DSP Report (15%)

Controllable Filter:

- Function:
  - Converts carrier signal from stereo to mono, if it has two dimensions.
  - Obtains interpolation factors by regularly sampling a sine wave control signal, oscillating at an input frequency, with the same sample rate and duration as the carrier signal.
  - Zero-pads carrier signal with $filter\ length\ - 1$ zeros at both ends.
  - Iterates through the interpolation factors, whilst moving the slice through the signal by one sample each time:
    - Calculates the interpolated filter coefficients using the formula, where B_n is the n^{th} set of filter coefficients:

$$Interpolated\ Filter\ Coefficients\ =\ factor\ *\ B_1 + (1 - factor) * B_2$$

    - Appends the dot product of the reversed filter coefficients and the slice of the signal to a list of filtered samples.
- Define low pass and band pass FIR filters, acting as arguments to the function with the carrier wav samples, sample rate and 2Hz control frequency.
- Original and filtered signals can be heard using AudioSignal.

Digit Recognition:

Correlation system:

- Iterates through an input test set, defining a correlation list each iteration.
  - Iterates through the training set, calculating the correlation for the test image with each training image by summing the dot product of the two images.
  - Appends correlations to the list until we have the correlations for the test image with every training image.
- Correlation list is converted to an array and the index of the maximum value is used to identify the corresponding training image label.
- Completing this process for all test images, a list of predicted labels is returned.

Cross-correlation system:

- Same function as the correlation system however, it uses cross-correlation, by taking the maximum value of signal.correlate2d, rather than correlation.
- Runtime for all test images is over an hour so, after calculating accuracy on the full test set, future runs of this model will use a subset of test images.

Prediction system variants:

Variant systems were implemented using altered image sets:

1. Negative training and test images.
2. Training images rotated anticlockwise by 10° and -10°.
3. Training images with random noise added.
4. Training and test images that have been smoothed using Gaussian filtering.
5. New training set containing a single image for each digit, acquired by averaging over all training images of the corresponding digit.

System accuracy:

- Iterates through an input list of predicted labels, adding 1 to a counter for correctly predicted labels if the label is correct.
- Returns the number of correctly predicted labels divided by the total number of labels.

Correlation system accuracy (to 6 s.f.)						
Standard	Negatives	Rotated training images		Noisy training images	Gaussian filtered images	Averaged training digits
		10°	-10°			
0.901379	0.908802	0.765642	0.827147	0.902439	0.786850	0.873807

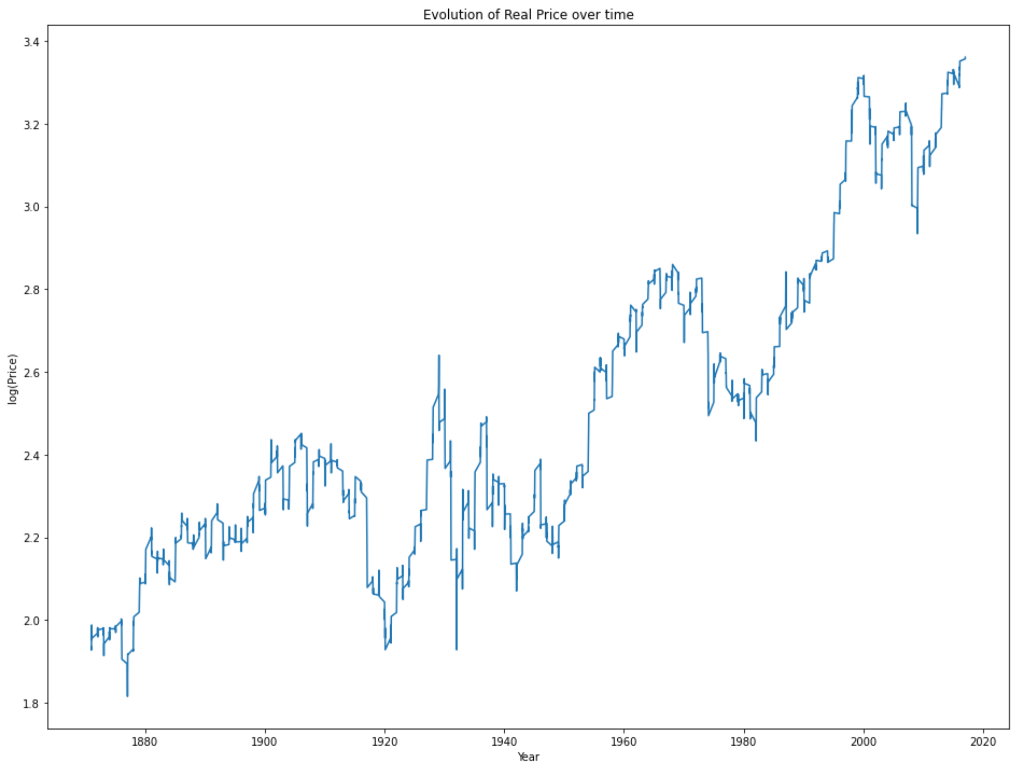
Cross-correlation system accuracy (to 6s.f.)						
Standard	Negatives	Rotated training images		Noisy training images	Gaussian filtered images	Averaged training digits
		10°	-10°			
0.917285	0.908802	0.761310	0.839873	0.915164	0.772004	0.887593

- The cross-correlation systems consistently achieves accuracy about 1% higher than the corresponding correlation systems, excluding systems run on negatives and training images rotated anticlockwise by 10°.
- Cross-correlation system follows a similar method as the correlation system but also accounts for image offset, improving accuracy.
- Training images rotated anticlockwise by 10° and -10° show large differences in accuracy, which could be due to 7s looking similar to 4s after 10° rotation.
- Using averaged training digits reduces accuracy by about 3%, due to reduced training data. Nonetheless, it greatly improves the runtime of both systems, allowing for testing on the full test set in reduced time.

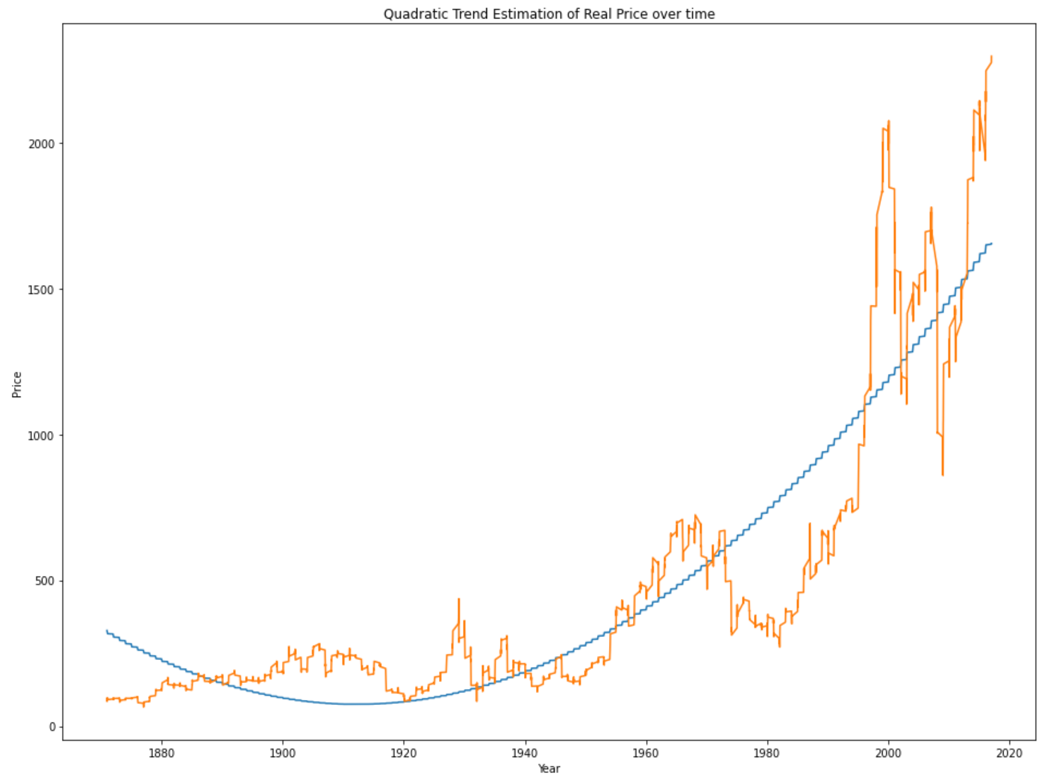
Time Series Prediction:

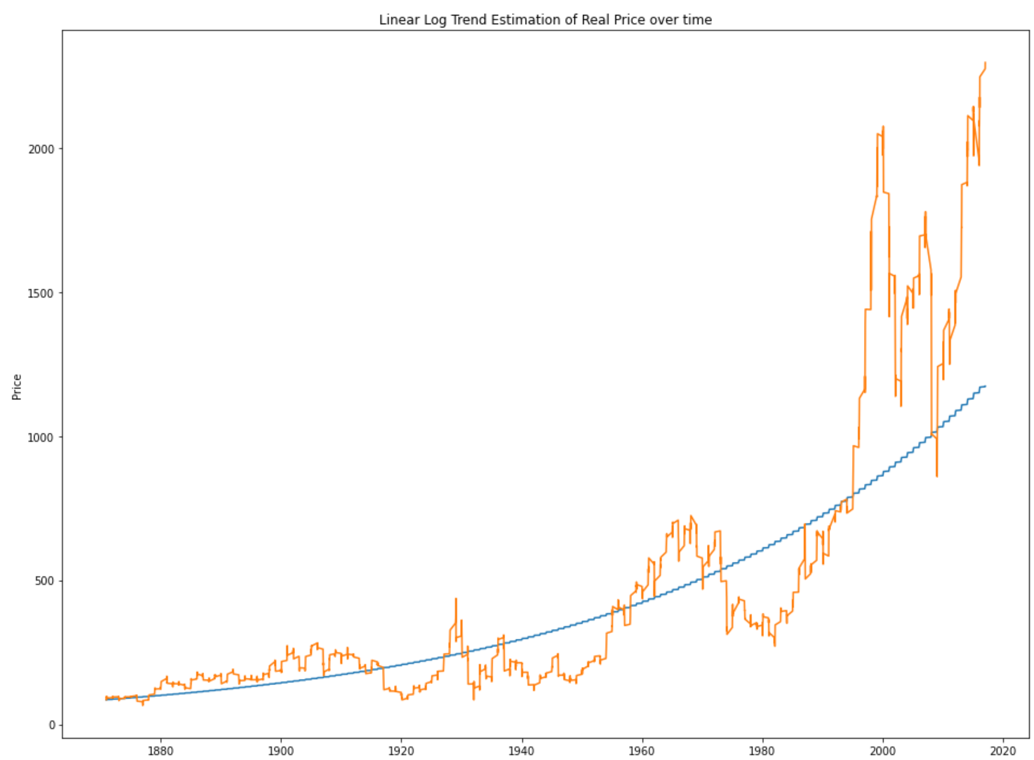
Trend estimation:

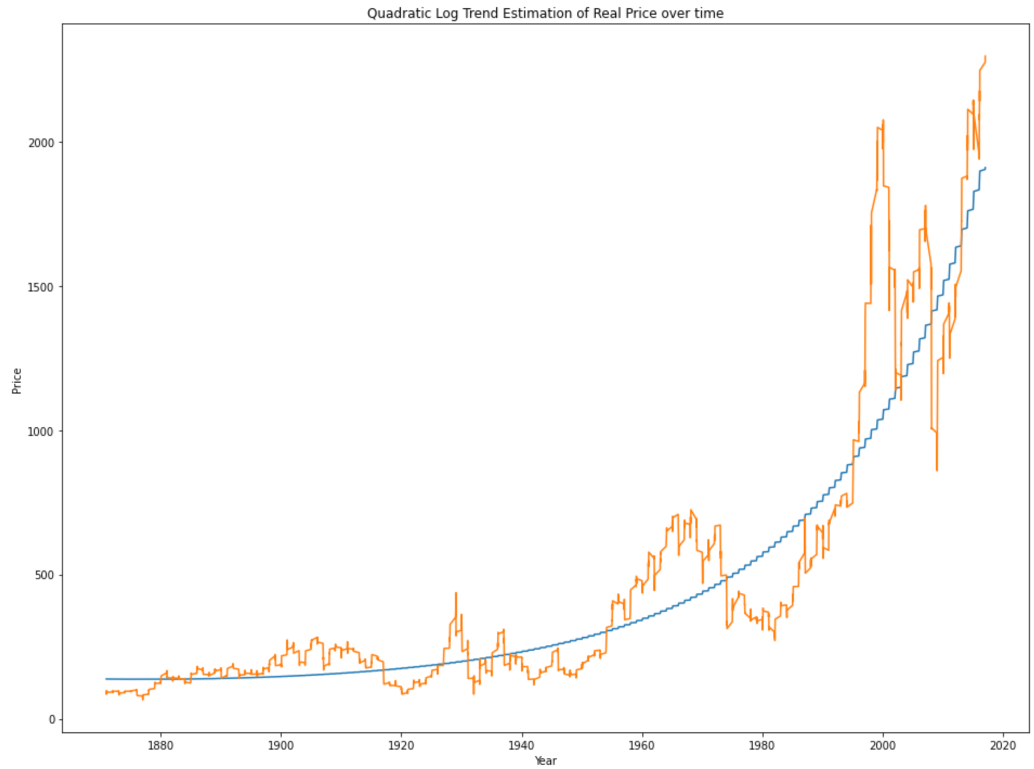
- Smoothing applied using a 5-year moving average, as well as an exponentially-weighted moving average.
- Shape of the resulting graphs suggested a quadratic or exponential trend, indicating that the time series is multiplicative. Thus, the time series is detrended through division by the trend.
- Converting to a log scale confirmed the shape of the trend:



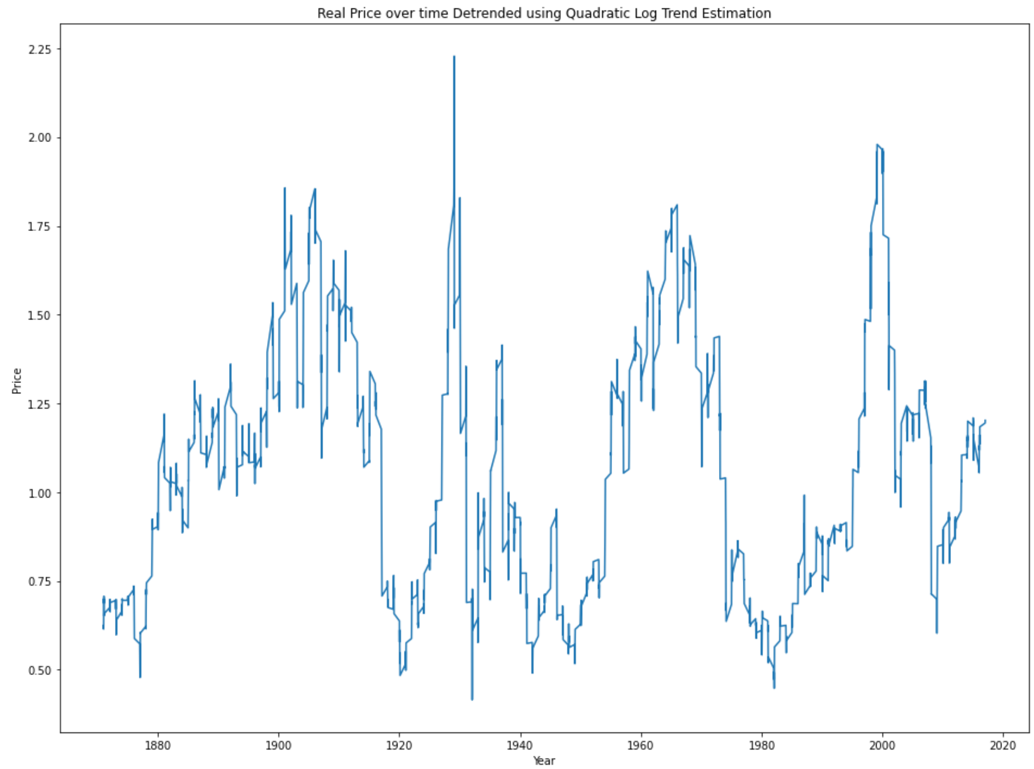
- Three trends are approximated using polyfit in the linear and log scales.
- Graphs and the variance of the three detrended time series are compared to find the best approximation:


Quadratic Trend Estimation of Real Price over time
Price
Year
0
500
1000
1500
2000
1880
1900
1920
1940
1960
1980
2000
2020


Linear Log Trend Estimation of Real Price over time
Price
Year
0
500
1000
1500
2000
1880
1900
1920
1940
1960
1980
2000
2020

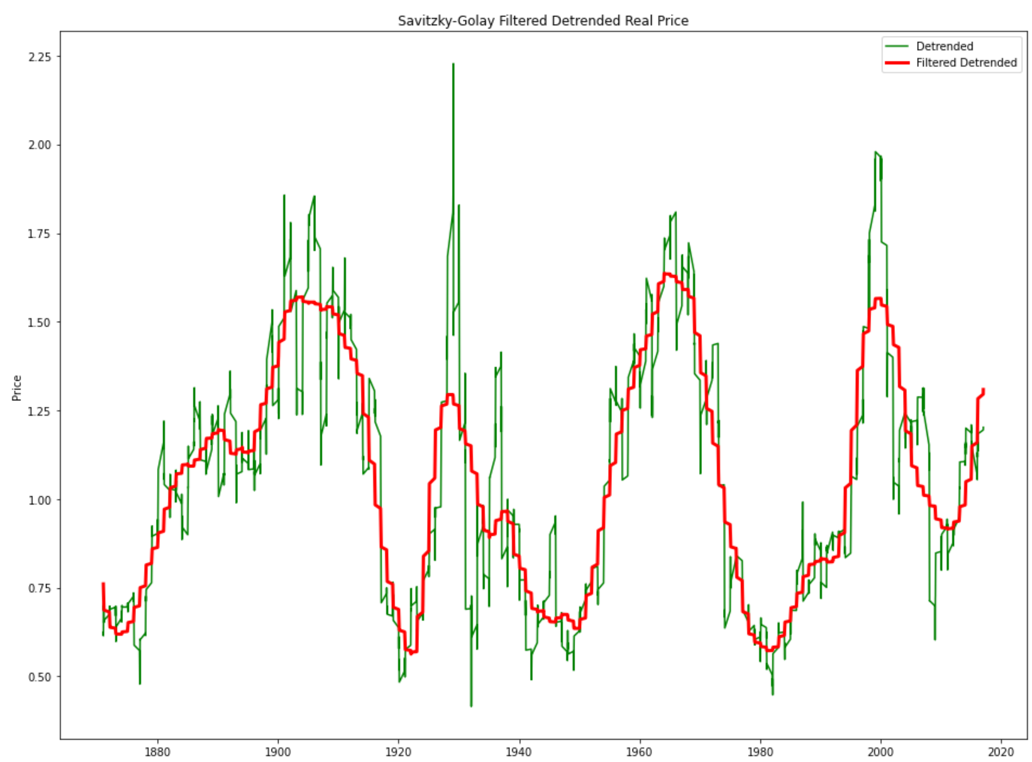


Variance for detrended time series (to 6 s.f.)		
Quadratic	Log Linear	Log Quadratic
0.549884	0.165457	0.125768

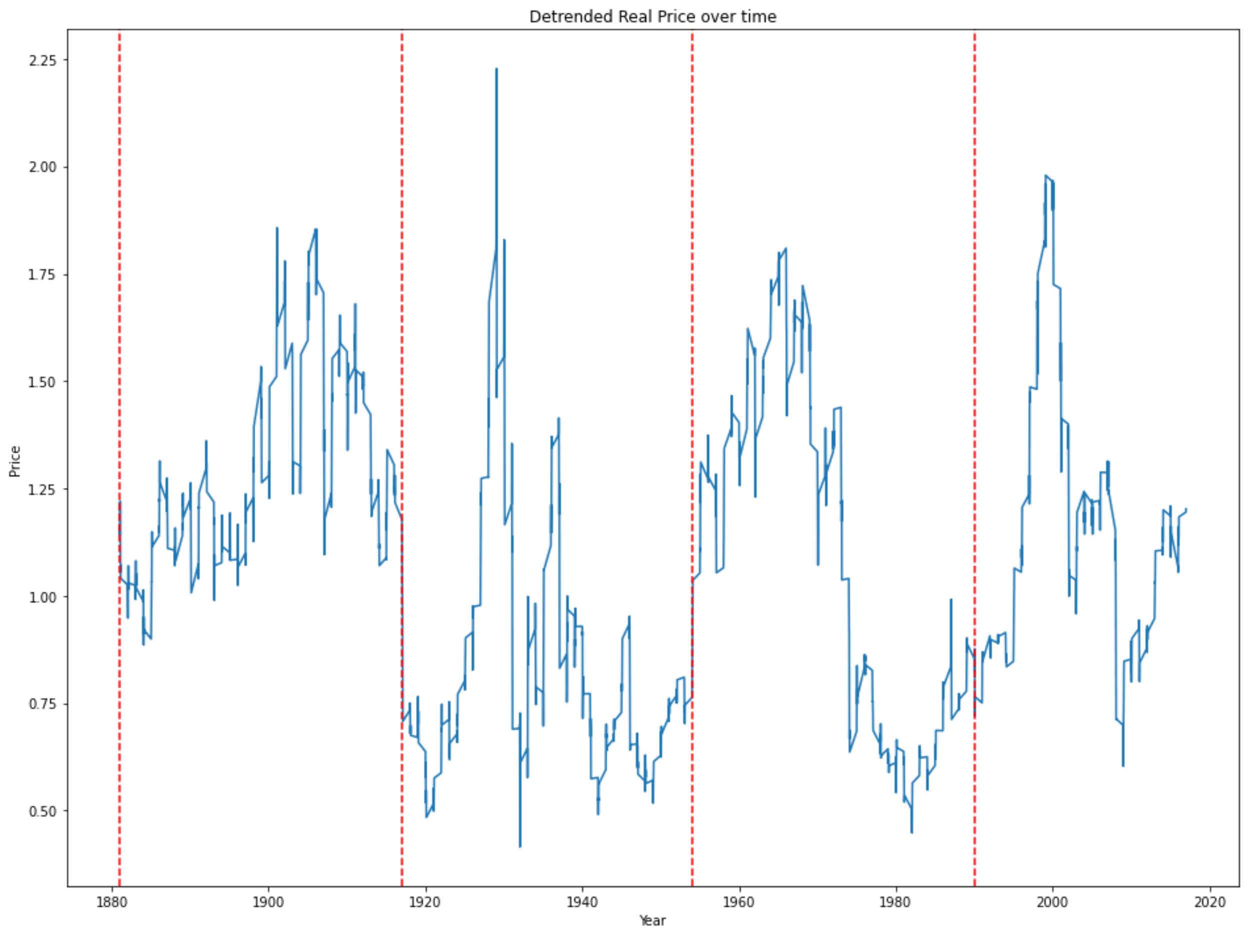


Periodicity estimation:

- Detrended time series is smoothed using Savitzky-Golay filtering to decrease noise.



- Sample rate: 12 a year, converted to Hz.
- FFTs of original and filtered detrended time series are calculated.
- Frequency peaks above a certain amplitude threshold are identified and converted to potential periodicities in the time domain.
- Prominent peaks are isolated, removing all other frequencies, and the inverse FFT is calculated.
- From, roughly, January 1881, the detrended data may contain a periodicity of approximately 36 years, corresponding to both FFTs' frequency of the highest amplitude.



- The approximate periodicity seems to be dictated by four key troughs in the detrended time series, occurring during:
    - First World War.
    - Wall Street Crash.
    - Second World War.
    - Early 80s recession.

Refined analysis:

- The process of acquiring periodicity predictions is implemented on each approximate period seen in the above graph.

- Results are varied but all periods contain an estimated periodicity in the range 3.32 - 4.06 years:

Micro-periodicity estimates within approximate macro-periods (years to 3 s.f.)							
Period 1: 01/1881			Period 2: 07/1917		Period 3: 01/1954	Period 4: 07/1990	
3.32	4.56	6.08	3.65	9.13	4.06	2.05	3.81

- An average year time series is created by taking the mean monthly values for each month, and duplicating it to form a time series the same length as the original time series.
- The prominent frequency of importance in the FFT of this signal corresponds to 6 months, which is likely to be governed by strategy employed for different financial quarters.

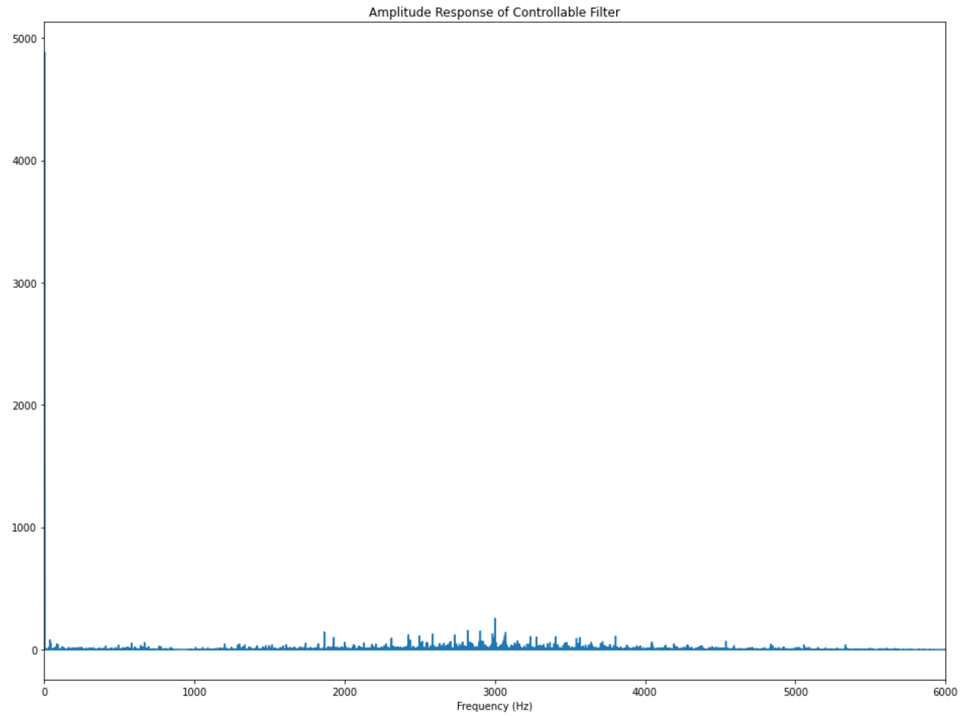
Controllable Filter Frequency Response:

Frequency Response Calculation:

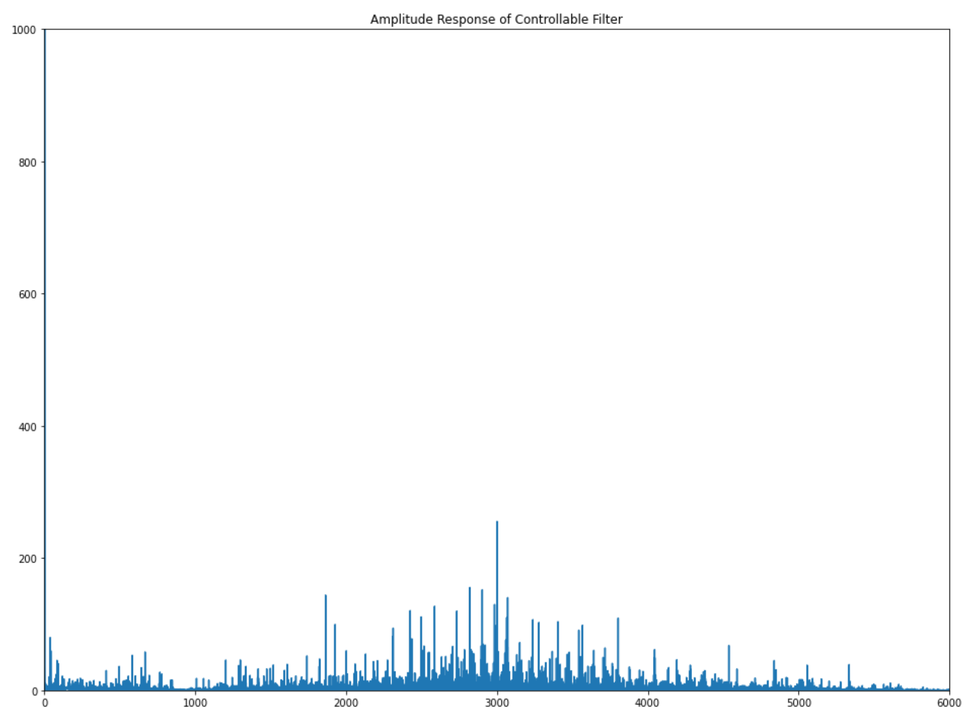
- The filter coefficients vary over time, meaning the frequency response cannot be calculated from a single set of filter coefficients.
- As a result, the frequency response of the controllable filter was calculated by dividing the FFT of the filter's output by the FFT of the filter's input.
- The FFT sample frequencies are calculated from the size and sample rate of the carrier samples, which is equivalent to that of the filtered samples.
- The absolute value is taken to plot the amplitude response, while the phase response is acquired by taking the angle values from the complex frequency response values.

Amplitude Response:

- The number of taps used for both filters is 21, resulting in a lower frequency resolution.



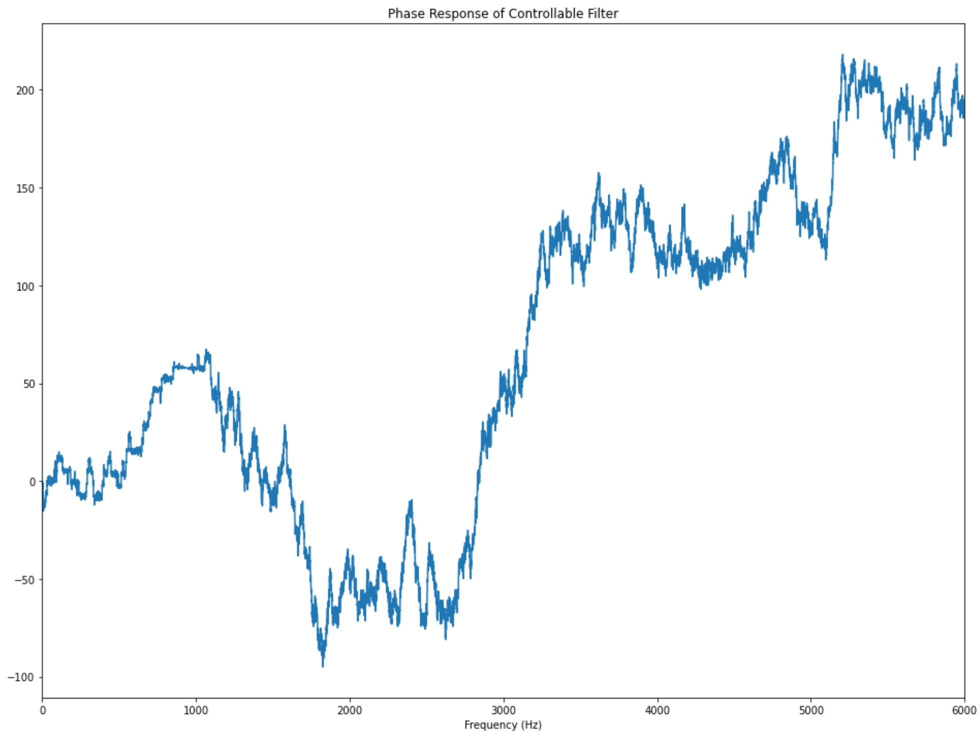
- This can be seen in the amplitude response, where we have amplitudes for frequencies, outside of the FIR filter frequency bands that have gain applied, that are similar to the amplitudes within the bands. This is caused by the more shallow run-offs that occur due to low frequency resolution.



- The tallest peak seen in the amplitude response occurs at the central frequency (3000Hz) of the pass band, 2000Hz-4000Hz, used in the band pass filter. This is

supported by the low frequency resolution, as shallow run-offs for a band pass filter result in a more narrow peak at the central frequency of the band
    - The run-off from 3000Hz, moving up in frequency, continues almost all the way to the 6000Hz Nyquist frequency. This leaves a 3000Hz gap between the central frequency of the pass band and the upper end of the run-off.
    - The run-off from 3000Hz, moving down in frequency, reaches 1000Hz, which is the cut-off for the low pass FIR filter. This leaves a 2000Hz gap between the central frequency and the lower end of the run-off.
- In the pass frequency band for the low pass filter (0Hz-1000Hz), we can see that the largest amplitude values occur at the lower frequencies and slightly above the band's central frequency.
    - Approaching the cut-off frequency (1000Hz) from the central frequency, we can see that the run-off is a lot more compact than that of the band pass filter.
    - The amplitudes appear to run-off to negligible levels by 900Hz, and remain at that level until 1000Hz.
- Higher amplitudes are seen in the frequency band of the band pass than in the band for the low pass, largely due to the fact that the band pass applies a gain of 2, whereas the low pass applies a gain of 1.5. These amplitudes will also be affected by the width of these frequency bands, as well as the number of taps used for each filter.
- The amplitude response demonstrates that the filter's could be made more effective, and precise, by finding a more optimal number of taps to use. One way to acquire this could be through the use of Bellenger's formula for estimating the required number of taps.

Phase Response:



- The phase response of the system is clearly not linear.
- Frequencies in the range of approximately 1700Hz to 2900Hz show negative phase, with phase growing to large positive values around the 3000Hz mark.
- This means that, within the filter, each frequency component is shifted in time by different amounts, which creates phase distortion.