Chapter 2. Approximation Methods for Solving Differential Equations

1. Given the following stiff initial-value problems

$$y' = -20y + 20cost - sint, \quad y(0) = 0, \quad h = 0.025$$

   Prove that it has the actual solution $y = -e^{-20t} + cost$

2. **Using Euler's method** and then **Improved Euler's method** to find the value of y when t=0.075, and compare the results with the actual solution. *(Note: unit of trigonometric function is in radian)*

$$y' = -20y + 20cost - sint, \quad y(0) = 0, \quad h = 0.025$$

3. Solve the Exercise 1 using **Runge-Kutta fourth-order method**.

t	**Exact**	**Euler method**	**Error**	**Improved Euler**	**Error**	**Runge-Kutta**	**Error**
0							
0.025							
0.050							
0.075							

4. Problem

An engineer whose major is in Heat & Refrigeration engineering is having a project of designing insulation walls for a building. The mathematical model of the rate of temperature change for the building could be described by the following ODE:

$$T'(t) = k[T(t) - T_A] \tag{1}$$

where $T(t)^oC$ is the instant temperature inside the building and T_A is the outside temperature (suppose that it is constant). In this model, k is the heat transfer coefficient which heavily depends on the material used for the wall. Assuming that the material he used has $k = -0.0603$. Suppose that in winter **the daytime temperature in a certain office of that building is always maintained at 21°C by a heating system**. This system is shut off at 10 P.M. and turned on again at 6 A.M. On a certain day the **outside temperature was approximately 7°C** throughout the night. Estimate the temperature inside the building when the heat was turned on at 6 A.M. on the following day by:

a) Using the 1st-order Euler approximation method with time step h = 2.0 hours.
b) Using the 3rd-order Runge-Kutta approximation method with time step h = 2.0 hours.
c) Compare the approximation results with the exact value if the exact solution of Eq. (1) is $T(t) = 7 + 14e^{kt}$

❖ **Notes:** the 3rd-order Runge-Kutta approximation formula for solving the ODE $y' = f(x, y)$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1\right)$$

$$k_3^* = hf(x_{n+1}, y_n - k_1 + 2k_2)$$

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 4k_2 + k_3^*)$$