The first step on the path to an all-orders calculation is to derive a factorization formula for the heavy hemisphere mass cross section. The basic process for doing so is laid out in technical detail in Ref. [1], and an example of a similar flavor to our calculation is provided by Frye et al. in Ref. [2].[1] There are two primary steps in developing a factorization formula:

1. **Power counting**: this involves determining the possible radiative modes of an event and their dominant momentum scales. The term 'power counting' refers to the fact that for some momentum scale λ, different radiative modes have momenta that scale as different powers of λ.

2. **Factorization and refactorization**: Once the different radiative modes and energy scales are identified, we can use the framework of SCET to split the cross section into a convolution of terms describing different radiative modes. These terms themselves must then be split (refactored) into convolutions of terms, each of which depends, to leading order, only on a single energy scale.

0.1 Setup

Throughout the following discussion, with n^μ the jet direction and $\bar{n}^\mu$ the direction opposite the jet, we will describe momenta in light-cone coordinates

$$p^\mu = \left(p^-, p^+, p_\perp\right) \tag{1}$$

with

$$p^- = \bar{n}\cdot p \qquad\qquad p^+ = n\cdot p \tag{2}$$

and $p_\perp$ the components of momentum transverse to n.

Recall that the hemisphere mass is defined to be

$$\rho = \frac{1}{E_J^2}\sum_{i<j} 2p_i\cdot p_j \tag{3}$$

with E_J the jet energy and the sum ranging over all pairs of particles in the jet. Expanding out the dot product, we have

$$\rho = \frac{2}{E_J^2}\sum_{i<j}(E_iE_j - \mathbf{p}_i\cdot\mathbf{p}_j) = \frac{2}{E_J^2}\sum_{i<j}E_iE_j(1-\cos\theta_{ij}) = \sum_{i<j}2z_iz_j(1-\cos\theta_{ij}). \tag{4}$$

Here, z_i and z_j are the relative energy fractions of each particle and θ_{ij} is the angle between particles i and j.

[1] Indeed, the calculation of Frye et al. is a more general factorization of mass-like variables in groomed jets. Setting $\alpha = 2, \beta = 0$ for their two-point energy correlation function $e_2^{(\alpha)}$ under soft drop grooming with angular exponent β yields the mMDT-groomed jet mass ρ. Their factorization is valid in the limit $\rho \ll \mathrm{z_{cut}} \ll 1$, whereas we are interested in the limit $\rho \sim \mathrm{z_{cut}} \ll 1$.

In an $e^+e^- \to$ jets event, there are two types of emission: resolved and unresolved. The essential difference is that a resolved emission is one which manifests itself as a jet at a particular scale of observation, while an unresolved emission, does not. The presence of unresolved emissions can, however, perturb observable values of a resolved emission. **[TODO: check that this is a reasonable description]**

Suppose now that we have applied an mMDT groomer with energy fraction cutoff $\mathrm{z}_{\mathrm{cut}}$. Then every *resolved* emission must satisfy

$$z_i > \mathrm{z}_{\mathrm{cut}}, \tag{5}$$

while other emissions with $z_i < \mathrm{z}_{\mathrm{cut}}$ can only pass the groomer if they are at a sufficiently small angle to a resolved emission.

0.2 Power counting

0.3 Factorization

References

[1] T. Becher, A. Broggio and A. Ferroglia, *Introduction to Soft-Collinear Effective Theory*, vol. 896 of *Lecture Notes in Physics*, Springer International Publishing, Cham (2015), 10.1007/978-3-319-14848-9.

[2] C. Frye, A.J. Larkoski, M.D. Schwartz and K. Yan, *Factorization for groomed jet substructure beyond the next-to-leading logarithm*, *J. High Energ. Phys.* **2016** (2016) 64.