CS 3653 – Discrete Mathematics for Computer Science

Assignment # 3	Due – Jan 31, 2022, 11:59pm (CST)
Chapter # 1.7 - 1.8	Max. Points # 25

SN	QUESTION	Pts
1	a) Use a direct proof to show that the sum of two even integers is even. b) Show that the additive inverse, or negative, of an even number is an even number using a direct proof. c) Use a direct proof to show that the product of two odd numbers is odd. d) Prove that if n is a perfect square, then $n + 2$ is not a perfect square. e) Use a direct proof to show that the product of two rational numbers is rational.	5 X 1
2	a) Prove that if m and n are integers and mn is even, then m is even or n is even. b) Prove the proposition $P(0)$, where $P(n)$ is the proposition "If n is a positive integer greater than 1, then $n^2 > n$." What kind of proof did you use?	2 1
3	a) Prove that if n is a positive integer, then n is even if and only if $7n + 4$ is even. b) Show that these three statements are equivalent, where a and b are real numbers: (i) a is less than b, (ii) the average of a and b is greater than a, and (iii) the average of a and b is less than b.	2 2
4	a) Prove that $n^2 + 1 \geq 2^n$ when n is a positive integer with $1 \leq n \leq 4$. b) Prove that there are 100 consecutive positive integers that are not perfect squares. Is your proof constructive or nonconstructive?	1 1
5	a) Prove that if x and y are real numbers, then $\max(x, y) + \min(x, y) = x + y$. [Hint: Use a proof by cases, with the two cases corresponding to $x \geq y$ and $x<y$, respectively.] b) Prove using the notion of without loss of generality that $5x + 5y$ is an odd integer when x and y are integers of opposite parity.	2 2
6	a) Prove that there is no positive integer n such that $n^2 + n^3 = 100$. b) Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.	1 2

7	a) Prove or disprove that you can use dominoes to tile the standard checkerboard with two adjacent corners removed (that is, corners that are not opposite). b) Prove or disprove that you can use dominoes to tile a standard checkerboard with all four corners removed.	2 2

1	a) Use a direct proof to show that the sum of two even integers is even. b) Show that the additive inverse, or negative, of an even number is an even number using a direct proof. c) Use a direct proof to show that the product of two odd numbers is odd. d) Prove that if n is a perfect square, then n + 2 is not a perfect square. e) Use a direct proof to show that the product of two rational numbers is rational.

a.) if two Integers are even, then the Sum of these two Integers is even

assume $n = 2k$, $m = 2l$, then $n + m = 4(k+l)$ is even

b.) assume $n = 2k$, then $-n = -2k$ is even

C.) if two Integers are odd, then the Product of them is odd

assume $n = 2k+1$, $m = 2l+1$ $\qquad (k+1)(l+1)$

then $n \cdot m = 4kl + 2k + 2l + 1$ is odd $\qquad kl + k + l + 1$

D.) if n is a Perfect Square, then n+2 is not a Perfect Square
Proof by Contradiction

assume n+2 is a Perfect Square when n is a Perfect Square

$n = a^2$, $n+2 = b^2$ $\qquad (b^2) - (a^2) = 2$

$(n+2) - (n) = 2$ $\qquad (b+a)(b-a) = 2$

$b+a = 2$, $b-a = 1$

$(b+a) + (b-a) = 3 \Rightarrow 2b = 3 \Rightarrow b = \frac{3}{2}$

$\frac{3}{2}$ is not an integer Contradicting the assumtion that n+2 is a Perfect Square.

E.) if n and m are rational, then the Product of the two will be rational.

assume n & m are rational. $n = \frac{a}{b}$, $m = \frac{c}{d}$ and $b \neq 0$, $d \neq 0$

$n \cdot m = \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, Since $b \neq 0$, $d \neq 0$ $bd \neq 0$

So $\frac{ac}{bd}$ is rational and $n \cdot m$ is rational.

2	a) Prove that if m and n are integers and mn is even, then m is even or n is even. b) Prove the proposition P(0), where P(n) is the proposition "If n is a positive integer greater than 1, then $n^2 > n$." What kind of proof did you use?	

a.) if n & m are integers and nm is even, the m or n is even.

assume 3n+2 is even and n is odd.
if n is odd and the Product of two odd numbers is odd
the 3n is odd and 3n+2 is odd.
So 3n+2 is even and n is odd is incorrect
So if n is an integer and 3n+2 is even then n is even

b.)

P(0) is Vacuously true because 0 is not a Positive integer.

Vacuous Proof.

3	a) Prove that if n is a positive integer, then n is even if and only if 7n + 4 is even. b) Show that these three statements are equivalent, where a and b are real numbers: (i) a is less than b, (ii) the average of a and b is greater than a, and (iii) the average of a and b is less than b.	

a.) assume n is even then $n = 2k$

$7n+4 = 7(2k)+4 = 14k+4$ hence $7n+4$ is even

if n is odd then $n = 2k+1$ <— Proving the Converse

$7n+4 = 7(2k+1)+4 = 14k+11 = 2(7k+5)+1$
hence $7n+4$ is odd

b.) To Prove (i) -> (ii)

if $a<b$ then $b>a$
$a+b > a+a$
$a+b > 2a$
$\frac{a+b}{2} > a$ thus Proving average of a&b is > a

To Prove (ii) -> (iii)

Suppose $\frac{a+b}{2} > a$
$a+b > 2a$
$b > 2a - a$
$b > a$
$a < b$

$a+b < b+b$
$a+b < 2b$
$\frac{a+b}{2} < b$ thus Proving average of a & b is $< b$

To Prove (iii) -> (i)

Suppose $\frac{a+b}{2} < b$

$a+b < b$
$a < 2b - b$
$a < b$ thus Proving $a < b$

and that (i), (ii), (iii) are equivalent.

4	a) Prove that $n^2 + 1 \geq 2^n$ when n is a positive integer with $1 \leq n \leq 4$. b) Prove that there are 100 consecutive positive integers that are not perfect squares. Is your proof constructive or nonconstructive?

a.) Solve by Cases

Case 1: $n = 1$

$(1)^2 + 1 \geq 2^1 = 2 \geq 2$ ✓

Case 2: $n = 2$

$(2)^2 + 1 \geq 2^2 = 5 \geq 4$ ✓

Case 3: $n = 3$

$(3)^2 + 1 \geq 2^3 = 10 \geq 8$ ✓

Case 4: $n = 4$

$(4)^2 + 1 \geq 2^4 = 17 \geq 16$ ✓

Thus it is True

b.)

10,001, 10,002 ... 10,100 are all non-Squares

$100^2 = 10{,}000$, $101^2 = 10{,}201$

Constructive

5	a) Prove that if x and y are real numbers, then max(x, y) + min(x, y) = x + y. [Hint: Use a proof by cases, with the two cases corresponding to $x \geq y$ and $x<y$, respectively.] b) Prove using the notion of without loss of generality that $5x + 5y$ is an odd integer when x and y are integers of opposite parity.	

a.) if x and y are real numbers, then $Max(x,y) + min(x,y) = x+y$

if $x \leq y$, then $Max(x,y) + min(x,y) = y + x = x + y$

if $x \geq y$, then $Max(x,y) + min(x,y) = x + y$

Since these are the only two Case the equality holds

b.) Case 1:

if x is even and y is odd

$x = 2m$, $y = 2n+1$

$5x + 5y = 5(2m) + 5(2n+1) = 10m + 10n + 5 = 2(5m + 5n + 2) + 1$

Thus $5x+5y$ is odd

Case 2:

if y is even and x is odd

$y = 2m$ $x = 2n+1$

$5x + 5y = 5(2n+1) + 5(2m) = 10n + 5 + 10m = 2(5n + 2 + 5m) + 1$

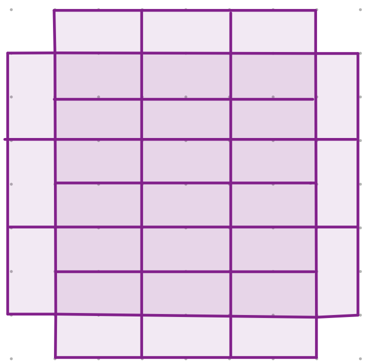
Thus $5x+5y$ is odd

6	a) Prove that there is no positive integer n such that $n^2 + n^3 = 100$. b) Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.	

7	a) Prove or disprove that you can use dominoes to tile the standard checkerboard with two adjacent corners removed (that is, corners that are not opposite). b) Prove or disprove that you can use dominoes to tile a standard checkerboard with all four corners removed.

a) It doesn't tell us the tiles so assume it is the top Left & top Right Squares

Place three dominoes in the top row horizontally and then 4 dominoes in the remeining 7 rows

b) 

Three dominoes in the boarders and then 3 dominoes horizondally in the remeining 6 rows