Indicies:
- $i = (1, n): packing\ orders$
- $j = (1, m): line\ numbers$
- $t = (1, T): time\ slots, given\ in\ terms\ of\ a\ base\ time\ interval\ \Delta T\ e.g. 15\ minutes$

Decision Variables:
- $x(i, j, t) \coloneqq packing\ order\ i\ starts\ on\ line\ j\ at\ time\ t$ (binary)
- $prod(i, t) \coloneqq number\ of\ units\ produced\ at\ time\ t$ (relaxed too non-negative real)

Decision Variables (Workforce tracking)

- $workers_{used}(t) \coloneqq total\ workers\ actually\ working\ at\ time\ t$ (relaxed too real $[0, W]$)
- $\mathrm{workers}_{\mathrm{max}} \coloneqq maximal\ workers\ used\ in\ any\ time\ slot$ (relaxed too real $[0, W]$)
- $\mathrm{workers}_{min} \coloneqq minimum\ workers\ used\ in\ any\ time\ slot$ (relaxed too real $[0, W]$)

Decision Variables (WIP tracking)

- $inv(i) \coloneqq number\ of\ packing\ order\ i\ in\ stock$ (relaxed too non-negative real)
- $ship(i, t) \coloneqq packing\ order\ i\ is\ shipped\ at\ time\ t$ (binary)

Input parameters:

- $p(i, j) \coloneqq processing\ time\ for\ packing\ order\ i\ on\ line\ j$ (positive real, dividable by base time interval)
- $s(i, k, j) \coloneqq setup\ time\ for\ packing\ order\ j\ and\ k\ on\ line\ j$ (positive real , dividable by base time interval)

  $inv_0(i) \coloneqq initial\ inventory\ stock\ for\ packing\ order\ i$ (positive integer)

Input parameters (OTIF):

- $due(i) \coloneqq due\ date\ for\ packing\ order\ i$ (integer $[1, T]$)
- $priority(i) \coloneqq priority\ weight\ for\ order\ i$ (positive integer, e.g. $[1,100]$)

Constraints

- One assignment: Each order assigned to only one line, one time

$$\sum_{j}\sum_{t} x(i,j,t) = 1 \quad \forall i$$

- Line capacity: No overlap of order on the same line

$$\sum_{i}\sum_{t\le\tau<t+p(i,j)} x(i,j,t) \le 1 \quad \forall j, \forall \tau$$

- Reserved line capacity

$$\sum_{i}\sum_{j}\sum_{\tau}\sum_{t\le\tau<t+p(i,j)} x(i,j,t) < (1-\alpha) * m * T$$

Constraints

Constraints (OTIF)

Constraints (Shipping)

- Each order ships exactly once

$$\sum_{t} ship(i,t) = 1 \quad \forall i$$

- Ship no earlier than due time

$$\sum_{t} t * ship(i,t) \geq due(i)$$

Constraints (WIP)

- Number of packing orders produced

$$prod(i,t) = \sum_j x(i,j,t-p(i,j))$$

- Inventory balance equation

$$inv(i,t) = inv(i,t-1) + prod(i,t) - ship(i,t) \quad \forall i, \forall t > 0$$

$$inv(i,0) = inv_0(i) \quad \forall i$$

Constraints (Workforce)

- Active workers per time slot

$$\mathrm{workers}_{\mathrm{used}}(\tau) = \sum_{i} \sum_{j} \sum_{t \leq \tau < t+p(i,j)} x(i,j,t) \ \forall \tau$$

- Maximum workforce

$$workers_{max} \geq workers_{used}(t) \ \forall t$$

- Minium workforce

$$\mathrm{workers}_{\mathrm{used}}(\mathrm{t}) \geq workers_{min}$$

- Workforce range

$$workforce_{range} = \mathrm{workers}_{\mathrm{max}} - \mathrm{workers}_{\mathrm{min}}$$

Objective function

Minimization objective function:

$$f = \beta * wip_{obj} + \gamma * workforce + \delta * total_{not_{utilized}}$$

where

- WIP term

$$wip_{obj} = \sum_t inv(t)$$

- Workforce term (reduces worker variance, deviations from target workforce and changes from one time instant to another)

$$workforce = workers_{range}$$