Change of coordinates? Yes.

I. 1-variable Recap.

II. A wrong Calculation.

III. How to Fix it!

I. Recall: "u-substitution"

Substitution.

"Transfor (Change) of variables"

Change of coordinates.

$$\int_{x=1}^{x=2} \sqrt{2x+5}\ \boxed{dx}$$

Yoga.

$u = \sin x$

$du = \boxed{\cos x}\, dx$

$$\underbrace{u} = \underbrace{2x+5}$$

change in the symbol dx. $\rightarrow du = \boxed{2} \cdot dx$

$\frac{du}{dx}$

Transfer whole problem into u's world!

correction factor!

$$\int_{u=7}^{u=9} \sqrt{u} \cdot \boxed{\frac{1}{2}} \cdot du$$

$$= \frac{1}{2} \int_{u=7}^{u=9} u^{1/2} \, du$$

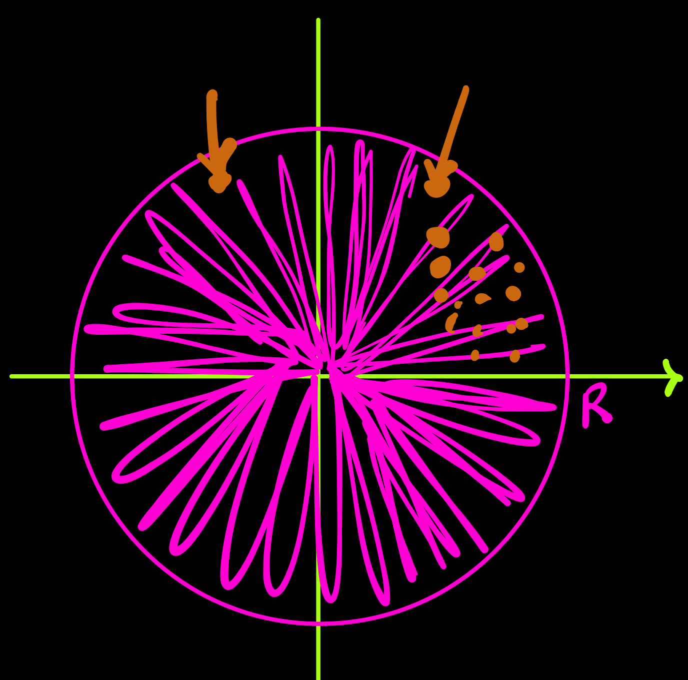
$$\star = \frac{1}{2} \left[\frac{2}{3} u^{3/2} \right]_{u=7}^{u=9}$$

$$= \frac{1}{2}\left[\tfrac{2}{3}(9)^{3/2} - \tfrac{2}{3}(7)^{3/2}\right]$$

II. Wrong Math!

Shouldn't we exploit Polar Coordinates when finding area of circle (radius R)?

Yeah! $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$



$$\iint 1 \, dA \qquad dA = r\,dr\,d\theta$$

Fubinify! (r, θ)

$$\int_0^{2\pi} \underbrace{\int_0^{R} 1 \, \boxed{\overset{\text{Jacobian}}{r}} \, \boxed{dr}}_{S1} \, \boxed{d\theta}$$

Correction factor!

$$\underline{S1}: \stackrel{*}{=} r \Big|_{r=0}^{r=R} = R$$

$$\underline{S2}: \int_0^{2\pi} R \, d\theta$$

$$\overset{*}{=} R\theta \Big|_{\theta=0}^{\theta=2\pi}$$

$$= \boxed{2\pi R}.$$

III. Fix IT: "Jacobian".

$$r$$

$$\underbrace{\int_0^{2\pi} \int_0^R 1 \cdot r \, dr \, d\theta}$$

$$S_1 \overset{\star}{=} \frac{1}{2} r^2 \Big|_{r=0}^{r=R}$$

$$= \frac{1}{2} R^2$$

$$\underline{S_2:} \quad \int_0^{2\pi} \frac{1}{2} R^2 \, d\theta \overset{\star}{=} \frac{1}{2} R^2 \theta \Big|_{\theta=0}^{\theta=2\pi}$$

$$= \cancel{\frac{1}{2}} R^2 \cdot (\cancel{2}\pi)$$

$$= \boxed{\pi R^2}.$$

Integrate $\boxed{x}$ over that circle!

$$\iint x \, dA$$

Go Polar!

$x = r\cos\theta$

$y = r\sin\theta$

$$\int_0^{2\pi}\int_0^{R} r\cos\theta \; r \; dr \, d\theta$$

$$\delta I = \int_0^{R} r^2\cos\theta \; dr$$

$$\overset{\star}{=} \frac{r^3}{3}\cos\theta \Big|_{r=0}^{r=R}$$

$$= \frac{R^3}{3}\cos\theta$$

*

2π

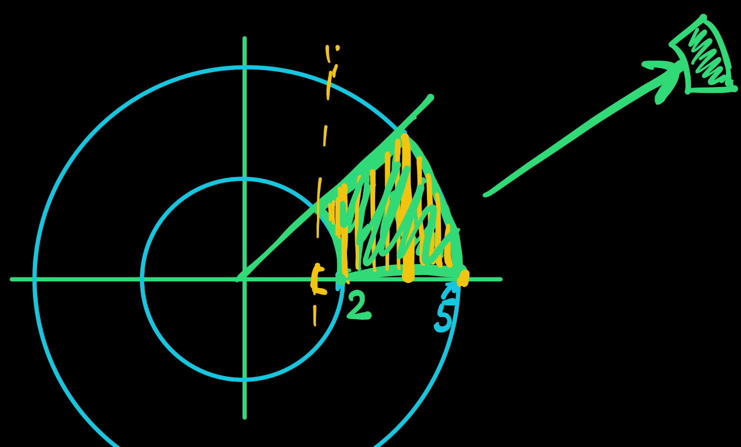
S2: $\int_{0}^{2\pi} \frac{R^3}{3} \cos\theta \, d\theta$

$$\stackrel{\star}{=} \frac{R^3}{3} \sin\theta \Big|_{\theta=0}^{\theta=2\pi}$$

$$= \left[\frac{R^3}{3} \underbrace{\sin(2\pi)}_{0} - \frac{R^3}{3} \underbrace{\sin(0)}_{0} \right]$$

$$= 0.$$

Integrate $\iint y \, \boxed{dA}$



Switch to Polar (Remember Jac.)

$$\int_0^{\pi/4} \underbrace{\int_2^5 \underbrace{r\sin\theta \; r}\cdot dr} \; d\theta$$

$$S_1 = \left.\frac{r^3}{3}\sin\theta\right|_{r=2}^{r=5}$$

$$= \frac{125}{3}\sin\theta - \frac{8}{3}\sin\theta$$

$$\boxed{39\sin\theta}$$

$\underline{S2}$: $$\int_0^{\pi/4} 39\sin\theta \; d\theta$$

$$\overset{\star}{=} -39\cos\theta \Big|_{\theta=0}^{\theta=\pi/4}$$

$$= -39\left(\frac{\sqrt{2}}{2}\right) + 39(1)$$

$$\boxed{39 - 39\left(\frac{\sqrt{2}}{2}\right)}$$