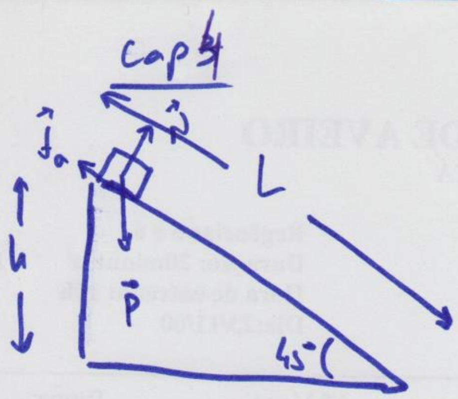
cap 4

(2)



$m = 10\ Kg$

$h = 20\ m$

$\mu_{est} = 0.2$

$\mu_{cin} = 0.1$

a) (~~se não houvesse atrito~~)

$$W_{Peso} = -\Delta E_{pg} = -(E_{pf} - E_{pi}) = -(mg.0 - mgh)$$

$$= mgh = 10 \times 9.8 \times 20 = 1960\ J$$

ou $$W = \int_{topo}^{base} \vec{F}.d\vec{r} = \int m\vec{g}.d\vec{r} = \int_0^L mg \underbrace{\cos 45}_{\text{ang. entre } \vec{g} \text{ e } \vec{x}} dx$$

$$= mg\cos 45 . L = mg\cancel{\cos 45} . \frac{h}{\cancel{\sin 45}} = mgh = 1960\ J$$

b)

$W_{\vec{N}} = 0$ pois $\vec{N}$ é perpendicular ao deslocamento

$$W_{\vec{N}} = \int \vec{N}.d\vec{r} = 0 \quad (\text{pois } \int N\, dr \cos 90 = 0)$$

c) $$W_{fa} = \underbrace{\int \vec{f}_a . d\vec{r}}_{= \int f_a\, dr \cos 180} = -f_a \overbrace{\frac{h}{\sin 45}}^{L} = -\mu_{cin} N . \frac{h}{\sin 45} = -\mu_{cin}\, mg \cancel{\cos 45} \frac{h}{\cancel{\sin 45}}$$

$$= -\frac{\mu_{cin}\, mgh}{\cancel{tg 45}} = -\frac{0.1 \times 10 \times 9.8 \times 20}{\cancel{tg 45}} = -196\ J$$

ou ~~$\Delta E_M =$~~

d) ~~$\Delta E_M = -W_{fa}$~~ $\quad \Delta E_c + \Delta E_p + \overbrace{\Delta U}^{-W_{fa}} = 0$

~~$\Delta E_M = -W_{fa}$~~

$$\Delta E_c = -(W_{fa} - \Delta E_p) = -(196 - (\overset{0,\ h=0}{E_{pf}} - E_{pi})) = -(196 - 1960)$$

$$= 1764\ J$$

(31)

② (cont.) ccp B4

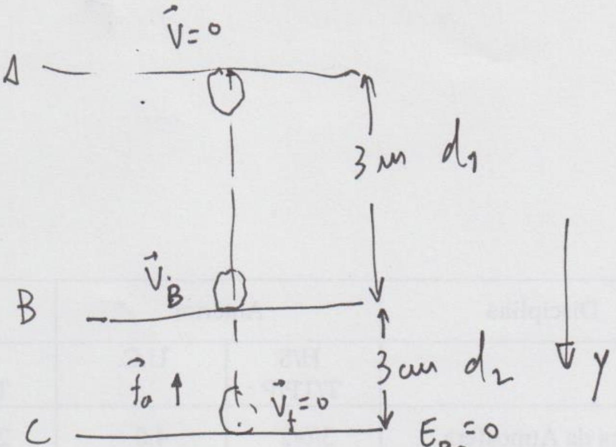
f) sem atrito

$\Delta E_M = 0 \Leftrightarrow \Delta E_c + \Delta E_p = 0 \Leftrightarrow \Delta E_c = -\Delta E_p$

$\Leftrightarrow \Delta E_c = -(E_{pf} - E_{pi}) = -(0 - mgh) = 1960\,J$

32

13



escolho $E_{pg}=0$ em C!

entre B e C

há força de atrito (não conservativa)

$$\Delta E_M = W_{fa} \Leftrightarrow (E_{cC}+E_{pC}) - (E_{cB}+E_{pB}) = \underbrace{\vec{f_a}\cdot\vec{d_2}}_{f_a d_2 \cos 180}$$

$$\Leftrightarrow (0+0) - \left(\frac{1}{2} m v_B^2 + mgd_2\right) = -f_a d_2$$

$$\Leftrightarrow f_a = \frac{\frac{1}{2} m v_B^2 + mgd_2}{d_2}$$

(?)

entre A e B (só há peso = Força conservativa)

$$\Delta E_M = 0 \Leftrightarrow (E_{cB}+E_{pB}) - (E_{cA}+E_{pA}) = 0$$

$$\Leftrightarrow \left(\frac{1}{2} \cancel{m} v_B^2 + \cancel{m}gd_2\right) - \left(0+\cancel{m}g(d_1+d_2)\right) = 0$$

$$\Leftrightarrow v_B = \sqrt{2\left(g(d_1+d_2) - gd_2\right)} = \sqrt{2gd_1 + \cancel{d_2} - \cancel{d_2}} = \sqrt{2gd_1}$$

$$f_a = \underbrace{\frac{\frac{1}{2} m\, 2gd_1 + mgd_2}{d_2}}_{\frac{mg(d_1+d_2)}{d_2}} = \frac{\frac{1}{2}\, 10\times10^{-3}\times 2\times 9.8\times 3 + 10\times10^{-3}\times 9.8\times 3\times10^{-2}}{3\times10^{-2}} = 9.9\ N$$

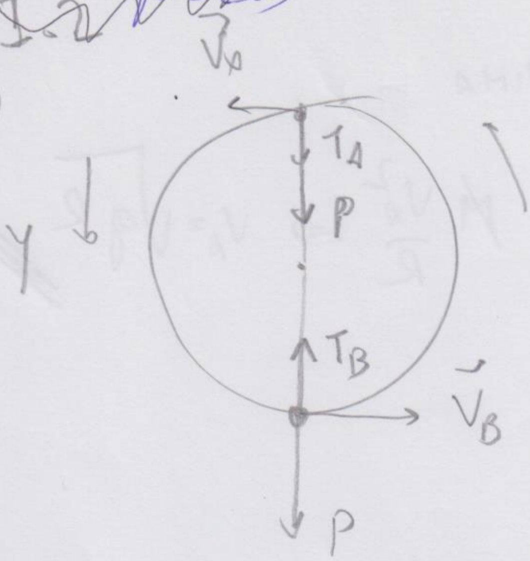
ou mais fácil ainda:

entre A e C: $\Delta E_M = W_{fa} \Leftrightarrow E_{MC} - E_{MA} = f_a d_2 \Leftrightarrow (E_{cC}+E_{pC}) - (E_{cA}+E_{pA}) = f_a d_2$

$$\Leftrightarrow mg(d_1+d_2) = f_a d_2 \Leftrightarrow f_a = \frac{mg(d_1+d_2)}{d_2} = 9.9\,N$$

I.2 NMEROS

16



$T_B - T_A = 6Mg$

$\underline{A}$ $\quad T_A + P = M \frac{V_A^2}{R}$

$\underline{B}$ $\quad -T_B + P = -M \frac{V_B^2}{R}$

$\Delta E_M = 0 \Leftrightarrow E_{MA} = E_{MB} \Leftrightarrow \frac{1}{2} \cancel{M} V_A^2 + \cancel{M} g 2R = \frac{1}{2} \cancel{M} V_B^2$

$\Leftrightarrow \frac{1}{2} V_B^2 - \frac{1}{2} V_A^2 = g 2R \Leftrightarrow \frac{1}{2}(V_B^2 - V_A^2) = 2gR$

$\downarrow$

$V_B^2 - V_A^2 = 4gR$

$T_B = +M \frac{V_B^2}{R} + P$

$T_A = M \frac{V_A^2}{R} - P$

$T_B - T_A = + \frac{M V_B^2}{R} + P - M \frac{V_A^2}{R} + P$

$= 2Mg + \frac{M}{R}(V_B^2 - V_A^2)$

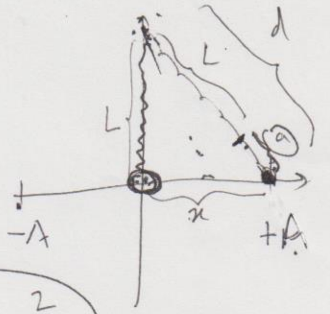
$= 2Mg + \frac{M}{\cancel{R}}(4g\cancel{R}) = 6Mg$ //

b) $T_A = 0 \Rightarrow V_A$ MÍNIMA

$$P = M\frac{V_A^2}{R} \Leftrightarrow \cancel{M}g = \cancel{M}\frac{V_A^2}{R} \Leftrightarrow V_A = \sqrt{gR}$$

(23) ~~[illegible]~~

a)



$$d^2 = L^2 + x^2$$

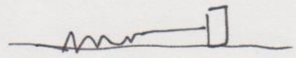
$$(L+a)^2 = L^2 + x^2$$

$$L + a = \sqrt{L^2 + x^2}$$

$$\underline{a} = \sqrt{L^2 + x^2} - L$$

$$E_p = \frac{1}{2} k a^2$$

$$E_p = \frac{1}{2} k x^2$$



w/o

$$E_p = \frac{1}{2} k \left(\sqrt{L^2 + x^2} - L\right)^2 = \frac{1}{2} k \left(L^2 + x^2 - 2L\sqrt{L^2 + x^2} + L^2\right)$$

$$\Leftrightarrow E_p = \frac{1}{2} k \left(x^2 + 2L^2 - 2L\sqrt{L^2 + x^2}\right)$$

b) $\vec{F} = -\nabla E_p \Leftrightarrow (1-D) \quad F = -\frac{dE_p}{dx}$

$$\Leftrightarrow F = -\frac{d}{dx}\left[\frac{1}{2}k\left(x^2 + 2L^2 - 2L\sqrt{L^2 + x^2}\right)\right] = -\frac{d}{dx}\left(\frac{1}{2}kx^2 + kL^2 - kL\sqrt{L^2 + x^2}\right)$$

$$\Leftrightarrow F = kx - kL\frac{d}{dx}\left(L^2 + x^2\right)^{1/2} = kx - kL\left(\frac{1}{2}\left(L^2 + x^2\right)^{-\frac{1}{2}} \cdot 2x\right)$$

$$\Leftrightarrow F = kx - kL\left(\frac{2x}{2\sqrt{L^2 + x^2}}\right) = kx - \frac{kLx}{\sqrt{L^2 + x^2}} = kx\left(1 - \frac{L}{\sqrt{L^2 + x^2}}\right)$$

c) $F(x) = 0 \Leftrightarrow kx\left(1 - \frac{L}{\sqrt{L^2 + x^2}}\right) = 0 \Leftrightarrow x = 0$

d) $\omega = \sqrt{\frac{k}{m}}$

$$x = A\cos(\omega t + \phi)$$

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$E_p = \frac{1}{2} k \left(\sqrt{L^2 + x^2} - L\right)^2$$

amplitude $(x = A)$

$$|v_{max}| = \omega A$$

$$|v_{max}| = \sqrt{\frac{k}{m}}\left(\sqrt{L^2 + A^2} - L\right)$$

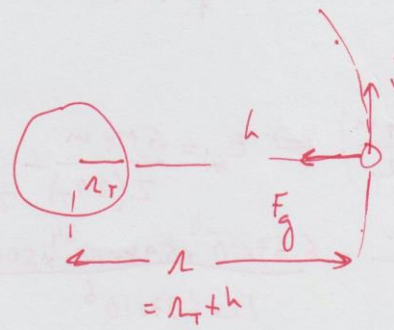
e) $v_{max} = \sqrt{\frac{40}{1}}\left(\sqrt{1^2 + 0.5^2} - 1\right) = 0.74\, m/s$

f) $v_{max} = \omega A = \sqrt{\frac{40}{1}} \cdot 0.5 = 3.16\, m/s$

48

27

Cap 3 Mec. 01-02



2ª Lei Newton

$\Sigma F = ma$

$\Rightarrow F_g = m \frac{v^2}{r}$

$\Leftrightarrow G \frac{M_T m}{(r_T+h)^2} = m \frac{v^2}{(r_T+h)}$

p/ Satélite geostacionário

$l = vt \Leftrightarrow v = \frac{l}{t} = \frac{2\pi(r_T+h)}{24\times3600}$

$\Leftrightarrow \frac{G M_T \cancel{m}}{(r_T+h)^{\cancel{2}}} = \cancel{m}\left(\frac{2\pi(r_T+h)}{24\times3600}\right)^2 \frac{1}{\cancel{(r_T+h)}}$

$\Leftrightarrow G M_T = \left(\frac{2\pi}{24\times3600}\right)^2 \cdot (r_T+h)^3$

$\Leftrightarrow h = \sqrt[3]{G M_T \left(\frac{24\times3600}{2\pi}\right)^2} - r_T = \sqrt[3]{6.67\times10^{-11}\times5.98\times10^{24}\left(\frac{24\times3600}{2\pi}\right)^2} - 6.37\times10^6$

$\Leftrightarrow h = 3.588\times10^7 \, m$

a) Conservação de energia

há conservação de E_M:

$E_{Mi} = E_{Mf}$

$E_{Mi} = E_{ci} + E_{pi} = \frac{1}{2} m v_i^2 + \left(-\frac{G M_T m}{r_T}\right)$

$E_{Mf} = 0 + E_{pf} = -\frac{G M_T m}{r_T + h}$

$\Leftrightarrow \frac{1}{2} m v_i^2 = \frac{G M_T m}{r_T} - \frac{G M_T m}{r_T+h} = G M_T m \left(\frac{1}{r_T} - \frac{1}{r_T+h}\right) = 6.67\times10^{-11}\times5.98\times10^{24}\times500\left(\frac{1}{6.37\times10^6} - \frac{1}{4.225\times10^7}\right)$

$= 2.66\times10^{10} \, J$

$\Leftrightarrow v_i = 10.3 \, km/s$

52

b) $E_{Mi} + E_{ext} = E_{Mf}$ $r_i = 2R_T$

$r_f = 3R_T$

$\Leftrightarrow E_{EXT} = E_{Mf} - E_{Mi}$

$= -\frac{GM_T m}{2r_f} - \left(-\frac{GM_T m}{2r_i}\right) \Leftrightarrow E_{ext} = \frac{GM_T m}{2(2R_T)} - \frac{GM_T m}{2(3R_T)}$

$\Leftrightarrow E_{ext} = \frac{GM_T m}{12 R_T} = \frac{6.67\times10^{-11}\times5.98\times10^{24}\times500}{12\times6.37\times10^{6}} = 2.61\times10^{9}\ J$

~~c) qual a diferença de velocidade do satélite entre a posição final e a posição inicial?~~

$\cancel{\Delta E_c = 2.6\times10^{9} J = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} m (v_f^2 - v_i^2)}$

$\cancel{v_f^2 - v_i^2 = 1.044\times10^{7}}$

c) $E_{Mi} = E_{Mf} \Leftrightarrow \frac{1}{2} m v_i^2 - \frac{GM_T m}{r_i} = \frac{1}{2} m v_f^2 - \frac{GM_T m}{r_f} = \text{const.}$

Para novo fora do campo gravítico da Terra: $v_f = 0 \quad r \to \infty$

$\Leftrightarrow \frac{1}{2} m v_i^2 = \frac{GM_T m}{R_T} = \frac{6.67\times10^{-11}\times5.98\times10^{24}\times1000}{6.37\times10^{6}} = 6.26\times10^{10}\ J$

$v_i = \sqrt{\frac{6.26\times10^{10}\times2}{1000}} = 11.2\ km/s$

$-\frac{GMm}{r} + \frac{GMm}{2r}$

$+\frac{1}{2}$

$E_M = -\frac{GMm}{2r}$

$E_p = -\frac{GMm}{r}$

$E_c = \frac{GMm}{2r}$

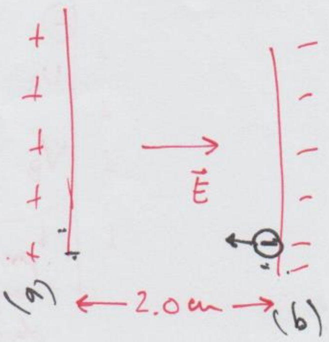
$E_c = E_M - E_p$

$= -\frac{GMm}{2r} - \left(-\frac{GMm}{r}\right)$

$= -\frac{GMm}{2r} + \frac{GMm}{r}$

$= -\frac{1}{2} \cancel{GM}$

(28)



$E = 2.0\times10^{4}\,(N/C)$

relembrar

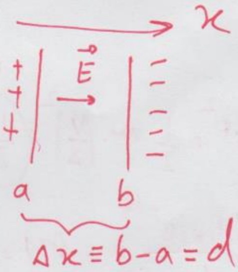
$$F_e = k\frac{Qq}{r^2} \quad ; \quad E = \frac{F_e}{q} = k\frac{Q}{r^2} \quad ; \quad \Delta E_{Pe} = -W_{F_e} = -\int_a^b \vec{F}_e\cdot d\vec{r}$$

$$\Delta V = \frac{\Delta E_p}{q} = \frac{-\int_a^b \vec{F}_e\cdot d\vec{r}}{q} = -\int_a^b \frac{\vec{F}_e}{q}\cdot d\vec{r} = -\int_a^b \vec{E}\cdot d\vec{r}$$

$$\Leftrightarrow \vec{E} = -\left(\frac{\partial V}{\partial r}\right)\hat{r} \equiv \vec{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

Voltando ao problema:

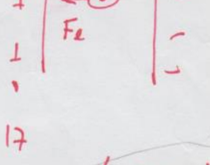
a) $\Delta V = -\int_a^b \vec{E}\cdot d\vec{x} \qquad \vec{E} = \text{const}$



$\Delta V = -E\cdot d$

$= 2.0\times10^{4} \times 2\times10^{-2} = -400\,(V) \rightarrow V_a > V_b$

b) $\vec{E} = \frac{\vec{F}}{q_e} \Leftrightarrow |\vec{F}| = |\vec{E}|\cdot q_e = 2{,}0\times10^{4} \times 1.6\times10^{-19} = 3.2\times10^{-15}\,(N)$



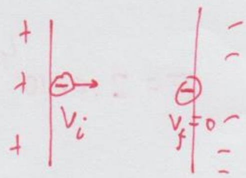
c) $\Delta V = \overbrace{\frac{\Delta E_p}{q_e} = \frac{-\Delta E_c}{q_e}}^{\text{conservação de energia mec}} \Leftrightarrow \Delta E_c = \Delta V\cdot q_e = 400\times1.6\times10^{-19} = 6.4\times10^{-17}\,J \equiv 400\,eV$

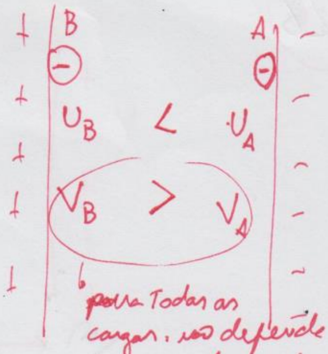
lembrar definição de eV → energia que electrão ganha ao ser acelerado por $\Delta V = 1$ Volt.

VIRE →

28 (cont)

P/ electrão





d)

$E_{ci} = \frac{1}{2} m v_i^2 \quad E_{cf} = 0$

$\underbrace{U_i \qquad U_f}$

$\Delta U = U_f - U_i > 0$

$\Delta E_c = E_{cf} - E_{ci} < 0$

$\Delta U = -\Delta E_c$
$\Delta V . q_e$

$\Rightarrow \Delta E_c = -\Delta V . q_e = E_{cf} - E_{ci}$

$= -(V_A - V_B) . q_e = 0 - \frac{1}{2} m v^2$

$= -(-400\,V)(-1.6 \times 10^{-19}) = -\frac{1}{2} m v^2$

$\Rightarrow v = \sqrt{\frac{2 \times 400 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 1.2 \times 10^{7}\ m/s$

e) Se $v' = \frac{v}{2}$

$\Delta E_c' = \frac{1}{2} m v'^2 = \frac{1}{2} m \left(\frac{v}{2}\right)^2 = \frac{\overbrace{\frac{1}{2} m v^2}^{\Delta E_c}}{4} = \frac{\Delta E_c}{4}$

de c) $\Delta E_c' = \frac{6.4 \times 10^{-17}}{4} = -\Delta U = -\Delta V . q_e = -E . d . q_e$

$\Rightarrow d = \frac{6.4 \times 10^{-17}}{4 \times E \times q_e} = 0.005 = 0.50\,cm$ ($\frac{1}{4}$ de 2.0 cm)