

Indices:

- $i = (1, n)$: *packing orders*
- $j = (1, m)$: *line numbers*
- $t = (1, T)$: *time slots, given in terms of a base time interval ΔT e. g. 15 minutes*
- $w = (1, W)$: *workers*

Decision Variables:

- $x(i, j, t, w) :=$ *packing order i starts on line j at time t with worker w* (binary)
- $y(i, k, j) :=$ *setup between orders i and k on line j* (binary)
- $b(i, k) :=$ *batch indicator, order i and k batched together* (binary)
- $w(w, t) :=$ *worker w is working during time t* (binary)
- $m(w, t) :=$ *worker w is working on another line that he did at $t - 1$*
- $prod(i, t) :=$ *number of units produced at time t* (positive integer)
- $inv(i) :=$ *number of packing order i in stock* (positive integer)
- $ship(i, t) :=$ *packing order i is shipped at time t* (binary)
- $time_{ship}(i) :=$ *time slot when order i is shipped* (integer $[0, T]$)
- $ship_{early}(i) :=$ *order completes before due date* (binary)
- $ship_{late}(i) :=$ *order completes after due date* (binary)
- $has_{inv}(i, t) :=$ *order i has available inventory at time t* (binary)
- $u(j) :=$ *line in use* (binary)

Decision Variables (OTIF tracking)

- $late(i) :=$ (binary)
- $lateness(i) :=$ (positive integer)
- $early(i) :=$ (positive integer)

Decision Variables (Workforce tracking)

- $workers_{used}(t) :=$ total workers actually working at time t (integer $[0, W]$)
- $workers_{max} :=$ maximal workers used in any time slot (integer $[0, W]$)
- $workers_{min} :=$ minimum workers used in any time slot (integer $[0, W]$)
- $deviation_{above}(t) :=$ workers above target at time t (non-negative integer)
- $deviation_{below}(t) :=$ workers below target at time t (non-negative integer)
- $workforce_{change}(t) :=$ absolute change in workforce from $t - 1$ to t (non-negative integer)
- $workforce_{increase}(t) :=$ increase in workforce from $t - 1$ to t (non-negative integer)
- $workforce_{decrease}(t) :=$ decrease in workforce from $t - 1$ to t (non-negative integer)

Decision Variables (WIP tracking)

- $time_{start}(i) :=$ actual start time for packing order i (integer $[1, T]$)
- $time_{completion}(i) :=$ actual completion time for packing order i (integer $[1, T]$)
- $time_{flow}(i) :=$ time from start to completion for packing order i (non-negative integer)
- $wip_{indicator}(i, t) :=$ order i is in processing at time t (binary)
- $wip(t) :=$ number of orders in process at time t (non-negative integer)
- $wip_{weighted}(t) :=$ value weighted WIP at time t (non-negative integer)



Input parameters:

- $p(i, j) :=$ processing time for packing order i on line j (positive real, dividable by base time interval)
- $s(i, k, j) :=$ setup time for packing order j and k on line j (positive real, dividable by base time interval)
- $a(w, t) :=$ worker w is available at time t (binary)
- $inv_0(i) :=$ initial inventory stock for packing order i (positive integer)
- $\alpha :=$ reserved capacity (positive real less than 1)

Input parameters (OTIF):

- $due(i) :=$ due date for packing order i (integer $[1, T]$)
- $priority(i) :=$ priority weight for order i (positive integer, e.g. $[1, 100]$)

Input parameters (Workforce):

- $workforce_{target} :=$ ideal steady – state workforce level (positive integer)

Constraints

- One assignment: Each order assigned to only one line, one time, and one worker

$$\sum_j \sum_t \sum_w x(i, j, t, w) = 1 \quad \forall i$$

- Line capacity: No overlap of order on the same line

$$\sum_i \sum_w \sum_{t \leq \tau < t + p(i, j) + \sum_k y(i, k, j) * s(i, k, j) * (1 - b(i, k))} x(i, j, t, w) \leq u(j) \quad \forall j, \forall \tau$$

- A worker is working on a given packing order

$$\sum_i \sum_j \sum_{t \leq \tau < t + p(i, j)} x(i, j, t, w) = w(w, t) \quad \forall \tau, \forall w$$

- Workers can only work during their availability windows

$$w(w, t) \leq a(w, t) \quad \forall w, \forall t$$

- Movement balance equation

$$m(w, t) \geq \sum_i (x(i, j, w, t) - x(i, j, w, t - 1)) \quad \forall j$$

- Reserved line capacity

$$\sum_i \sum_j \sum_\tau \sum_w \sum_{t \leq \tau < t + p(i, j) + \sum_k y(i, k, j) * s(i, k, j) * (1 - b(i, k))} x(i, j, t, w) < (1 - \alpha) * m * T$$

Constraints

- Reserved worker capacity

$$\sum_w \sum_t w(w, t) < (1 - \alpha) \sum_w \sum_t a(w, t)$$

- Line in use

$$\sum_i \sum_w \sum_{t \leq \tau < t+p(i,j)} x(i, j, t, w) \leq u(j) \quad \forall \tau$$

- Each order ships exactly once

$$\sum_t ship(i, t) = 1 \quad \forall i$$

- Shipping time

$$time_{ship}(i) = \sum_t t * ship(i, t) \quad \forall i$$

Constraints (OTIF)

- Start time

$$\text{time}_{\text{start}}(i) = \sum_j \sum_w \sum_t t * x(i, j, t, w) \quad \forall i$$

- Completion time

$$\text{time}_{\text{complete}}(i) = \sum_j \sum_w \sum_t (t + p(i, j)) * x(i, j, t, w) \quad \forall i$$

- Lateness

$$\begin{aligned} \text{lateness}(i) &\geq \text{time}_{\text{complete}}(i) - \text{due}(i) \quad \forall i \\ \text{lateness}(i) &\geq 0 \quad \forall i \end{aligned}$$

- Earliness

$$\begin{aligned} \text{early}(i) &\geq \text{due}(i) - \text{time}_{\text{complete}}(i) \quad \forall i \\ \text{early}(i) &\geq 0 \quad \forall i \end{aligned}$$

- Identify late orders (forces $\text{late}(i) = 1$ if order is late)

$$\begin{aligned} \text{lateness}(i) &\leq T * \text{late}(i) \quad \forall i \\ \text{time}_{\text{complete}}(i) &> \text{due}(i) - T(1 - \text{late}(i)) \quad \forall i \end{aligned}$$



Constraints (Shipping)

- Cannot ship before order complete

$$time_{ship}(i) \geq time_{complete}(i) \quad \forall i$$

- Has inventory

$$inv(i, t) \geq due(i, t) - M * (1 - has_{inv}(i, t)) \quad \forall i \forall t$$

$$inv(i, t) \leq M * has_{inv}(i, t) \quad \forall i \forall t$$

- Can only ship when inventory is sufficient

$$ship(i, t) \leq has_{inv}(i, t) \quad \forall i \forall t$$

Constraints (WIP)

- Flow time

$$time_{flow}(i) = time_{ship}(i) - time_{start}(i) \quad \forall i$$

- Number of packing orders produced

$$prod(i, t) = \sum_j \sum_w x(i, j, t - p(i, j), w)$$

- Inventory balance equation

$$inv(i, t) = inv(i, t - 1) + prod(i, t) - ship(i, t) \quad \forall i, \forall t > 0$$

$$inv(i, 0) = inv_0(i) \quad \forall i$$

- WIP indicator (order i is being worked on at time t or in inventory)

$$wip_{indicator}(i, t) = \sum_j \sum_w \sum_{t-p(i, j) \leq \tau < t} x(i, j, \tau, w) + inv(i, t)$$

- WIP at time t

$$wip(t) = \sum_i wip_{indicator}(i, t) \quad \forall t$$

Constraints (Workforce)

- Active workers per time slot

$$workers_{used}(t) = \sum_w w(w, t) \quad \forall t$$

- Maximum workforce

$$workers_{max} \geq workers_{used}(t) \quad \forall t$$

- Minimum workforce

$$workers_{used}(t) \geq workers_{min}$$

- Absolute deviation (the solver will choose only one deviation to be positive at a given time)

$$workers_{used}(t) = workforce_{target} + deviation_{above}(t) - deviation_{below}(t) \quad \forall t$$

$$deviation_{total} = \sum_t (deviation_{above}(t) + deviation_{below}(t))$$

- Workforce changes between periods (the solver will choose only one deviation to be positive at a given time)

$$workers_{used}(t) = workers_{used}(t-1) + workforce_{increase}(t) - workforce_{decrease}(t)$$

$$workforce_{change}(t) = workforce_{increase}(t) + workforce_{decrease}(t)$$

$$\forall t > 1$$

- Workforce range

$$workforce_{range} = workers_{max} - workers_{min}$$

Objective function

Minimization objective function:

$$f = \alpha * otif + \beta * wip_{obj} + \gamma * workforce + \delta * total_{not_utilized}$$

where

- OTIF term (heavy penalty for late order but also penalizes according to lateness)

$$otif = \sum_i priority(i) * (7 * late(i) + 3 * lateness(i))$$

- WIP term

$$wip_{obj} = 4 * \sum_t wip(t) + 6 * \sum_t time_{flow}(t)$$

- Workforce term (reduces worker variance, deviations from target workforce and changes from one time instant to another)

$$workforce = 5 * workers_{range} + 3 * deviation_{total} + 2 * \sum_t workforce_{change}(t)$$

- Number of lines fully utilized

$$total_{not_utilized} = \sum_j u(j)$$