Indicies:

- $i = (1, n)$: $packing\ orders$
- $j = (1, m)$: $line\ numbers$
- $t = (1, T)$: $time\ slots, given\ in\ terms\ of\ a\ base\ time\ interval\ \Delta T\ e.g. 15\ minutes$
- $w = (1, W)$: $workers$

Decision Variables:

- $x(i, j, t, w) \coloneqq packing\ order\ i\ starts\ on\ line\ j\ at\ time\ t\ with\ worker\ w$ (binary)
- $y(i, k, j) \coloneqq setup\ between\ orders\ i\ and\ k\ on\ line\ j$ (binary)
- $b(i, k) \coloneqq batch\ indicator, order\ i\ and\ k\ batched\ together$ (binary)
- $w(w, t) \coloneqq worker\ w\ is\ working\ during\ time\ t$ (binary)
- $m(w, t) \coloneqq worker\ w\ is\ working\ on\ another\ line\ that\ he\ did\ at\ t - 1$
- $prod(i, t) \coloneqq number\ of\ units\ produced\ at\ time\ t$ (positive integer)
- $inv(i) \coloneqq number\ of\ packing\ order\ i\ in\ stock$ (positive integer)
- $ship(i, t) \coloneqq packing\ order\ i\ is\ shipped\ at\ time\ t$ (binary)
- $time_{ship}(i) \coloneqq time\ slot\ when\ order\ i\ is\ shipped$(integer $[0, T]$)
- $has_{inv}(i, t) \coloneqq order\ i\ has\ available\ inventory\ at\ time\ t$ (binary)
- $u(j) \coloneqq line\ in\ use$ (binary)

Decision Variables (OTIF tracking)

- $late(i) \coloneqq$ (binary)
- $lateness(i) \coloneqq$ (positive integer)
- $early(i) \coloneqq$ (positive integer)

Decision Variables (Workforce tracking)

- $workers_{used}(t) \coloneqq total\ workers\ actually\ working\ at\ time\ t$ (integer $[0, W]$)
- $\mathrm{workers_{max}} \coloneqq maximal\ workers\ used\ in\ any\ time\ slot$ (integer $[0, W]$)
- $\mathrm{workers_{min}} \coloneqq minimum\ workers\ used\ in\ any\ time\ slot$ (integer $[0, W]$)
- $deviation_{above}(t) \coloneqq workers\ above\ targer\ at\ time\ t$ (non-negative integer)
- $deviation_{below}(t) \coloneqq workers\ below\ targer\ at\ time\ t$ (non-negative integer)
- $workforce_{change}(t) \coloneqq absolute\ change\ in\ workforce\ from\ t-1\ to\ t$ (non-negative integer)
- $workforce_{increase}(t) \coloneqq increase\ in\ workforce\ from\ t-1\ to\ t$ (non-negative integer)
- $workforce_{decrease}(t) \coloneqq decrease\ in\ workforce\ from\ t-1\ to\ t$ (non-negative integer)

Decision Variables (WIP tracking)

- $time_{start}(i) \coloneqq actual\ start\ time\ for\ packing\ order\ i$ (integer $[1, T]$)
- $time_{completion}(i) \coloneqq actual\ completion\ time\ for\ packing\ order\ i$ (integer $[1, T]$)
- $time_{flow}(i) \coloneqq time\ from\ start\ to\ completion\ for\ packing\ order\ i$ (non-negative integer)
- $wip_{indicator}(i, t) \coloneqq order\ i\ is\ in\ processing\ at\ time\ t$ (binary)
- $wip(t) \coloneqq number\ of\ orders\ in\ process\ at\ time\ t$ (non-negative integer)
- $wip_{weighted}(t) \coloneqq value\ weighted\ WIP\ at\ time\ t$ (non-negative integer)

Input parameters:

- $p(i,j) \coloneqq processing\ time\ for\ packing\ order\ i\ on\ line\ j$ (positive real, dividable by base time interval)
- $s(i,k,j) \coloneqq setup\ time\ for\ packing\ order\ j\ and\ k\ on\ line\ j$ (positive real , dividable by base time interval)
- $a(w,t) \coloneqq worker\ w\ is\ available\ at\ time\ t$ (binary)
- $inv_0(i) \coloneqq initial\ inventory\ stock\ for\ packing\ order\ i$ (positive integer)
- $\alpha \coloneqq reserved\ capacity$ (positive real less than 1)

Input parameters (OTIF):

- $due(i) \coloneqq due\ date\ for\ packing\ order\ i$ (integer $[1, T]$)
- $priority(i) \coloneqq priority\ weight\ for\ order\ i$ (positive integer, e.g. $[1{,}100]$)

Input parameters (Workforce):

- $workforce_{target} \coloneqq ideal\ steady - state\ workforce\ level$ (positive integer)

Constraints

- One assignment: Each order assigned to only one line, one time, and one worker

$$\sum_j \sum_t \sum_w x(i,j,t,w) = 1 \quad \forall i$$

- Line capacity: No overlap of order on the same line

$$\sum_i \sum_w \sum_{t \le \tau < t + p(i,j) + \sum_k y(i,k,j) * s(i,k,j) * (1 - b(i,k))} x(i,j,t,w) \le u(j) \quad \forall j, \forall \tau$$

- A worker is working on a given packing order

$$\sum_i \sum_j \sum_{t \le \tau < t + p(i,j)} x(i,j,t,w) = w(w,t) \quad \forall \tau, \forall w$$

- Workers can only work during their availability windows

$$w(w,t) \le a(w,t) \quad \forall w, \forall t$$

- Movement balance equation

$$m(w,t) \ge \sum_i \left(x(i,j,w,t) - x(i,j,w,t-1) \right) \quad \forall j$$

- Reserved line capacity

$$\sum_i \sum_j \sum_\tau \sum_w \sum_{t \le \tau < t + p(i,j) + \sum_k y(i,k,j) * s(i,k,j) * (1 - b(i,k))} x(i,j,t,w) < (1 - \alpha) * m * T$$

Constraints

- Reserved worker capacity
  $\sum_w \sum_t w(w,t) < (1-\alpha) \sum_w \sum_t a(w,t)$
- Line in use

$$\sum_i \sum_w \sum_{t \le \tau < t + p(i,j)} x(i,j,t,w) \le u(j) \quad \forall \tau$$

- Each order ships exactly once

$$\sum_t ship(i,t) = 1 \quad \forall i$$

- Shipping time

$$time_{ship}(i) = \sum_t t * ship(i,t) \quad \forall i$$

Constraints (OTIF)

- Start time

$$\text{time}_{\text{start}}(i) = \sum_j \sum_w \sum_t t \ * x(i,j,t,w) \quad \forall i$$

- Completion time

$$\text{time}_{complete}(i) = \sum_j \sum_w \sum_t (t + p(i,j)) \ * x(i,j,t,w) \quad \forall i$$

- Lateness

$$lateness(i) \geq time_{complete}(i) - due(i) \quad \forall i$$
$$lateness(i) \geq 0 \quad \forall i$$

- Earliness

$$\text{early}(i) \geq due(i) - time_{complete}(i) \quad \forall i$$
$$\text{early}(i) \geq 0 \quad \forall i$$

- Identify late orders (forces $late(i) = 1$ if order is late)

$$lateness(i) \leq T * late(i) \quad \forall i$$
$$time_{complete}(i) > due(i) - T\big(1 - late(i)\big) \quad \forall i$$

Constraints (Shipping)

- Cannot ship before order complete

$$time_{ship}(i) \geq time_{complete}(i) \quad \forall i$$

- Has inventory

$$inv(i,t) \geq due(i,t) - M * \left(1 - has_{inv}(i,t)\right) \;\; \forall i \; \forall t$$
$$inv(i,t) \leq M * has_{inv}(i,t) \quad \forall i \; \forall t$$

- Can only ship when inventory is sufficient

$$ship(i,t) \leq has_{inv}(i,t) \quad \forall i \; \forall t$$

Constraints (WIP)

- Flow time

$$time_{flow}(i) = time_{ship}(i) - time_{start}(i) \quad \forall \text{i}$$

- Number of packing orders produced

$$prod(i,t) = \sum_{j} \sum_{w} x(i,j,t-p(i,j),w)$$

- Inventory balance equation

$$inv(i,t) = inv(i,t-1) + prod(i,t) - ship(i,t) \quad \forall i, \forall t > 0$$
$$inv(i,0) = inv_0(i) \quad \forall i$$

- WIP indicator (order i is being worked on at time t or in inventory)

$$\text{wip}_{\text{indicator}}(i,t) = \sum_{j} \sum_{w} \sum_{t-p(i,j) \leq \tau < t} x(i,j,\tau,w) \quad + inv(i,t)$$

- WIP at time t

$$\text{wip(t)} = \sum_{i} wip_{indicator}(i,t) \quad \forall t$$

Constraints (Workforce)

- Active workers per time slot

$$\text{workers}_{\text{used}}(t) = \sum_{w} w(w,t) \quad \forall t$$

- Maximum workforce

$$workers_{max} \geq workers_{used(t)} \;\; \forall t$$

- Minium workforce

$$\text{workers}_{\text{used}}(\text{t}) \geq workers_{min}$$

- Absolute deviation (the solver will choose only one deviation to be positive at a given time)

$$\text{workers}_{\text{used}}(\text{t}) = workforce_{target} + deviation_{above}(t) - deviation_{below}(t) \; \forall t$$

$$deviation_{total} = \sum_{t} (deviation_{above}(t) + deviation_{below}(t))$$

- Workforce changes between periods (the solver will choose only one deviation to be positive at a given time)

$$wokers_{used}(t) = workers_{used}(t-1) + workforce_{increase}(t) - workforce_{decrease}(t)$$
$$workforce_{change}(t) = workforce_{increase}(t) + workforce_{decrease}(t)$$
$$\forall t > 1$$

- Workforce range

$$workforce_{range} = \text{workers}_{\text{max}} - \text{workers}_{\text{min}}$$

Objective function

Minimization objective function:

$$f = \alpha * otif + \beta * wip_{obj} + \gamma * workforce + \delta * total_{not_{utilized}} + \omega worker_{move}$$

where

- OTIF term (heavy penalty for late order but also penalizes according to lateness)

$$\text{otif} = \sum_{i} priority(i) * (7 * late(i) + 3 * lateness(i))$$

- WIP term

$$wip_{obj} = 4 * \sum_{t} wip(t) + 6 * \sum_{t} time_{flow}(t)$$

- Workforce term (reduces worker variance, deviations from target workforce and changes from one time instant to another)

$$workforce = 5 * workers_{range} + 3 * deviation_{total} + 2 * \sum_{t} workforce_{change}(t)$$

- Number of lines fully utilized

$$total_{not\_utilized} = \sum_{j} u(j)$$

- Total worker movements

$$worker_{move} = \sum_{w} \sum_{t} m(w,t)$$