




Indices:

- $i = (1, n)$: *packing orders*
- $j = (1, m)$: *line numbers*
- $t = (1, T)$: *time slots, given in terms of a base time interval ΔT e. g. 15 minutes*

Decision Variables:

- $x(i, j, t) :=$ *packing order i starts on line j at time t* (binary)
- $prod(i, t) :=$ *number of units produced at time t* (positive integer)



Decision Variables (Workforce tracking)

- $workers_{used}(t) := \text{total workers actually working at time } t$ (integer $[0, W]$)
- $workers_{max} := \text{maximal workers used in any time slot}$ (integer $[0, W]$)
- $workers_{min} := \text{minimum workers used in any time slot}$ (integer $[0, W]$)

Decision Variables (WIP tracking)

- $inv(i) := \text{number of packing order } i \text{ in stock}$ (positive integer)
- $ship(i, t) := \text{packing order } i \text{ is shipped at time } t$ (binary)



Input parameters:

- $p(i, j) :=$ *processing time for packing order i on line j* (positive real, dividable by base time interval)
- $s(i, k, j) :=$ *setup time for packing order j and k on line j* (positive real, dividable by base time interval)
- $inv_0(i) :=$ *initial inventory stock for packing order i* (positive integer)

Input parameters (OTIF):

- $due(i) :=$ *due date for packing order i* (integer $[1, T]$)
- $priority(i) :=$ *priority weight for order i* (positive integer, e.g. $[1, 100]$)



Constraints

- One assignment: Each order assigned to only one line, one time

$$\sum_j \sum_t x(i, j, t) = 1 \quad \forall i$$

- Line capacity: No overlap of order on the same line

$$\sum_i \sum_{t \leq \tau < t+p(i,j)} x(i, j, t) \leq 1 \quad \forall j, \forall \tau$$

- Reserved line capacity

$$\sum_i \sum_j \sum_{\tau} \sum_{t \leq \tau < t+p(i,j)} x(i, j, t) < (1 - \alpha) * m * T$$



Constraints

- Each order ships exactly once

$$\sum_t ship(i, t) = 1 \quad \forall i$$



Constraints (OTIF)



Constraints (Shipping)

- Ship no earlier than due time

$$ship(i) \geq due(i)$$



Constraints (WIP)


- Number of packing orders produced

$$prod(i, t) = \sum_j x(i, j, t - p(i, j))$$

- Inventory balance equation

$$inv(i, t) = inv(i, t - 1) + prod(i, t) - ship(i, t) \quad \forall i, \forall t > 0$$

$$inv(i, 0) = inv_0(i) \quad \forall i$$



Constraints (Workforce)

- Active workers per time slot

$$\text{workers}_{\text{used}}(\tau) = \sum_i \sum_j \sum_{t \leq \tau < t+p(i,j)} x(i,j,t) \quad \forall \tau$$

- Maximum workforce

$$\text{workers}_{\text{max}} \geq \text{workers}_{\text{used}}(t) \quad \forall t$$

- Minimum workforce

$$\text{workers}_{\text{used}}(t) \geq \text{workers}_{\text{min}}$$

- Workforce range

$$\text{workforce}_{\text{range}} = \text{workers}_{\text{max}} - \text{workers}_{\text{min}}$$



Objective function

Minimization objective function:

$$f = \beta * wip_{obj} + \gamma * workforce + \delta * total_{not_{utilized}}$$

where

- WIP term

$$wip_{obj} = \sum_t inv(t)$$

- Workforce term (reduces worker variance, deviations from target workforce and changes from one time instant to another)

$$workforce = workers_{range}$$