

# CS 3653 – Discrete Mathematics for Computer Science

Assignment # 11	Due – Apr 11, 2022, 11:59pm (CST)
Chapter # 9.1 & 9.3	Max. Points # 25

SN	QUESTION	Pts
1	<p>List the ordered pairs in the relation R from <math>A = \{0, 1, 2, 3, 4\}</math> to <math>B = \{0, 1, 2, 3\}</math>, where <math>(a, b) \in R</math> if and only if</p> <ul style="list-style-type: none"><li>a) <math>a = b</math>.</li><li>b) <math>a + b = 4</math>.</li><li>c) <math>a &gt; b</math>.</li><li>d) <math>a   b</math>.</li></ul>	2
2	<ul style="list-style-type: none"><li>a) List all the ordered pairs in the relation <math>R = \{(a, b) \mid a \text{ divides } b\}</math> on the set <math>\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}</math>.</li><li>b) Display this relation graphically.</li><li>c) Display this relation in tabular form.</li></ul>	3
3	<p>For each of these relations on the set <math>\{1, 2, 3, 4\}</math>, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive. Justify your answer.</p> <ul style="list-style-type: none"><li>a) <math>\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}</math></li><li>b) <math>\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}</math></li><li>c) <math>\{(2, 4), (4, 2)\}</math></li><li>d) <math>\{(1, 2), (2, 3), (3, 4)\}</math></li><li>e) <math>\{(1, 1), (2, 2), (3, 3), (4, 4)\}</math></li><li>f) <math>\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}</math></li></ul>	3
4	<p>Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where <math>(x, y) \in R</math> if and only if</p> <ul style="list-style-type: none"><li>a) <math>x + y = 0</math>.</li><li>b) <math>x = 2y</math>.</li><li>c) <math>xy \geq 0</math>.</li><li>d) <math>xy = 0</math>.</li></ul>	4
5	<p>Let <math>R_1 = \{(1, 2), (2, 3), (3, 4)\}</math> and <math>R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}</math> be relations from <math>\{1, 2, 3\}</math> to <math>\{1, 2, 3, 4\}</math>. Find</p> <ul style="list-style-type: none"><li>a) <math>R_1 \cup R_2</math>.</li><li>b) <math>R_1 \cap R_2</math>.</li><li>c) <math>R_1 - R_2</math>.</li><li>d) <math>R_2 - R_1</math>.</li></ul>	2

6	Let R be the relation $\{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$ , and let S be the relation $\{(2, 1), (3, 1), (3, 2), (4, 2)\}$ . Find $S \circ R$ , will it be similar to $R \circ S$ , justify your answer.	2
7	Represent each of these relations on $\{1, 2, 3, 4\}$ with a matrix (with the elements of this set listed in increasing order). a) $\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$ b) $\{(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)\}$ c) $\{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)\}$ d) $\{(2, 4), (3, 1), (3, 2), (3, 4)\}$	2
8	Let $R_1$ and $R_2$ be relations on a set A represented by the matrices $\mathbf{M}_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$ Find the matrices that represent a) $R_1 \cup R_2$ . b) $R_1 \cap R_2$ . c) $R_2 \circ R_1$ . d) $R_1 \circ R_1$ . e) $R_1 \oplus R_2$ .	5
9	Draw the directed graphs representing each of the relations from: a) $\{(1, 1), (1, 2), (1, 3)\}$ b) $\{(1, 2), (2, 1), (2, 2), (3, 3)\}$ c) $\{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$ d) $\{(1, 3), (3, 1)\}$	2

	List the ordered pairs in the relation R from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$ , where $(a, b) \in R$ if and only if	
1	a) $a = b$ . b) $a + b = 4$ . c) $a > b$ . d) $a \mid b$ .	2

$$a.) \quad R = \{(a,b) \mid a=b\} \\ = \{(0,0), (1,1), (2,2), (3,3)\}$$

$$b.) R = \{ (a,b) \mid a+b=4 \} \\ = \{ (1,3), (2,2), (3,1), (4,0) \}$$

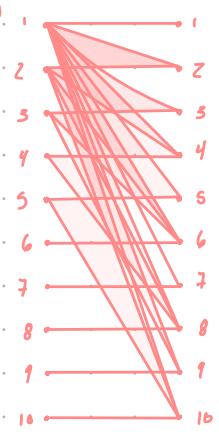
$$C.) \quad R = \{(a, b) \mid a > b\} \\ = \{(1, 0), (2, 0), (2, 1), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 2), (4, 3)\}$$

$$D.) \quad R = \{ (a,b) \mid a|b \} \\ = \{ (1,0), (1,1), (1,2), (1,3), (2,0), (2,2), (3,0), (3,3), (4,0) \}$$

2 a) List all the ordered pairs in the relation  $R = \{(a, b) \mid a \text{ divides } b\}$  on the set  $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ .  
b) Display this relation graphically.  
c) Display this relation in tabular form.

3

$$\begin{aligned}
 a.) \quad R &= \{(a,b) \mid a \text{ divides } b\} \\
 &= \{(1,1), (1,2), \dots, (1,10), (2,2), (2,4), \dots, (2,10), (3,3), (3,6), (3,9), \\
 &\quad (4,4), (4,8), (5,5), (5,10), (6,6), (7,7), (8,8), (9,9), (10,10)\}
 \end{aligned}$$



For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

Justify your answer.

- 3
- a) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}
  - b) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}
  - c) {(2, 4), (4, 2)}
  - d) {(1, 2), (2, 3), (3, 4)}
  - e) {(1, 1), (2, 2), (3, 3), (4, 4)}
  - f) {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)}

3

- a.) Transitive  
b.) Reflexive, Symmetric, Transitive  
c.) Symmetric  
d.) AntiSymmetric  
e.) Reflexive, Symmetric, AntiSymmetric, Transitive  
f.) None of These Properties

4 Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where  $(x, y) \in R$  if and only if

- a)  $x + y = 0$ .
- b)  $x = 2y$ .
- c)  $xy \geq 0$ .
- d)  $xy = 0$ .

4

a, d, c, f

- a.) Symmetric  
b.) Anti-Symmetric  
c.) Reflexive, Symmetric  
d.) Symmetric

5 Let  $R_1 = \{(1, 2), (2, 3), (3, 4)\}$  and  $R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$  be relations from {1, 2, 3} to {1, 2, 3, 4}. Find

- a)  $R_1 \cup R_2$ .
- b)  $R_1 \cap R_2$ .
- c)  $R_1 - R_2$ .
- d)  $R_2 - R_1$ .

2

- a.)  $R_1 \cup R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$   
b.)  $R_1 \cap R_2 = \{(1, 2), (2, 3), (3, 4)\}$   
c.)  $R_1 - R_2 = \{\}$   
d.)  $R_2 - R_1 = \{(1, 1), (2, 1), (2, 2), (3, 1), (3, 2), (3, 3)\}$

6 Let R be the relation  $\{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$ , and let S be the relation  $\{(2, 1), (3, 1), (3, 2), (4, 2)\}$ . Find  $S \circ R$ . Will it be similar to  $R \circ S$ , justify your answer.

2

7 Represent each of these relations on  $\{1, 2, 3, 4\}$  with a matrix (with the elements of this set listed in increasing order).

- a)  $\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$
- b)  $\{(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)\}$
- c)  $\{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)\}$
- d)  $\{(2, 4), (3, 1), (3, 2), (3, 4)\}$

2

a.)

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 1 & 1 \\ 2 & 0 & 0 & 1 & 1 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

c.)

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 1 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 1 & 1 & 1 & 0 \end{array}$$

b.)

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 & 0 \\ 4 & 1 & 0 & 0 & 0 \end{array}$$

d.)

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

Let  $R_1$  and  $R_2$  be relations on a set A represented by the matrices

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

- 8 Find the matrices that represent
- a)  $R_1 \cup R_2$ .
  - b)  $R_1 \cap R_2$ .
  - c)  $R_2 \circ R_1$ .
  - d)  $R_1 \circ R_1$ .
  - e)  $R_1 \oplus R_2$ .

5

a.)  $M_{R_1 \cup R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

c.)  $M_{R_2 \circ R_1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

b.)  $M_{R_1 \cap R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

d.)  $M_{R_1 \circ R_1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

e.)  $M_{R_1 \oplus R_2} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Draw the directed graphs representing each of the relations from:

- a)  $\{(1, 1), (1, 2), (1, 3)\}$
- b)  $\{(1, 2), (2, 1), (2, 2), (3, 3)\}$
- c)  $\{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$
- d)  $\{(1, 3), (3, 1)\}$

