MATH 2163 **Name (Print):** _____
Fall 2021
Exam 1
9/17/21
Time Limit: 1 hour

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing.

No notes, no calculators, only use your brain. You may direct-message me on Discord for clarifications, though I may refuse to answer some questions. **You should show your work whenever possible, so that I believe that you did not cheat.**
You may **not** consult the internet or textbooks or other people.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided.
- **Unsupported answers will not receive full credit**.
- **If you are quarantining, hope you get better and all that. But submit your solutions all together as a single file on Canvas.**

Do not write in the table to the right.

Good luck, and make your professor PROUD!

Problem	Points	Score
1	12	
2	12	
3	8	
4	10	
5	8	
Total:	50	

1. (12 points) (a) What is the terminal point Q of the vector $\mathbf{v} = \langle 2, 4\rangle$ based at $P = (1, 6)$?

(b) The vector $\langle 1, 7\rangle$ is parallel to the vector $\langle 5, ?\rangle$. Find the question mark.

(c) Do all three points $(2, 4), (3, 6)$ and $(5, 9)$ lie on the same line?

(d) Find the unit vector in the direction of $\langle 2, 3, 5\rangle$.

(e) Find the vector $\vec{PQ}$. $P = (-1, -3, 5), Q = (1, -5, 5)$.

(f) Find b and c which make the vector $\langle 2, 3, 5\rangle$ parallel to $\langle 3, b, c\rangle$.

2. (12 points) (a) Find a parametrization of the line joining the points $(-3, 0, 1)$ and $(3, 5, 5)$.

(b) Do the lines $r(t) = (1 + t, 4 - 2t, 3t)$ and $s(t) = (2 - t, 2 + 15t, 3 - 2t)$ intersect? If yes, where. If no, why.

(c) Compute the dot product $\langle 0, 1, 3\rangle \cdot \langle -2, -2, 0\rangle$ Are these two vectors perpendicular?

(d) Do the vectors $\langle 2, 3, 4\rangle$ and $\langle -4, 2, 1\rangle$ form an acute angle? Why.

(e) $\mathbf{u}, \mathbf{v}$ are two vectors in space, with magnitudes 3 and 7 respectively. The angle between them is $\pi/4$ radians. What is their dot product?

(f) Calculate the 2×2 determinant of $\begin{bmatrix} 1 & 7 \\ 8 & 2 \end{bmatrix}$.

3. (8 points) (a) Find the volume of the parallelopiped created by the three vectors $\langle 1, 0, 0\rangle, \langle 1, 2, 3\rangle, \langle 2, 0, 3\rangle$. (Remember what determinants measure?)

(b) Calculate $\mathbf{v} \times \mathbf{w}$ if $\mathbf{v} = \langle 4, 3, 2\rangle, \mathbf{w} = \langle 1, -1, 2\rangle$.

(c) Calculate $\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v})$ if $\mathbf{u} = \langle 2020, 2021, 2022\rangle$ and $\mathbf{v} = \langle 69, 420, 80085\rangle$.

(d) Find an equation ($ax + by + cz = d$ style) of the plane passing through the three points given. $P = (3, -1, 2), Q = (1, 1, 1), R = (4, 1, -4)$.

4. (10 points) (a) Find parametric equations for the line through $P_0 = (2, -1, 1)$ perpendicular to the plane $2x+5y-3z = 34$. (Enter your answers as a comma-separated list of equations.)

(b) Find a parametrization of the horizontal circle (meaning, parallel to (x, y)-plane) of radius 7 with center $(1, -9, 6)$.

(c) Convert from rectangular to cylindrical coordinates: $(4, 4, 2021)$.

(d) Convert from rectangular to spherical coordinates: $(\sqrt{3}/2, 3/2, -1)$.

(e) Write the equation of the sphere of radius 10 in spherical coordinates.

5. (8 points) (a) Compute the derivative of: $r(t) = (t, t^6, t^4)$.

(b) Compute the tangent vector for $r(t) = (t^4, 2t^3)$ at $t = 2$. Then, parametrize the tangent line of the path $r(t)$ at that point.

(c) Compute the exact arc-length of the path $r(t) = (4t^{1/2}, \ln(t), 2t)$ between times $t = 1$ and $t = 2$. (Make sure you clearly set up the calculation, so I can give some points, even if you can't execute the whole thing.)

Scratchwork, if necessary: