Exercícios MC - CAP 1

1)

$(\rho, \phi, z) = (2, \pi/3, 1)$

$$\begin{cases} x = \rho \cdot \cos\phi \\ y = \rho \cdot \sin\phi \\ z = z \end{cases} \Leftrightarrow \begin{cases} x = 2 \cdot \cos\left(\frac{\pi}{3}\right) \\ y = 2 \cdot \sin\left(\frac{\pi}{3}\right) \\ z = 1 \end{cases} \Leftrightarrow \begin{cases} x = 1 \\ y = \sqrt{3} \\ z = 1 \end{cases}$$

$(x, y, z) = (1, \sqrt{3}, 1)$

2)

$(r, \theta, \phi) = (2, \pi/3, \pi/2)$

$$\begin{cases} x = r \cdot \sin\theta \cdot \cos\phi \\ y = r \cdot \sin\theta \cdot \sin\phi \\ z = r \cdot \cos\theta \end{cases} \Leftrightarrow \begin{cases} x = 2 \cdot \sin\frac{\pi}{3} \cdot \cos\frac{\pi}{2} \\ y = 2 \cdot \sin\frac{\pi}{3} \cdot \sin\frac{\pi}{2} \\ z = 2 \cdot \cos\frac{\pi}{3} \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y = \sqrt{3} \\ z = 1 \end{cases}$$

3)

$(x, y, z) = (\sqrt{2}, -\sqrt{2}, 2)$

$$\begin{cases} \rho = \sqrt{x^2 + y^2} \\ \operatorname{tg}\theta = y/x \\ z = z \end{cases} \Leftrightarrow \begin{cases} \rho = \sqrt{2+2} \\ \operatorname{tg}\theta = -\sqrt{2}/\sqrt{2} \\ z = 2 \end{cases} \Leftrightarrow \begin{cases} \rho = 2 \\ \operatorname{tg}\theta = -1 \\ z = 2 \end{cases} \Leftrightarrow \begin{cases} \rho = 2 \\ \theta = -\pi/4 \\ z = 2 \end{cases}$$

$(\rho, \theta, z) = (2, -\pi/4, 2)$

4)

$(x, y, z) = (\sqrt{2}, 0, -\sqrt{2})$

$$\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \phi = \tan^{-1}\left(y/x\right) \\ \theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) \end{cases} \Leftrightarrow \begin{cases} r = \sqrt{2+2} \\ \phi = \tan^{-1}(0) \\ \theta = \tan^{-1}\left(\frac{\sqrt{2+0}}{-\sqrt{2}}\right) \end{cases} \Leftrightarrow \begin{cases} r = 2 \\ \phi = 0 \\ \theta = -\pi/4 \end{cases}$$

$(r, \theta, \phi) = (2, -\pi/4, 0)$

5) $\vec{r} = (x,y,z) = (\rho\cos\theta, \rho\sin\theta, z)$

$$\hat{e}_\rho = \frac{\left(\frac{\partial \vec{r}}{\partial \rho}\right)}{\left|\frac{\partial \vec{r}}{\partial \rho}\right|} = \frac{(\cos\theta, \sin\theta, 0)}{1} = (\cos\theta, \sin\theta, 0)$$

$$\hat{e}_\theta = \frac{\left(\frac{d\vec{r}}{d\theta}\right)}{\left|\frac{d\vec{r}}{dz}\right|} = \frac{\rho(-\sin\theta, \cos\theta, 0)}{\rho} = (-\sin\phi, \cos\phi, 0)$$

$$\hat{e}_z = \frac{\left(\frac{d\vec{r}}{d\theta}\right)}{\left|\frac{d\vec{r}}{dz}\right|} = \frac{(0,0,1)}{1} = (0,0,1)$$

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\hat{i} = \cos\theta\, \hat{e}_\rho - \sin\theta\, \hat{e}_\phi$

$\hat{j} = \sin\theta\, \hat{e}_\rho + \cos\theta\, \hat{e}_\phi$

$\hat{k} = \hat{e}_z$

6) $\vec{r} = (x,y,z) = (r\cdot\sin\theta\cdot\cos\phi, r\sin\theta\sin\phi, r\cos\theta)$

$$\vec{e}_r = \frac{\left(\frac{\partial \vec{r}}{\partial r}\right)}{\left|\frac{d\vec{r}}{dr}\right|} = \frac{(\sin\theta\cdot\cos\phi, \sin\theta\sin\phi, \cos\theta)}{1} = (\sin\theta\cdot\cos\phi, \sin\theta\cdot\sin\phi, \cos\theta)$$

$$\vec{e}_\theta = \frac{\left(\frac{d\vec{r}}{\partial\theta}\right)}{\left|\frac{d\vec{r}}{d\theta}\right|} = \frac{(r\cdot\cos\theta\cdot\cos\phi, r\cos\theta\sin\phi, -r\sin\theta)}{r} = (\cos\theta\cdot\cos\phi, \cos\theta\sin\phi, -\sin\theta)$$

$$\vec{e}_\phi = \frac{\left(\frac{d\vec{r}}{\partial\phi}\right)}{\left|\frac{d\vec{r}}{d\phi}\right|} = \frac{(-r\sin\theta\sin\phi, r\sin\theta\cos\phi, 0)}{r\sin\theta} = (-\sin\phi, \cos\phi, 0)$$

$$\begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix}$$

$\hat{i} = \sin\theta\cos\phi\, \vec{e}_r + \cos\theta\cos\phi\, \vec{e}_\theta - \sin\phi\, \vec{e}_\phi$

$\hat{j} = \sin\theta\sin\phi\, \hat{e}_r + \cos\theta\sin\phi\, \hat{e}_\theta + \cos\phi\, \hat{e}_\phi$

$\hat{k} = \cos\theta\, \hat{e}_r - \sin\theta\, \hat{e}_\theta$

7) $\vec{r} = (x,y,z) = (\rho\cos\theta, \rho\sin\theta, z) \rightarrow$ coordenadas cilíndricas

$= (r\sin\theta\cos\phi, r\sin\theta\sin\phi, r\cos\theta) \rightarrow$ coordenadas esféricas

$\vec{r} = \rho\cos\theta \times (\cos\theta\, \hat{e}_\rho - \sin\theta\, \hat{e}_\theta) + \rho\sin\theta(\sin\theta\, \hat{e}_\rho + \cos\theta\, \hat{e}_\theta) + z \times \hat{e}_z =$

$= \rho\cos^2\theta\, \hat{e}_\rho \underbrace{- \rho\cos\theta\sin\theta\, \hat{e}_\theta + \rho\sin^2\theta\, \hat{e}_\rho}_{(1)\cdot\rho} + \rho\sin\theta\cos\theta\, \hat{e}_\theta + z \times \hat{e}_z$

$= \rho\hat{e}_\rho + z\,\hat{e}_z$

8) $V = xz\hat{i} + yz\hat{j} + xy\hat{k}$

· cilíndricos:

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \\ z = z \end{cases} \qquad \begin{cases} \hat{i} = \cos\theta\,\hat{e\rho} - \sin\theta\,\hat{e\theta} \\ \hat{j} = \sin\theta\,\hat{e\rho} + \cos\theta\,\hat{e\theta} \\ \hat{k} = \hat{ez} \end{cases}$$

$$V = (\rho\cos\theta)(z)(\cos\theta\,\hat{e\rho} - \sin\theta\,\hat{e\theta}) + (\rho\sin\theta)z(\sin\theta\,\hat{e\rho} + \cos\theta\,\hat{e\theta}) + (\rho\cos\theta)(\rho\sin\theta)\,\hat{ez}$$
$$= (\rho\cos\theta^2 z + \rho\sin\theta^2 z)\,\hat{e\rho} + (-\rho\cos\theta\sin\theta z + \rho\sin\theta\cos\theta z)\,\hat{e\theta} + (\rho^2\cos\theta\sin\theta)\,\hat{ez}$$
$$= (\rho z)\,\hat{e\rho} + 0\,\hat{e\theta} + (\rho^2\cos\theta\sin\theta)\,\hat{ez}$$

· esféricos:

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases} \qquad \begin{cases} \hat{i} = \sin\theta\cos\phi\,\hat{er} + \cos\theta\cos\phi\,\hat{e\theta} - \sin\phi\,\hat{e\phi} \\ \hat{j} = \sin\theta\sin\phi\,\hat{er} + \cos\theta\sin\phi\,\hat{e\theta} + \cos\phi\,\hat{e\phi} \\ \hat{k} = \cos\theta\,\hat{er} - \sin\theta\,\hat{e\theta} \end{cases}$$

$$V = (r\sin\theta\cos\phi)(r\cos\theta)(\sin\theta\cos\phi\,\hat{er} + \cos\theta\cos\phi\,\hat{e\theta} - \sin\phi\,\hat{e\phi}) +$$
$$+ (r\sin\theta\sin\phi)(r\cos\theta)(\sin\theta\sin\phi\,\hat{er} + \cos\theta\sin\phi\,\hat{e\theta} + \cos\phi\,\hat{e\phi}) +$$
$$+ (r\sin\theta\sin\phi)(r\sin\theta\sin\phi)(\cos\theta\,\hat{er} - \sin\theta\,\hat{e\theta}) =$$
$$= (r^2\sin\theta^2\cos\phi^2\cos\theta + r^2\sin\theta^2\sin\phi^2\cos\theta + r^2\sin\theta^2\sin\phi^2\cos\theta)\,\hat{er} +$$
$$+ (r^2\sin\theta\cos\phi^2\cos\theta^2 + r^2\sin\theta\sin\phi^2\cos\theta^2 - r^2\sin\theta^3\sin\phi^2)\,\hat{e\theta} +$$
$$+ (-r^2\sin\theta\cos\phi\cos\theta\sin\phi + r^2\sin\theta\sin\phi\cos\theta\cos\phi)\,\hat{e\phi}$$
$$= [(r^2\sin\theta^2\cos\theta)(\cos^2\phi + 2\sin^2\phi)]\,\hat{er} + [(r^2\sin\theta)(\cos^2\phi + \sin^2\phi - \sin\theta\sin^2\phi)]\,\hat{e\theta} + 0\,\hat{e\phi} =$$
$$= [(r^2\sin\theta^2\cos\theta)(1 + \sin\phi)]\,\hat{er} + [(r^2\sin\theta)(1 - \sin\theta\sin^2\phi)]\,\hat{e\theta}$$

9) $V = z\hat{i} + 2x\hat{j} + y\hat{k}$

· cilíndricos:

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \\ z = z \end{cases} \qquad \begin{cases} \hat{i} = \cos\theta\,\hat{e\rho} - \sin\theta\,\hat{e\theta} \\ \hat{j} = \sin\theta\,\hat{e\rho} + \cos\theta\,\hat{e\theta} \\ \hat{k} = \hat{ez} \end{cases}$$

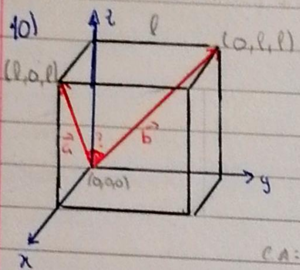
$$V = z(\cos\theta\,\hat{e\rho} - \sin\theta\,\hat{e\theta}) + 2(\rho\cos\theta)(\sin\theta\,\hat{e\rho} + \cos\theta\,\hat{e\theta}) + (\rho\sin\theta)(\hat{ez})$$
$$= (z\cos\theta + 2\rho\cos\theta\sin\theta)\,\hat{e\rho} + (-z\sin\theta + 2\rho\cos^2\theta)\,\hat{e\theta} + (\rho\sin\theta)\,\hat{ez}$$
$$= [(z + 2\rho\sin\theta)(\cos\theta)]\,\hat{e\rho} + (-z\sin\theta + 2\rho\cos^2\theta)\,\hat{e\theta} + (\rho\sin\theta)\,\hat{ez}$$

· esféricas:

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases} \qquad \begin{cases} \hat{i} = \sin\theta\cos\phi\,\hat{e_r} + \cos\theta\cos\phi\,\hat{e_\theta} - \sin\phi\,\hat{e_\phi} \\ \hat{j} = \sin\theta\sin\phi\,\hat{e_r} + \cos\theta\sin\phi\,\hat{e_\theta} + \cos\phi\,\hat{e_\phi} \\ \hat{k} = \cos\theta\,\hat{e_r} - \sin\theta\,\hat{e_\theta} \end{cases}$$

$$V = (r\cos\theta)(\sin\theta\cos\phi\,\hat{e_r} + \cos\theta\cos\phi\,\hat{e_\theta} - \sin\phi\,\hat{e_\phi}) +$$
$$+ 2(r\sin\theta\cos\phi)(\sin\theta\sin\phi\,\hat{e_r} + \cos\theta\sin\phi\,\hat{e_\theta} + \cos\phi\,\hat{e_\phi}) + (r\sin\theta\sin\phi)(\cos\theta\,\hat{e_r} - \sin\theta\,\hat{e_\theta}) =$$
$$= (r\cos\theta\sin\theta\cos\phi + 2r\sin\theta^2\cos\phi\sin\phi + r\sin\theta\sin\phi\cos\theta)\,\hat{e_r} +$$
$$+ (r\cos\theta^2\cos\phi + 2r\sin\theta\cos\phi\cos\theta\sin\phi - r\sin\theta^2\sin\phi)\,\hat{e_\theta} + (-r\cos\theta\sin\phi + 2r\sin\theta\cos\phi^2)\,\hat{e_\phi}$$

10)



$\vec{a} = (l,0,l) - (0,0,0) = (l,0,l) \rightarrow |\vec{a}| = \sqrt{2}\,l$

$\vec{b} = (0,l,l) - (0,0,0) = (0,l,l) \rightarrow |\vec{b}| = \sqrt{2}\,l$

$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\cos(\theta) \Leftrightarrow \cos\theta = (\vec{a}\cdot\vec{b}) / (|\vec{a}|\cdot|\vec{b}|)$

C.A.:

$\vec{a}\cdot\vec{b} = (l,0,l)\cdot(0,l,l)$

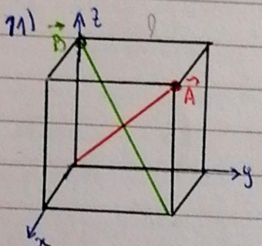
$= 0 + 0 + l^2$

$\Leftrightarrow \cos\theta = l^2 / (\sqrt{2}\,l)^2$

$\Leftrightarrow \cos\theta = 1/2$

$\Leftrightarrow \theta = \pi/3$ //

11)



$(\vec{A}) = (l,l,l) \Rightarrow |\vec{A}| = l\cdot\sqrt{3l}$

$\vec{B} = (0,0,l) - (l,l,0) = (-l,-l,l) = l\sqrt{3l}$

$\vec{A}\cdot\vec{B} = (l,l,l)\cdot(-l,-l,l) = -l^2 - l^2 + l^2 = -l^2$

$\cos\theta = \vec{A}\cdot\vec{B} / (|\vec{A}|\cdot|\vec{B}|) \Leftrightarrow \cos\theta = -l^2 / (l\cdot\sqrt{3l})^2$

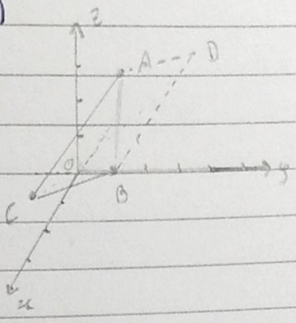
$\Leftrightarrow \cos\theta = -l^2 / (l^2\cdot 3l)$

$\Leftrightarrow \cos\theta = -1/3$ //

12)

$|\vec{A}| = 10$

13)



$\vec{CB} = B - C = (0,1,0) - (1,-1,0) = (-1,2,0)$

$\vec{CA} = A - C = (-1,0,2) - (1,-1,0) = (-2,1,2)$

$\vec{D} = \vec{CA} + \vec{CB} = (-1,2,0) + (-2,1,2) = (-3,3,2)$

14)

$|\vec{A}| = \sqrt{3^2+4^2+1^2} = \sqrt{26}$

$|\vec{B}| = \sqrt{1^2+(-1)^2+1^2} = \sqrt{3}$

$\vec{A}\cdot\vec{B} = (3,4,1)\cdot(1,-1,1) =$

$= 3 - 4 + 1$

$= 0$

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}|\cdot|\vec{B}|} \Leftrightarrow \cos\theta = \dfrac{0}{\sqrt{26}\times\sqrt{3}}$

$\Leftrightarrow \cos\theta = 0$

$\Leftrightarrow \theta = \pi/2$

15) $A = 2\hat{i} + 4\hat{j} + 6\hat{k}$

$B = 3\hat{i} - 3\hat{j} - 5\hat{k}$

$\vec{A}\cdot\vec{B} = (2,4,6)\cdot(3,-3,-5) = 6 - 12 - 30 = -36$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 6 \\ 3 & -3 & -5 \end{vmatrix} = -20\hat{i} + 18\hat{j} - 6\hat{k} - (12\hat{k} - 18\hat{i} - 10\hat{j})$$

$= -20\hat{i} + 18\hat{i} + 18\hat{j} + 10\hat{j} - 6\hat{k} - 12\hat{k}$

$= -2\hat{i} + 28\hat{j} - 18\hat{k}$

16) $\vec{P} = 3\hat{i} + 2\hat{j} - \hat{k} \rightarrow |\vec{P}| = \sqrt{3^2+2^2+(-1)^2} = \sqrt{14}$

$\vec{Q} = -6\hat{i} - 4\hat{j} + 2\hat{k} \rightarrow |\vec{Q}| = \sqrt{(-6)^2+(-4)^2+2^2} = \sqrt{56}$

$\vec{R} = \hat{i} - 2\hat{j} - \hat{k} \rightarrow |\vec{R}| = \sqrt{1^2+(-2)^2+(-1)^2} = \sqrt{6}$

Antiparalelos: $\theta = 180° \rightarrow \cos\theta = -1$

Paralelos: $\theta = 90° \rightarrow \vec{a}\cdot\vec{b} = 0$

$\vec{P}\cdot\vec{Q} = (3,2,-1)\cdot(-6,-4,2) = -18 - 8 - 2 = -28$

$\vec{P}\cdot\vec{R} = (3,2,-1)\cdot(1,-2,-1) = 3 - 4 + 1 = 0$ perpendiculares

$\vec{Q}\cdot\vec{R} = (-6,-4,2)\cdot(1,-2,-1) = -6 + 8 - 2 = 0$

$\cos\theta = \dfrac{\vec{P}\cdot\vec{Q}}{|\vec{P}|\cdot|\vec{Q}|} = \dfrac{-28}{\sqrt{14}\cdot\sqrt{56}} = \dfrac{-28}{\sqrt{784}} = \dfrac{-28}{28} = -1 \rightarrow$ antiparalelos

17)

1.º) $\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$

$\begin{cases} \vec{A} \cdot \vec{U} = 0 \\ \vec{A} \cdot \vec{V} = 0 \end{cases} \Leftrightarrow \begin{cases} (x,y,z)\cdot(2,1,-1) = 0 \\ (x,y,z)\cdot(1,-1,1) = 0 \end{cases} \Leftrightarrow \begin{cases} 2x + y - z = 0 \\ x - y + z = 0 \end{cases} \Leftrightarrow \begin{cases} 2(y-z) + y - z = 0 \\ x = y - z \end{cases} \Leftrightarrow \begin{cases} 2y - 2z + y - z = 0 \\ x = y - z \end{cases}$

$\Leftrightarrow \begin{cases} 3y = 3z \\ x = y - z \end{cases} \Leftrightarrow \begin{cases} y = z \\ x = z - z \end{cases} \Leftrightarrow \begin{cases} y = z \\ x = 0 \end{cases}$

$\vec{A} = (0, z, z) = z(0,1,1) = (0,1,1) \rightarrow |\vec{A}| = \sqrt{0^2+1^2+1^2} = \sqrt{2}$

2.º) $\vec{A} = \vec{U} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i} - \hat{j} - 2\hat{k} - (\hat{k} + \hat{i} + 2\hat{j}) = -3\hat{j} - 3\hat{k}$

$|\vec{A}| = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = 2\sqrt{3}$ //

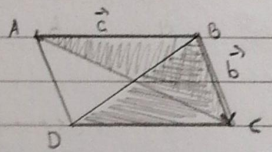
18)

A(1,0,0)

B(2,-1,0)

C(0,-1,1)

D(-1,0,1)



$\vec{a} = B - A = (2,-1,0) - (1,0,0) = (1,-1,0)$

$\vec{b} = C - B = (0,-1,1) - (2,-1,0) = (-2,0,1)$

→ área do paralelogramo

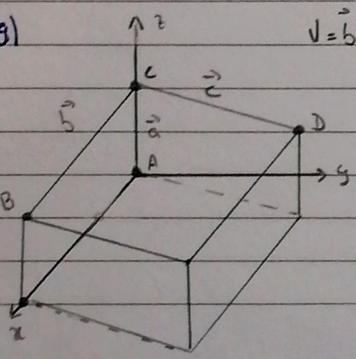
$A[ABC] = A[BCD] = \frac{1}{2} |\vec{a} \times \vec{b}| = \frac{1}{2} \cdot \sqrt{6} = \frac{\sqrt{6}}{2}$ //

C.A.:

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ -2 & 0 & 1 \end{vmatrix} = -\hat{i} - 2\hat{k} - \hat{j} = (-1,-1,-2)$

$|\vec{c} \times \vec{b}| = \sqrt{(-1)^2 + (-1)^2 + (-2)^2} = \sqrt{6}$

19)



$V = \vec{b} \cdot (\vec{a} \times \vec{c}) = (3,0,-2) \cdot (6,0,0) = 18 \text{ cm}^3$

$\vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -2 \\ 0 & 3 & -1 \end{vmatrix} = 6i$

C.A.:

$\vec{a} = A - C = (0,0,0) - (0,0,2) = (0,0,-2)$

$\vec{b} = B - C = (3,0,0) - (0,0,2) = (3,0,-2)$

$\vec{c} = D - C = (0,3,1) - (0,0,2) = (0,3,-1)$

20)

a)

$$\vec{B}\times\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{vmatrix} = -\hat{i} + \hat{j} - \hat{k}$$

$\vec{A}\cdot(\vec{B}\times\vec{C}) = (1,1,0)\cdot(-1,1,-1) = -1+1+0 = 0 \rightarrow$ A está no mesmo plano que B e que C, mas é perpendicular ao plano B×C.

b)

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\hat{i}+\hat{j})\times(-\hat{i}+\hat{j}-\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ -1 & 1 & -1 \end{vmatrix} =$$

$$= -\hat{i} + \hat{k} + \hat{k} + \hat{j}$$

$$= -\hat{i} + \hat{j} + 2\hat{k}$$

21)

1º) $$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 2 \\ 6 & 4 & -2 \end{vmatrix} = 4\hat{i} + 12\hat{j} + 12\hat{k} - (-12\hat{k} + 8\hat{i} - 6\hat{j}) = -4\hat{i} + 18\hat{j} + 24\hat{k}$$

$$(\vec{A}\times\vec{B})\times\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 18 & 24 \\ -3 & -2 & -4 \end{vmatrix} = -72\hat{i} - 72\hat{j} + 8\hat{k} - (-54\hat{k} - 48\hat{i} + 16\hat{j}) = -24\hat{i} - 88\hat{j} + 62\hat{k}$$

2º) $$\vec{B}\times\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & -2 \\ -3 & -2 & -4 \end{vmatrix} = -16\hat{i} + 6\hat{j} - 12\hat{k} - (-12\hat{k} + 4\hat{i} - 24\hat{j}) = -12\hat{i} - 18\hat{j} + 0\hat{k}$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 2 \\ -12 & -18 & 0 \end{vmatrix} = 0\hat{i} - 24\hat{j} - 54\hat{k} - (+24\hat{k} - 36\hat{i} + 0\hat{j}) = -36\hat{i} - 24\hat{j} - 78\hat{k}$$

R: O produto vetorial não é associativo