CS 3653 – Discrete Mathematics for Computer Science

Assignment # 10	Due – Apr 4, 2022, 11:59pm (CST)
Chapter # 7.3 & 7.4	Max. Points # 25

SN	QUESTION	Pts
1	a) Suppose that Ann selects a ball by first picking one of two boxes at random and then selecting a ball from this box. The first box contains three orange balls and four black balls, and the second box contains five orange balls and six black balls. What is the probability that Ann picked a ball from the second box if she has selected an orange ball? b) When a test for steroids is given to soccer players, 98% of the players taking steroids test positive and 12% of the players not taking steroids test positive. Suppose that 5% of soccer players take steroids. What is the probability that a soccer player who tests positive takes steroids?	4
2	Suppose that 4% of the patients tested in a clinic are infected with avian influenza. Furthermore, suppose that when a blood test for avian influenza is given, 97% of the patients infected with avian influenza test positive and that 2% of the patients not infected with avian influenza test positive. What is the probability that: a) a patient testing positive for avian influenza with this test is infected with it? b) a patient testing positive for avian influenza with this test is not infected with it? c) a patient testing negative for avian influenza with this test is infected with it? d) a patient testing negative for avian influenza with this test is not infected with it?	4
3	Suppose that E, F_1, F_2, and F_3 are events from a sample space S and that F_1, F_2, and F_3 are pairwise disjoint and their union is S. a) Find $p(F_1 \mid E)$ if $p(E \mid F_1) = 1/8$, $p(E \mid F_2) = 1/4$, $p(E \mid F_3) = 1/6$, $p(F_1) = 1/4$, $p(F_2) = 1/4$, and $p(F_3) = 1/2$. b) Find $p(F_2 \mid E)$ if $p(E \mid F_1) = 2/7$, $p(E \mid F_2) = 3/8$, $p(E \mid F_3) = 1/2$, $p(F_1) = 1/6$, $p(F_2) = 1/2$, and $p(F_3) = 1/3$.	3
4	a) Suppose that a Bayesian spam filter is trained on a set of 500 spam messages and 200 messages that are not spam. The word "exciting" appears in 40 spam messages and in 25 messages that are not spam. Would an incoming message be rejected as spam if it contains the word "exciting" and the threshold for rejecting spam is 0.9? b) Suppose that a Bayesian spam filter is trained on a set of 1000 spam messages and 400 messages that are not spam. The word "opportunity" appears in 175 spam messages and 20 messages that are not spam. Would an incoming message be	3

	rejected as spam if it contains the word “opportunity” and the threshold for rejecting a message is 0.9?	
5	a) What is the expected number of heads that come up when a fair coin is flipped 10 times? b) A coin is biased so that the probability a head comes up when it is flipped is 0.6. What is the expected number of heads that come up when it is flipped 10 times?	3
6	What is the expected sum of the numbers that appear when three fair dice are rolled?	2
7	Suppose that we roll a fair die until a 6 comes up. a) What is the probability that we roll the die n times? b) What is the expected number of times we roll the die?	3
8	A dodecahedral die has 12 faces that are numbered 1 through 12. a) What is the expected value of the number that comes up when a fair dodecahedral die is rolled? b) What is the variance of the number that comes up when a fair dodecahedral die is rolled?	3

1	a) Suppose that Ann selects a ball by first picking one of two boxes at random and then selecting a ball from this box. The first box contains three orange balls and four black balls, and the second box contains five orange balls and six black balls. What is the probability that Ann picked a ball from the second box if she has selected an orange ball? b) When a test for steroids is given to soccer players, 98% of the players taking steroids test positive and 12% of the players not taking steroids test positive. Suppose that 5% of soccer players take steroids. What is the probability that a soccer player who tests positive takes steroids?	4

a.) $\boxed{\begin{matrix} 3O \\ 4B \end{matrix}}$ $\boxed{\begin{matrix} 5O \\ 6B \end{matrix}}$

$$P(F|E) = \frac{(5/11)(1/2)}{(5/11)(1/2) + (3/7)(1/2)} = \frac{35}{68} = 51.47\%$$

b.) S = uses Steroids E = Test Positive

$P(E|S) = .98$ $P(\bar{E}|S) = .02$ $P(E|\bar{S}) = .12$ $P(\bar{E}|\bar{S}) = .88$ $P(S) = .05$

$$P(S|E) = \frac{(.98)(.05)}{(.98)(.05) + (.12)(.95)} = 0.3006 = 30.06\%$$

2	Suppose that 4% of the patients tested in a clinic are infected with avian influenza. Furthermore, suppose that when a blood test for avian influenza is given, 97% of the patients infected with avian influenza test positive and that 2% of the patients not infected with avian influenza test positive. What is the probability that: a) a patient testing positive for avian influenza with this test is infected with it? b) a patient testing positive for avian influenza with this test is not infected with it? c) a patient testing negative for avian influenza with this test is infected with it? d) a patient testing negative for avian influenza with this test is not infected with it?	4

a = infected E = Test Positive

$P(a) = .04$ $P(\bar{a}) = .96$

$P(E|a) = (.97)$ $P(\bar{E}|a) = (.03)$ $P(E|\bar{a}) = (.02)$ $P(\bar{E}|\bar{a}) = (.98)$

a.) $$P(a|E) = \frac{(.97)(.04)}{(.97)(.04) + (.02)(.96)} = 0.66897 = 66.897\%$$

b.) $$P(\bar{a}|E) = \frac{(.03)(.96)}{(.03)(.96) + (.97)(.04)} = 0.4260 = 42.6\%$$

c.) $$P(a|\bar{E}) = \frac{(.03)(.04)}{(.03)(.04) + (.02)(.96)} = 0.05882 = 5.882\%$$

d.) $$P(\bar{a}|\bar{E}) = \frac{(.98)(.96)}{(.98)(.96) + (.03)(.04)} = 0.9987 = 99.87\%$$

3	Suppose that E, F_1, F_2, and F_3 are events from a sample space S and that F_1, F_2, and F_3 are pairwise disjoint and their union is S. a) Find $p(F_1 \mid E)$ if $p(E \mid F_1) = 1/8$, $p(E \mid F_2) = 1/4$, $p(E \mid F_3) = 1/6$, $p(F_1) = 1/4$, $p(F_2) = 1/4$, and $p(F_3) = 1/2$. b) Find $p(F_2 \mid E)$ if $p(E \mid F_1) = 2/7$, $p(E \mid F_2) = 3/8$, $p(E \mid F_3) = 1/2$, $p(F_1) = 1/6$, $p(F_2) = 1/2$, and $p(F_3) = 1/3$.	3

a.)

$$P(F_1 \mid E) = \frac{\left(\frac{1}{8}\right)\left(\frac{1}{4}\right)}{\left(\frac{1}{8}\right)\left(\frac{1}{4}\right) + \left(\frac{7}{8}\right)\left(\frac{3}{4}\right)} = \frac{1}{22} = 0.045$$

b.)

$$P(F_2 \mid E) = \frac{\left(\frac{3}{8}\right)\left(\frac{1}{2}\right)}{\left(\frac{3}{8}\right)\left(\frac{1}{2}\right) + \left(\frac{5}{8}\right)\left(\frac{1}{2}\right)} = \frac{3}{8} = 0.375$$

4	a) Suppose that a Bayesian spam filter is trained on a set of 500 spam messages and 200 messages that are not spam. The word "exciting" appears in 40 spam messages and in 25 messages that are not spam. Would an incoming message be rejected as spam if it contains the word "exciting" and the threshold for rejecting spam is 0.9? b) Suppose that a Bayesian spam filter is trained on a set of 1000 spam messages and 400 messages that are not spam. The word "opportunity" appears in 175 spam messages and 20 messages that are not spam. Would an incoming message be rejected as spam if it contains the word "opportunity" and the threshold for rejecting a message is 0.9?	3

a.)

$$r(\text{exciting}) = \frac{P(\text{exciting})}{P(\text{exciting}) + r(\text{exciting})} = \frac{(40/500)}{(40/500) + (25/200)} = 0.39$$

b.)

$$r(\text{opportunity}) = \frac{P(\text{OPP})}{P(\text{OPP}) - 9(\text{OPP})} = \frac{(175/1000)}{(175/1000) + (20/100)} = .777$$

5	a) What is the expected number of heads that come up when a fair coin is flipped 10 times? b) A coin is biased so that the probability a head comes up when it is flipped is 0.6. What is the expected number of heads that come up when it is flipped 10 times?	3

a.) $\frac{1}{2} \cdot 10 = \frac{10}{2} = 5$

b.) $0.6 \cdot 10 = 6$

6	What is the expected sum of the numbers that appear when three fair dice are rolled?	2

$$E(x) = \frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 2 + \dots + \frac{1}{6} \cdot 6 = \frac{7}{2}$$

$$\left(\frac{7}{2}\right) 3 = 10.5$$

7	Suppose that we roll a fair die until a 6 comes up. a) What is the probability that we roll the die n times? b) What is the expected number of times we roll the die?	3

a.) $P(6) = \frac{1}{6}$

$P(\text{Not } 6) = \frac{5}{6}$

$P(x = n) = \left(\frac{5}{6}\right)^{n-1} \cdot \left(\frac{1}{6}\right)$

$b = \frac{1}{P(6)} = \frac{1}{(1/6)} = 6$

8	A dodecahedral die has 12 faces that are numbered 1 through 12. a) What is the expected value of the number that comes up when a fair dodecahedral die is rolled? b) What is the variance of the number that comes up when a fair dodecahedral die is rolled?	3

a.)

$$E(x) = \frac{1}{12}(1+2+3+4+5+6+7+8+9+10+11+12) = \frac{13}{2}$$

$$E(x^2) = \frac{1}{12}(1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2+10^2+11^2+12^2) = \frac{325}{6} - \left(\frac{13}{2}\right)^2 = \frac{143}{12}$$