**H1.1 Determine the dot product, cross product, and angle between P = 2ax – 6ay + 5az and Q = 3ay + az.**

$$\vec{P} = (2, -6, 5)$$
$$\vec{Q} = (0, 3, 1)$$

$$\vec{P}\cdot\vec{Q} = [(2\cdot 0) + ((-6)\cdot 3) + (5\cdot 1)]$$
$$= 0 + (-18) + 5$$
$$\boxed{= -13}$$

$$\vec{P}\times\vec{Q} = \begin{vmatrix} a_x & a_y & a_z \\ 2 & -6 & 5 \\ 0 & 3 & 1 \end{vmatrix} = (-6-15)a_x + (0-2)a_y + (6-0)a_z$$
$$= (-21)a_x + (-2)a_y + 6a_z$$
$$\boxed{\begin{aligned} &= -21a_x - 2a_y + 6a_z \\ &= (-21, -2, 6)\end{aligned}}$$

$$|\vec{P}| = \sqrt{(2)^2 + (-6)^2 + (5)^2} = \sqrt{65}$$
$$|\vec{Q}| = \sqrt{(0)^2 + (3)^2 + (1)^2} = \sqrt{10}$$

$$\cos\theta_{AB} = \frac{\vec{P}\cdot\vec{Q}}{|\vec{P}||\vec{Q}|} = \frac{-13}{\sqrt{65}\sqrt{10}}$$
$$= -0.5099019514$$
$$\theta_{AB} = \cos^{-1}(-0.5099)$$
$$\boxed{= 120.6573^\circ}$$

**H1.2 Points *P*, *Q* and *R* are located at (-1, 4, 8), (2, -1, 3) and (-1, 2, 3), respectively. Determine: (a) the distance between *P* and *Q*, (b) the distance vector from *P* to *R*, (c) the angle between *QP* and *QR*, (d) the area of triangle *PQR*, (e) the perimeter of triangle *PQR*.**

a)
$$\vec{r}_{PQ} = \vec{r}_Q - \vec{r}_P = (2,-1,3) - (-1,4,8) = (3,-5,-5)$$
$$d = |\vec{r}_{PQ}| = \sqrt{(3)^2 + (-5)^2 + (-5)^2} = \boxed{7.6811}$$

b)
$$\vec{r}_{PR} = \vec{r}_R - \vec{r}_P = (-1,2,3) - (-1,4,8)$$
$$\boxed{\begin{aligned} &= (0, -2, -5) \\ &= -2a_y - 5a_z\end{aligned}}$$

$$\cos\theta_{QR} = \frac{\vec{Q}\cdot\vec{R}}{|\vec{Q}||\vec{R}|} = \frac{5}{\sqrt{14}\sqrt{14}} = 0.35714$$
$$\vec{Q}\cdot\vec{R} = (2\cdot -1) + (-1\cdot 2) + (3\cdot 3)$$
$$= -2 - 2 + 9$$
$$= 5$$

$$|\vec{R}| = \sqrt{(-1)^2 + (2)^2 + (3)^2} = \sqrt{14}$$

C)
$$\cos\theta_{QP} = \frac{\vec{Q}\cdot\vec{P}}{|\vec{Q}||\vec{P}|} = \frac{18}{\sqrt{14}\cdot 9} = 0.5345$$
$$\vec{Q}\cdot\vec{P} = (2\cdot(-1)) + ((-1)\cdot 4) + (3\cdot 8)$$
$$= (-2 + (-4) + 24)$$
$$= 18$$
$$|\vec{Q}| = \sqrt{(2)^2 + (-1)^2 + (3)^2} = \sqrt{14}$$
$$|\vec{P}| = \sqrt{(-1)^2 + (4)^2 + (8)^2} = \sqrt{81} = 9$$

$$\theta_{QP} = \cos^{-1}(0.5345) = \boxed{57.6885^\circ}$$

$$\theta_{QR} = \cos^{-1}(0.35714) = 69.0752^\circ$$

b.) $\vec{PQ} = (2-(-1), (-1)-4, 3-8)$ $\quad A = \frac{1}{2}\|\vec{v}\times\vec{w}\|$

$= (3, -5, -5) = \vec{v}$

$\vec{PR} = (-1-(-1), 2-4, 3-8)$

$= (0, -2, -5) = \vec{w}$

$$\vec{v}\times\vec{w} = \begin{vmatrix} a_x & a_y & a_z \\ 3 & -5 & -5 \\ 0 & -2 & -5 \end{vmatrix} = (25-10)a_x + (0-(-15))a_y + (-6-0)a_z$$

$= (15, 15, -6)$

$\|\vec{v}\times\vec{w}\| = \sqrt{(15)^2+(15)^2+(-6)^2} = \sqrt{486}$

$A = \frac{1}{2}|\vec{v}\times\vec{w}| = \frac{1}{2}\sqrt{486}$ $\boxed{= 11.0227}$

$\vec{PQ}$ $\quad \vec{QR}$ $\quad \vec{RP}$ $\qquad P = |\vec{PQ}| + |\vec{QR}| + |\vec{RP}|$

$\vec{PQ} = (3, -5, -5)$

$\vec{QR} = ((-1-(2)), (2-(-1)), (3-3)) = (-3, 3, 0)$

$\vec{RP} = ((-1)-(-1)), (4-2), (8-3) = (0, 2, 5)$

$|\vec{PQ}| = \sqrt{(3)^2+(-5)^2+(-5)^2} = \sqrt{59}$

$\boxed{P = 17.30895}$

$|\vec{QR}| = \sqrt{(-3)^2+(3)^2+(0)^2} = \sqrt{18}$

$|\vec{RP}| = \sqrt{(0)^2+(2)^2+(5)^2} = \sqrt{29}$