Bootcamp Day 3, TODAY. 8 pm.

Optimization in Several Variables.

← This HW is postponed to Tuesday!

I. General Situation, General Strategy.

II. $f(x,y) = x^3 + 7xy - 7y^2 - 19x$.

III. An "actual" problem...

I. General Situation:

You have some function

$$f(x,y).$$

Goal: Figure out the inputs (x,y) which MINIMIZE/MAXIMIZE

$f(x,y)$.

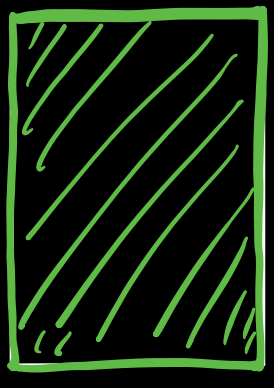
BAGGAGE: $2 \leq x \leq 8$.

$3 \leq y \leq 9$

Some sort of Restrictions on inputs.

~ Domain Over which you need to optimize.

Domain:



Dream Strategy: Step 1: Optimize $f(x,y)$ on the Inside of the Domain.

"Find Critical Points,

and classify as local min, local max, neither."

Step 2: Check the ~~"endpoints"~~ boundary.

Either: Just wing it.
or.
use the Method of Lagrange Multipliers.

II. Execute Step 1 for

$$f(x,y) = x^3 + 7xy - 7y^2 - 19x.$$

$$f_x = 3x^2 + 7y - 19$$

$$f_y = 7x - 14y$$

Find the critical Points of this function.

$$\nabla f = \langle 0, 0 \rangle$$

i.e.

$$\frac{\partial f}{\partial x} = 0$$

} Solve this System!

$$\frac{\partial f}{\partial y} = 0 \Bigg\}$$

(x,y)'s which make these true.

$$\begin{cases} f_x = 3x^2 + 7y - 19 \overset{\text{SET}}{=} 0 \\ f_y = 7x - 14y \overset{\text{SET}}{=} 0 \end{cases}$$

$$7x = 14y$$

$$\boxed{x = 2y}$$

$$3(2y)^2 + 7y - 19 = 0$$

$$12y^2 + 7y - 19 = 0$$

$$y = \frac{-7 \pm \sqrt{49 + 4(12)(19)}}{2(12)}$$

$$49 + 912$$

$$\frac{-7 \pm \sqrt{961}}{2(12)}$$

$$y = \frac{-7 \pm 31}{2(12)}$$

$$y = \frac{24}{24} \text{ or } \frac{-38}{24}$$

24 24

$$\begin{bmatrix} y = 1 \\ \downarrow \\ x = 2 \end{bmatrix} \quad \text{or} \quad \begin{matrix} -19 \\ \overline{12} \end{matrix} \; . \qquad \begin{matrix} \downarrow \\ x = \frac{-19}{6} \; . \end{matrix}$$

So:

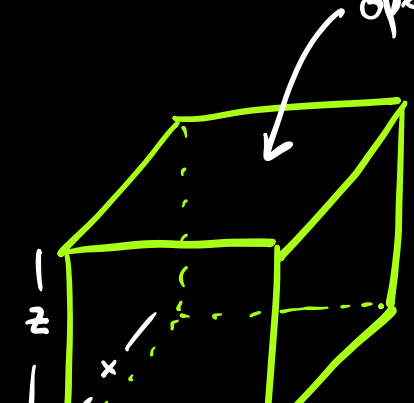
$(2, 1)$ AND $\left(\frac{-19}{6}, \frac{-19}{12}\right)$.

Now... How To Classify ?

(Mystery for now.)

III. You're making a box out of cardboard, open top.

Need it to be 100 Volumes.



Minimize Cardboard.

$= y$

Volume $= 100$

$$\begin{cases} xyz = 100 \rightsquigarrow z = \dfrac{100}{xy} \\ SA = xy + 2xz + 2yz. \end{cases}$$

$$SA = xy + 2x\frac{100}{xy} + 2y\cdot\frac{100}{xy}$$

$$SA = xy + \frac{200}{y} + \frac{200}{x}$$

Huge

(1) Critical Point Hunting:

$$\frac{\partial SA}{\partial x} \overset{\text{set}}{=} 0 = y - \frac{200}{x^2}$$

$$\frac{\partial SA}{\partial y} \overset{\text{set}}{=} 0 = x - \frac{200}{y^2}$$

$$\begin{cases} 0 = xy - \dfrac{200}{x} \\ \Updownarrow \\ 0 = xy - \dfrac{200}{y} \end{cases}$$

$$\Rightarrow \frac{200}{x} = \frac{200}{y}.$$

$$\boxed{x = y}.$$

$$x - \frac{200}{x^2} = 0$$

$$x^3 = 200$$

$$x = \sqrt[3]{200}$$

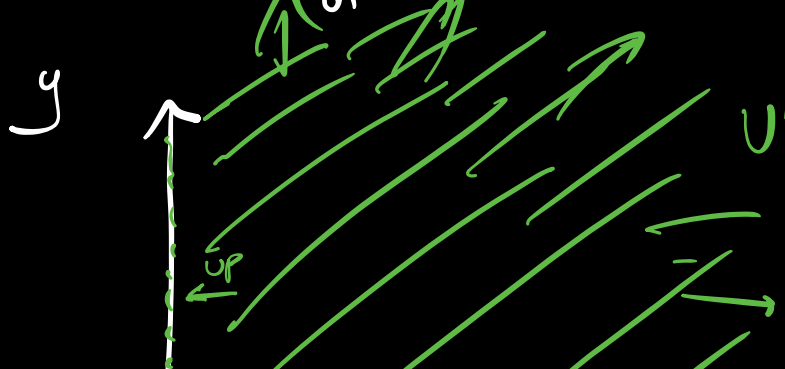
$$y = \sqrt[3]{200}$$

$$z = \frac{100}{\left(\sqrt[3]{200}\right)^2}$$

} Critical Point

(x, y) has to be the Global min of SA function.

Domain:




UPS
x