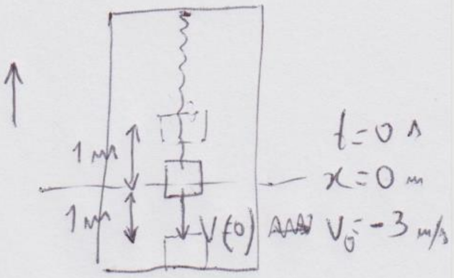
(6)

$m = 1\,kg$

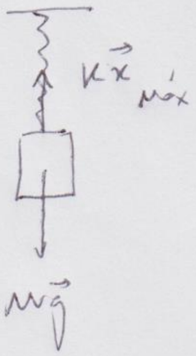
$K = 9\,N/m$

$$x = A\cos(\omega t + \delta)$$

$$V = \frac{dx}{dt} = -\omega A \,\text{sen}(\omega t + \delta)$$



cálculo de $A \equiv x_{máx}$



$x_{máx} = .$

$\sum \vec{F} = 0 \Leftrightarrow K x_{máx} = mg$

$\Leftrightarrow x_{máx} \equiv A = \frac{mg}{K} = \frac{1 \times 9.8}{9}$

$\Leftrightarrow A \sim 1\,m$

cálculo de ω.

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{9}{1}} = 3\ rad\,s^{-1}$$

então:

$x = 1\cos(3t + \delta)$

$V = -3\,\text{sen}(\omega t + \delta)$

em $t = 0$

$x(0) = \cos\delta = 0$

$V(0) = -3\,\text{sen}\,\delta = -3$

$\delta < \frac{\pi}{2}\ rad$ ou $\cancel{\frac{3}{2}\pi}\ rad$

$\text{sen}\,\delta = 1 \Leftrightarrow \delta = \frac{\pi}{2}\ rad$

$x(t) = \cos\left(3t + \frac{\pi}{2}\right)$ (~~rad~~)

$V(t) = -3\,\text{sen}\left(3t + \frac{\pi}{2}\right)\ m/s^{-1}$

em que instantes $V = 0$?

$x = x_{máx} \equiv \cos\left(3t + \frac{\pi}{2}\right) = \pm 1 \Leftrightarrow 3t + \frac{\pi}{2} = m\pi$

$\Leftrightarrow t = \left(m\pi - \frac{\pi}{2}\right)/3\ (s)$

$m = 1, 2, \dots, m$

$\omega = 2\pi f = \frac{2\pi}{T} = 3\ rad\,s^{-1}$

$\Leftrightarrow f = \frac{3}{2\pi} = 0.478\ Hz$

$T = 2.094\ s$

ou

$V = 0 = -3\,\text{sen}\left(3t + \frac{\pi}{2}\right) \Leftrightarrow 3t + \frac{\pi}{2} = m\pi$

(7)

$$x = A\cos(\omega t + \delta)$$

$t=0 \Rightarrow v=0 \quad x = 6\,cm$ $\qquad T = 2\,s$

logo $A = 6\,cm$ $\qquad \omega = \frac{2\pi}{T} = \pi\,rad\,s^{-1}$ //

$$x = 6\cos(\pi t + \delta)\ cm$$

Falta conhecer δ:

$x(0) = 6\cos(\pi . 0 + \delta) = 6 \Rightarrow \cos\delta = 1 \rightarrow \delta = 0$ ✓ $\vee$ ~~$\delta = \pi$~~

$v(0) = -6\pi\,sen(\pi . 0 + \delta) = 0 \Rightarrow sen\,\delta = 0 \rightarrow \delta = 0$ ✓ $\vee$ ~~$\delta = \pi$~~

$x(t) = 6\cos(\pi t)\ (cm)$ //

$v(t) = -6\pi\,sen(\pi t)\ (cm/s)$ //

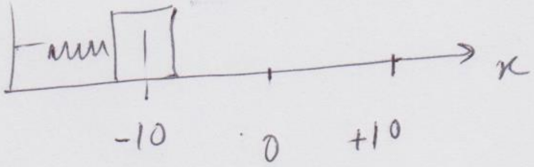
$a(t) = -6\pi^2\cos(\pi t)\ (cm/s^2)$ //

(11) $m = 4.0\ Kg \qquad k = 100\ N/m \qquad A = 10\ cm$

a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{40}} = 5\ rad\ s^{-1}$

b) $\omega = \frac{2\pi}{T} \Leftrightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{5} = 1.256\ s$

c) $x(0) = 10 \cos(5\cancel{\pi}\overset{0}{t} + \delta) = -10 \qquad t = 0$



$\Leftrightarrow \cos\delta = -1$

$\Leftrightarrow \delta = \pi$

$x(t) = 10 \cos(5t + \pi)\ (cm)$

d) $E_M = \frac{1}{2} k A^2 \qquad E_c = E_T - E_p = \frac{1}{2} k (A^2 - x^2)$

$E_p = \frac{1}{2} k x^2$

$x(1) = 10 \cos(5 \cdot 1 + \pi) = 10 \cos(5 + \pi) = -2.84\ cm$

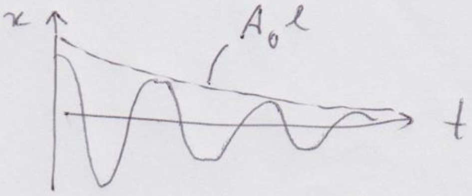
$\cancel{E_c(1) = \frac{1}{2} 100 (10^2 - 10^2)}$

$E_c(1) = \frac{1}{2} 100 \left[(10 \times 10^{-2})^2 - (-2.84 \times 10^{-2})^2 \right] = 0.46\ J$

(20) $K = 250\ N/m \quad m = 10\ Kg \quad c = 40\ Ns/m$

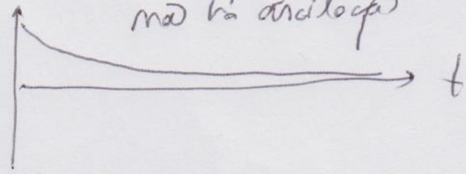
$$x = A_0 e^{-\cancel{\gamma}t\left(\frac{-c}{2m}\right)t} \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{K}{m} - \left(\frac{c}{2m}\right)^2} \qquad \omega_0 = \sqrt{\frac{K}{m}}$$



sub-amortecimento $\gamma^2 < \omega_0^2$

amortecimento critico $\gamma^2 = \omega_0^2$



$\omega^2 < 0$

Sobre-amortecimento $\gamma^2 > \omega_0^2$

$$\omega^2 < 0 \Rightarrow \omega = \pm i|\omega|$$

a) $\omega_0 = \sqrt{\frac{K}{m}} = \sqrt{\frac{250}{10}} = 5\ rad\,s^{-1}$; $\gamma = \frac{c}{2m} = \frac{40}{2 \times 10} = 2$

logo $\gamma^2 < \omega_0^2 \rightarrow$ Sub-amortecido

b)

paralelo $\left[\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} = \right.$

$K = \frac{K_1 K_2}{K_1 + K_2}$

serie $\left[K = K_1 + K_2 \cdots \right.$

(23) $m = 1 kg$

$k = 100 \ N/m$

$A = 10 cm$

a) $E_c = ?$ $\quad E_M = \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

$E_p = ?$

$\Leftrightarrow E_p = E_c = \frac{1}{4}kA^2 = \frac{1}{2}kx^2$

$\Leftrightarrow x = \sqrt{\frac{1}{2}A^2} = A\cos(\omega t + \delta)$

$\omega = \sqrt{\frac{k}{m}} = 10 \ rad\,s^{-1}$

$10\cos 10t = \sqrt{\frac{1}{2}10^2}$

$x(0) = A = A\cos\delta \Leftrightarrow \delta = 0$

$\cancel{1}0\cos 10t = 1\cancel{0}\sqrt{\frac{1}{2}}$

logo

$10t = \cos^{-1}\sqrt{\frac{1}{2}}$

$x(t) = 10\cos 10t$

$t = \cos^{-1}\left(\sqrt{\frac{1}{2}}\right) / 10$

$t = 0.0785 \ s$ $\quad x(0.0785) = 7{,}07 \ cm$ $\quad \gamma = \frac{b}{2m} = \frac{2}{2} = 1$

b) $b = 2 \ kg/s$ $\quad A(t) = A_0 e^{-\gamma t}$

$E_M = \frac{1}{2}kA^2$ $\quad \Delta E_M = E_{M,1} - E_{M,0} = \frac{1}{2}k\left(A_0 e^{-1 \times 1}\right)^2 - \frac{1}{2}kA_0^2$

$\Delta E_M = \frac{1}{2} \times 100\left[0.1^2 e^{-2} - 0.1^2\right] = -0.43 \ J$

c) $W_F = -\Delta E_M = +0.49 \ J$ em $\Delta t = 1 \ s$

$\overline{P} = \frac{W_F}{\Delta t} = 0.43 \ Watt$

(25)

a) $F(t) = F_0 \cos(6t + \delta)$

b) $$A = \frac{\frac{F_0}{m}}{\sqrt{(\omega_f^2 - \omega_0^2)^2 + \left(\frac{b\,\omega_f}{m}\right)^2}}$$

$\omega_0^2 = \frac{k}{m} = 100$

$\Rightarrow \omega_0 = 10$ rad/s

$$A = \frac{\frac{10}{1}}{\sqrt{(6^2 - 10^2)^2 + \left(\frac{2\times 6}{1}\right)^2}} = 0.154 \text{ m}$$

c) $\omega_f = \omega_0 = 10$ rad/s

d) mov. amortecido

$A(t) = A_0 e^{-\gamma t}$

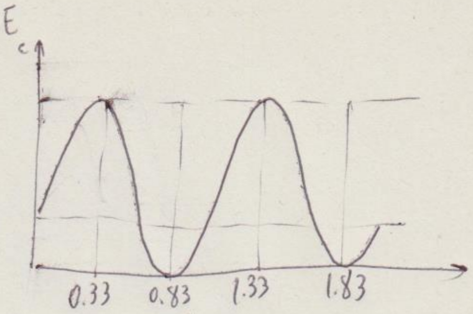
$\gamma = \frac{\cancel{2}b}{2m} = \frac{\cancel{2}\times 2}{2\times 1} = 1$

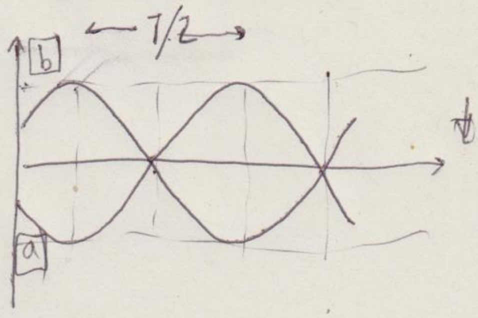
$$\frac{A(t)}{A_0} = \frac{1}{2} = e^{-\gamma} \Rightarrow t = -\ln\frac{1}{2} = 0.69 \text{ s}$$

(28) a) $T = 2\,s$

$$\omega = \frac{2\pi}{T} = \pi \text{ rad } s^{-1} = \sqrt{\frac{k}{m}}$$

$$\Leftrightarrow k = \omega^2 m = 4.9 \text{ N/m}$$





b) $E_c(0) = \frac{1}{4} E_{c\,máx}$

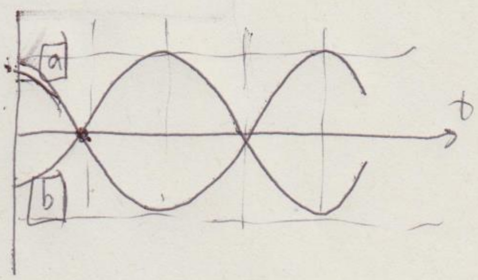
como $E_c \propto v^2$

$v(0) = \frac{1}{2} v_{max} =$ e $v_{máx} = \pm \omega A$

então $v(0) = \pm \omega A \operatorname{sen}(\pi \times 0 + \delta) = \frac{1}{2} \omega A$

$\pm \operatorname{sen} \delta = \frac{1}{2} \Leftrightarrow \operatorname{sen} \delta = \pm \frac{1}{2}$

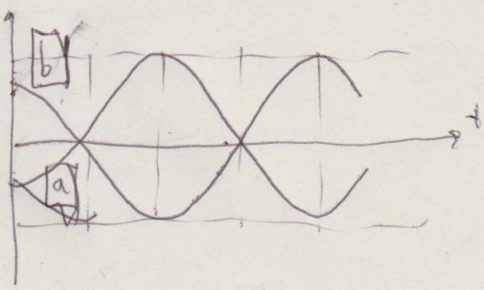
$\Leftrightarrow \delta = \pm \frac{\pi}{6}$ rad   2 soluções possíveis $\boxed{a}$ e $\boxed{b}$



$v(t) = -0.5\pi \operatorname{sen}(\pi t + \pi/6)$ $\boxed{a}$

ou

$v(t) = -0.5\pi \operatorname{sen}(\pi t - \pi/6)$ $\boxed{b}$



$a(t) = -0.5\pi^2 \cos(\pi t + \frac{\pi}{6})$ $\boxed{a}$

ou

$a(t) = -0.5\pi^2 \cos(\pi t - \frac{\pi}{6})$ $\boxed{b}$

$x(t) = 0.5 \cos(\pi t + \frac{\pi}{6})$ $\boxed{a}$

ou

$x(t) = 0.5 \cos(\pi t - \frac{\pi}{6})$ $\boxed{b}$

c) gráfico de $a(t) \equiv F(t)$

d) $$A = \frac{\frac{F_0}{m}}{\sqrt{(\omega_f^2 - \omega_0^2)^2 + \underbrace{\left(\frac{b\omega_f}{m}\right)^2}_{=0}}} \qquad b = 0$$

$$A = \frac{\frac{2}{0.5}}{\sqrt{(5^2 - \pi^2)^2}} = 0.26 \text{ m}$$