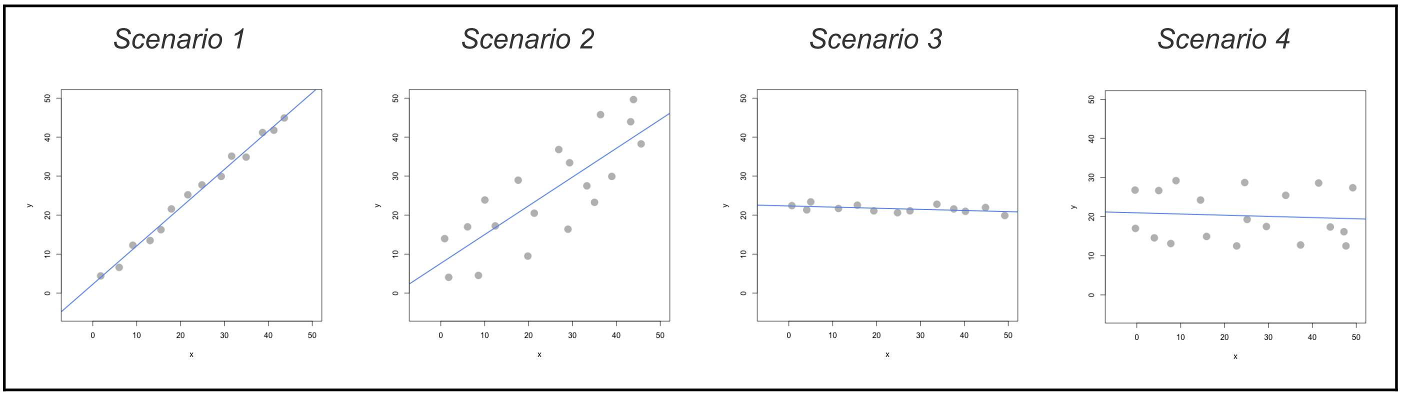
BACS HW (Week 10)

Question 1) Download `demo_simple_regression_rsq.R` from Canvas – it has a function that runs a regression simulation. This week, the simulation also reports R^2 along with the other metrics from last week.

To answer the questions below, understand each of these four scenarios by simulating them:
Scenario 1: Consider a very narrowly dispersed set of points that have a negative or positive steep slope
Scenario 2: Consider a widely dispersed set of points that have a negative or positive steep slope
Scenario 3: Consider a very narrowly dispersed set of points that have a negative or positive shallow slope
Scenario 4: Consider a widely dispersed set of points that have a negative or positive shallow slope



a. Comparing scenarios 1 and 2, which do we expect to have a stronger R^2 ?
b. Comparing scenarios 3 and 4, which do we expect to have a stronger R^2 ?
c. Comparing scenarios 1 and 2, which do we expect has bigger/smaller SSE, SSR, and SST? (intuitively)
d. Comparing scenarios 3 and 4, which do we expect has bigger/smaller SSE, SSR, and SST? (intuitively)

Question 2) Let's perform regression ourselves on the `programmer_salaries.txt` dataset we saw in class
You can read the file using `read.csv("programmer_salaries.txt", sep="\t")`

a. First, use the `lm()` function to estimate the model `Salary ~ Experience + Score + Degree`
   (show the beta coefficients, R^2 and the first 5 values of $\hat{y}$ (`$fitted.values`) and ε (`$residuals`)
b. Use only linear algebra (and the geometric view of regression) to estimate the regression yourself:
   i. Create an `X` matrix that has a first column of 1s followed by columns of the independent variables *(only show the code)*
   ii. Create a `y` vector with the Salary values *(only show the code)*
   iii. Compute the `beta_hat` vector of estimated regression coefficients *(show the code and values)*
   iv. Compute a `y_hat` vector of estimated $\hat{y}$ values, and a `res` vector of residuals
   (show the code and the first 5 values of `y_hat` and `res`)
   v. Using only the results from (i) – (iv), compute SSR, SSE and SST *(show the code and values)*
c. Compute R^2 for scenario 2 in two ways, and confirm you get the same results *(show code and values)*:
   i. Use any combination of SSR, SSE, and SST
   ii. Use the squared correlation of vectors y and $\hat{y}$

(see question 3 on next page)

Question 3) We're going to take a look back at the early heady days of global car manufacturing, when American, Japanese, and European cars competed to rule the world. Take a look at the data set in file `auto-data.txt`. We are interested in explaining what kind of cars have higher fuel efficiency (`mpg`).

```
1. mpg:           miles-per-gallon (dependent variable)
2. cylinders:     cylinders in engine
3. displacement:  size of engine
4. horsepower:    power of engine
5. weight:        weight of car
6. acceleration:  acceleration ability of car
7. model_year:    year model was released
8. origin:        place car was designed (1: USA, 2: Europe, 3: Japan)
9. car_name:      make and model names
```

Note that the data has missing values ('?' in data set), and lacks a header row with variable names:

```
auto <- read.table("auto-data.txt", header=FALSE, na.strings = "?")
names(auto) <- c("mpg", "cylinders", "displacement", "horsepower", "weight",
                 "acceleration", "model_year", "origin", "car_name")
```

a. Let's first try exploring this data and problem:
   i. Visualize the data in any way you feel relevant (report only relevant/interesting ones)
   ii. Report a correlation table of all variables, rounding to two decimal places (in the `cor()` function, set `use="pairwise.complete.obs"` to handle missing values)
   iii. From the visualizations and correlations, which variables seem to relate to `mpg`?
   iv. Which relationships might not be linear? *(don't worry about linearity for rest of this HW)*
   v. Are there any pairs of independent variables that are highly correlated (*r > 0.7*)?
b. Let's create a linear regression model where `mpg` is dependent upon all other suitable variables *(Note: `origin` is categorical with three levels, so use `factor(origin)` in `lm(...)` to split it into two dummy variables)*
   i. Which independent variables have a 'significant' relationship with mpg at 1% significance?
   ii. Looking at the coefficients, is it possible to determine which independent variables are the *most effective* at increasing mpg? If so, which ones, and if not, why not? (hint: units!)
c. Let's try to resolve some of the issues with our regression model above.
   i. Create fully standardized regression results: are these slopes easier to compare? (note: consider if you should standardize `origin`)
   ii. Regress mpg over each *nonsignificant* independent variable, individually. Which ones become significant when we regress `mpg` over them individually?
   iii. Plot the density of the *residuals*: are they normally distributed and centered around zero? (get the residuals of a fitted linear model, e.g. `regr <- lm(...)`, using `regr$residuals`