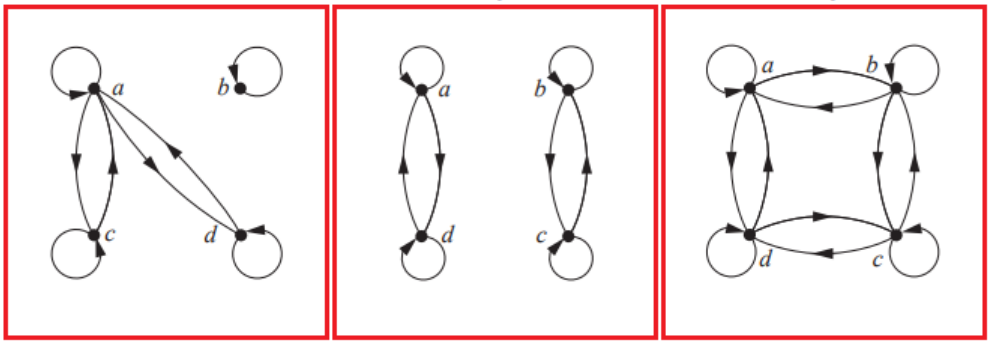
CS 3653 – Discrete Mathematics for Computer Science

Assignment # 12	Due – Apr 18, 2022, 11:59pm (CST)
Chapter # 9.5 & 9.6	Max. Points # 25

SN	QUESTION	Pts
1	Which of these relations on {0, 1, 2, 3} are equivalence relations? Determine the properties of an equivalence relation that the others lack. a) {(0, 0), (1, 1), (2, 2), (3, 3)} b) {(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)} c) {(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)} d) {(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)} e) {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)}	2.5
2	Determine whether the relation with the following directed graphs shown is an equivalence relation. Justify your answer. a) b) c) 	3
3	Determine whether the relations represented by these zero–one matrices are equivalence relations a) $\begin{bmatrix}1&1&1\\0&1&1\\1&1&1\end{bmatrix}$ b) $\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&1&0\\0&1&0&1\end{bmatrix}$ c) $\begin{bmatrix}1&1&1&0\\1&1&1&0\\1&1&1&0\\0&0&0&1\end{bmatrix}$	3
4	What is the congruence class $[4]_m$ when m is: a) 2? b) 3? c) 6? d) 8?	2
5	Let R be the equivalence relation consisting of all pairs (x, y) such that x and y are strings of uppercase and lowercase English letters with the property that for every positive integer n, the nth characters in x and y are the same letter, either uppercase or lowercase. What is the equivalence class of each of these strings with respect to the equivalence	3

	relation given above? a) No b) Yes c) Help	
6	Which of these collections of subsets are partitions of {−3, −2, −1, 0, 1, 2, 3}? a) {−3, −1, 1, 3}, {−2, 0, 2} b) {−3, −2, −1, 0}, {0, 1, 2, 3} c) {−3, 3}, {−2, 2}, {−1, 1}, {0} d) {−3, −2, 2, 3}, {−1, 1}	2
7	List the ordered pairs in the equivalence relations produced by these partitions of {a, b, c, d, e, f, g}. a) {a, b}, {c, d}, {e, f, g} b) {a}, {b}, {c, d}, {e, f}, {g} c) {a, b, c, d}, {e, f, g} d) {a, c, e, g}, {b, d}, {f}	2
8	Which of these relations on {0, 1, 2, 3} are partial orderings? Determine the properties of a partial ordering that the others lack. a) {(0, 0), (2, 2), (3, 3)} b) {(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 3)} c) {(0, 0), (1, 1), (1, 2), (2, 2), (3, 1), (3, 3)} d) {(0, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (2, 3), (3, 0), (3, 3)} e) {(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (3, 3)}	2.5
9	Which of these are posets? a) (R, =) b) (R, <) c) (R, ≤) d) (R, =)	2
10	Find the lexicographic ordering of these strings of lowercase English letters: a) quack, quick, quicksilver, quicksand, quacking b) open, opener, opera, operand, opened c) zoo, zero, zoom, zoology, zoological	3

1	Which of these relations on {0, 1, 2, 3} are equivalence relations? Determine the properties of an equivalence relation that the others lack. a) {(0, 0), (1, 1), (2, 2), (3, 3)} b) {(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)} c) {(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)} d) {(0, 0), (1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)} e) {(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)}	2.5

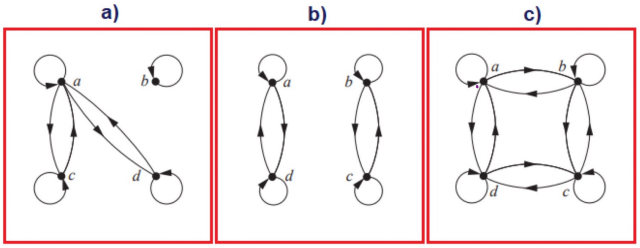
A.) Yes, this relation is equivalence Relation

B.) No, this relation is not an equivalence Relation

C.) Yes, this relation is equivalence Relation

D.) No, this relation is not an equivalence Relation

E.) No, this relation is not an equivalence Relation

2	Determine whether the relation with the following directed graphs shown is an equivalence relation. Justify your answer. a) b) c) 	3

A.) No, this is not an equivalence relation

B.) Yes, this is an equivalence relation

C.) No, this is Not an equivalence relation

3	Determine whether the relations represented by these zero–one matrices are equivalence relations a) $\begin{bmatrix}1&1&1\\0&1&1\\1&1&1\end{bmatrix}$ b) $\begin{bmatrix}1&0&1&0\\0&1&0&1\\1&0&1&0\\0&1&0&1\end{bmatrix}$ c) $\begin{bmatrix}1&1&1&0\\1&1&1&0\\1&1&1&0\\0&0&0&1\end{bmatrix}$	3

A.) No, this Matrix is not an equivalence relation

B) Yes, this Matrix is an equivalence relation

C.) Yes, this Matrix is an equivalence relation

4	What is the congruence class $[4]_m$ when m is: a) 2? b) 3? c) 6? d) 8?	2

A.) $\{\ldots, 4-3(2), 4-2(2), 4-(2), 4+(2), 4+2(2), \ldots\}$
$\{\ldots, -2, 0, 2, 4, 6, 8, \ldots\}$

B.) $\{\ldots, -5, -2, 1, 4, 7, 10, \ldots\}$

C.) $\{\ldots, -14, -8, -2, 4, 10, 16, \ldots\}$

D.) $\{\ldots, -20, -12, -4, 4, 12, 20\}$

5	Let R be the equivalence relation consisting of all pairs (x, y) such that x and y are strings of uppercase and lowercase English letters with the property that for every positive integer n, the nth characters in x and y are the same letter, either uppercase or lowercase. What is the equivalence class of each of these strings with respect to the equivalence relation given above? a) No b) Yes c) Help	3

C.) Hello

6	Which of these collections of subsets are partitions of {−3, −2, −1, 0, 1, 2, 3}? a) {−3, −1, 1, 3}, {−2, 0, 2} b) {−3, −2, −1, 0}, {0, 1, 2, 3} c) {−3, 3}, {−2, 2}, {−1, 1}, {0} d) {−3, −2, 2, 3}, {−1, 1}	2

A.) $P_1 \cup P_2 = \{-3, -2, -1, 0, 1, 2, 3\} = A$
$P_1 \cap P_2 = \emptyset$
$\{P_1, P_2\}$ is a Partition of A

B.) $P_1 \cap P_2 = \{\emptyset\} \neq \emptyset$
$\{P_1, P_2\}$ is not a Partition

C.) $P_1 \cup P_2 \cup P_3 \cup P_4 = \{-3, -2, -1, 0, 1, 2, 3\}$
$P_i \cap P_j = \emptyset$, $i \neq j$
$\{P_1, P_2, P_3, P_4\}$ is a Partition of A

D.) $P_1 \cup P_2 = \{-3, -2, -1, 1, 2, 3\} \neq A$
$\{P_1 \cup P_2\}$ is not a Partition of A

7	List the ordered pairs in the equivalence relations produced by these partitions of {a, b, c, d, e, f, g}. a) {a, b}, {c, d}, {e, f, g} b) {a}, {b}, {c, d}, {e, f}, {g} c) {a, b, c, d}, {e, f, g} d) {a, c, e, g}, {b, d}, {f}	2

A.) $R = \{(a,b), (a,a), (b,a), (b,b), (c,d), (d,c), (c,c), (d,d), (e,f), (c,g), (e,e), (f,e), (f,g), (f,f), (g,e), (g,f), (g,g)\}$

B.) $R = \{(a,a), (b,b), (c,c), (c,d), (d,c), (d,d), (e,e), (e,f), (f,e), (f,f), (g,g)\}$

C.) $R = \{(a,a), (a,b), (a,c), (a,d), (b,a), (b,b), (b,c), (b,d), (c,e), (c,b), (c,c), (c,d), (d,a), (d,b), (d,c), (d,d), (e,e), (e,f), (e,g), (f,e), (f,f), (f,g), (g,e), (g,f), (g,g)\}$

D.) $R = \{(a,a), (a,c), (a,e), (a,g), (c,a), (c,e), (c,e), (c,g), (e,a), (e,c), (e,e), (e,g), (g,a), (g,c), (g,e), (g,g), (b,b), (b,d), (d,b), (d,d), (f,f)\}$

8	Which of these relations on {0, 1, 2, 3} are partial orderings? Determine the properties of a partial ordering that the others lack. a) {(0, 0), (2, 2), (3, 3)} b) {(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 3)} c) {(0, 0), (1, 1), (1, 2), (2, 2), (3, 1), (3, 3)} d) {(0, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (2, 3), (3, 0), (3, 3)} e) {(0, 0), (0, 1), (0, 2), (0, 3), (1, 0), (1, 1), (1, 2), (1, 3), (2, 0), (2, 2), (3, 3)}	2.5

A.) Not a Partial Order

B.) Partial Order

C.) Not a Partial Order

D.) Not a Partial Order

E.) Partial Order

9	Which of these are posets? a) (R, =) b) (R, <) c) (R, ≤) d) (R, =)	2

A.) Posets

C.) Posets

B) Not a Posets

D.) Posets

10	Find the lexicographic ordering of these strings of lowercase English letters: a) quack, quick, quicksilver, quicksand, quacking b) open, opener, opera, operand, opened c) zoo, zero, zoom, zoology, zoological	3

A.) quack < quacking

B) open < opened < opener < opera < operand

C.) zero < zoo < zoological < zoology < zoom