Groomed heavy hemisphere mass in $e^+e^- \to$ jets events

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Thesis Mini-Orals

1. Background
   - Events of interest
   - Observables
2. Work so far
   - First-order calculation
   - Zooming in on intermediate mass
3. Looking forward

Electron-positron annihilation

- Electron-positron annihilation experiments enable precision probes of the Standard Model
  - Theoretically simpler than proton-proton collisions (as at the LHC)
  - Some results carry over to *pp* collisions
- In events of interest, e^+ and e^- produce a photon, which splits into a quark-antiquark pair
- One or more gluons are produced off of q or $\bar{q}$

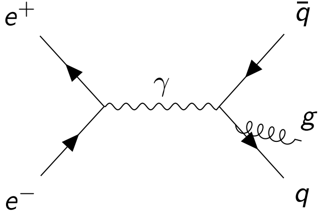

Figure: Feynman diagram for $e^+e^- \to q\bar{q}g$

Jets

- In quantum chromodynamics (QCD), the gluon carries color charge just like quarks
- Interesting nonlinear dynamics:
    - **Self-coupling**: gluons beget gluons
    - **Scale-invariance**: QCD events have approximately no intrinsic scale
    - **Confinement**: particles with color charge don't like to live alone
- Result: producing a quark or gluon yields a collimated spray of hadronic radiation called a **jet**

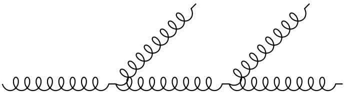

Figure: QCD self-coupling as gluon propagates

Heavy hemisphere mass

- We are interested in properties of these jets
- Key observable: **heavy hemisphere mass**
  - Split the $e^+e^- \to$ jets events into two hemispheres
  - Look at the normalized mass of the heaviest hemisphere
- For a heavy hemisphere with four-momentum P^μ, hemisphere mass is given by

$$\rho = \frac{m_h^2}{E_h^2}$$

with $m_h^2 = P \cdot P$ and $E_h^2 = \left(P^0\right)^2$.

Grooming

- We almost have a good quantity to measure in experiments
- Problem: in high-luminosity colliders, significant background radiation can contaminate jets
- This contamination is almost exclusively low-energy (**soft**)
- **Jet grooming**: removes soft emissions in the jet in order to focus on features of interest
- We use **mMDT** (modified Mass Drop Tagger) grooming [1, 2]
    - Set a cutoff energy fraction z_{cut}
    - For two emissions i and j, only keep them if their energies satisfy

$$\frac{\min[E_i, E_j]}{E_i + E_j} > z_{\mathrm{cut}}.$$

Reproduction of first-order calculation

- To leading order, ρ is generated by $e^+e^- \to q\bar{q}g$ events. For momenta p_i and total momentum Q, introduce phase space variables

$$x_i = \frac{2p_i \cdot Q}{Q^2}$$

- With α_s the strong coupling, $C_F = 4/3$ the fundamental Casimir of color, and σ_0 the cross section for $e^+e^- \to q\bar{q}$ events, the cross section is given by [3]:

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\rho} = \frac{\alpha_s C_F}{2\pi}\int_0^1 dx_1 \int_0^1 dx_2 \overbrace{\Theta(x_1 + x_2 - 1)}^{\text{kinematic requirement}} \overbrace{\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}}^{\text{matrix element}}$$
$$\times \underbrace{\delta\left(\rho - \frac{4(1-\max\{x_i\})}{(2-\max\{x_i\})^2}\right)}_{\text{measurement}} \underbrace{\Theta\left(\frac{\min\{x_i\}}{2-\max\{x_i\}} - z_{\text{cut}}\right)}_{\text{jet grooming}}$$

Reproduction of first-order calculation

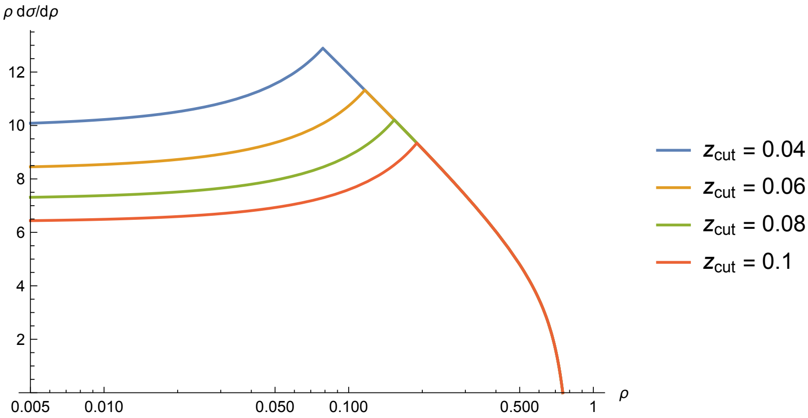

Figure: Groomed heavy hemisphere mass to first order. Note the cusp around $\rho \sim z_{\text{cut}}$

Cusp physics

Goal

Understand the cusp by performing an all-orders calculation of the distribution in that regime

- Suppose there are many labeled emissions with energy fractions z_i and angles θ_{ij}
- Emissions relevant for the cusp have $z_i \sim \rho$ and $z_i \sim z_{\mathrm{cut}}$. Assuming $z_{\mathrm{cut}} \ll 1$, this means $z_i \ll 1$ (so the emission is soft)
- Since $\rho \simeq \sum_{i,j} z_i z_j \theta_{ij}^2$, the leading contribution from emission i will be its interaction with the **hard** (high-energy) quark j, so that $\rho \sim z_i \theta_{ij}^2$
- Therefore, $\rho \sim z_{\mathrm{cut}} \theta_{ij}^2 \sim z_{\mathrm{cut}}$, so $\theta_{ij} \sim 1$
- Thus, we are looking for **soft emissions at a wide angle to the quark**

Soft limit

- In the limit of soft gluon emissions, the matrix element is known [4]
- For one gluon with momentum k^μ, the matrix element is

$$|\mathcal{M}|^2 = 4\pi\alpha_s C_F \frac{2}{k^+ k^-} \tag{1}$$

with light-cone coordinates

$$k^+ = k^0 - k^3 \qquad\qquad k^- = k^0 + k^3. \tag{2}$$

- Problem: matrix element diverges in soft limit $k^- \to 0$
- Solution: **dimensional regularization** [5]
  - Basically analytic continuation of the dimension of the problem: work in $d = 4 - 2\epsilon$ dimensions with $\epsilon > 0$
  - Introduces $(k^+ k^-)^{-\epsilon}$ term which allows integration; also introduces an energy scale $\mu^{2\epsilon}$
  - Divergences are collected in terms which diverge as $\epsilon \to 0$

Killing divergences

- Dimensional regularization helps us find divergences — so what?
  - Divergences remain, only change is they are now explicit
  - Also, what is μ?
- Degeneracy saves the day: adding the remaining singular contributions (i.e., the collinear limit) cancels divergent terms
- Can then set $\epsilon = 0$
- Adding all degenerate regions of phase space also eliminates terms containing μ
  - Physical result must not depend on an arbitrary energy scale

Results: $z_{\mathrm{cut}} = 0.04$

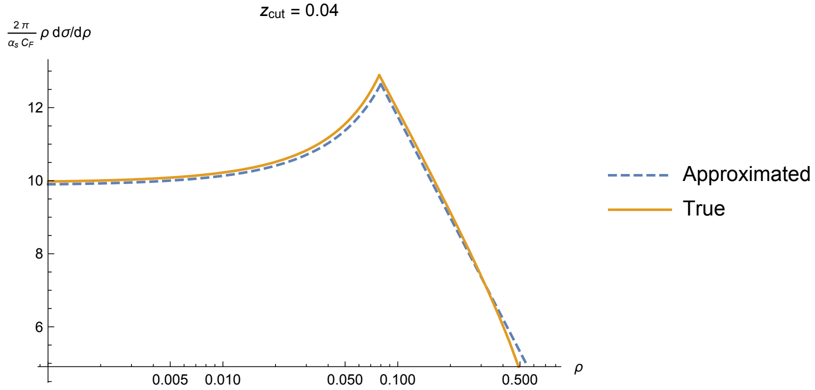

Figure: Groomed heavy hemisphere mass to first order, alongside an approximation around the cusp region, with $z_{\mathrm{cut}} = 0.04$

Results: $z_{\mathrm{cut}} = 0.04$

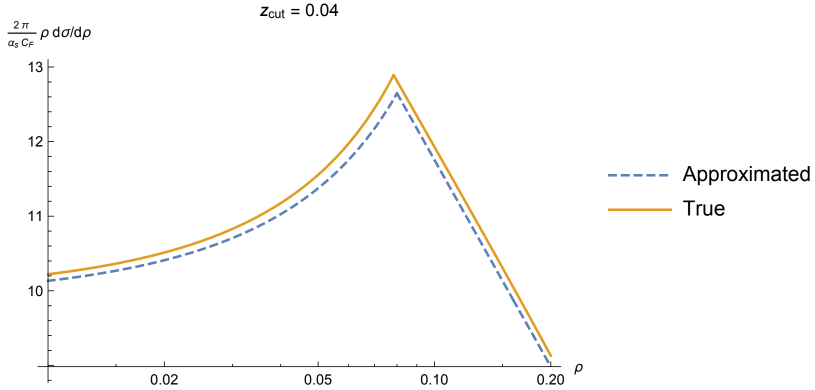

Figure: Groomed heavy hemisphere mass to first order, alongside an approximation around the cusp region, with $z_{\mathrm{cut}} = 0.04$

Results: $z_{\mathrm{cut}} = 0.001$

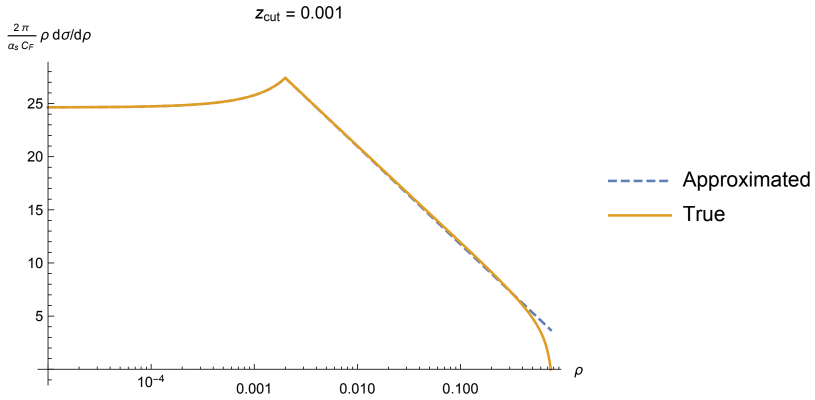

Figure: Groomed heavy hemisphere mass to first order, alongside an approximation around the cusp region, with $z_{\mathrm{cut}} = 0.001$

Results: $z_{\text{cut}} = 0.001$

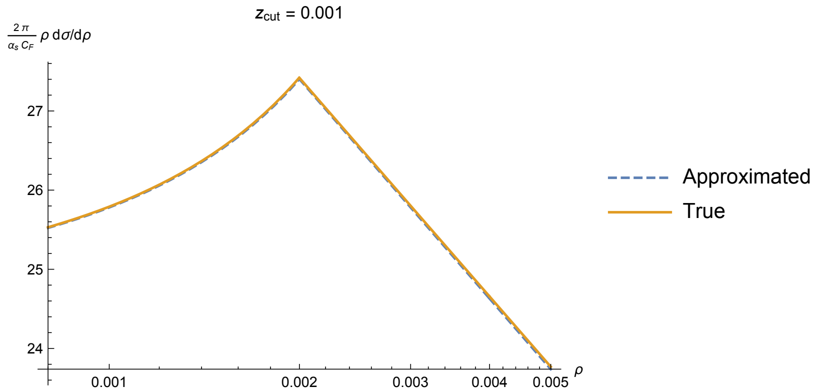

Figure: Groomed heavy hemisphere mass to first order, alongside an approximation around the cusp region, with $z_{\text{cut}} = 0.001$

Conclusion and next steps

- I have familiarized myself with previous first-order results
  - Has required learning lots of quantum field theory
- Main forward thrust will be to derive a new factorization formula describing the distribution to all orders, as in [6]
  - This formula was valid for $\rho \ll z_{\mathrm{cut}} \ll 1$
  - Factorization formula for $\rho \sim z_{\mathrm{cut}} \ll 1$ would enable calculating the cusp to arbitrary accuracy
  - The name of the game: ensuring no dependence on arbitrary energy scales μ
- With factorization formula in hand, will push to next-order accuracy
- Will enable precision understanding of intermediate regions of the distribution

References

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[2] Adam Kardos, Andrew J. Larkoski, and Zoltán Trócsányi. Two- and three-loop data for the groomed jet mass. *Phys. Rev. D*, 101(11):114034, June 2020.

[3] Andrew J. Larkoski. Improving the Understanding of Jet Grooming in Perturbation Theory. *arXiv:2006.14680 [hep-ex, physics:hep-ph]*, August 2020.

[4] S. Catani and M. Grazzini. Infrared factorization of tree-level QCD amplitudes at the next-to-next-to-leading order and beyond. *Nuclear Physics B*, 570(1-2):287–325, March 2000.

[5] Matthew Dean Schwartz. *Quantum Field Theory and the Standard Model*. Cambridge University Press, New York, 2014.

[6] Christopher Frye, Andrew J. Larkoski, Matthew D. Schwartz, and Kai Yan. Factorization for groomed jet substructure beyond the next-to-leading logarithm. *J. High Energ. Phys.*, 2016(7):64, July 2016.