3.11)$$

Since this emission is sensitive to $z_{\text{cut}} \ll 1$, we can also conclude that $z_i \ll 1$. Thus, we see that the leading contribution to the cusp region comes from a **resolved soft, wide-angle** gluon. Its momentum scales like

$$p_R \sim z_{\text{cut}} Q(1, 1, 1). \quad (3.12)$$

For future reference, let this gluon have energy fraction z_R and angle θ_R from the quark axis.

3.2.2 Ungroomed extra-soft unresolved radiation

Unresolved soft emissions can also contribute to the jet mass if they are sufficiently close to the resolved emission or the quark. Suppose there is another emission i with energy fraction z_i at angle θ_i from the jet axis and angle θ_{iR} from the resolved gluon. If $\theta_i < \theta_R$ or $\theta_{iR} < \theta_R$, a situation displayed in Fig. 3.1, then emission i will pass the groomer.

What does this mean for the hemisphere mass? Well, first notice that since $\theta_R \sim \pi/2$, most soft radiation in the hemisphere passes this cut. The effect of these extra-soft emissions, which have $z_i \ll z_R$, is to perturb the mass of the resolved emission, so that the hemisphere mass is approximately

$$\rho \sim z_R + \sum_i z_i (1 - \cos \theta_i). \quad (3.13)$$

The dominant contributions come again from the wide-angle emissions with $1 - \cos \theta_i \sim 1$; these must evidently have an energy scale set by

$$\rho - z_R \sim \sum_i z_i. \quad (3.14)$$

Hence, since $z_R \sim z_{\text{cut}}$, the unresolved extra-soft emissions must scale as

$$p_{S_R} \sim (\rho - z_{\text{cut}})Q(1, 1, 1). \quad (3.15)$$

3.2.3 Groomed soft radiation

Radiation outside of the region displayed in Fig. 3.1 gets groomed away if its energy fraction is

$$z_i < z_{\text{cut}}. \quad (3.16)$$

If $z_i > z_{\text{cut}}$, then we would have a second resolved emission, which we have already decided to ignore. These emissions must be at a very wide angle in order to not be within the region of mMDT acceptance. Since this **global soft** radiation is removed by the groomer, it is not sensitive to ρ and contributes only to the normalization. It must scale as

$$p_{S_G} \sim z_{\text{cut}}Q(1, 1, 1). \quad (3.17)$$

3.2.4 Collinear radiation

Finally, we consider radiation collinear to a jet axis with angle $\theta_i \ll 1$. This radiation has $p^- \gg p^+$, which means from Eq. 3.5 that

$$z \approx \frac{p^+}{2Q}. \quad (3.18)$$

Then because the particle must satisfy

$$z_i \theta_i^2 \lesssim \rho, \quad (3.19)$$

we find that [13]

$$\rho \sim \frac{p^+}{Q}. \quad (3.20)$$

If $z \sim 1$, we know that $z \gg z_{\text{cut}}$, so the momentum scales independently of z_{cut} . Hence, the scaling of these **collinear** modes is [13]

$$p_c \sim Q(1, \rho, \rho^{1/2}). \quad (3.21)$$

In a hemisphere where the resolved emission stops the mMDT groomer, this is the only collinear radiation that contributes **[TODO: ask Andrew: is this correct?]**.

If on the other hand $z \sim z_{\text{cut}} \ll 1$, the result is **collinear-soft** radiation with $p^- \sim z_i Q$ and $p^+ \sim \theta_i^2 z_i Q$. From [13], these momenta scale like

$$p_{cs} \sim z_{\text{cut}}Q\left(1, \frac{\rho}{z_{\text{cut}}}, \left(\frac{\rho}{z_{\text{cut}}}\right)^{1/2}\right) \quad (3.22)$$

and depend on the single energy scale $\sqrt{\rho z_{\text{cut}}}$. This scale matters in the hemisphere which does not contain the resolved soft gluon.

3.3 Factorization

With the power counting in hand, we are now ready to derive a factorization formula describing the hemisphere mass distribution in the limit $\rho \sim z_{\text{cut}} \ll 1$. First, we should note that it is most straightforward to compute the double differential cross section in the masses of the individual hemispheres, then integrate over them to get the heavy hemisphere mass [14]:

$$\frac{d\sigma}{d\rho} = \int \frac{d^2\sigma}{d\rho_1 d\rho_2} [\delta(\rho - \rho_1)\Theta(\rho_1 - \rho_2) + \delta(\rho - \rho_2)\Theta(\rho_2 - \rho_1)]. \quad (3.23)$$

The integral simply breaks up the two cases $\rho_1 > \rho_2$ and $\rho_2 > \rho_1$ and assigns the correct value of ρ in each case.

Now, in the limit $\rho_1, \rho_2 \ll 1$, we can apply the technology of SCET to factorize the double-differential cross section into a product of hard, soft, and jet contributions [13, 15]. The basic form is

$$\frac{d^2\sigma}{d\rho_1 d\rho_2} = H(Q^2) \otimes S(\rho_1, \rho_2, z_{\text{cut}}) \otimes J(\rho_1) \otimes J(\rho_2). \quad (3.24)$$

The symbol \otimes represents convolution. Here, Q^2 is the squared center-of-mass energy of the collision. $H(Q^2)$ is hard function representing the cross section of $e^+e^- \rightarrow q\bar{q}$ events, $S(\rho_1, \rho_2, z_{\text{cut}})$ is the function representing soft contributions (which are sensitive to z_{cut}), and $J(\rho)$ is a function describing the production of jets with hemisphere mass ρ .

As this factorization currently stands, several terms depend on multiple scales and must be refactorized. In the limit $\rho \sim z_{\text{cut}} \ll 1$ after mMDT grooming, the soft function consists of global soft emissions which contribute only to the normalization; a resolved soft, wide-angle emission generated by a fixed-order function; and soft radiation which passes the groomer due to the resolved emission. Therefore, we can write

$$\begin{aligned} \frac{d^2\sigma}{d\rho_1 d\rho_2} = H(Q^2) \times S_G(z_{\text{cut}}) \times R(\rho_1, \rho_2, z_{\text{cut}}) \times S_R(\rho - z_{\text{cut}}) \\ \otimes J_c(\rho_1, z_{\text{cut}}) \otimes J_c(\rho_2, z_{\text{cut}}). \end{aligned} \quad (3.25)$$

Here, $S_G(z_{\text{cut}})$ describes the groomed soft wide-angle radiation, $R(\rho_1, \rho_2, z_{\text{cut}})$ describes the resolved emission, and $S_R(\rho - z_{\text{cut}})$ describes radiation which passes the groomer in the presence of the resolved emission. We have also re-written the jet functions as $J_c(\rho, z_{\text{cut}})$ to make explicit their dependence on multiple scales.

Now, as we established in Sec. 3.2.4, radiation collinear to the jet with energy fraction of order 1 depends only on ρ , and radiation with energy fraction much less than 1 depends on the scale $\sqrt{\rho z_{\text{cut}}}$. However, this soft-collinear radiation only matters in the absence of a resolved gluon emission (which stops the mMDT groomer), in which case the jet function factorizes as

$$J_c(\rho, z_{\text{cut}}) = J(\rho) \otimes S_C(\sqrt{\rho z_{\text{cut}}}) \quad (3.26)$$

The result is that we must treat each hemisphere separately. Assuming, without loss of generality, that $\rho_1 > \rho_2$, the cross section then becomes

$$\boxed{\frac{d^2\sigma}{d\rho_1 d\rho_2} = 2H(Q^2) \times S_G(z_{\text{cut}}) \times R(\rho_1, \rho_2, z_{\text{cut}}) \times S_R(\rho - z_{\text{cut}}) \otimes J(\rho_1) \otimes [J(\rho) \otimes S_C(\sqrt{\rho} z_{\text{cut}})]}. \quad (3.27)$$

This is our final factorization formula, with the heavy hemisphere mass then given by Eq. 3.23.

Chapter 4

All-orders calculation

Test

Conclusion

Conclusion here

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