TITLE TBD

A Thesis
Presented to
The Division of Mathematics and Natural Sciences
Reed College

In Partial Fulfillment
of the Requirements for the Degree
Bachelor of Arts

Kees Benkendorfer

May 2021

Approved for the Division
(Physics)

Andrew Larkoski

Acknowledgements

I want to thank a few people.

Preface

This is an example of a thesis setup to use the reed thesis document class.

List of Abbreviations

QCD	Quantum chromodynamics
SCET	Soft collinear effective theory

Table of Contents

List of Tables

List of Figures

Abstract

The preface pretty much says it all.

Dedication

You can have a dedication here if you wish.

Introduction

Introduction here

0.1 Technical and notational conventions

First, we hold Planck's constant and the speed of light to be equal to unity: $\hbar = c = 1$. It turns out that non-unity values of these quantities are, for our purposes, redundant; when converting a given quantity to SI units, the appropriate factors of c and $\hbar$ can be intuited from context. The result is that all quantities will be measured in units of energy. Physics where we will be working is at the GeV scale and higher. Therefore, to a high degree of accuracy, we will assume all particles to be massless.

Unless otherwise stated (and we *will* eventually state otherwise), we will work in 4 dimensions, comprising the usual three spatial dimensions and one temporal dimension. Vectors in 4 dimensions (called four-vectors) are denoted by a Greek-letter index and take the form

$$p^\mu = (p^0, p^1, p^2, p^3). \tag{1}$$

The 0-th component of a four-vector is its 'time' (or equivalent) component, and the others are the 'spatial' (or equivalent) components. Thus, for example, a four-vector representing position would be

$$x^\mu = (t, x, y, z), \tag{2}$$

while a four-momentum has the components

$$p^\mu = (E, p_x, p_y, p_z) \tag{3}$$

with energy taking the place of time. It is sometimes convenient to refer to lower-dimensional pieces of a four-vector (usually two or three of the spatial components). When doing so, we will denote the sub-vector using a bold-face letter:

$$p^\mu = (E, \mathbf{p}), \quad \mathbf{p} = (p_x, p_y, p_z). \tag{4}$$

As is standard in high-energy physics, we will neglect the effects of gravity and assume we are working in a flat space-time. When combining four-vectors, we will therefore use the 'mostly minus' metric[1]

$$\eta^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{5}$$

We will also employ the Einstein summation notation, in which one sums over repeated indices in an expression (known as 'contracting' the index). Hence, for $p^\mu = (p^0, \mathbf{p})$ and $k_\mu = (k_0, \mathbf{k})$, we have

$$k_\mu p^\mu = k_0 p^0 + k_1 p^1 + k_2 p^2 + k_3 p^3. \tag{6}$$

With our choice of metric, there is little mechanical difference between a contravariant and a covariant four-vector; one picks up a formal minus sign in the spatial components, but that is all. We will, therefore, not distinguish between the two, and we will interchange upper and lower indices freely, bearing in mind that contracting an index negates the spatial terms of the sum. Hence, for $p^\mu = (p^0, \mathbf{p})$ and $k^\mu = (k^0, \mathbf{k})$, we will write[2]

$$k^\mu p_\mu = k_\mu p^\mu = k^\mu p^\mu = k_\mu p_\mu = k^0 p^0 - \mathbf{k} \cdot \mathbf{p}. \tag{7}$$

The final term is the standard dot product between the three-vectors. This choice enables us to abuse notation in a convenient manner: we will often drop the Greek sub/superscript on four-vectors, and use the standard notation of linear algebra to indicate their contraction:

$$k \cdot p = k^0 p^0 - \mathbf{k} \cdot \mathbf{p}. \tag{8}$$

Let us end with a reminder about the connection between these four-vectors and the physical world. Suppose a particle has a momentum four-vector p^μ. Transforming our frame of reference to the particle's rest frame, we could write $p^\mu = (E, 0, 0, 0)$, where E is the particle's energy. But then, recalling the famous relation $E = mc^2 = m$ (since we set $c = 1$), we have

$$p^2 = p \cdot p = E^2 = m^2. \tag{9}$$

Thus, the square of a particle's four-momentum yields its squared mass. Recall now that we are assuming all particles to be massless; therefore, for any 'on-shell' particle (that is, a particle that could exist on its own and not just in some quantum fluctuation), we see that $p^2 = 0$, and also that $E^2 = \mathbf{p} \cdot \mathbf{p}$.[3] This will greatly simplify our calculations later on.

[1] Also known as the 'West Coast' metric, among other names. The 'East Coast' metric takes the opposite sign convention. Our convention here is clearly the correct one, as it results in naturally positive masses.

[2] Sorry, Joel.

[3] This is not strictly an accurate proof of these properties, since massless particles move at the speed of light and one cannot boost into a light-like reference frame using Lorentz transformations. But the spirit of the argument is right, and the result is the same regardless.

Chapter 1

QCD, jet grooming, and SCET

First chapter here

1.1 Quantum Chromodynamics (QCD)

1.2 Jets

1.3 mMDT Grooming

1.4 Soft Collinear Effective Theory (SCET)

Chapter 2

Leading-order calculation

2.1 Setup

2.2 Dimensional regularization

2.3 Putting it all together

Chapter 3

Factorization formula

The first step on the path to an all-orders calculation is to derive a factorization formula for the heavy hemisphere mass cross section. The basic process for doing so is laid out in technical detail in Ref. [1], and an example of a similar flavor to our calculation is provided by Frye et al. in Ref. [2].[1] There are two primary steps in developing a factorization formula:

1. **Power counting**: this involves determining the possible radiative modes of an event and their dominant momentum scales. The term 'power counting' refers to the fact that for some momentum scale λ, different radiative modes have momenta that scale as different powers of λ.

2. **Factorization and refactorization**: Once the different radiative modes and energy scales are identified, we can use the framework of SCET to split the cross section into a convolution of terms describing different radiative modes. These terms themselves must then be split (refactored) into convolutions of terms, each of which depends, to leading order, only on a single energy scale.

3.1 Setup

Throughout the following discussion, with n^μ the jet direction and $\bar{n}^\mu$ the direction opposite the jet, we will describe momenta in light-cone coordinates

$$p^\mu = \left(p^-, p^+, p_\perp\right) \tag{3.1}$$

with

$$p^- = \bar{n} \cdot p \qquad\qquad p^+ = n \cdot p \tag{3.2}$$

and $p_\perp$ the components of momentum transverse to n.

[1] Indeed, the calculation of Frye et al. is a more general factorization of mass-like variables in groomed jets. Setting $\alpha = 2, \beta = 0$ for their two-point energy correlation function $e_2^{(\alpha)}$ under soft drop grooming with angular exponent β yields the mMDT-groomed jet mass ρ. Their factorization is valid in the limit $\rho \ll \mathrm{z_{cut}} \ll 1$, whereas we are interested in the limit $\rho \sim \mathrm{z_{cut}} \ll 1$.

Recall that the hemisphere mass is defined to be

$$\rho = \frac{1}{E_J^2} \sum_{i<j} 2p_i \cdot p_j \tag{3.3}$$

with E_J the jet energy and the sum ranging over all pairs of particles in the jet. Expanding out the dot product, we have

$$\rho = \frac{2}{E_J^2} \sum_{i<j} (E_i E_j - \mathbf{p}_i \cdot \mathbf{p}_j) = \frac{2}{E_J^2} \sum_{i<j} E_i E_j (1 - \cos\theta_{ij}) = \sum_{i<j} 2 z_i z_j (1 - \cos\theta_{ij}). \tag{3.4}$$

Here, z_i and z_j are the relative energy fractions of each particle and θ_{ij} is the angle between particles i and j.

In an $e^+e^- \to$ jets event, there are two types of emission: resolved and unresolved. The essential difference is that a resolved emission is one which manifests itself as a jet at a particular scale of observation, while an unresolved emission does not. The presence of unresolved emissions can, however, perturb observable values of a resolved emission. **[TODO: check that this is a reasonable description]**

Suppose now that we have applied an mMDT groomer with energy fraction cutoff $\mathrm{z_{cut}}$. Then every *resolved* emission must satisfy

$$z_i > \mathrm{z_{cut}}, \tag{3.5}$$

while other emissions with $z_i < \mathrm{z_{cut}}$ can only pass the groomer if they are at a sufficiently small angle to a resolved emission.

3.2 Power counting

3.3 Factorization

Chapter 4

All-orders calculation

Test

Conclusion

Conclusion here

References

[1] T. Becher, A. Broggio and A. Ferroglia, *Introduction to Soft-Collinear Effective Theory*, vol. 896 of *Lecture Notes in Physics*, Springer International Publishing, Cham (2015), 10.1007/978-3-319-14848-9.

[2] C. Frye, A.J. Larkoski, M.D. Schwartz and K. Yan, *Factorization for groomed jet substructure beyond the next-to-leading logarithm*, *J. High Energ. Phys.* **2016** (2016) 64.