CS 3653 – Discrete Mathematics for Computer Science

Assignment # 2	Due – Jan 24, 2022, 11:59pm (CST)
Chapter # 1.4 - 1.6	Max. Points # 25

SN	QUESTION	Pts
1	Let N (x) be the statement “x has visited North Dakota,” where the domain consists of the students in your school. Express each of these quantifications in English. a) $\exists x\, N(x)$ b) $\forall x\, N(x)$ c) $\neg\exists x\, N(x)$ d) $\exists x\, \neg N(x)$ e) $\neg\forall x\, N(x)$ f) $\forall x \neg\, N(x)$	6 X 0.5
2	Let C(x) be the statement “x has a cat,” let D(x) be the statement “x has a dog,” and let F (x) be the statement “x has a ferret.” Express each of these statements in terms of C(x), D(x), F (x), quantifiers, and logical connectives. Let the domain consist of all students in your class. a) A student in your class has a cat, a dog, and a ferret. b) All students in your class have a cat, a dog, or a ferret. c) Some student in your class has a cat and a ferret, but not a dog. d) No student in your class has a cat, a dog, and a ferret. e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.	5 X 0.5
3	Determine the truth value of each of these statements if the domain of each variable consists of all real numbers. a) $\exists x\, (x^2 = 2)$ b) $\exists x\, (x^2 = -1)$ c) $\forall x\, (x^2 + 2 \geq 1)$ d) $\forall x\, (x^2 \neq x)$	4 X 0.5
4	Suppose that the domain of the propositional function P (x) consists of −5, −3, −1, 1, 3, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions. a) $\exists x\, P(x)$ b) $\forall x\, P(x)$ c) $\forall x\, ((x \neq 1) \rightarrow P(x))$ d) $\exists x\, ((x \geq 0) \wedge P(x))$ e) $\exists x\, (\neg P(x)) \wedge \forall x\, ((x < 0) \rightarrow P(x))$	5 X 0.5

5	Suppose that the domain of $Q(x, y, z)$ consists of triples x, y, z, where $x = 0, 1$, or 2, $y = 0$ or 1, and $z = 0$ or 1. Write out these propositions using disjunctions and conjunctions. a) $\forall y\, Q(0, y, 0)$ b) $\exists x\, Q(x, 1, 1)$ c) $\exists z\, \neg Q(0, 0, z)$ d) $\exists x\, \neg Q(x, 0, 1)$	4 X 0.5
6	If the domain consists of all integers, what are the truth values of these statements? a) $\exists! x\, (x > 1)$ b) $\exists! x\, (x^2 = 1)$ c) $\exists! x\, (x + 3 = 2x)$ d) $\exists! x\, (x = x + 1)$	4 X 0.5
7	Let $C(x, y)$ mean that student x is enrolled in class y, where the domain for x consists of all students in your school and the domain for y consists of all classes being given at your school. Express each of these statements by a simple English sentence. a) C(Randy Goldberg, CS 252) b) $\exists x\, C(x, \text{Math } 695)$ c) $\exists y\, C(\text{Carol Sitea}, y)$ d) $\exists x\, (C(x, \text{Math } 222) \land C(x, \text{CS } 252))$ e) $\exists x \exists y \forall z\, ((x \neq y) \land (C(x, z) \rightarrow C(y, z)))$ f) $\exists x \exists y \forall z\, ((x \neq y) \land (C(x, z) \leftrightarrow C(y, z)))$	6 X 0.5
8	A discrete mathematics class contains 1 mathematics major who is a freshman, 12 mathematics majors who are sophomores, 15 computer science majors who are sophomores, 2 mathematics majors who are juniors, 2 computer science majors who are juniors, and 1 computer science major who is a senior. Express each of these statements in terms of quantifiers and then determine its truth value. a) There is a student in the class who is a junior. b) Every student in the class is a computer science major. c) There is a student in the class who is neither a mathematics major nor a junior. d) Every student in the class is either a sophomore or a computer science major. e) There is a major such that there is a student in the class in every year of study with that major.	5 X 0.5
9	Rewrite each of these statements so that negations appear only within predicates	5 X

	(that is, so that no negation is outside a quantifier or an expression involving logical connectives). a) $\neg\exists y\, \exists x\, P(x, y)$ b) $\neg\forall x\, \exists y\, P(x, y)$ c) $\neg\exists y\, (Q(y) \land \forall x\, \neg R(x, y))$ d) $\neg\exists y\, (\exists x\, R(x, y) \lor \forall x\, S(x, y))$ e) $\neg\exists y\, (\forall x\, \exists z\, T(x, y, z) \lor \exists x\, \forall z\, U(x, y, z))$	0.5
10	a) Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained." b) Identify the error or errors in this argument that supposedly shows that: If $\forall x\, (P(x) \lor Q(x))$ is true then $\forall x\, P(x) \lor \forall x\, Q(x)$ is true. 1. $\forall x\, (P(x) \lor Q(x))$ Premise 2. $P(c) \lor Q(c)$ Universal instantiation from (1) 3. $P(c)$ Simplification from (2) 4. $\forall x\, P(x)$ Universal generalization from (3) 5. $Q(c)$ Simplification from (2) 6. $\forall x\, Q(x)$ Universal generalization from (5) 7. $\forall x\, (P(x) \lor \forall x\, Q(x))$ Conjunction from (4) and (6)	2 1

1	Let N (x) be the statement "x has visited North Dakota," where the domain consists of the students in your school. Express each of these quantifications in English. a) $\exists x\, N(x)$ b) $\forall x\, N(x)$ c) $\neg\exists x\, N(x)$ d) $\exists x\, \neg N(x)$ e) $\neg\forall x\, N(x)$ f) $\forall x \neg\, N(x)$

a.) Some Students have Visited North Dakota

b.) Every Student in this School has visited North Dakota

c.) None of the Students have visited North Dakota

d.) Most of the Students have visited North Dakota

E.) At least one Student has not visited North Dakota

F.) Every Student in this School has Not Visited North Dakota

2	Let C(x) be the statement "x has a cat," let D(x) be the statement "x has a dog," and let F (x) be the statement "x has a ferret." Express each of these statements in terms of C(x), D(x), F (x), quantifiers, and logical connectives. Let the domain consist of all students in your class. a) A student in your class has a cat, a dog, and a ferret. b) All students in your class have a cat, a dog, or a ferret. c) Some student in your class has a cat and a ferret, but not a dog. d) No student in your class has a cat, a dog, and a ferret. e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

a.) $\exists x\,(C(x) \wedge D(x) \wedge F(x))$ b.) $\forall x\,(C(x) \vee D(x) \vee F(x))$

C.) $\exists x\,((C(x) \wedge F(x)) \wedge \neg D(x))$ D.) $\forall x\, \neg(C(x) \wedge D(x) \wedge F(x))$

E.) $\exists x\, C(x) \wedge \exists x\, D(x) \wedge \exists x\, F(x)$

3	Determine the truth value of each of these statements if the domain of each variable consists of all real numbers. a) $\exists x\,(x^2 = 2)$ b) $\exists x\,(x^2 = -1)$ c) $\forall x\,(x^2 + 2 \geq 1)$ d) $\forall x\,(x^2 \neq x)$

a.) False b.) False C.) True D.) True

4	Suppose that the domain of the propositional function P (x) consists of −5, −3, −1, 1, 3, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions. a) $\exists x\, P(x)$ b) $\forall x\, P(x)$ c) $\forall x\,((x \neq 1) \rightarrow P(x))$ d) $\exists x\,((x \geq 0) \wedge P(x))$ e) $\exists x\,(\neg P(x)) \wedge \forall x\,((x < 0) \rightarrow P(x))$

a.) $P(-5) \vee P(-3) \vee P(-1) \vee P(1) \vee P(3) \vee P(5)$

b.) $P(-5) \wedge P(-3) \wedge P(-1) \wedge P(1) \wedge P(3) \wedge P(5)$

C.) $P(-5) \wedge P(-3) \wedge P(-1) \wedge P(3) \wedge P(5)$

d.) $P(1) \vee P(3) \vee P(5)$

e.) $(\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)) \wedge (P(-5) \wedge P(-3) \wedge P(-1))$

5	Suppose that the domain of Q(x, y, z) consists of triples x, y, z, where x = 0, 1, or 2, y = 0 or 1, and z = 0 or 1. Write out these propositions using disjunctions and conjunctions. a) $\forall y\, Q(0, y, 0)$ b) $\exists x\, Q(x, 1, 1)$ c) $\exists z\, \neg Q(0, 0, z)$ d) $\exists x\, \neg Q(x, 0, 1)$

a.) $Q(0,0,0) \wedge Q(0,1,0)$ b.) $Q(0,1,1) \vee Q(1,1,1) \vee Q(2,1,1)$

C.) $\neg Q(0,0,0) \vee \neg Q(0,0,1)$ d.) $\neg Q(0,0,1) \vee \neg Q(1,0,1) \vee \neg Q(2,0,1)$

6	If the domain consists of all integers, what are the truth values of these statements? a) $\exists! x\,(x > 1)$ b) $\exists! x\,(x^2 = 1)$ c) $\exists! x\,(x + 3 = 2x)$ d) $\exists! x\,(x = x + 1)$

a.) False, There are many Numbers that Satisfy $x > 1$

b.) True, $x = 1$ C.) True, $x = 3$

d.) False

7	Let C(x, y) mean that student x is enrolled in class y, where the domain for x consists of all students in your school and the domain for y consists of all classes being given at your school. Express each of these statements by a simple English sentence. a) C(Randy Goldberg, CS 252) b) $\exists x\, C(x, \text{Math } 695)$ c) $\exists y\, C(\text{Carol Sitea}, y)$ d) $\exists x\, (C(x, \text{Math } 222) \land C(x, \text{CS } 252))$ e) $\exists x \exists y \forall z\, ((x \neq y) \land (C(x, z) \rightarrow C(y, z)))$ f) $\exists x \exists y \forall z\, ((x \neq y) \land (C(x, z) \leftrightarrow C(y, z)))$

a.) Randy Goldberg is enrolled in CS252

b.) At least 1 Student is enrolled in Math 695

C.) Carol Sitea is enrolled in at least one Class

d.) At least one Student is enrolled in both Math 222 and CS 252

e.) If one Student is enrolled in a Certian Class then another Student is also in that Class

f.) One Student is enrolled in a Class if and only if another Student is enrolled as well

8	A discrete mathematics class contains 1 mathematics major who is a freshman, 12 mathematics majors who are sophomores, 15 computer science majors who are sophomores, 2 mathematics majors who are juniors, 2 computer science majors who are juniors, and 1 computer science major who is a senior. Express each of these statements in terms of quantifiers and then determine its truth value. a) There is a student in the class who is a junior. b) Every student in the class is a computer science major. c) There is a student in the class who is neither a mathematics major nor a junior. d) Every student in the class is either a sophomore or a computer science major. e) There is a major such that there is a student in the class in every year of study with that major.

a.) $\exists C(\text{Junior}, y)$ - True

b.) $\forall_x (C(x, \text{Computer Science}))$ - False

C.) $\exists \neg C(\text{Junior}, \text{Mathematics})$ - True

d.) $\forall_y (C(\text{Sophomore}, y)) \lor \forall_x (C(x, \text{Computer Science}))$ - False

e.) $\exists_x \exists_y \forall_z ((x \neq y) \land (C(x, z) \rightarrow C(y, z))$

9	Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives). a) $\neg\exists y\, \exists x\, P(x, y)$ b) $\neg\forall x\, \exists y\, P(x, y)$ c) $\neg\exists y\, (Q(y) \wedge \forall x\, \neg R(x, y))$ d) $\neg\exists y\, (\exists x\, R(x, y) \vee \forall x\, S(x, y))$ e) $\neg\exists y\, (\forall x\, \exists z\, T(x, y, z) \vee \exists x\, \forall z\, U(x, y, z))$

a.) $\forall_y \forall_x \neg P(x,y)$ b.) $\exists_x \forall_y \neg P(x,y)$

c.) $\forall_x (\neg Q(y) \wedge \exists_x R(x,y))$ d.) $\forall_y (\forall_x \neg R(x,y) \wedge \exists_x \neg S(x,y))$

e.) $\forall_y (\exists_x \forall_z \neg T(x,y,z) \wedge \forall_x \exists_z \neg U(x,y,z))$

10	a) Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained." b) Identify the error or errors in this argument that supposedly shows that: If $\forall x\, (P(x) \vee Q(x))$ is true then $\forall x\, P(x) \vee \forall x\, Q(x)$ is true. 1. $\forall x\, (P(x) \vee Q(x))$ — Premise 2. $P(c) \vee Q(c)$ — Universal instantiation from (1) 3. $P(c)$ — Simplification from (2) 4. $\forall x\, P(x)$ — Universal generalization from (3) 5. $Q(c)$ — Simplification from (2) 6. $\forall x\, Q(x)$ — Universal generalization from (5) 7. $\forall x\, (P(x) \vee \forall x\, Q(x))$ — Conjunction from (4) and (6)

r = rain
f = foggy
s = sailing
L = lifesaving
t = trophy

a.) $((((\neg r \vee \neg f) \to (s \wedge L)) \wedge (s \to T)) \wedge \neg T) \to r$

$(\neg r \vee \neg f) \to (s \wedge L)$
$s \to T$
$\neg T$
$\therefore r$

$(\neg r \vee \neg f) \to (s \wedge L)$	Premise 1
$s \to t$	Premise 2
$\neg t$	Premise 3
$\neg s$	Modus Tollens with Premise 2 & 3
$\neg(s \wedge d) \to \neg(\neg r \vee \neg f)$	equivalency $p \to q \equiv \neg q \to \neg p$ With Premise 1
$(\neg s \vee \neg d) \to (r \wedge f)$	De Morgan's Law

b.) The Error Occurs in Step 7.