

Indices:

- $i = (1, n)$ : packing orders
- $j = (1, m)$ : line numbers
- $t = (1, T)$ : time slots, given in terms of a base time interval  $\Delta T$  e.g. 15 minutes
- $w = (1, W)$ : workers

Decision Variables:

- $x(i, j, t, w) :=$  packing order  $i$  starts on line  $j$  at time  $t$  with worker  $w$  (binary)
- $y(i, k, j) :=$  setup between orders  $i$  and  $k$  on line  $j$  (binary)
- $b(i, k) :=$  batch indicator, order  $i$  and  $k$  batched together (binary)
- $w(w, t) :=$  worker  $w$  is working during time  $t$  (binary)
- $m(w, t) :=$  worker  $w$  is working on another line that he did at  $t - 1$
- $prod(i, t) :=$  number of units produced at time  $t$  (positive integer)
- $inv(i) :=$  number of packing order  $i$  in stock (positive integer)
- $ship(i, t) :=$  packing order  $i$  is shipped at time  $t$  (binary)
- $time_{ship}(i) :=$  time slot when order  $i$  is shipped (integer  $[0, T]$ )
- $ship_{early}(i) :=$  order completes before due date (binary)
- $ship_{late}(i) :=$  order completes after due date (binary)
- $has_{inv}(i, t) :=$  order  $i$  has available inventory at time  $t$  (binary)
- $u(j) :=$  line in use (binary)

Decision Variables (OTIF tracking)

- $late(i) :=$  (binary)
- $lateness(i) :=$  (positive integer)
- $early(i) :=$  (positive integer)

## Decision Variables (Workforce tracking)

- $\text{workers}_{\text{used}}(t) := \text{total workers actually working at time } t$  (integer  $[0, W]$ )
- $\text{workers}_{\text{max}} := \text{maximal workers used in any time slot}$  (integer  $[0, W]$ )
- $\text{workers}_{\text{min}} := \text{minimum workers used in any time slot}$  (integer  $[0, W]$ )
- $\text{deviation}_{\text{above}}(t) := \text{workers above target at time } t$  (non-negative integer)
- $\text{deviation}_{\text{below}}(t) := \text{workers below target at time } t$  (non-negative integer)
- $\text{workforce}_{\text{change}}(t) := \text{absolute change in workforce from } t - 1 \text{ to } t$  (non-negative integer)
- $\text{workforce}_{\text{increase}}(t) := \text{increase in workforce from } t - 1 \text{ to } t$  (non-negative integer)
- $\text{workforce}_{\text{decrease}}(t) := \text{decrease in workforce from } t - 1 \text{ to } t$  (non-negative integer)

## Decision Variables (WIP tracking)

- $\text{time}_{\text{start}}(i) := \text{actual start time for packing order } i$  (integer  $[1, T]$ )
- $\text{time}_{\text{completion}}(i) := \text{actual completion time for packing order } i$  (integer  $[1, T]$ )
- $\text{time}_{\text{flow}}(i) := \text{time from start to completion for packing order } i$  (non-negative integer)
- $\text{wip}_{\text{indicator}}(i, t) := \text{order } i \text{ is in processing at time } t$  (binary)
- $\text{wip}(t) := \text{number of orders in process at time } t$  (non-negative integer)
- $\text{wip}_{\text{weighted}}(t) := \text{value weighted WIP at time } t$  (non-negative integer)

Input parameters:

- $p(i, j) :=$  processing time for packing order  $i$  on line  $j$  (positive real, dividable by base time interval)
- $s(i, k, j) :=$  setup time for packing order  $j$  and  $k$  on line  $j$  (positive real , dividable by base time interval)
- $a(w, t) :=$  worker  $w$  is available at time  $t$  (binary)
- $inv_0(i) :=$  initial inventory stock for packing order  $i$  (positive integer)
- $\alpha :=$  reserved capacity (positive real less than 1)

Input parameters (OTIF):

- $due(i) :=$  due date for packing order  $i$  (integer  $[1, T]$ )
- $priority(i) :=$  priority weight for order  $i$  (positive integer, e.g.  $[1,100]$ )

Input parameters (Workforce):

- $workforce_{target} :=$  ideal steady – state workforce level (positive integer)

# Constraints

- One assignment: Each order assigned to only one line, one time, and one worker

$$\sum_j \sum_t \sum_w x(i, j, t, w) = 1 \quad \forall i$$

- Line capacity: No overlap of order on the same line

$$\sum_i \sum_w \sum_{t \leq \tau < t + p(i, j) + \sum_k y(i, k, j) * s(i, k, j) * (1 - b(i, k))} x(i, j, t, w) \leq u(j) \quad \forall j, \forall \tau$$

- A worker is working on a given packing order

$$\sum_i \sum_j \sum_{t \leq \tau < t + p(i, j)} x(i, j, t, w) = w(w, t) \quad \forall \tau, \forall w$$

- Workers can only work during their availability windows

$$w(w, t) \leq a(w, t) \quad \forall w, \forall t$$

- Movement balance equation

$$m(w, t) \geq \sum_i (x(i, j, w, t) - x(i, j, w, t - 1)) \quad \forall j$$

- Reserved line capacity

$$\sum_i \sum_j \sum_\tau \sum_w \sum_{t \leq \tau < t + p(i, j) + \sum_k y(i, k, j) * s(i, k, j) * (1 - b(i, k))} x(i, j, t, w) < (1 - \alpha) * m * T$$

# Constraints

- Reserved worker capacity

$$\sum_w \sum_t w(w, t) < (1 - \alpha) \sum_w \sum_t a(w, t)$$

- Line in use

$$\sum_i \sum_w \sum_{t \leq \tau < t + p(i, j)} x(i, j, t, w) \leq u(j) \quad \forall \tau$$

- Each order ships exactly once

$$\sum_t ship(i, t) = 1 \quad \forall i$$

- Shipping time

$$time_{ship}(i) = \sum_t t * ship(i, t) \quad \forall i$$

# Constraints (OTIF)

- Start time

$$\text{time}_{\text{start}}(i) = \sum_j \sum_w \sum_t t * x(i, j, t, w) \quad \forall i$$

- Completion time

$$\text{time}_{\text{complete}}(i) = \sum_j \sum_w \sum_t (t + p(i, j)) * x(i, j, t, w) \quad \forall i$$

- Lateness

$$\begin{aligned} \text{lateness}(i) &\geq \text{time}_{\text{complete}}(i) - \text{due}(i) \quad \forall i \\ \text{lateness}(i) &\geq 0 \quad \forall i \end{aligned}$$

- Earliness

$$\begin{aligned} \text{early}(i) &\geq \text{due}(i) - \text{time}_{\text{complete}}(i) \quad \forall i \\ \text{early}(i) &\geq 0 \quad \forall i \end{aligned}$$

- Identify late orders (forces  $\text{late}(i) = 1$  if order is late)

$$\begin{aligned} \text{lateness}(i) &\leq T * \text{late}(i) \quad \forall i \\ \text{time}_{\text{complete}}(i) &> \text{due}(i) - T(1 - \text{late}(i)) \quad \forall i \end{aligned}$$

# Constraints (Shipping)

- Cannot ship before order complete

$$time_{ship}(i) \geq time_{complete}(i) \quad \forall i$$

- Has inventory

$$inv(i, t) \geq due(i, t) - M * (1 - has_{inv}(i, t)) \quad \forall i \forall t$$

$$inv(i, t) \leq M * has_{inv}(i, t) \quad \forall i \forall t$$

- Can only ship when inventory is sufficient

$$ship(i, t) \leq has_{inv}(i, t) \quad \forall i \forall t$$

# Constraints (WIP)

- Flow time

$$time_{flow}(i) = time_{ship}(i) - time_{start}(i) \quad \forall i$$

- Number of packing orders produced

$$prod(i, t) = \sum_j \sum_w x(i, j, t - p(i, j), w)$$

- Inventory balance equation

$$inv(i, t) = inv(i, t - 1) + prod(i, t) - ship(i, t) \quad \forall i, \forall t > 0$$

$$inv(i, 0) = inv_0(i) \quad \forall i$$

- WIP indicator (order  $i$  is being worked on at time  $t$  or in inventory)

$$wip_{indicator}(i, t) = \sum_j \sum_w \sum_{t-p(i,j) \leq \tau < t} x(i, j, \tau, w) + inv(i, t)$$

- WIP at time  $t$

$$wip(t) = \sum_i wip_{indicator}(i, t) \quad \forall t$$

# Constraints (Workforce)

- Active workers per time slot

$$\text{workers}_{\text{used}}(t) = \sum_w w(w, t) \quad \forall t$$

- Maximum workforce

$$\text{workers}_{\text{max}} \geq \text{workers}_{\text{used}}(t) \quad \forall t$$

- Minimum workforce

$$\text{workers}_{\text{used}}(t) \geq \text{workers}_{\text{min}}$$

- Absolute deviation (the solver will choose only one deviation to be positive at a given time)

$$\text{workers}_{\text{used}}(t) = \text{workforce}_{\text{target}} + \text{deviation}_{\text{above}}(t) - \text{deviation}_{\text{below}}(t) \quad \forall t$$

$$\text{deviation}_{\text{total}} = \sum_t (\text{deviation}_{\text{above}}(t) + \text{deviation}_{\text{below}}(t))$$

- Workforce changes between periods (the solver will choose only one deviation to be positive at a given time)

$$\text{workers}_{\text{used}}(t) = \text{workers}_{\text{used}}(t - 1) + \text{workforce}_{\text{increase}}(t) - \text{workforce}_{\text{decrease}}(t)$$

$$\text{workforce}_{\text{change}}(t) = \text{workforce}_{\text{increase}}(t) + \text{workforce}_{\text{decrease}}(t)$$

$$\forall t > 1$$

- Workforce range

$$\text{workforce}_{\text{range}} = \text{workers}_{\text{max}} - \text{workers}_{\text{min}}$$

# Objective function

Minimization objective function:

$$f = \alpha * otif + \beta * wip_{obj} + \gamma * workforce + \delta * total_{not\_utilized}$$

where

- OTIF term (heavy penalty for late order but also penalizes according to lateness)

$$otif = \sum_i priority(i) * (7 * late(i) + 3 * lateness(i))$$

- WIP term

$$wip_{obj} = 4 * \sum_t wip(t) + 6 * \sum_t time_{flow}(t)$$

- Workforce term (reduces worker variance, deviations from target workforce and changes from one time instant to another)

$$workforce = 5 * workers_{range} + 3 * deviation_{total} + 2 * \sum_t workforce_{change}(t)$$

- Number of lines fully utilized

$$total_{not\_utilized} = \sum_j u(j)$$