Problem-set 2

Winning strategies in two-player games

Problem 1. *In Lecture 3 and Theorem 1.10 of the notes, the following theorem is stated.*

1.10. *For any **two-player, zero-sum** game with **perfect information** and **no chance**, that ends after a finite number of moves, one of the following is true.*

1. *Player 1 has a winning strategy;*
2. *Player 2 has a winning strategy;*
3. *Player 1 and Player 2 both have strategies which ensure a draw.*

Are all the assumptions needed? For each assumption, give a counter-example where the assumption is dropped and the theorem is false, or else an argument that the assumption is not needed.

The pie rule

Problem 2. *In games with a strong first-player advantage, such as Hex, it is common to use an addition rule called "the pie rule" (also called "the swap rule") to make the game more fair. Let Alice be Player 1 and Bob be Player 2. After Player 1 (Alice) makes her first move, Player 2 (Bob) has a choice: either Bob accepts the move and continue to play as Player 2 and makes a move in response, or Bob can "swap". If Bob swaps, he becomes Player 1 and Alice becomes Player 2, and needs to move in response to her original move. If this is confusing, perhaps* `https://www.academickids.com/encyclopedia/Pie_rule` *or* `https://sites.google.com/site/boardandpieces/terminology/pie-rule` *Will help. We use the pie rule in the project. The pie rule can only be used by Player 2 and only after the very first move of the game.*

Suppose Alice and Bob are playing a version of Hex which is weakly solved, and assume both Alice and Bob know the winning strategy.

(a) *Should Alice make the winning move as her first move? What happens if she does?*

(b) *Suppose Alice makes a non-winning move as her opening move. Does this mean that Bob has a guaranteed win? Speculate why Bob might have a winning move.*

Equilibria in normal form games

Problem 3. *Below is a pay-off matrix for a zero-sum, two-player game with perfect information and no chance. The two players are Alice and Bob. Alice has three actions: A–C; Bob has four actions: a–d. The table shows the pay-off to Alice.*

		Bob: a	b	c	d
Alice	A	-2	0	-3	1
	B	-1	1	1	-1
	C	-2	-1	2	3

(a) *Find a Nash equilibrium for the game using* dominance. *If it takes more than one step to get to the equilibrium, be sure to write down every step in reducing the matrix until the equilibrium is found. (Remember, a dominant strategy for Player 2 has lower values than other strategies, for all opponent actions.)*

(b) *Now find the equilibrium using the minimax approach, again giving the steps in your reasoning.*

Problem 4. *This question involves the following game. The two players move simultaneously.*

		Brian: a	b
Amy	A	$(1,-1)$	$(-1,1)$
	B	$(-1,1)$	$(2,-2)$

Now answer the following questions.

(a) *What type of game is this? Name all types of games it is.*

(b) *Find all of the pure strategy equilbria. Justify that they are equilibria.*

(c) *Is there a fully mixed strategy equilibrium? If so, find it. Otherwise, give a convincing argument that one does not exist.*

Problem 5. *This question involves the following game. The two players move simultaneously.*

		Benjamin: a	b
Amelia	A	$(1,1)$	$(-1,-1)$
	B	$(-1,-1)$	$(3,2)$

Now answer the following questions.

(a) *What type of game is this? Name all types of games it is.*

(b) *Find all of the pure strategy equilbria. Justify that they are equilibria.*

(c) *Is there a fully mixed strategy equilibrium? If so, find it. Otherwise, give a convincing argument that one does not exist.* ***Hint:*** *In general, Amelia wants to chose her strategy so that Benjamin is indifferent using Benjamin's payoff, so that he will have no incentive to deviate. Likewise, Benjamin wants to chose his strategy, so that Amelia is indifferent using her payoff, so she will have no incentive to deviate. Otherwise, it will not be a Nash equilibrium. This question is tricky.*

Game trees

Problem 6. *Figure 1 shows a game tree. In what order are the nodes values determined? Nodes can be evaluated left to right or right to left, but say which you are using.*

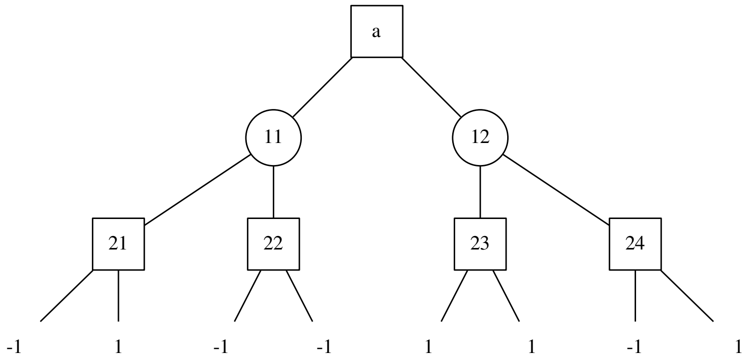

Figure 1: A game tree used for Problem 6. Squares are `MAX` nodes and circles are `MIN` nodes.

Problem 7. *Figure 2 shows a win-loss tree. Apply win-loss pruning (either left to right or right to left). What nodes are* not *evaluated.*

Problem 8. *Figure 3 shows a game tree. Apply alpha-beta pruning (either left to right or right to left). What nodes are* not *evaluated. What are the final alpha-beta values of nodes C1, C2, C3 and the root?*

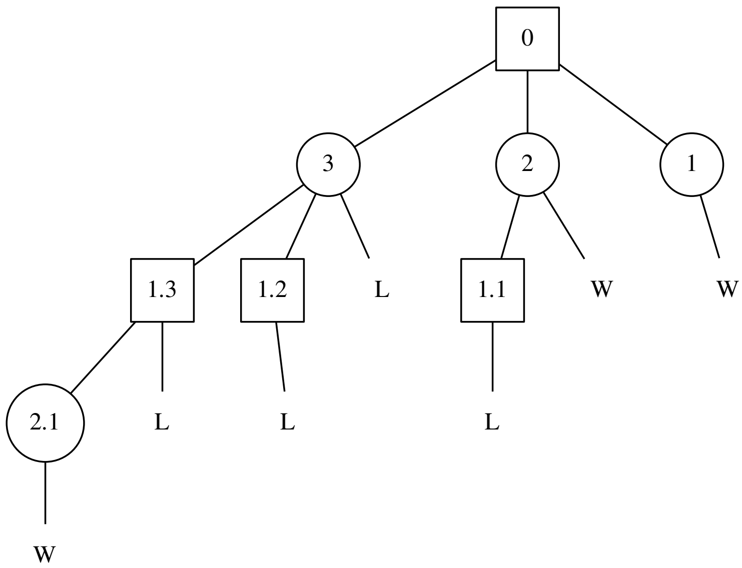

Figure 2: A win-loss tree associated with Problem 7. You might recognize this tree.

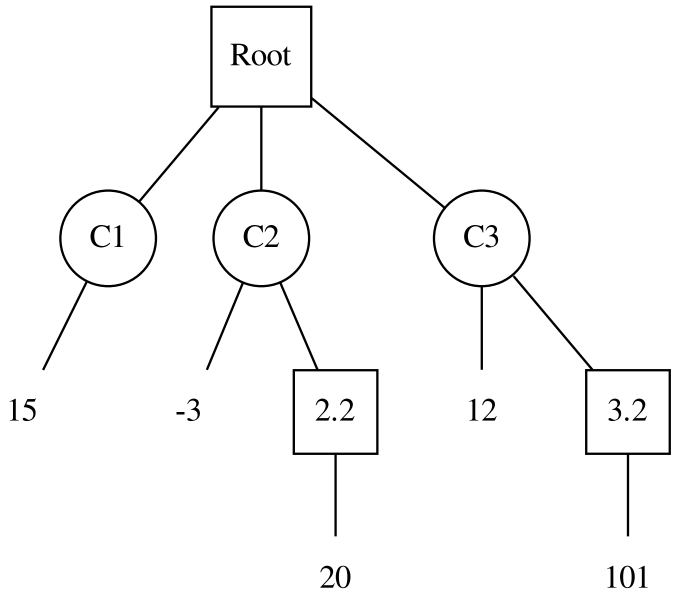

Figure 3: A game tree associated with Problem 8. To practice alpha-beta pruning. You might recognize this tree.