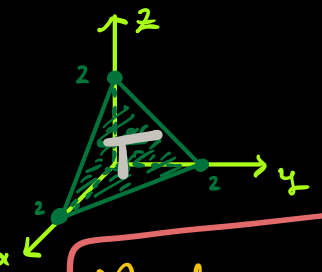
$\int\int\int$?? ?? ?!

More Integration.

∫ Bootcamp Next Week MWF 8pm DISCORD.

I. Finish Set-up of last problem.

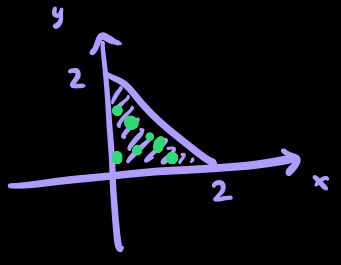
Suspense!

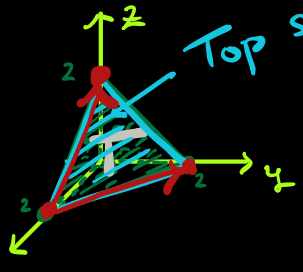


II. Swapping the order; Easy example, Medium if time.

III. A Riddle.

Recall: $\iiint_T (x+y+z)\,dV$





"Name".

Equation of this surface...

"$ax+by+cz=d$"

... "$z = 2-x-y$"

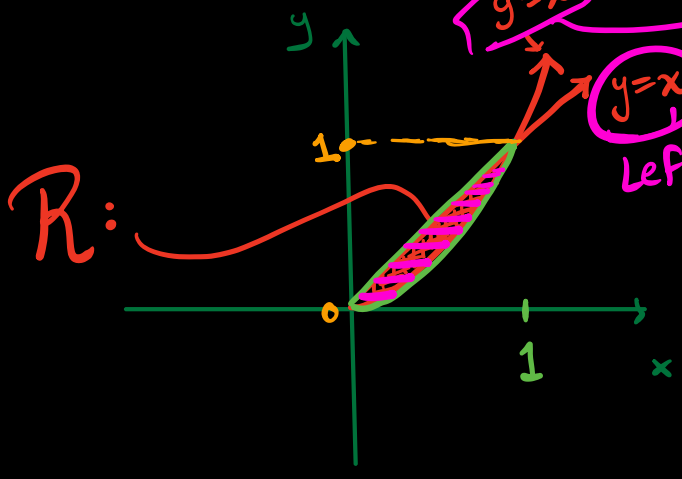
$$= \int_0^2 \int_0^{2-x} \int_0^{2-x-y} (x+y+z)\,dz\,dy\,dx$$

Execution is No problem.

Tedious ✓

II. Swapping the order of integration.

Ex: Compute $\iint (x+y)\, dA$



$y=x^2$ Rightest. $\rightsquigarrow y=x^2 \rightsquigarrow x=\sqrt{y}$

$y=x$ Leftest $\rightsquigarrow x=y$.

R

Which order?

$$\int_0^1 \int_{x^2}^{x} (x+y)\, dy\, dx$$

$$\int_0^1 \int_{x=y}^{x=\sqrt{y}} (x+y)\, dx\, dy$$

uglier.

SI

S1: $\overset{\star}{=} \frac{x^2}{2} + yx \Big|_{x=y}^{x=\sqrt{y}}$

$$\left(\frac{y}{2} + y\sqrt{y}\right) - \left(\frac{y^2}{2} + y^2\right)$$

$$= \boxed{-\frac{3}{2}y^2 + y^{3/2} + \frac{y}{2}}$$

S2: $$\int_0^1 \left(-\frac{3}{2}y^2 + y^{3/2} + \frac{y}{2}\right) dy$$

$$\overset{\star}{=} -\frac{3}{2}\cdot\frac{y^3}{3} + \frac{y^{5/2}}{5/2} + \frac{y^2}{4}\Big|_{y=0}^{y=1}$$

$$= \left(-\frac{1}{2} + \frac{2}{5} + \frac{1}{4}\right) - (0)$$

$$\boxed{2/5 - 1/4}$$ ↶ Answer!

$$\int_0^1 \int_{x^2}^{x} (x+y)\,dy\,dx$$

S1

S1: $\overset{\star}{=} xy + y^2/2 \Big|_{y=x^2}^{y=x}$

$$\left(x^2 + \frac{x^2}{2}\right) - \left(x^3 + \frac{x^4}{2}\right)$$

$$= \boxed{-\frac{x^4}{2} - x^3 + \frac{3}{2}x^2}$$

S2: $$\int_0^1 -\frac{x^4}{2} - x^3 + \frac{3}{2}x^2 \, dx$$

$$\overset{\star}{=} -\frac{x^5}{10} - \frac{x^4}{4} + \frac{1}{2}x^3 \Big|_{x=0}^{x=1}$$

$$= -\frac{1}{10} - \frac{1}{4} + \frac{1}{2} - 0$$

$$= \frac{1}{4} - \frac{1}{10}$$

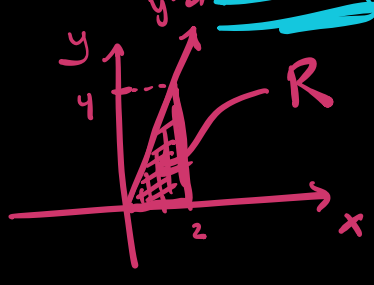
$$\frac{2}{5} - \frac{1}{4} = \frac{1}{4} - \frac{1}{10}$$

$$\frac{2}{5} + \frac{1}{10} \overset{?}{=} \frac{2}{4}$$

✓✓.

III. Riddle.

Set up THIS integral in the opposite Way :

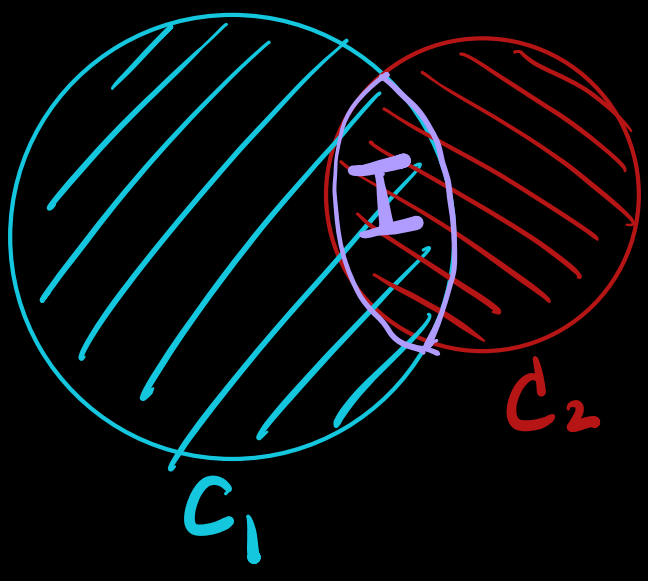


$$\int_0^2 \int_{0}^{2x} x^2 \tan(xy)\, dy\, dx$$

CORRECT:

$$\int_0^4 \int_{\frac{1}{2}y}^{2} \text{BLAH} \; dx \; dy$$

Medium Fun Riddle:

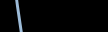


$f(x,y)$ some mystery function.

$$\iint_{C_1} f\,dA = 4$$

$$+ \iint_{C_2} f\,dA = 2$$

$$\iint_{\text{Whole Thing}} f\,dA = 10$$



$$+ \iint_{I} f\,dA = ?$$

Q:

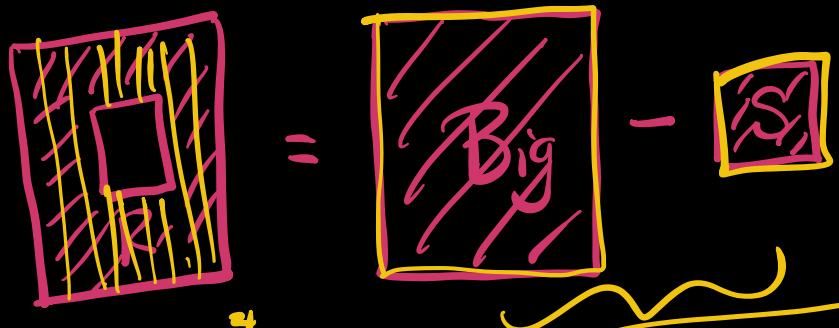
$$= 6$$

$$6 = 10 + \iint_{I} f\,dA$$

$$-4 = \iint_{I} f\,dA$$

Last little "Trick":

$$\iint_{R} f\,dA$$



$$\int_2^4 \int_?^? \, dx$$

Way easier to set up.

Monday Treat: "Volumes of Spheres"