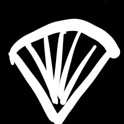
Changing Variables 3D?

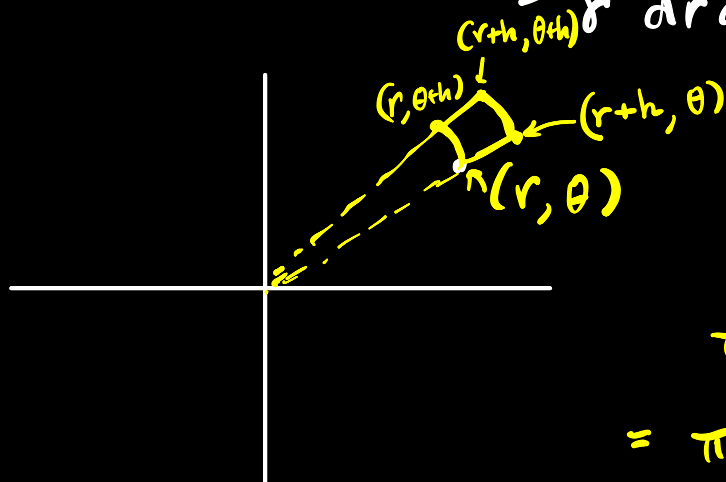
I. Some BS explanation of $r\,\underline{dr}\,\underline{d\theta}$ (which is good enough for us.)

II. Cylindrical & Spherical Jacobians?

$$\iint F(x,y)\,\boxed{dA}$$



"$r\,dr\,d\theta$"



Area: between huge circles:

$$\pi(r+h)^2 - \pi r^2$$

$$= \pi(r^2+2rh+h^2) - \pi r^2$$

$$= \underline{2\pi r h + \pi h^2}$$

$\frac{h}{2\pi}$ of this

$$\frac{h}{2\pi}\left(2\pi r h + \pi h^2\right)$$

Dolphin's $h \times h$ SQ Area

$$\frac{\boxed{r h^2 + \frac{1}{2} h^3}}{h^2}$$

Human's $h \times h$ square

$$\underbrace{\qquad\qquad}$$

$$\lim_{h \to 0} \xleftarrow{r + \frac{1}{2}h} = \boxed{r}$$

II. Switching from (x, y, z) to Cylindrical is:

"Polar coordinate switch on (x, y)"

+ Nothing new in z.

$$\left\{\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{array}\right\}$$

$$\iiint f(x,y,z)\, \boxed{dV.}$$

$$\int_0^{2\pi}\int_0^{1}\int_0^{\text{BLAH}} \underbrace{f(x,y,z)}_{\text{TRANSLATE!}}\; r \quad dz\, dr\, d\theta$$

OR whatever ends up being friendlier.

Change to Spherical?

(x,y,z) $\qquad$ $(\rho, \underline{\underline{\theta}}, \underline{\underline{\varphi}})$

$$\left\{\begin{aligned} x &= \rho\sin\varphi\cos\theta \\ y &= \rho\sin\varphi\sin\theta \\ z &= \rho\underbrace{\cos\varphi}_{} \end{aligned}\right\}$$

EASY For us...

$$\iiint f(x,y,z)\;\underbrace{dV}_{}$$

$$\boxed{\rho^2\sin\varphi}\,d\rho\,d\theta\,d\varphi$$

Check:

$$\iiint 1\,dV$$

$$\frac{4}{3}\pi\left(3^3\right)$$

$$= 9\cdot 4\cdot\pi$$

$$= \underline{36\pi}.$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{3} 1\ \underbrace{\rho^2 \sin\varphi}\ \boxed{d\rho}\ d\varphi\ d\theta$$

S1:

$$\overset{\star}{=} \frac{\rho^3}{3}\sin\varphi \Big|_0^3$$

$$= \boxed{9\sin\varphi}$$

S2:

$$\int_0^{\pi} 9\sin\varphi\ d\varphi$$

$$\overset{\star}{=} -9\cos\varphi \Big|_{\varphi=0}^{\varphi=\pi}$$

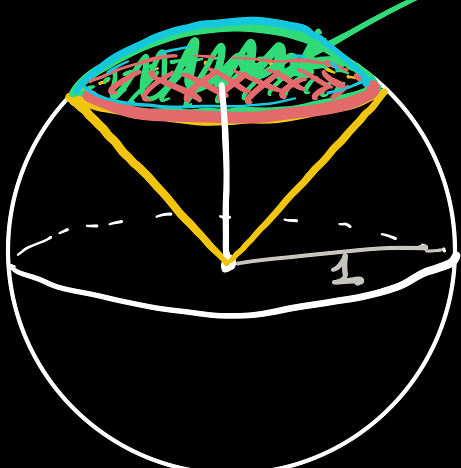
$$= [9] - [-9]$$

$$= 18.$$

S3: $$\int_0^{2\pi} 18 \, d\theta$$

$$\overset{\star}{=} 18\theta \Big|_{\theta=0}^{\theta=2\pi}$$

$$\boxed{36\pi} - 0$$



Volume.

$$\iiint 1 \, dV$$

$$\int^{2\pi}\int^{\pi/4}\int^{\rho=1} 1 \cdot \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

choose order

$0 \quad 0 \quad \frac{\sqrt{2}}{2}\sec\varphi$

$$\boxed{z = \frac{\sqrt{2}}{2}}$$

$$\underbrace{\rho\cos\varphi = \frac{\sqrt{2}}{2}}_{\rho = \frac{\sqrt{2}}{2}\sec\varphi}$$

$$\boxed{\text{I won't do integral: You!}}$$