Hyperuniform structures, rigid point processes and related topics

International Workshop

20 – 22 February 2023

Lille, France

Laboratoire Paul Painlevé

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Université de Lille

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SPP2265 Random Geometric Systems

https://www.mathconf.org/hsrpp2023

Monday, February 20

13:45–14:00	**Welcome remarks**	
14:00–14:45	**Hartmut Löwen** Universität Düsseldorf	Active circle swimmers as hyperuniform systems
14:50–15:35	**Jeanne Boursier** École normale sup. de Lyon	Dipole transition for the two-component plasma
15:35–16:00	**Coffee break**	
16:00–17:00	**Salvatore Torquato** Princeton University	Hyperuniformity of Point Processes and Two-phase Media
17:05–17:50	**Raphael Lachieze-Rey** Université Paris Cité	Linearity of Gaussian excursions variance
18:00–20:00	City walk	

Tuesday, February 21

9:00–9:45	**Thomas Leblé** Université de Paris	Charge Fluctuations in 2d Coulomb (and related) Systems
9:45–10:20	**Coffee break**	
10:20–11:05	**Tobias Hartnick** Karlsruhe Institute of Tech.	Hyperuniformity and non-hyperuniformity of quasicrystals
11:10–11:55	**Michael Björklund** Chalmers and Gothenburg Universities	A spectral approach to isotropic point processes in “curved” geometries – with a view towards hyperuniformity
11:55–14:00	**Lunch break**	
14:00–14:45	**Simon Coste** Université de Paris	A survey on the rigidity of point processes
14:50–15:35	**Daniela Flimmel** Université de Lille	On the hyperuniformity of short range Gibbs point processes
15:35–16:10	**Coffee break**	
16:10–16:55	**Peter Grabner** Graz University of Tech.	Hyperuniformity on the sphere
17:00–17:45	**Diala Hawat** Université de Lille	On estimating the structure factor of a point process, with applications to hyperuniformity
17:50–18:00	**Günter Last** Karlsruhe Institute of Tech.	A genuine test for hyperuniformity
19:30	Conference Dinner	

Wednesday, February 22

9:00–9:45	**Sophie Marbach** Sorbonne University	Fancy counting: dynamic fluctuations in finite volumes
9:45–10:20	**Coffee break**	
10:20–11:05	**Thibaut Vasseur** Université Paris Cité	Gibbs point processes and number-rigidity
11:10–11:55	**Daniel Hug** Karlsruhe Institute of Tech.	Poisson flats in constant curvature spaces
11:55–14:00	**Lunch & Time to say goodbye**	

Contents

General information

The workshop focuses on recent developments in hyperuniformity and rigidity phenomena of point processes, random measures, and random fields. Related topics of interest are stochastic geometry (geometric point processes and tessellations) as well as applications in physics, materials science, and image analysis. It will take place in the building of **Laboratoire Paul Painlevé** (Cité scientifique 59650 Villeneuve-d'Ascq). We hope you will enjoy the workshop.

Practical information

- We booked rooms for all the speakers in the hotel *Brueghel* (3 Parvis Saint-Maurice, near railway and metro stations).
- For the sake of lively discussion, please respect the following maximal talk times allocated to your presentations: 40 minutes of talk time + 5 minutes for discussion.
- The lecture hall is equipped with a computer and a video projector. There is also some black or white board available. Please upload the slides of your talk (preferably in pdf format) to the computer in the lecture room well before the start of the session allocated to your presentation.
- During **coffee breaks**, water, coffee, and tea will be available.
- On Tuesday evening you are cordially invited to a **conference dinner** in the restaurant *Le Compostelle* (4 rue Saint Etienne Lille) starting at 7 : 30 pm to celebrate a hopefully inspiring and enjoyable workshop.

Scientific committee

David Dereudre
University of Lille, France

Günter Last
Karlsruhe Institute of Technology, Germany

Organising committee

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Abstracts of talks

Michael Björklund

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A spectral approach to isotropic point processes in "curved" geometries – with a view towards hyperuniformity

Joint works with Tobias Hartnick (KIT) and Mattias Bylehn (Chalmers/GU).

Gelfand pairs allow us to bring the Euclidean, Heisenberg and hyperbolic (as well as higher-rank symmetric) geometries under the same umbrella and to formulate a coherent Fourier analysis for all of these geometries. During my talk I will discuss what Gelfand pairs are, and how familiar concepts like auto-correlations and spectral density functions can be formulated in this general setting – giving plenty of examples (Poisson processes, determinantal processes, quasi-crystals). Some surprises (concerning hyperuniformity) in hyperbolic geometries will be presented, as well as (if time permits) some intriguing problems in Heisenberg geometries. No previous knowledge of Gelfand pairs will be assumed.

Jeanne Boursier

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Dipole transition for the two-component plasma

Joint work with Sylvia Serfaty.

We study the two-dimensional two-component plasma or 2D Coulomb gas. The system exhibits a transition between a situation where particles are free and a situation where they form dipoles of opposite signs. This "dipole transition" appears at beta=2, in contrast with the KT transition which is expected to happen at beta=4 in our setting. We give a proof of the "dipole transition" by using energetic-entropic arguments via large deviations and electrostatic estimates on the energy. The fluctuations of the system will also be discussed.

Simon Coste

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A survey on the rigidity of point processes

In this informal talk, I will give a high-level overview of the mathematics related to rigidities of point processes, and formulate a few potentially intriguing questions, mostly coming from physics and machine learning.

Daniela Flimmel

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On the hyperuniformity of short range Gibbs point processes

A stationary point process is called hyperuniform if the number of points that fall into a given bounded set fluctuates in lower order than the volume of the set. Usually, proving hyperuniformity rigorously is a rather difficult task and therefore, a lot of attention is given to estimating the so called structure factor instead. However, we show that Gibbsian models are not hyperuniform if the interaction among points become negligible with the distance. This class includes for instance finite range pair potentials and many non-pairwise interactions depending on the geometrical structure such as Voronoi tessellation or k–nearest neighbour graph, where the range of interaction may be random.

Peter Grabner

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Hyperuniformity on the sphere

Joint work with J. Brauchart, W. Kusner, and J. Ziefle.

The concept of hyperuniformity had been introduced by S. Torquato and F. Stillinger to measure regularity of distributions of infinite particle systems in $\mathbb{R}^d$. An infinite particle system $X \subset \mathbb{R}^d$ is called hyperuniform, if the variance (with respect to the thermodynamic limit) of the number of points in a **large** ball is smaller than "usual":

$$\mathbb{V}\#(X \cap B(\mathbf{x}, R)) = \mathcal{O}(R^{d-1}) \text{ for } R \to \infty.$$

Notice that this variance is of order R^d for Poisson point processes.
We generalise this concept to the sphere and the torus by considering sequences of **finite** point sets $(X_N)_N$ (with $\#X_N = N$). The phenomenon of a "smaller than usual" variance of the point counting function is then be observed for geodesic balls with $N\mathrm{vol}(B_R) \to \infty$ but $\mathrm{vol}(B_R) \to 0$. We will discuss several examples of hyperuniform sequences of point sets.

Tobias Hartnick

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Hyperuniformity and non-hyperuniformity of quasicrystals

We discuss hyperuniformity for a class of stationary jammed hardcore point processes in $\mathbb{R}^n$ known as cut-and-project (CUP) processes which serve as the main mathematical models of quasicrystals and depend on two geometric parameters, an underlying lattice and a window. While hyperuniformity of some CUP processes has been established both numerically and rigorously in the literature, no general hyperuniformity results were known until recently.
In joint work with Michael Björklund we showed that for almost all lattices the corresponding CUP process is hyperuniform provided the window is sufficiently Fourier smooth. At the same time we provided the first examples of anti-hyperuniform quasicrystals. We also showed that the rate of hyperuniformity of a CUP process depends entirely on certain diophantine (i.e. number theoretic) properties of the underlying lattice. Our approach is based on a spectral characterization of hyperuniformity which was previously established under the assumption that the point process admits a structure factor (which is not satisfied in the CUP case), but turns out to be equally valid without this assumption.
In this talk we are going to introduce CUP processes and the general spectral criterion for hyperuniformity. We are then going to illustrate the dichotomy between hyperuniform and anti-hyperuniform quasicrystals in a one-dimensional toy model which depends on a single parameter. We will explain why in this toy model the rate of hyperuniformity only depends on how well the parameter can be approximated by rational numbers.

Diala Hawat

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On estimating the structure factor of a point process, with applications to hyperuniformity

Hyperuniformity is the study of stationary point processes with a sub-Poisson variance in a large window. Hyperuniform point processes have received a lot of attention in statistical physics, both for the investigation of naturally organized structures and the synthesis of materials. Unfortunately, rigorously proving that a point process is hyperuniform is usually difficult. A common practice in statistical physics and chemistry is to use a few samples to estimate a spectral measure called the structure factor. Its decay around zero provides a diagnostic of hyperuniformity.
The talk starts with a survey of some of the estimators of the structure factor. Then, we construct an asymptotically valid statistical test of hyperuniformity based on these estimators. Finally, we benchmark the estimators and the hyperuniformity test on a set of examples using the Python toolbox structure-factor, containing all the needed tools.

Preprint: https://arxiv.org/abs/2203.08749
GitHub: https://github.com/For-a-few-DPPs-more/structure-factor
Documentation: https://for-a-few-dpps-more.github.io/structure-factor/

Daniel Hug

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Poisson flats in constant curvature spaces

Joint work with Carina Betken and Christoph Thäle.

Poisson hyperplane processes in Euclidean, spherical and more recently also in hyperbolic space have been investigated. It has been found that certain mean values of Poisson functionals in these spaces do not differ too much, but the asymptotic behaviour of variances and the asymptotic fluctuations may depend on the signature of the constant curvature $\kappa \in \{-1, 0, 1\}$ in an essential way, depending on the specific asymptotic setup. For instance, for fixed intensity and a growing spherical observation window we have previously shown (in joint work with Felix Herold and Christoph Thäle) that a central limit theorem holds in hyperbolic space $\mathbb{H}^d$ for the volume of intersection processes within the observation window only if $d \in \{2, 3\}$, which is in stark contrast to the corresponding behaviour in Euclidean spaces. Several questions arise naturally:
1) What is so special about dimension 4 in hyperbolic space?
2) What is the limit behaviour if no CLT holds?
We present some answers to these questions and also discuss asymptotic and extremal properties of related variances.

Raphael Lachieze-Rey

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Linearity of Gaussian excursions variance

In this talk, we study variance linearity for random sets obtained by thresholding a random Gaussian field at the level 0. After briefly discussing some results in various contexts, we will focus on the case of the excursion volume for real Euclidean stationary Gaussian fields. In particular, we show that the variance magnitude on a spherical window growing to infinity is directly related to the recurrence properties for the random walk associated to the spectral measure. It yields that for fields which spectral measure has a one-dimensional component, and in particular for isotropic fields, the variance is always at least linear in the volume. Hence we focus on fields with a singular spectral measure, and we show that in this case the variance can be of any order, depending on the diophantine properties of the spectral atoms.

Günter Last

Karlsruhe Institute of Technology, Germany

Joint work with Michael Klatt (Düsseldorf) and Norbert Henze (Karlsruhe).

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A genuine test for hyperuniformity

This talk introduces a rigorous significance test for hyperuniformity with sensitive results even for a single sample and at practically relevant system sizes. The test is based on the asymptotic distribution of the (empirical) scattering intensity, which is (suitably normalized) a standard estimator of the structure factor. We can then use the likelihood ratio principle to test for hyperuniformity. Remarkably, the asymptotic distribution of the resulting test statistic is universal under the null hypothesis of hyperuniformity. We obtain its explicit form from simulations with very high accuracy. The novel test precisely keeps a nominal significance level for hyperuniform models, and it rejects non-hyperuniform examples with high power, even in borderline cases.

Thomas Leblé

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Charge Fluctuations in 2d Coulomb (and related) Systems

I will review some results concerning number statistics for particles in 2D Coulomb gases (and related systems) in the mathematics and the physics literature and sketch a proof of hyperuniformity for the two-dimensional one-component plasma.

Hartmut Löwen

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Active circle swimmers as hyperuniform systems

Recently active systems of Brownian circle swimmers have been found to exhibit hyperuniformity at low temperatures [1]. In this talk I shall discuss the physical background of the model used to describe single circle swimmers [1,2] and dense interacting many-body systems of the latter [4]. Some attempts to use a theoretical understanding of the underlying hyperuniformity and other active system with hyperuniform structures are also discussed.

1. Q. L. Lei, M. P. Ciamarra, R. Ni (2019). Nonequilibrium strongly hyperuniform fluids of circle active particles with large local density fluctuations. *Science Advances 5*, eaau7423.

2. S. van Teeffelen, H. Löwen (2008). Dynamics of a Brownian circle swimmer. *Physical Review E 78*, 020101.

3. F. Kümmel, B. ten Hagen, R. Wittkowski, I. Buttinoni, R. Eichhorn, G. Volpe, H. Löwen and C. Bechinger (2013). Circular motion of asymmetric self-propelling particles. *Phys. Rev. Letters 110*, 198302.

4. V. E. Debets, H. Löwen, L. M. C. Janssen (in press). Glassy dynamics in chiral fluids. *Physical Review Letters*, see also arXiv:2210.03196.

Sophie Marbach

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Fancy counting: dynamic fluctuations in finite volumes

Probing thermodynamic properties from fluctuations in finite observation volumes is common in molecular simulations, but less attention has been paid to analyzing kinetic properties. Here we study fluctuations in finite observation volumes with a range of experiments, simulations and theory. We quantify particle number and charge fluctuations, distinguishing between steric, hydrodynamic and electrostatic effects. We first show how fluctuations may be used to measure kinetic properties such as the diffusion coefficient. Further, we find a rich phenomenology of timescales affecting both particle number and charge number fluctuations. Overall, we find that the power spectrum of these fluctuations decays as $1/f^{3/2}$ where f is the frequency. This decay is a signature of fractional noise (or more precisely, that these random numbers are fractional brownian walks with Hurst index 1/4), and we show that it is a universal feature when observing diffusing particles in finite domains. We also find that the hyperuniform behavior of charge fluctuations, namely that correlations scale with the area of the observation volume, is preserved in time. Correlations even become proportional to the box perimeter at long enough times. We will show how our results pave the way for a better understanding of fluctuations in more complex contexts such as biological or synthetic nanopores, especially for applications in sequencing.

Salvatore Torquato

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Hyperuniformity of Point Processes and Two-phase Media

The study of hyperuniform states of matter is an emerging multidisciplinary field, influencing and linking developments across the physical sciences, mathematics and biology. A hyperuniform point process in d-dimensional Euclidean space is characterized by an anomalous suppression of large-scale density fluctuations relative to those in typical disordered systems, such as liquids and amorphous solids. As such, the hyperuniformity concept generalizes the traditional notion of long-range order to include not only all perfect crystals and quasicrystals, but also exotic disordered states of matter, thus providing a unified framework to quantitatively categorize such phases of matter. Disordered hyperuniform states have attracted great attention across many fields over the last two decades because they have the character of crystals on large length scales but are isotropic like liquids.
I will begin by reviewing the hyperuniformity concept, including generalizations to two-phase media, scalar fields and vector fields. Then I will give an overview of how hyperuniformity is linked to the Epstein zeta function of number theory, sphere packing problems, eigenvalues of random matrices, nontrivial zeros of the Riemann zeta function, spatial distribution of the prime numbers, free fermions, Laughlin's incompressible quantum states, and photoreceptor mosaics in avian retina. Finally, I will show how to generate disordered hyperuniform two-phase media that are rigorously disordered and hyperuniform.

Thibaut Vasseur

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Gibbs point processes and number-rigidity

Gibbs point processes are an important and rich class of point processes with interactions between points. It is a natural tool in the study of the microscopic thermodynamic limit of interacting particle systems. In this talk, we will investigate the property of number-rigidity for Gibbs point processes with pair-wise interaction, which raises a natural question about the relationship between the range of the interaction and the possibility of being number-rigid. We will briefly introduce the DLR (Dobrushin-Lanford-Ruelle) equations, which describe the local conditional laws of Gibbs point processes, and show how they can be used in conjunction with stationarity to determine whether such a process is cardinal rigid or not.

Participant list

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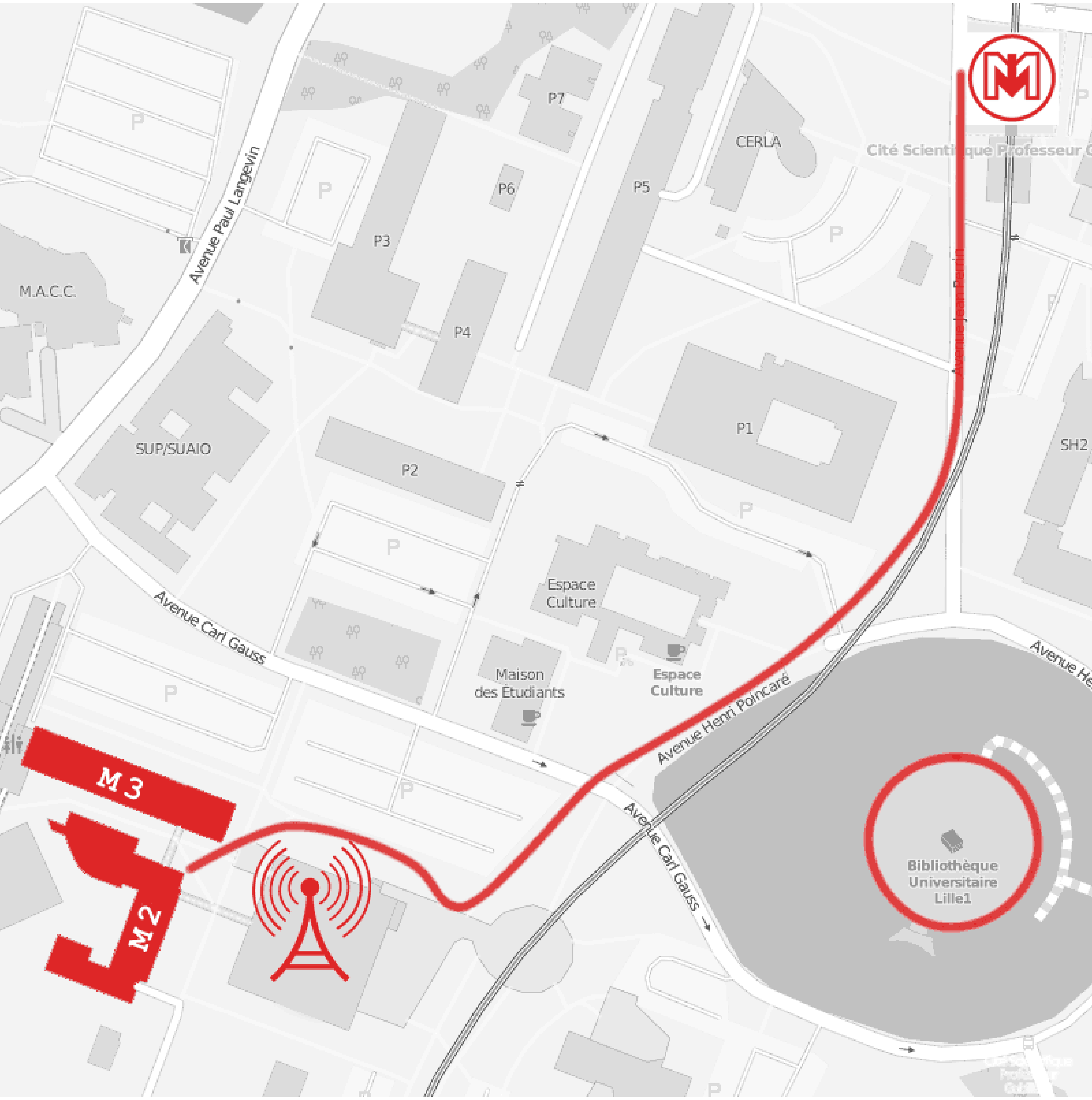

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