HW3 106061218 李駿恩

Problem 1.

DTFT: $x[n]=\int_{-\frac{1}{2}}^{+\frac{1}{2}}\sum_{n=-\infty}^{+\infty}x[n]e^{-j2\pi fn}\cdot e^{+j2\pi fn}\,df$

$$\begin{cases} X(f)=\sum_{n=-\infty}^{+\infty}x[n]e^{-j2\pi fn} \\ x[n]=\int_{-\frac{1}{2}}^{+\frac{1}{2}}X(f)e^{+j2\pi fn}\,df \end{cases}$$

DTFS: $x[n]=\sum_{k=0}^{N-1}\frac{1}{N}\sum_{n=0}^{N-1}x[n]e^{-j\frac{k2\pi n}{N}}e^{j\frac{k2\pi n}{N}}$

$$\begin{cases} X[k]=\sum_{n=0}^{N-1}x[n]e^{-j\frac{k2\pi n}{N}} \\ x[n]=\sum_{k=0}^{N-1}\frac{1}{N}X[k]e^{+j\frac{k2\pi n}{N}} \end{cases}$$

Problem 2.

Claim: $X_p[k]=X_1(f=k\cdot\frac{1}{N})$

pf: $X_1(f)=\sum_{n=-\infty}^{+\infty}x_1[n]e^{-j2\pi fn}$

$=\sum_{n=0}^{N-1}x_1[n]e^{-j2\pi fn}$ (Since $x_1[n]$ is finite-length)

$X_p[k]=\sum_{n=0}^{N-1}x_1[n]e^{-j\frac{k2\pi n}{N}}$

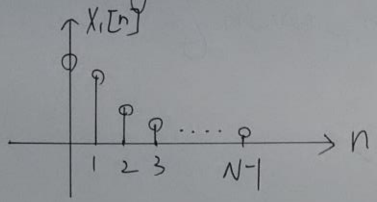
Compare two equations above, we have

$X_p[k]=X_1(f=k\cdot\frac{1}{N})$

Q.E.D.

Problem 3.

Scatching $x_1[n]$, we have:



1. $X_1(f) = \sum_{n=-\infty}^{+\infty} x_1[n] e^{-j2\pi fn}$

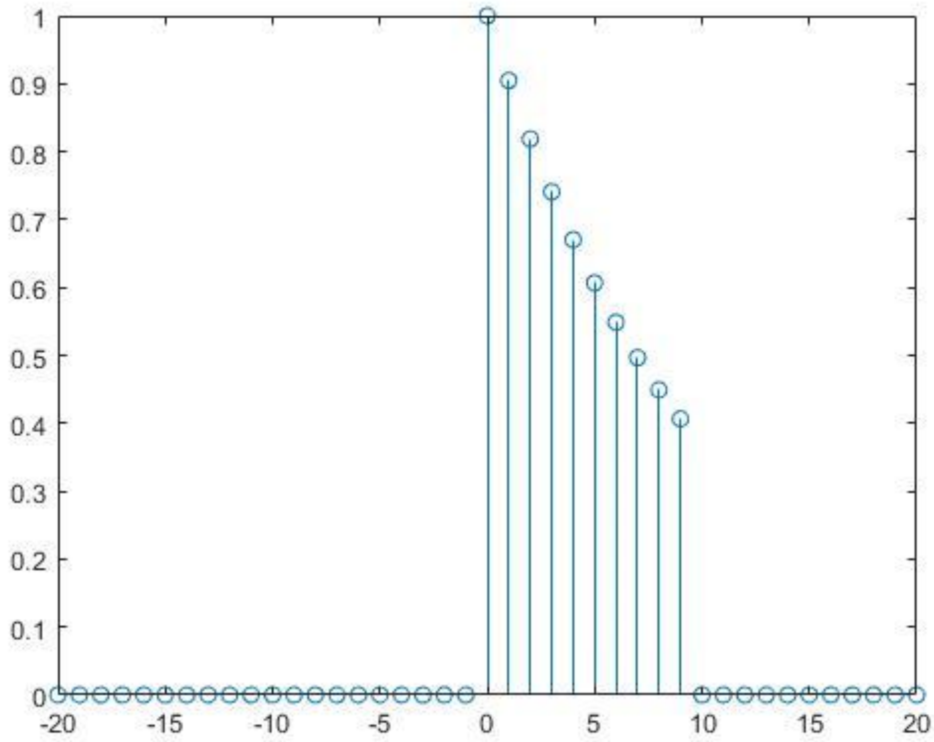
$= \sum_{n=0}^{N-1} e^{-an} e^{-j2\pi fn}$ (since $x_1[n]$ is finite-length)

$= \sum_{n=0}^{N-1} e^{-(a+j2\pi f)n}$

$= \frac{1 \times (1 - e^{-(a+j2\pi f)\times N})}{1 - e^{-(a+j2\pi f)}}$ (since it is a geometric series)

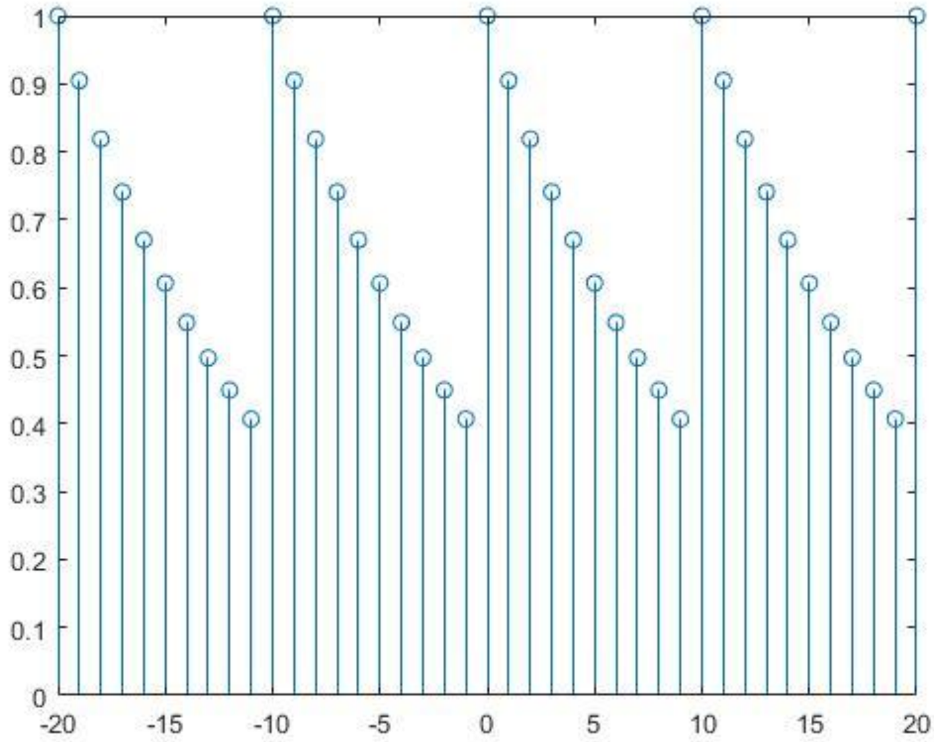
$X_p[k] = X_1(f = k \cdot \frac{1}{N})$

$= \frac{1 - e^{-(a+j2\pi \frac{k}{N})\times N}}{1 - e^{-(a+j2\pi \frac{k}{N})}}$



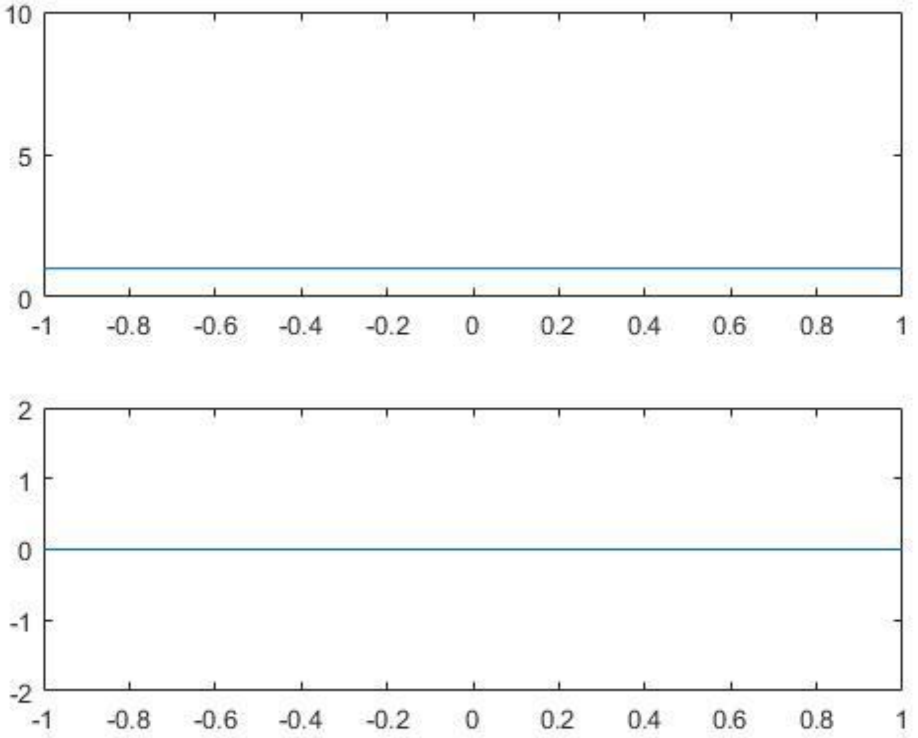
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-20
-15
-10
-5
0
5
10
15
20

▲x_1[n]



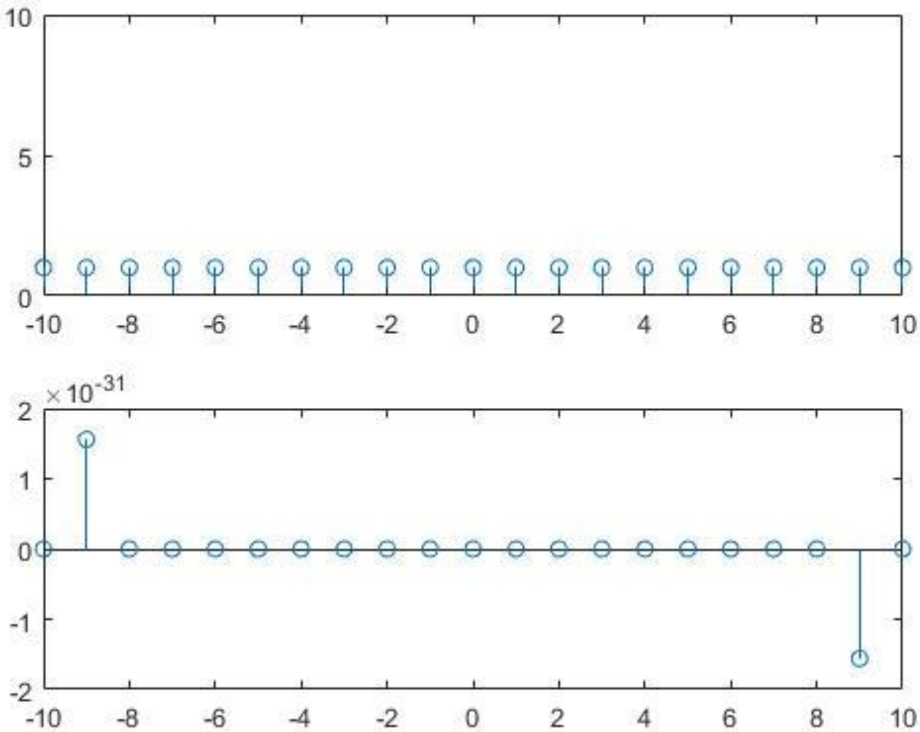
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-20
-15
-10
-5
0
5
10
15
20

▲x_p[n]



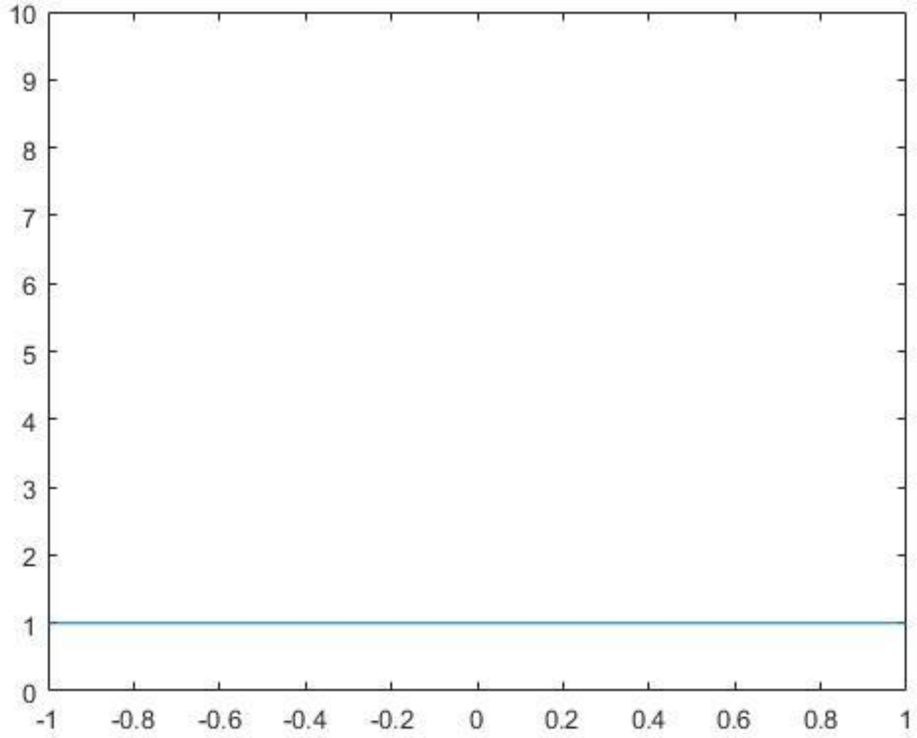
10
5
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
2
1
0
-1
-2

▲X_1(f)

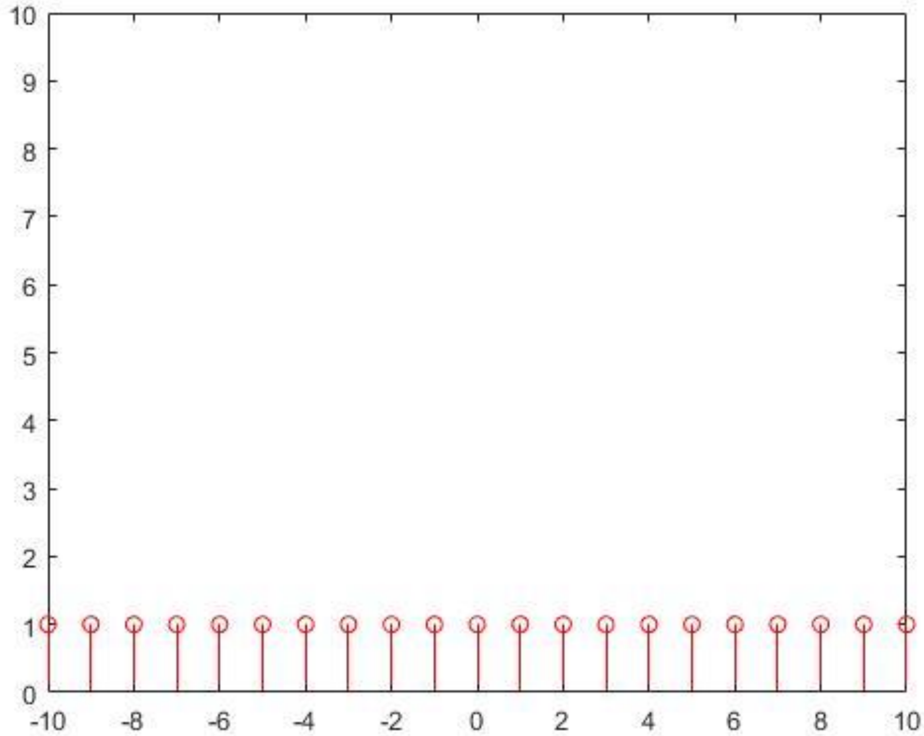


10
5
0
-10
-8
-6
-4
-2
0
2
4
6
8
10
×10^-31
2
1
0
-1
-2

▲X_p[k]



▲$|X_1(\mathrm{f})|$



▲$|X_p[\mathrm{k}]|$