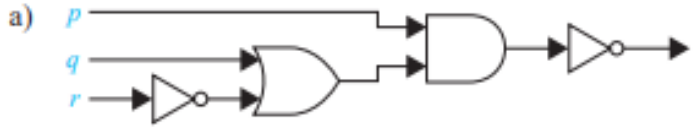
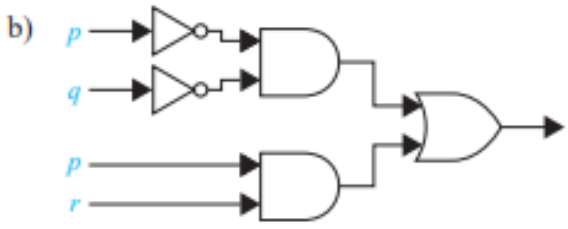
CS 3653 – Discrete Mathematics for Computer Science

Assignment # 1	Due – Jan 17, 2022, 11:59pm (CST)
Chapter # 1.1 - 1.3	Max. Points # 25

SN	QUESTION	Pts
1	What is the negation of each of these propositions? a) Jennifer and Teja are friends. b) There are 13 items in a baker's dozen. c) Abby sent more than 100 text messages yesterday. d) 121 is a perfect square.	4 X 0.5
2	Let p and q be the propositions "The election is decided" and "The votes have been counted," respectively. Express each of these compound propositions as an English sentence. a) $\neg p$ b) $p \vee q$ c) $\neg p \wedge q$ d) $q \rightarrow p$ e) $\neg q \rightarrow \neg p$ f) $\neg p \rightarrow \neg q$ g) $p \leftrightarrow q$ h) $\neg q \vee (\neg p \wedge q)$	8 X 0.5
3	Let p, q, and r be the propositions p: You get an A on the final exam. q: You do every exercise in this book. r: You get an A in this class. Write these propositions using p, q, and r and logical connectives (including negations). a) You get an A in this class, but you do not do every exercise in this book. b) You get an A on the final, you do every exercise in this book, and you get an A in this class. c) To get an A in this class, it is necessary for you to get an A on the final. d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class. e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class. f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.	6 X 0.5

4	State the converse, contrapositive, and inverse of each of these conditional statements. a) If it snows tonight, then I will stay at home. b) I go to the beach whenever it is a sunny summer day. c) When I stay up late, it is necessary that I sleep until noon.	3 X 1
5	Construct a truth table for each of these compound propositions. a) $p \rightarrow \neg p$ b) $p \leftrightarrow \neg p$ c) $p \oplus (p \vee q)$ d) $(p \wedge q) \rightarrow (p \vee q)$ e) $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$ f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$	6 X 0.5
6	Evaluate each of these expressions. a) 1 1000 ∧ (0 1011 ∨ 1 1011) b) (0 1111 ∧ 1 0101) ∨ 0 1000 c) (0 1010 ⊕ 1 1011) ⊕ 0 1000 d) (1 1011 ∨ 0 1010) ∧ (1 0001 ∨ 1 1011)	4 X 0.5
7	Following exercise relates to inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the way described below: A says "The two of us are both knights" and B says "A is a knave." If you cannot determine what these two people are, can you draw any conclusions?	2
8	Find the output of each of these combinatorial circuits. a)  b)  	2 X 1
9	Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output $((\neg p \vee \neg r) \wedge \neg q) \vee (\neg p \wedge (q \vee r))$ from input bits p, q, and r.	1

10	Show that each of these conditional statements is a tautology by using truth tables. a) $(p \wedge q) \rightarrow p$ b) $p \rightarrow (p \vee q)$ c) $\neg p \rightarrow (p \rightarrow q)$ d) $(p \wedge q) \rightarrow (p \rightarrow q)$ e) $\neg (p \rightarrow q) \rightarrow p$ f) $\neg (p \rightarrow q) \rightarrow \neg q$	6 X 0.5

1	What is the negation of each of these propositions? a) Jennifer and Teja are friends. b) There are 13 items in a baker's dozen. c) Abby sent more than 100 text messages yesterday. d) 121 is a perfect square.

a.) Jennifer and Teja are NOT friends
b.) There are NOT 13 items in a baker's dozen.
C.) Abby did NOT Send more then 100 text messages yesterday { Would Saying "Abby Sent LESS than 100 messages" also work? }
d.) 121 is NOT a Perfect Square

2	Let p and q be the propositions "The election is decided" and "The votes have been counted," respectively. Express each of these compound propositions as an English sentence. a) $\neg p$ b) $p \vee q$ c) $\neg p \wedge q$ d) $q \rightarrow p$ e) $\neg q \rightarrow \neg p$ f) $\neg p \rightarrow \neg q$ g) $p \leftrightarrow q$ h) $\neg q \vee (\neg p \wedge q)$

a.) The Election is NOT decided

b.) The Votes have been Counted OR The Election is decided

C.) The Votes have been Counted AND The Election is Still NOT Decided

D.) If, the Votes have been Counted, Then The election is decided

E.) IF, The Votes have NOT been Counted, THEN, The Electin is NOT Decided.

F.) If, The Election is NOT decided, Then The Votes have NOT been Counted.

G.) The Election will be decided When the Votes have been Counted and If, the Election is NOT decided, then the Votes have NOT been Counded.

h.) Either the Votes have NOT been Counted OR The Election is NOT decided And The Votes have been Counded.

3	Let p, q, and r be the propositions p: You get an A on the final exam. q: You do every exercise in this book. r: You get an A in this class. Write these propositions using p, q, and r and logical connectives (including negations). a) You get an A in this class, but you do not do every exercise in this book. b) You get an A on the final, you do every exercise in this book, and you get an A in this class. c) To get an A in this class, it is necessary for you to get an A on the final. d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class. e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class. f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

A.) $r \wedge \neg q$ B.) $p \wedge q \wedge r$ C.) $r \rightarrow p$ D.) $p \wedge \neg q \wedge r$ E.) $(p \wedge q) \rightarrow r$

F.) $r \leftrightarrow (q \oplus p)$

4	State the converse, contrapositive, and inverse of each of these conditional statements. a) If it snows tonight, then I will stay at home. b) I go to the beach whenever it is a sunny summer day. c) When I stay up late, it is necessary that I sleep until noon.

a.) Converse: I will Stay home IF Snows tonight

Contrapositive: IF, I don't Stay home, then, It did not Snow tonight

Inverse: If it is NOT Snowing tonight, then I will NOT Stay home

b.) Converse: When it is a Sunny Summer day I go to the beach

Contrapositive: When it is NOT a Sunny Summer day I do NOT go to the beach

Inverse: I do Not go to the beach when it is NOT a Sunny Summer day

C.) Converse: It is necessary that I Sleep until noon, when I Stay up late

Contrapositive: It is NOT Necessary that I Sleep until noon, when I don't Stay up late

Inverse: When I don't Stay up late, It is NOT necessary that I Sleep until noon

5	Construct a truth table for each of these compound propositions. a) $p \rightarrow \neg p$ b) $p \leftrightarrow \neg p$ c) $p \oplus (p \vee q)$ d) $(p \wedge q) \rightarrow (p \vee q)$ e) $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$ f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

a.)

P	¬P	P->¬P
T	F	F
F	T	T

b.)

P	¬P	P<->¬P
T	F	F
F	T	F

c.)

P	q	PVq	P ⊕ (PVq)
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

d.)

P	q	P∧q	PVq	(P∧q) ->(PVq)
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

e.)

P	q	¬P	q->¬P	P<->q	(q->¬P)<->(P<->q)
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

f.)

P	q	¬q	P<->q	P<->¬q	(P<->q) ⊕ (P<->¬q)
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

6	Evaluate each of these expressions. a) 1 1000 ∧ (0 1011 ∨ 1 1011) b) (0 1111 ∧ 1 0101) ∨ 0 1000 c) (0 1010 ⊕ 1 1011) ⊕ 0 1000 d) (1 1011 ∨ 0 1010) ∧ (1 0001 ∨ 1 1011)

a.)
```
  0 1011
  1 1011
  ------
  1 1011
  1 1000
  ------
  1 1000
```
1 1000

b.)
```
  0 1111
  1 0101
  ------
  0 0101
  0 1000
  ------
  0 1101
```
0 1101

c.)
```
  0 1010
  1 1011
  ------
  1 0001
  0 1000
  ------
  1 1001
```
1 1001

d.)
```
  1 1011   1 0001
  0 1010   1 1011
  ---------------
  1 1011   1 1011
  1 1011   1 1011
  ---------------
  1 1011   1 1011
```
1 1011

7	Following exercise relates to inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the way described below: A says "The two of us are both knights" and B says "A is a knave." If you cannot determine what these two people are, can you draw any conclusions?

B is a Knight, A is a Knave

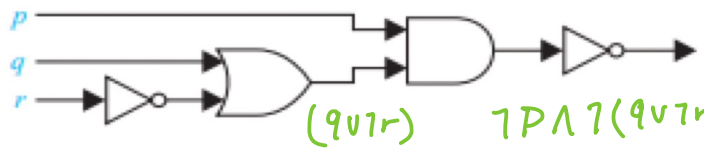
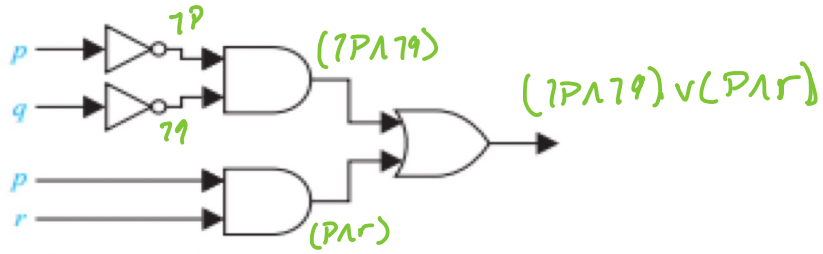
P: A, Knight; q: B, Knight
¬P: A, Knave; ¬q: B, Knave

P: true; A, Knight

A & B both Knights
if B, Knight: B's Statement: A, Knave, "Contradiction"
Conclude: A, Not Knight; P: False

A, is Knave

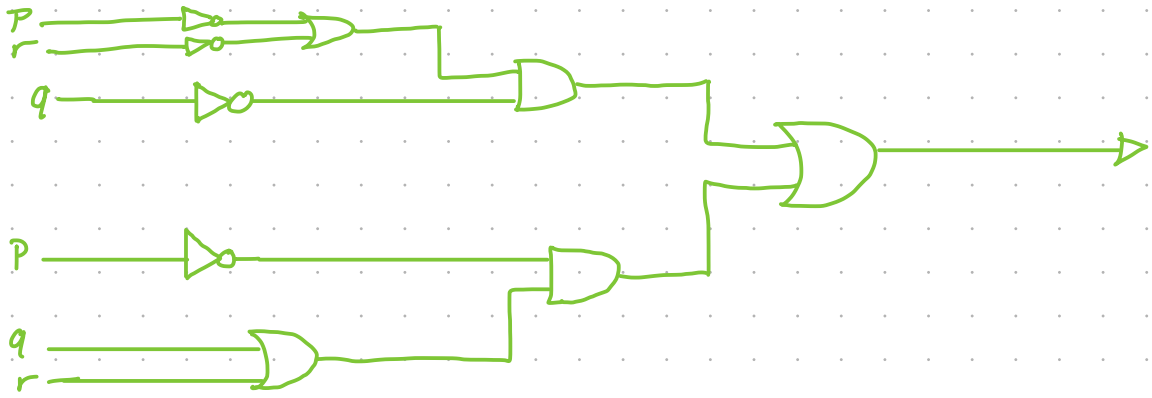
A: "both Knights" is False
So B Saying "A is Knave" TRUE
So B, Knight; A Knave

8	**Find the output of each of these combinatorial circuits.** a)  b) 

a.) $\neg p \wedge \neg(q \vee \neg r)$

b.) $(\neg p \wedge \neg q) \vee (p \wedge r)$

9	Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output $((\neg p \vee \neg r) \wedge \neg q) \vee (\neg p \wedge (q \vee r))$ from input bits p, q, and r.



10	Show that each of these conditional statements is a tautology by using truth tables. a) $(p \wedge q) \rightarrow p$ b) $p \rightarrow (p \vee q)$ c) $\neg p \rightarrow (p \rightarrow q)$ d) $(p \wedge q) \rightarrow (p \rightarrow q)$ e) $\neg (p \rightarrow q) \rightarrow p$ f) $\neg (p \rightarrow q) \rightarrow \neg q$

a.)

P	q	P∧q	(P∧q)->P
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

b.)

P	q	P∨q	P->(P∨q)
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

c.)

P	q	¬P	P->q	¬P->(P->q)
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

d.)

P	q	P∧q	P->q	(P∧q)->(P->q)
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

e.)

P	q	P->q	¬(P->q)	¬(P->q)->P
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

f.)

P	q	¬q	P->q	¬(P->q)	¬(P->q)->¬q
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	T	F	T
F	F	T	T	F	T