CS 3653 – Discrete Mathematics for Computer Science

Assignment # 3	Due – Jan 31, 2022, 11:59pm (CST)
Chapter # 1.7 - 1.8	Max. Points # 25

SN	QUESTION	Pts
1	a) Use a direct proof to show that the sum of two even integers is even. b) Show that the additive inverse, or negative, of an even number is an even number using a direct proof. c) Use a direct proof to show that the product of two odd numbers is odd. d) Prove that if n is a perfect square, then n + 2 is not a perfect square. e) Use a direct proof to show that the product of two rational numbers is rational.	5 X 1
2	a) Prove that if m and n are integers and mn is even, then m is even or n is even. b) Prove the proposition $P(0)$, where $P(n)$ is the proposition "If n is a positive integer greater than 1, then $n^2 > n$." What kind of proof did you use?	2 1
3	a) Prove that if n is a positive integer, then n is even if and only if $7n + 4$ is even. b) Show that these three statements are equivalent, where a and b are real numbers: (i) a is less than b, (ii) the average of a and b is greater than a, and (iii) the average of a and b is less than b.	2 2
4	a) Prove that $n^2 + 1 \geq 2^n$ when n is a positive integer with $1 \leq n \leq 4$. b) Prove that there are 100 consecutive positive integers that are not perfect squares. Is your proof constructive or nonconstructive?	1 1
5	a) Prove that if x and y are real numbers, then $\max(x, y) + \min(x, y) = x + y$. [Hint: Use a proof by cases, with the two cases corresponding to $x \geq y$ and $x<y$, respectively.] b) Prove using the notion of without loss of generality that $5x + 5y$ is an odd integer when x and y are integers of opposite parity.	2 2
6	a) Prove that there is no positive integer n such that $n^2 + n^3 = 100$. b) Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.	1 2

| 7 | a) Prove or disprove that you can use dominoes to tile the standard checkerboard with two adjacent corners removed (that is, corners that are not opposite). | 2 |
| | b) Prove or disprove that you can use dominoes to tile a standard checkerboard with all four corners removed. | 2 |