**ISyE 6501 HW 8**

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**Question 13.1**

For each of the following distributions, give an example of data that you would expect to follow this distribution (besides the examples already discussed in class).

a. Binomial
b. Geometric
c. Poisson
d. Exponential
e. Weibull

**a. Binomial Distribution**

We can use the binomial distribution to describe the pass rate of students in a particular exam. For example, if there are 100 students, with scores greater than 60 considered "pass" and scores below 60 considered "fail," and the probability of passing is fixed.

**b. Geometric Distribution**

The geometric distribution can be used to describe how many times a student needs to retake a course exam before passing. If the probability of passing the exam is fixed, the geometric distribution can model the number of attempts needed to succeed.

**c. Poisson Distribution**

The Poisson distribution can be used to describe the number of times a popular SKU is ordered in a warehouse within a day. The orders for the SKU are random and independent, and the order rate is fixed over short periods.

**d. Exponential Distribution**

The exponential distribution can be used to describe the time interval between two different orders arriving at the warehouse's queue system. The arrival of orders is independent and random, and this scenario can be modeled using the exponential distribution.

**e. Weibull Distribution**

We can describe an athlete's endurance performance at a particular stage using the Weibull distribution. Given that athletes have varying physical conditions and environmental factors, the endurance distribution may differ, and this can be adjusted using the parameters of the Weibull distribution.

**Question 13.2**

In this problem you, can simulate a simplified airport security system at a busy airport. Passengers arrive according to a Poisson distribution with $\lambda_1 = 5$ per minute (i.e., mean interarrival rate $\mu_1 = 0.2$ minutes) to the ID/boarding-pass check queue, where there are several servers who each have exponential service time with mean rate $\mu_2 = 0.75$ minutes. [Hint: model them as one block that has more than one resource.] After that, the passengers are assigned to the shortest of the several personal-check queues, where they go through the personal scanner (time is uniformly distributed between 0.5 minutes and 1 minute).

Use the Arena software (PC users) or Python with SimPy (PC or Mac users) to build a simulation of the system, and then vary the number of ID/boarding-pass checkers and personal-check queues to determine how many are needed to keep average wait times below 15 minutes. [If you're using SimPy, or if you have access to a non-student version of Arena, you can use $\lambda_1 = 50$ to simulate a busier airport.]

To set up the simulation environment using SimPy, begin by modeling passenger arrivals according to a Poisson process with an arrival rate of 5 passengers per minute.
For the ID check, define multiple servers using SimPy's Resource object, where each server has an exponential service time with a mean of 0.75 minutes. Passengers wait in line until a server becomes available. After passing the ID check, passengers move to the personal check, where they are assigned to the shortest queue. Each personal check queue is also modeled as a Resource, with a uniform service time between 0.5 and 1 minute.
To simulate the system, run it for a set amount of time, tracking each passenger's total waiting time, from arrival to completing both checks. Vary the number of ID checkers and personal-check queues, testing configurations with 1 to 5 servers for each, to observe how these changes affect the average wait time. Finally, analyze the results by calculating the average wait time for each configuration, and identify the minimum number of servers required to keep the average wait time below 15 minutes.

**Code:**

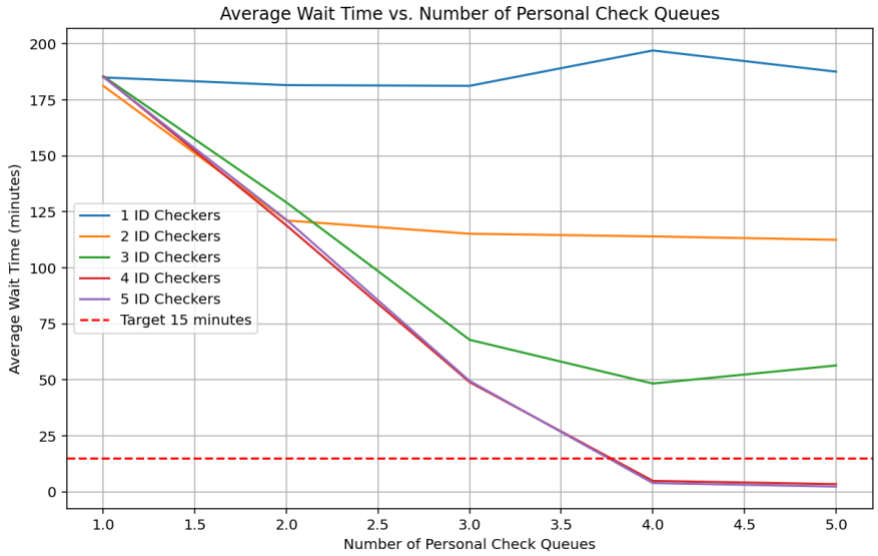
```
import simpy
import random
import numpy as np
import matplotlib.pyplot as plt

# Parameters

LAMBDA_ARRIVAL = 50  # Poisson arrival rate: 50 passengers per minute (busier airport)
MU_ID_CHECK = 0.75  # Mean service time for ID check in minutes
MU_PERSONAL_CHECK_MIN = 0.5  # Minimum time for personal check in minutes
MU_PERSONAL_CHECK_MAX = 1.0  # Maximum time for personal check in minutes
SIM_TIME = 500  # Simulation time in minutes
TARGET_WAIT_TIME = 15  # Target maximum average wait time in minutes


# Define passenger arrival process
def passenger_arrival(env, id_checkers, personal_queues):
    passenger_id = 0
    while True:
        interarrival_time = random.expovariate(LAMBDA_ARRIVAL)
        yield env.timeout(interarrival_time)
        passenger_id += 1
        env.process(passenger_process(env, passenger_id, id_checkers, personal_queues))

# Passenger process
def passenger_process(env, passenger_id, id_checkers, personal_queues):
```

```
    arrival_time = env.now

    # ID/Boarding-pass check
    with id_checkers.request() as request:
        yield request
        id_check_time = random.expovariate(1.0 / MU_ID_CHECK)
        yield env.timeout(id_check_time)

    # Personal check queue (choose shortest queue)
    shortest_queue = min(personal_queues, key=lambda x: len(x.queue))
    with shortest_queue.request() as request:
        yield request
        personal_check_time = random.uniform(MU_PERSONAL_CHECK_MIN, MU_PERSONAL_CHECK_MAX)
        yield env.timeout(personal_check_time)

    # Record waiting time
    total_wait_time = env.now - arrival_time
    waiting_times.append(total_wait_time)

# Run the simulation
def run_simulation(num_id_checkers, num_personal_queues):
    global waiting_times
    waiting_times = []

    # Create environment
    env = simpy.Environment()

    # Create resources (ID checkers and personal queues)
    id_checkers = simpy.Resource(env, capacity=num_id_checkers)
    personal_queues = [simpy.Resource(env, capacity=1) for _ in range(num_personal_queues)]

    # Start passenger arrival process
    env.process(passenger_arrival(env, id_checkers, personal_queues))

    # Run simulation
    env.run(until=SIM_TIME)

    # Calculate average waiting time
    avg_wait_time = np.mean(waiting_times)
    return avg_wait_time

# Main part of the code
def main():
    # Store results for visualization
    results = []

    # Experiment with different numbers of ID checkers and personal check queues
    for id_checkers in range(1, 50):  # Test with 1 to 5 ID checkers
        row = []
        for personal_queues in range(1, 50):  # Test with 1 to 5 personal check queues
            avg_wait_time = run_simulation(id_checkers, personal_queues)
            row.append(avg_wait_time)
            print(f"ID Checkers: {id_checkers}, Personal Check Queues: {personal_queues}, "
                  f"Avg Wait Time: {avg_wait_time:.2f} minutes")
        results.append(row)

    # Convert results to a numpy array for plotting
    results = np.array(results)

    # Plot the results
    plt.figure(figsize=(10, 6))
    for i in range(5):
        plt.plot(range(1, 51), results[i], label=f'{i + 1} ID Checkers')

    plt.axhline(y=TARGET_WAIT_TIME, color='r', linestyle='--', label=f'Target {TARGET_WAIT_TIME} minutes')
    plt.xlabel('Number of Personal Check Queues')
    plt.ylabel('Average Wait Time (minutes)')
    plt.title('Average Wait Time vs. Number of Personal Check Queues')
    plt.legend()
    plt.grid(True)
```

```
    plt.show()

if __name__ == "__main__":
    main()
```

## Result:

*ID Checkers: 1, Personal Check Queues: 1, Avg Wait Time: 184.99 minutes*
*ID Checkers: 1, Personal Check Queues: 2, Avg Wait Time: 181.56 minutes*
*ID Checkers: 1, Personal Check Queues: 3, Avg Wait Time: 181.24 minutes*
*ID Checkers: 1, Personal Check Queues: 4, Avg Wait Time: 197.02 minutes*
*ID Checkers: 1, Personal Check Queues: 5, Avg Wait Time: 187.57 minutes*
*ID Checkers: 2, Personal Check Queues: 1, Avg Wait Time: 181.23 minutes*
*ID Checkers: 2, Personal Check Queues: 2, Avg Wait Time: 121.15 minutes*
*ID Checkers: 2, Personal Check Queues: 3, Avg Wait Time: 115.24 minutes*
*ID Checkers: 2, Personal Check Queues: 4, Avg Wait Time: 114.05 minutes*
*ID Checkers: 2, Personal Check Queues: 5, Avg Wait Time: 112.53 minutes*
*ID Checkers: 3, Personal Check Queues: 1, Avg Wait Time: 185.53 minutes*
*ID Checkers: 3, Personal Check Queues: 2, Avg Wait Time: 129.21 minutes*
*ID Checkers: 3, Personal Check Queues: 3, Avg Wait Time: 67.91 minutes*
*ID Checkers: 3, Personal Check Queues: 4, Avg Wait Time: 48.39 minutes*
*ID Checkers: 3, Personal Check Queues: 5, Avg Wait Time: 56.45 minutes*
*ID Checkers: 4, Personal Check Queues: 1, Avg Wait Time: 185.46 minutes*
*ID Checkers: 4, Personal Check Queues: 2, Avg Wait Time: 118.95 minutes*
*ID Checkers: 4, Personal Check Queues: 3, Avg Wait Time: 49.02 minutes*
*ID Checkers: 4, Personal Check Queues: 4, Avg Wait Time: 4.97 minutes*
*ID Checkers: 4, Personal Check Queues: 5, Avg Wait Time: 3.52 minutes*
*ID Checkers: 5, Personal Check Queues: 1, Avg Wait Time: 185.17 minutes*
*ID Checkers: 5, Personal Check Queues: 2, Avg Wait Time: 121.49 minutes*
*ID Checkers: 5, Personal Check Queues: 3, Avg Wait Time: 49.56 minutes*
*ID Checkers: 5, Personal Check Queues: 4, Avg Wait Time: 3.99 minutes*
*ID Checkers: 5, Personal Check Queues: 5, Avg Wait Time: 2.43 minutes*




When you have 4 or 5 ID checkers, the average wait time dramatically drops.
With 4 ID checkers and 4 personal check queues, the average wait time is only **4.97 minutes**, and with 5 ID checkers and 5 personal check queues, the wait time further decreases to **2.43 minutes**.
These combinations (4-5 ID checkers and 4-5 personal queues) are **well below** the target of 15 minutes.

## In case of lambda =50:

Because the data is so massive, we select some of the results.

## When the number of ID checkers is 37 and more, the target time can be meet, the Avg Wait Time: 7.43 minutes.

*ID Checkers: 36, Personal Check Queues: 1, Avg Wait Time: 244.72 minutes*
*ID Checkers: 36, Personal Check Queues: 2, Avg Wait Time: 236.98 minutes*
*ID Checkers: 36, Personal Check Queues: 3, Avg Wait Time: 229.55 minutes*
*ID Checkers: 36, Personal Check Queues: 4, Avg Wait Time: 223.94 minutes*
*ID Checkers: 36, Personal Check Queues: 5, Avg Wait Time: 215.71 minutes*
*ID Checkers: 36, Personal Check Queues: 6, Avg Wait Time: 210.69 minutes*
*ID Checkers: 36, Personal Check Queues: 7, Avg Wait Time: 205.08 minutes*
*ID Checkers: 36, Personal Check Queues: 8, Avg Wait Time: 197.09 minutes*
*ID Checkers: 36, Personal Check Queues: 9, Avg Wait Time: 191.31 minutes*
*ID Checkers: 36, Personal Check Queues: 10, Avg Wait Time: 181.28 minutes*
*ID Checkers: 36, Personal Check Queues: 11, Avg Wait Time: 176.95 minutes*
*ID Checkers: 36, Personal Check Queues: 12, Avg Wait Time: 169.14 minutes*
*ID Checkers: 36, Personal Check Queues: 13, Avg Wait Time: 164.64 minutes*
*ID Checkers: 36, Personal Check Queues: 14, Avg Wait Time: 157.06 minutes*
*ID Checkers: 36, Personal Check Queues: 15, Avg Wait Time: 150.46 minutes*
*ID Checkers: 36, Personal Check Queues: 16, Avg Wait Time: 144.42 minutes*
*ID Checkers: 36, Personal Check Queues: 17, Avg Wait Time: 137.97 minutes*
*ID Checkers: 36, Personal Check Queues: 18, Avg Wait Time: 130.67 minutes*
*ID Checkers: 36, Personal Check Queues: 19, Avg Wait Time: 123.84 minutes*
*ID Checkers: 36, Personal Check Queues: 20, Avg Wait Time: 115.05 minutes*
*ID Checkers: 36, Personal Check Queues: 21, Avg Wait Time: 112.45 minutes*
*ID Checkers: 36, Personal Check Queues: 22, Avg Wait Time: 103.31 minutes*
*ID Checkers: 36, Personal Check Queues: 23, Avg Wait Time: 95.88 minutes*
*ID Checkers: 36, Personal Check Queues: 24, Avg Wait Time: 92.12 minutes*
*ID Checkers: 36, Personal Check Queues: 25, Avg Wait Time: 85.84 minutes*
*ID Checkers: 36, Personal Check Queues: 26, Avg Wait Time: 77.61 minutes*
*ID Checkers: 36, Personal Check Queues: 27, Avg Wait Time: 72.11 minutes*
*ID Checkers: 36, Personal Check Queues: 28, Avg Wait Time: 65.62 minutes*
*ID Checkers: 36, Personal Check Queues: 29, Avg Wait Time: 58.97 minutes*
*ID Checkers: 36, Personal Check Queues: 30, Avg Wait Time: 54.19 minutes*
*ID Checkers: 36, Personal Check Queues: 31, Avg Wait Time: 44.58 minutes*
*ID Checkers: 36, Personal Check Queues: 32, Avg Wait Time: 37.26 minutes*
*ID Checkers: 36, Personal Check Queues: 33, Avg Wait Time: 28.49 minutes*
*ID Checkers: 36, Personal Check Queues: 34, Avg Wait Time: 24.75 minutes*
*ID Checkers: 36, Personal Check Queues: 35, Avg Wait Time: 18.69 minutes*
***ID Checkers: 36, Personal Check Queues: 36, Avg Wait Time: 16.58 minutes***
*ID Checkers: 37, Personal Check Queues: 1, Avg Wait Time: 243.87 minutes*
*ID Checkers: 37, Personal Check Queues: 2, Avg Wait Time: 236.97 minutes*
*ID Checkers: 37, Personal Check Queues: 3, Avg Wait Time: 230.71 minutes*
*ID Checkers: 37, Personal Check Queues: 4, Avg Wait Time: 222.49 minutes*
*ID Checkers: 37, Personal Check Queues: 5, Avg Wait Time: 216.17 minutes*
*ID Checkers: 37, Personal Check Queues: 6, Avg Wait Time: 209.96 minutes*
*ID Checkers: 37, Personal Check Queues: 7, Avg Wait Time: 204.61 minutes*
*ID Checkers: 37, Personal Check Queues: 8, Avg Wait Time: 197.78 minutes*
*ID Checkers: 37, Personal Check Queues: 9, Avg Wait Time: 190.40 minutes*
*ID Checkers: 37, Personal Check Queues: 10, Avg Wait Time: 183.71 minutes*
*ID Checkers: 37, Personal Check Queues: 11, Avg Wait Time: 177.10 minutes*
*ID Checkers: 37, Personal Check Queues: 12, Avg Wait Time: 169.13 minutes*
*ID Checkers: 37, Personal Check Queues: 13, Avg Wait Time: 164.90 minutes*
*ID Checkers: 37, Personal Check Queues: 14, Avg Wait Time: 155.50 minutes*
*ID Checkers: 37, Personal Check Queues: 15, Avg Wait Time: 150.82 minutes*
*ID Checkers: 37, Personal Check Queues: 16, Avg Wait Time: 145.48 minutes*
*ID Checkers: 37, Personal Check Queues: 17, Avg Wait Time: 140.17 minutes*
*ID Checkers: 37, Personal Check Queues: 18, Avg Wait Time: 130.45 minutes*
*ID Checkers: 37, Personal Check Queues: 19, Avg Wait Time: 124.72 minutes*
*ID Checkers: 37, Personal Check Queues: 20, Avg Wait Time: 115.86 minutes*
*ID Checkers: 37, Personal Check Queues: 21, Avg Wait Time: 111.61 minutes*
*ID Checkers: 37, Personal Check Queues: 22, Avg Wait Time: 107.02 minutes*
*ID Checkers: 37, Personal Check Queues: 23, Avg Wait Time: 95.60 minutes*
*ID Checkers: 37, Personal Check Queues: 24, Avg Wait Time: 89.22 minutes*
*ID Checkers: 37, Personal Check Queues: 25, Avg Wait Time: 83.64 minutes*
*ID Checkers: 37, Personal Check Queues: 26, Avg Wait Time: 74.79 minutes*
*ID Checkers: 37, Personal Check Queues: 27, Avg Wait Time: 70.51 minutes*
*ID Checkers: 37, Personal Check Queues: 28, Avg Wait Time: 66.09 minutes*
*ID Checkers: 37, Personal Check Queues: 29, Avg Wait Time: 56.63 minutes*
*ID Checkers: 37, Personal Check Queues: 30, Avg Wait Time: 52.50 minutes*
*ID Checkers: 37, Personal Check Queues: 31, Avg Wait Time: 47.09 minutes*

*ID Checkers: 37, Personal Check Queues: 32, Avg Wait Time: 38.30 minutes*
*ID Checkers: 37, Personal Check Queues: 33, Avg Wait Time: 30.81 minutes*
*ID Checkers: 37, Personal Check Queues: 34, Avg Wait Time: 24.44 minutes*
*ID Checkers: 37, Personal Check Queues: 35, Avg Wait Time: 17.88 minutes*
*ID Checkers: 37, Personal Check Queues: 36, Avg Wait Time: 17.46 minutes*
***ID Checkers: 37, Personal Check Queues: 37, Avg Wait Time:*** ***7.43 minutes***