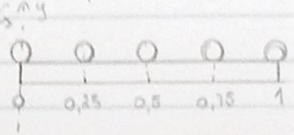
Exercícios MC - capítulo 7

1) $l = 1\,m$ a)

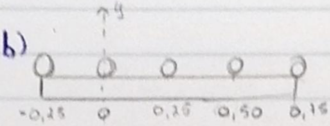


$m = 1\,Kg$

$I = ?$

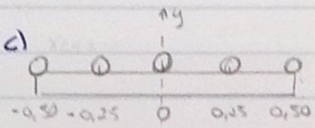
$I = m_1 x_1^2 + m_2 x_2^2 + m_3 x_3^2 + m_4 x_4^2 + m_5 x_5^2 =$
$= 1 \times 0,25^2 + 1 \times 0,50^2 + 1 \times 0,75^2 + 1 \times 1 =$
$= 1,875\ Kg \cdot m^2$

b)



$I = 1 \times (-0,25)^2 + 1 \times 0,25^2 + 1 \times 0,5^2 + 1 \times 0,75^2 =$
$= 0,9375\ Kg \cdot m^2$

c)



$I = 1 \times (-0,5)^2 + 1 \times (-0,25)^2 + 1 \times 0,5^2 + 1 \times 0,25^2$
$= 0,625\ Kg \cdot m^2$

d) $I = I_{CM} + Mx^2$ (Teorema de Steiner)
Considerando:
$I_e = I_{CM} + Mx^2 \Leftrightarrow 1,875 = 0,625 + 5 \times (0,50)^2$
$\Leftrightarrow 1,875 = 1,875$ c.q.d. //

2) $-d$ $\quad y$ $\quad d$

$O = C = O$ $\qquad d = 1,13 \times 10^{-10}\ m$ $\quad ; m(O) = 2,66 \times 10^{-26}\ Kg$

$I = 2m(O) \times d^2 = 2 \times (2,66 \times 10^{-26}) \times (1,13 \times 10^{-10})^2 = 6,79 \times 10^{-46}\ Kg \cdot m^2$

3) $dm = (M/l)\,dx$; $\quad I = \int r^2\, dm$

a) $I_e = \int r^2\, dm = \int x^2 \left(\frac{M}{l}\right) dx = \frac{M}{l}\int_0^l x^2\, dx = \frac{M}{l}\left[\frac{x^3}{3}\right]_0^l = \frac{Ml^2}{3}$

b) $I_c = \int r^2\, dm = \frac{M}{l}\int_{-\frac{l}{2}}^{\frac{l}{2}} x^2\, dx = \frac{M}{l}\left[\frac{x^3}{3}\right]_{-l/2}^{l/2} = \frac{ML^2}{24} + \frac{ML^2}{24} = \frac{ML^2}{12}$

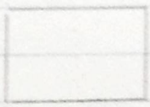
4) $m_h = 3\,Kg$

a) $I = I_1 + I_2 + I_3 + I_4 + I_5 + I_h = 1,875 + (ML^2)/3 = 1,875 + 1 = 2,875\ Kg \cdot m^2$

b) $I = I_1 + I_2 + I_3 + I_4 + I_5 + I_h = 0,9375 + 0,4375 = 1,375\ Kg \cdot m^2$

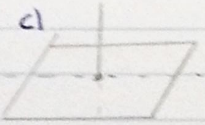
$I_h = \int r^2\, dm = \int x^2 \frac{M}{l}\, dx = \frac{M}{l}\left[\frac{x^3}{3}\right]_{-0,25}^{0,75} = 3(0,75^3/3 + 0,25^3/3) = 0,4375\ Kg\ m^2$

c) $I = I_1 + I_2 + I_3 + I_4 + I_5 + I_h = 0,625 + \frac{Ml^2}{12} = 0,625 + 0,25 = 0,875\ Kg\ m^2$

a) 

$$I = \int r^2 dm = \frac{M}{a}\int x^2 dx = \frac{M}{a}\int_0^a x^2 dx = \frac{M}{a}\left[\frac{x^3}{3}\right]_0^a = \frac{Ma^2}{3}$$

b) $$I = \int r^2 dm = \frac{M}{a}\int_{-\frac{a}{2}}^{\frac{a}{2}} x^2 dx = \frac{M}{a}\left[\frac{x^3}{3}\right]_{-\frac{a}{2}}^{\frac{a}{2}} = \frac{Ma^2}{24} + \frac{Ma^2}{24} = \frac{Ma^2}{12}$$

c) 

$$I = \int r^2 dm = \frac{M}{a}\int x^2 dx + \frac{M}{b}\int y^2 dy = \frac{M}{a}\int_{-\frac{a}{2}}^{\frac{a}{2}} x^2 dx + \frac{M}{b}\int_{-\frac{b}{2}}^{\frac{b}{2}} y^2 dy =$$

$$= \frac{Ma^2}{12} + \frac{Mb^2}{12} = \frac{M}{12}(a^2+b^2)$$

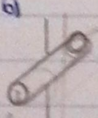
6) $$dm = \sigma dA = \left(\frac{M}{\pi R^2}\right)(2\pi r dr) = \frac{2M}{R^2} r\, dr$$

$$I = \int r^2 dm = \frac{2M}{R^2}\int_0^R r^3 dr = \frac{2M}{R^2}\left[\frac{r^4}{4}\right]_0^R = \frac{2MR^2}{4} = \frac{1}{2}(MR^2)$$

7) a) $$I = \int r^2 dm = \int_0^R r^2 \rho\, dV = \int_0^R r^2 \rho\, \ell\, dA = \int_0^R r^3 \rho\, \ell\, 2\pi\, dr = \rho \ell 2\pi \left[\frac{r^4}{4}\right]_0^R = \frac{m\ell 2\pi}{\pi R^2 \ell} \times \frac{R^4}{4} = \frac{mR^2}{2}$$

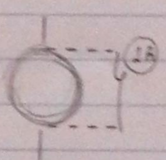
onde: $\rho = \frac{m}{V} = \frac{dm}{dV} \Rightarrow dm = \rho\, dV$, $dV = \ell\, dA$, $dA = 2\pi r\, dr$

$$\rho = \frac{m}{V} = \frac{m}{\pi R^2 \ell}$$

b) 

$$I = I_{CM} + Mx^2 \Leftrightarrow I = \frac{mR^2}{2} + mR^2 = \frac{3}{2} mR^2$$

(A) $I_{CM} = \frac{1}{2}(I_x + I_y + I_z) =$
$= \frac{3}{2} I_x \quad (I_x = I_y = I_z)$
$\Leftrightarrow I_x = \frac{2}{3} I_{CM} = \frac{2}{3}\left(\frac{3}{5} mR^2\right) =$
$= \frac{2}{5} mR^2$

8) 

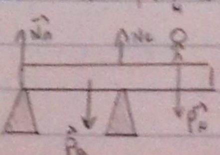
$$\rho = \frac{m}{V} = \frac{3m}{4\pi R^3}$$

$$I_{CM} = \int r^2 dm = \int_0^R r^2 \rho\, dV = \int_0^R \rho\, r^2 \times 4\pi r^2 dr =$$

$$= \rho 4\pi \int_0^R r^4 dr = \rho 4\pi \left[\frac{r^5}{5}\right]_0^R = \frac{3m}{4\pi R^3} \times 4\pi \times \frac{R^5}{5} = \frac{3}{5} mR^2$$

(A)

9) $\ell = 4m$; $m_h = 75\,kg$, $m_B = 50\,kg$; $x_C = 2{,}5\,m$; $x_B = x_{CM_B} = 2\,m$



Para que o sistema esteja em equilíbrio é necessário que $x_{CM} \leq x_C$:

$$\frac{m_B \cdot x_B + m_h \cdot x_h}{m_B + m_h} \leq 2{,}5 \Leftrightarrow \frac{50 \times 2 + 75 \times x_h}{50 + 75} \leq 2{,}5 \Leftrightarrow 0{,}8 + 0{,}6\, x_h \leq 2{,}5 \Leftrightarrow$$

$$\Leftrightarrow x_h \leq (2{,}5 - 0{,}8)/0{,}6 \Leftrightarrow x_h \leq 2{,}83$$

10) $m_A = 10\,Kg$; $m_B = 16\,Kg$

a)

$$x m_A g - (l-x) m_B g = 0 \Leftrightarrow x(m_A + m_B) - l m_b = 0$$

$$\Leftrightarrow x = \frac{+ l m_B}{m_A + m_B}$$

$$\Leftrightarrow x = \frac{l \times 16}{16+10}$$

$$\Leftrightarrow x = 0{,}62\,l.$$

b) $m_A g + m_B g = N \Leftrightarrow N = 10 \times 9{,}8 + 16 \times 9{,}8 \Leftrightarrow N = 254{,}8\,N$

11) $m = 40\,Kg$

Forças $(\Sigma \vec{F} = \vec{0})$
$$\begin{cases} F_3 - F_1 = 0 \\ 0{,}3 F_3 - P + F_2 = 0 \end{cases}$$

$$\downarrow$$

$$F_2 = -0{,}3 F_3 + mg \Leftrightarrow$$

$$\Leftrightarrow F_2 = -0{,}3 \times 96 + 40 \times 9{,}8$$

$$\Leftrightarrow F_2 = 363\,N$$

Momentos: $(\Sigma \vec{\tau}_i = \vec{0})$ $\tau(0{,}3F_3) + \tau(F_3) + \tau(P) = 0 \Leftrightarrow$

$$\Leftrightarrow -0{,}3 F_3 l \cos(60°) - F_3 l \sin(60°) + P\frac{l}{2}\cos(60°) = 0$$

$$\Leftrightarrow F_3(-0{,}3\cos(60°) - \sin(60°)) \frac{P}{2}\cos(60°) = 0$$

$$\Leftrightarrow F_3 = [P\cos(60°)] / [(0{,}3\cos(60°) - \sin(60°)) \times 2]$$

$$\hookrightarrow F_3 = F_1 = 96\,N$$

12) $P = 2N$; $T_{máx} = 1{,}1N$ $(N_2 = T \text{ e } N_1 = P)$

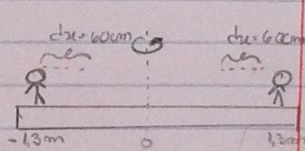
Momentos: $\tau(\cancel{N_2})^{0} - \tau(P) + \tau(T) - \tau(N_1) = 0$

$$\Leftrightarrow -P\frac{l}{2}\sin(47°) + T\frac{l}{2}\cos(47°) - N_1 l \cos(47°) = 0$$

$$\Leftrightarrow T = -\frac{P}{2}\sin(47°) + 2P\cos(47°).$$

$$\Leftrightarrow T = 4{,}19\,N$$

Como $T > T_{máx}$; o equilibrio não é possível



13) $m_c = 25\,Kg$; $l = 2{,}6\,m$; $m_p = 10\,Kg$; $f = 5\,rpm = 0{,}083\,Hz$

$$\hookrightarrow \omega_i = 2\pi f = 0{,}52\,rad/s$$

a) $\Delta L = 0 \Leftrightarrow L_f = L_i \Leftrightarrow I_f \omega_f = I_i \omega_i \Leftrightarrow$

$$\Leftrightarrow \omega_f = (I_i \omega_i)/I_f = 1{,}57\,rad/s$$

$I_p = \frac{m l^2}{12} = \frac{10 \times (2{,}6)^2}{12} = 5{,}63\,Kg \cdot m^2$; $I_i = m_c x_1^2 + m_c x_2^2 + I_p = 2 \times [25 \times (1{,}3)^2] + 5{,}63 = 90{,}13\,Kg \cdot m$

$I_f = m_c x_1^2 + m_c x_2^2 + I_p = 25 \times (1,3-0,6)^2 + 23 \times (1,3-0,6)^2 + 5,63 = 30,13 \ Kg \cdot m^2$

b) $E_c = \frac{1}{2} I \omega^2$

$\Delta E_c = E_{cf} - E_{ci} \Leftrightarrow \Delta E_c = \frac{1}{2} \times 30,13 \times (1,57)^2 - \frac{1}{2} \times 30,13 \times (0,52)^2$

$\Leftrightarrow \Delta E_c = 24,95 \ J$

14) Δ momento angular $= 0 \Leftrightarrow \vec{L_i} = \vec{L_f} \Rightarrow I_i \omega_i = I_f \omega_f \Leftrightarrow (I_H + I_{Hi})\omega_i = (I_H + I_{Hf})\omega_f$

$(L = I\omega)$

$\Leftrightarrow \omega_f = \frac{(I_H + I_{Hi})\omega_i}{(I_H + I_{Hf})}$

$m_m = 5,0 \ Kg$

$I_H = 5,0 \ Kg \cdot m^2$ (const)

$T = 2s \Rightarrow \omega_i = 2\pi/2 = \pi$ rad/s

$I_{Hi} = 2m \times (0,9)^2 = 8,1 \ Kg \cdot m^2$

$I_{Hf} = 2m \times (0,15)^2 = 0,225 \ Kg \cdot m^2$

$\Leftrightarrow \omega_f = \frac{(5 + 8,1)\pi}{(5 + 0,225)}$

$\Leftrightarrow \omega_f = 7,9$ rad/s

15) m_H - massa do homem

m_m - massa da mesa

R - raio da mesa

v_H - velocidade do homem

a) $I_{mesa} = \frac{1}{2} m R^2$

$I_{homem} = m_h \times x^2$

$\Delta L = 0 \Leftrightarrow \cancel{L_i}^{0} = I_f \omega_f \Leftrightarrow 0 = \frac{1}{2} m_m R^2 \omega_m + m_H R^2 (v_H/R) \Leftrightarrow$

$\Leftrightarrow \omega_m = -m_H v_H R / \frac{1}{2} m_m R^2$

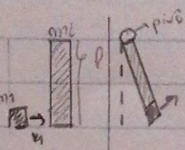
$\Leftrightarrow \omega_m = -2 m_H \times v_H / m_m R$

b) $W_H = \Delta E_c \Leftrightarrow W_H = \Delta E_{cH} + \Delta E_{cm}$

$\Leftrightarrow W_H = \frac{1}{2} m_H v$

16) A cadeira roda no sentido contrário ao roda da bicicleta, devido à conservação do momento angular.

17) $l = 1m$; $m = m_1 = m_2$

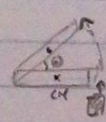


$I_{barra} = \frac{ml^2}{3}$; $I_{bola} = ml^2 \Rightarrow I = I_{barra} + I_{bola} = \frac{ml^2}{3} + ml^2 = \frac{4}{3} ml^2$

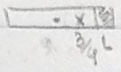
$\Delta L = 0 \Leftrightarrow L_f = L_i \Leftrightarrow I\omega = m l v_1 \Leftrightarrow \frac{4}{3} m l^2 \omega = m l v_1 \Leftrightarrow \omega = \frac{3}{4}\left(\frac{v_1}{l}\right)$

18) a) $\tau_{at} = \alpha I \Leftrightarrow \alpha = \tau / I \Leftrightarrow \alpha = \frac{3\tau}{4ml^2}$

b) $W = -\tau\theta \Rightarrow \theta = \frac{\frac{1}{2} I \omega_i^2}{\tau} \Leftrightarrow \theta = \frac{\frac{1}{2} \times \left(\frac{4}{3} ml^2\right)\left(\frac{3}{4} \frac{v_1}{l}\right)^2}{\tau} \Leftrightarrow \theta = \frac{3}{8} \frac{m v_1^2}{\tau}$



$ds = l\theta \Leftrightarrow s = \frac{3}{8}\left(\frac{m l v_1^2}{\tau}\right)$

c) 

19) $R = 0,5\,m$; $M = 20\,Kg$; $T = 9,8\,N$

Nota: $I_{disco} = \frac{1}{2} mR^2$

$a = \alpha R \Leftrightarrow \alpha = a/R$

$\omega(t) = \omega_0 + \alpha t$

$\tau = I \times \alpha \Leftrightarrow TR = \frac{1}{2} mR^2 \times \alpha$

$\Leftrightarrow T = \frac{1}{2} mR \times \alpha$

$\Leftrightarrow \alpha = 2T / mR$

$\Leftrightarrow \alpha = (2 \times 9,8) / (20 \times 0.5)$

$\Leftrightarrow 1,96$ rad/s

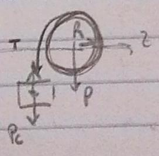
$\alpha = \frac{d\omega}{dt} \Leftrightarrow d\omega = \alpha dt \Rightarrow \omega = \int_0^2 \alpha dt \Leftrightarrow \omega = (\alpha t]_0^2$

$\Leftrightarrow \omega = 2 \times \alpha - 0$

$\Leftrightarrow \omega = 2 \times 1,96$

$\Leftrightarrow \omega = 3,92$ rad/s

20)



$L = I\omega + rp \Leftrightarrow L = \frac{1}{2} MR^2 \omega + mRv$

$\frac{dL}{dt} = r \times F_{ext} = R \times P_c = Rmg$ e $\frac{dL}{dt} = \frac{1}{2} MR^2 \frac{d\omega}{dt} + mR \frac{dv}{dt}$

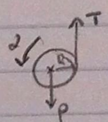
$\frac{d\omega}{dt} \to \alpha = a/R$, $\frac{dv}{dt} \to a$

logo: $Rmg = \frac{MRa}{2} + mRa \Leftrightarrow a = \frac{Rmg}{\left(\frac{MR}{2} + mR\right)} \Leftrightarrow$

$\Leftrightarrow a = \frac{mg}{\frac{M}{2} + m}$

$\Leftrightarrow a = 0,89\,m/s^2 \Rightarrow \alpha = \frac{a}{R} = 1,78$ rad/s

21)



$\tau = I\alpha \Leftrightarrow TR = \frac{1}{2} mR^2 \times a/R$

$\Leftrightarrow \cancel{m}(g-a) = \frac{1}{2} \cancel{m} a$

$\Leftrightarrow g - a = \frac{1}{2} a$

$\Leftrightarrow \frac{3}{2} a = g$

$\Leftrightarrow a = \frac{2}{3} g$

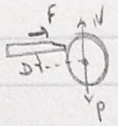
$\Leftrightarrow a = 6,53\,m/s^2 \Rightarrow a_{CM}$

CA: $ma = mg - T \Leftrightarrow$

$\Leftrightarrow T = m(g-a)$

R: $\alpha = a/R = 6,53/0,5 = 13,08$ rad/s

22)



$F + N - P = m a_{CM}$ $(N = P) \Rightarrow a_{CM} = F/m$

$\tau = I_{CM} \times \alpha \Leftrightarrow Fd = \left(\frac{2}{5} m R^2\right)\alpha \Leftrightarrow \alpha = \frac{5}{2}\left(\frac{Fd}{mR^2}\right)$

$a_{CM} = R\alpha \Leftrightarrow \frac{F}{m} = R \times \frac{5}{2}\left(\frac{Fd}{mR^2}\right) \Leftrightarrow d = \frac{2}{5} R$

Nota: Para que a bola rode sem deslizar: $a_{CM} = R\alpha$ e $v = \omega R$

23) $\tau(F) = 10\ Nm$; $R = 0{,}6\ m$; $m = 100\ kg$; $v_{angular} = 175\ rad/s$ (ω)

$I_{disco} = \frac{1}{2} m R^2 = \frac{1}{2} \times 100 \times (0{,}6)^2 = 18\ kg \cdot m^2$

$\tau = I \times \alpha \Leftrightarrow \alpha = \tau / I$

$\Leftrightarrow \alpha = 10/18$

$\Leftrightarrow \alpha = 0{,}56\ rad/s^2$

$\alpha = d\omega/dt \Leftrightarrow \Delta t = d\omega/\alpha$

$\Leftrightarrow \Delta t = (0 - 175)/0{,}56$

$\Leftrightarrow \Delta t = 315\ s$

$\Leftrightarrow \Delta t = 5{,}25\ min$

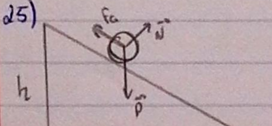
24) $R = 0{,}01\ m$; $m = 0{,}005\ kg$; $f = 6\ r.p.s \Rightarrow \omega = 2\pi f = 12\pi\ rad/s$

a) $Ec_{rot} = \frac{1}{2} I_{CM} \omega^2 = \frac{1}{2}\left(\frac{1}{2} m R^2\right)\omega^2 = \frac{1}{4} \times 0{,}005 \times 0{,}01^2 \times 12\pi^2 = 1{,}78 \times 10^{-4}\ J$

b) $Ec_{trans} = \frac{1}{2} m v_{CM}^2 = \frac{1}{2} \times m(\omega R)^2 = \frac{1}{2} \times 0{,}005 \times (12\pi \times 0{,}01)^2 = 3{,}55 \times 10^{-4}\ J$

c) $Ec_t = Ec_{rot} + Ec_{trans} = 5{,}33 \times 10^{-4}\ J$

25)



$(\tau(N) = \tau(P) = 0)$

$Em = Ep + \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \Leftrightarrow$

$\Leftrightarrow mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \left(\frac{v}{R}\right)^2$

$\Leftrightarrow v^2 = \dfrac{2gh}{1 + \frac{I}{mR^2}}$

Nota: como roda sem deslizar: $W_{Fat} = 0$; $v = \omega R$; $a = \alpha R$.

Como: $I_{esfera} = \frac{2}{5} m R^2$; $I_{cilindro} = \frac{1}{2} m R^2$; $I_{anel} = m R^2$

$v_{esfera} = \sqrt{\frac{10}{7} gh}$

$v_{cilindro} = \sqrt{\frac{4}{3} gh}$

$v_{anel} = \sqrt{gh}$