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Problem 1.

CTFS: $x(t) = \sum_{k=-\infty}^{+\infty} \frac{1}{T} \int_0^T x(t) e^{-j\frac{k2\pi t}{T}} dt \, e^{+j\frac{k2\pi t}{T}}$

$$\begin{cases} X[k] = \int_0^T x(t) e^{-j\frac{k2\pi t}{T}} dt \\ x(t) = \sum_{k=-\infty}^{+\infty} \frac{1}{T} X[k] e^{+j\frac{k2\pi t}{T}} \end{cases}$$

DTFS: $x[n] = \sum_{k=0}^{N-1} \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\frac{k2\pi n}{N}} e^{+j\frac{k2\pi n}{N}}$

$$\begin{cases} X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{k2\pi n}{N}} \\ x[n] = \sum_{k=0}^{N-1} \frac{1}{N} X[k] e^{+j\frac{k2\pi n}{N}} \end{cases}$$

Problem 2

$x_{p\Delta}(t) = x_p(t) \times \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$

$x_p[n] = x_p(nT_s)$

$$\Rightarrow X_{p\Delta}[k] = \int_0^T x_{p\Delta}(t) e^{-j\frac{k2\pi t}{T}} dt$$

$$= \int_0^T x_p(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) e^{-j\frac{k2\pi t}{T}} dt$$

$$= \sum_{n=-\infty}^{+\infty} x(nT_s) e^{-j\frac{k2\pi nT_s}{T}} \int_0^T \delta(t - nT_s) dt$$

$$= \sum_{n=0}^{N-1} x(nT_s) e^{-j\frac{k2\pi n}{N}}$$

$\because \tilde{X}_p[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{k2\pi n}{N}}$

$\therefore X_{p\Delta}[k] = \tilde{X}_p[k]$

On the other hand, recall that

$$\sum_{n=-\infty}^{+\infty} \delta(t-nT_s) = \frac{1}{T_s} \sum_{m=-\infty}^{+\infty} 1 \cdot e^{j\frac{m2\pi t}{T_s}}$$

Hence $X_{ps}[k] = \int_0^T x_{ps}(t) e^{-j\frac{k2\pi t}{T}} dt$

$$= \int_0^T x(t) \sum_{n=-\infty}^{+\infty} \delta(t-nT_s) e^{-j\frac{k2\pi t}{T}} dt, \text{ let } f_s = \frac{1}{T_s}$$

$$= \int_0^T x(t) f_s \sum_{m=-\infty}^{+\infty} 1 \cdot e^{j\frac{mN2\pi t}{NT_s}} e^{-j\frac{k2\pi t}{T}} dt$$

$$= f_s \sum_{m=-\infty}^{+\infty} \int_0^T x(t) e^{-j\frac{(k-mN)2\pi t}{T}} dt$$

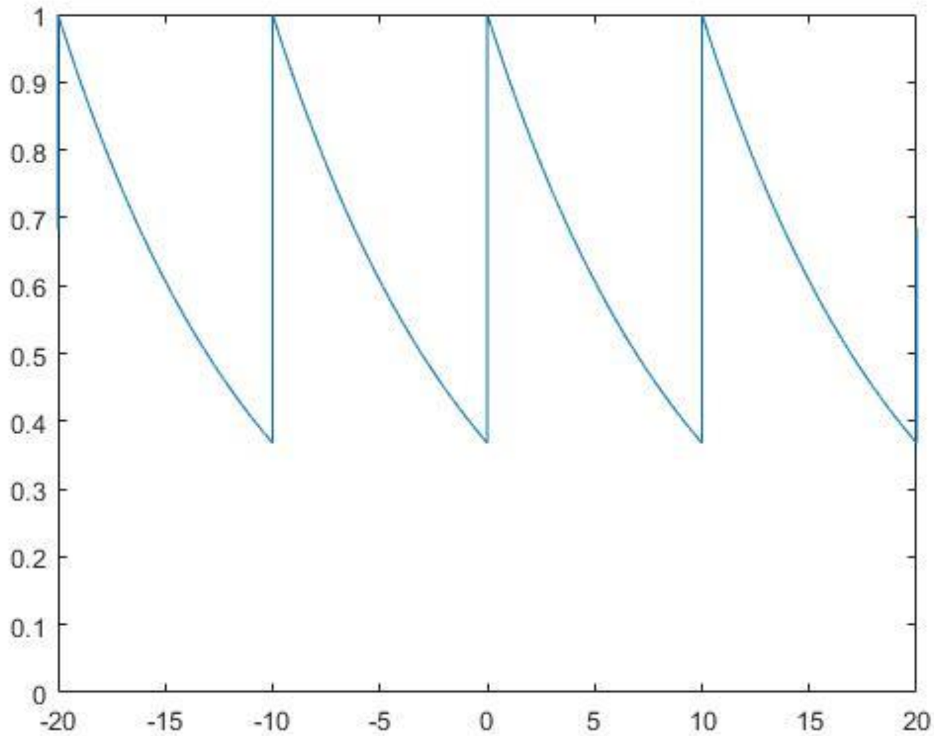
$$= f_s \sum_{m=-\infty}^{+\infty} X_p[k-mN]$$

$$\therefore X_{ps}[k] = \tilde{X}_p[k] = f_s \sum_{m=-\infty}^{+\infty} X_p[k-mN]$$

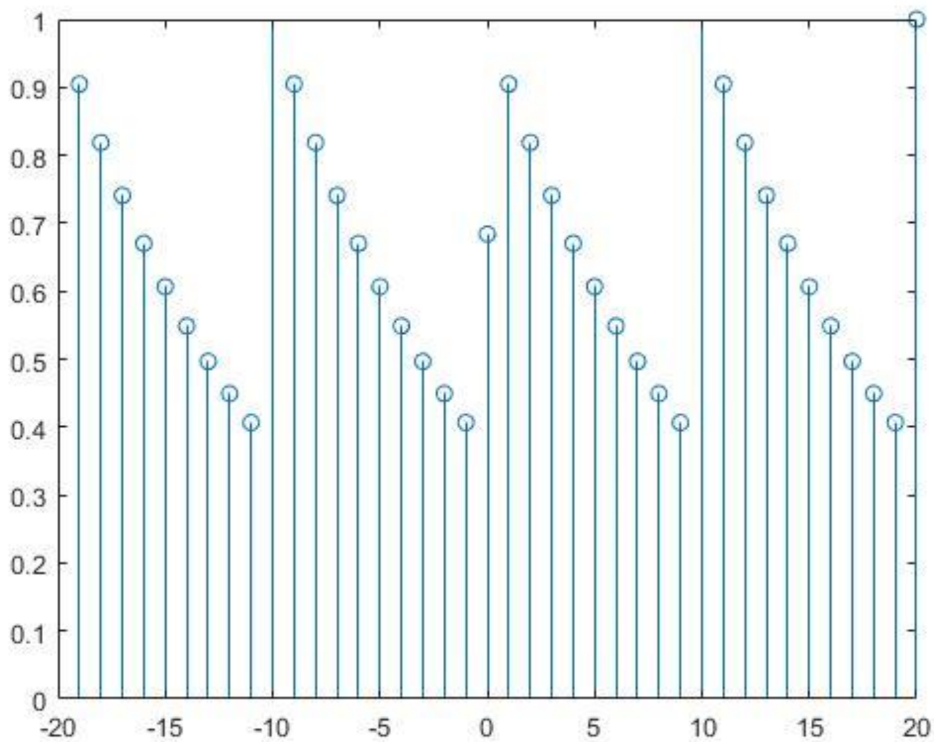
Q.E.D.

Problem3

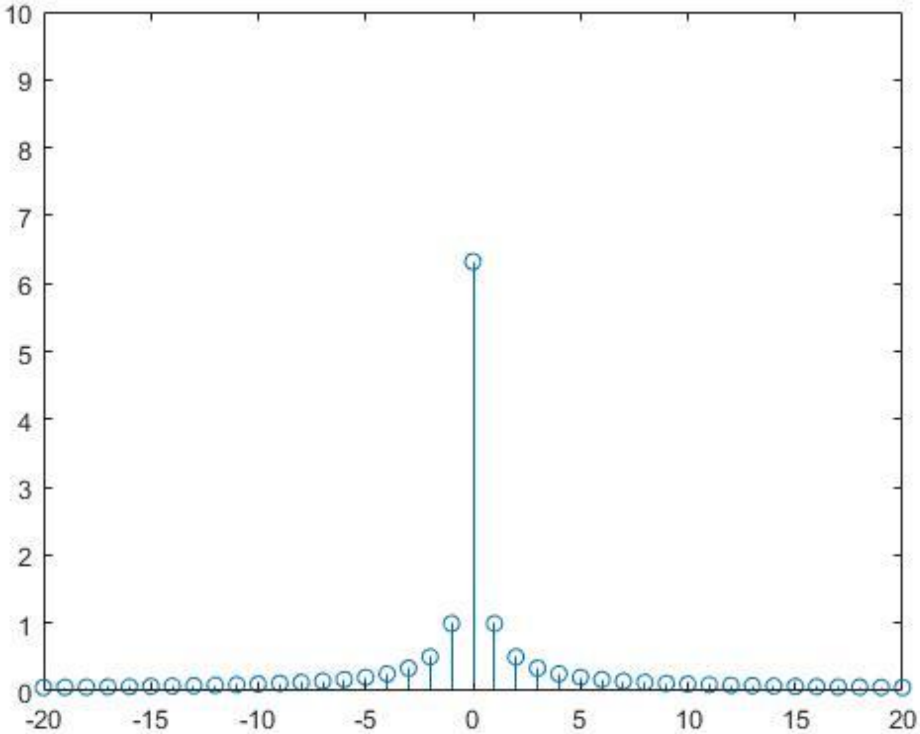
▼$x_p(t)$



▼$x_p[n]$



▼$|X_p[k]|$



▼$|\tilde{X}_p[k]|$