10606l218 李承恩 HW7

1. $y[n] = x[n] + ay[n-1]$

$\Rightarrow y[n] - ay[n-1] = x[n]$

2. 令 $h[n] = w[n]u[n]$ 代入 $h[n] - ah[n-1] = \delta[n]$

$\Rightarrow w[n]u[n] - aw[n-1]u[n-1] = \delta[n]$

$\Rightarrow w[n]u[n] - w[n]u[n-1] + w[n]u[n-1] - aw[n-1]u[n-1] = \delta[n]$

$\Rightarrow w[n](u[n] - u[n-1]) + u[n-1](w[n] - aw[n-1]) = \delta[n]$

$\Rightarrow w[0]\delta[n] + (w[n] - aw[n-1])u[n-1] = 0$

$\therefore w[n] - aw[n-1] = 0,\ w[0] = 1$

$\Rightarrow$ 令 $w[n] = C \cdot a^n$，则 $C = 1$

$\therefore h[n] = a^n u[n]$.

3. $H(f) = \sum_{n=-\infty}^{+\infty} h[n] e^{-j2\pi fn}$

$= \sum_{n=0}^{\infty} (ae^{-j2\pi f})^n$

$= \frac{1}{1 - ae^{-j2\pi f}}, \quad a < 1$

$$\begin{cases} |H(f)| = \dfrac{1}{\sqrt{(a^2+1) - 2a\cos 2\pi f}} \\ \angle H(f) = \tan^{-1} \dfrac{a\sin 2\pi f}{1 - a\cos 2\pi f} \end{cases}$$

4. $x[n]$ is a pulse train

Consider the DTFS of $x[n]$

$$X[k] = \sum_{k=0}^{N-1} x[n] e^{-j\frac{k2\pi n}{N}}$$

$$= x[0]e^{-j\frac{0\cdot 2\pi n}{8}} + x[1]e^{-j\frac{1\cdot 2\pi n}{8}} + \cdots + x[7]e^{-j\frac{7\cdot 2\pi n}{8}}$$

$$= 1$$

$$\therefore x[n] = \sum_{k=0}^{7} \frac{1}{8} e^{j\frac{k2\pi n}{8}}$$

$$= \frac{1}{8}\left(\begin{matrix} e^{j2\pi\frac{0}{8}n} + e^{j2\pi\cdot\frac{4}{8}n} \\ e^{j2\pi\frac{1}{8}n} + e^{j2\pi\frac{7}{8}n} \\ e^{j2\pi\frac{2}{8}n} + e^{j2\pi\frac{6}{8}n} \\ e^{j2\pi\frac{3}{8}n} + e^{j2\pi\frac{5}{8}n} \end{matrix}\right)$$

$$= \frac{1}{8}\left(1 + (-1)^n + 2\cos 2\pi\cdot\frac{1}{8}n + 2\cos 2\pi\cdot\frac{3}{8}n\right)$$

$$= \frac{1}{8}\left(1+(-1)^n\right) + \frac{1}{4}\cos\frac{\pi n}{4} + \frac{1}{4}\cos\frac{\pi n}{2} + \frac{1}{4}\cos\frac{3\pi n}{4}$$

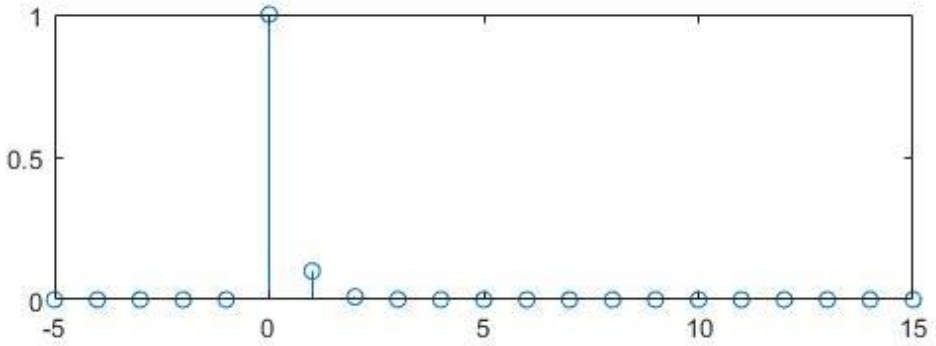
$$\Rightarrow y[n] = \frac{1}{8}\left|H(0)\right|\left(1+(-1)^n\right) + \frac{1}{4}\left|H(\tfrac{1}{8})\right|\cos\left(\frac{\pi n}{4} + \angle H(\tfrac{1}{8})\right)$$

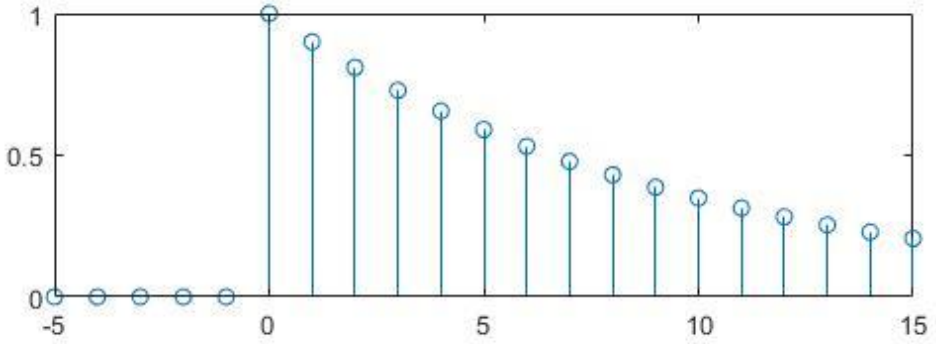
$$+ \frac{1}{4}\left|H(\tfrac{1}{4})\right|\cos\left(\frac{\pi n}{2} + \angle H(\tfrac{1}{4})\right)$$

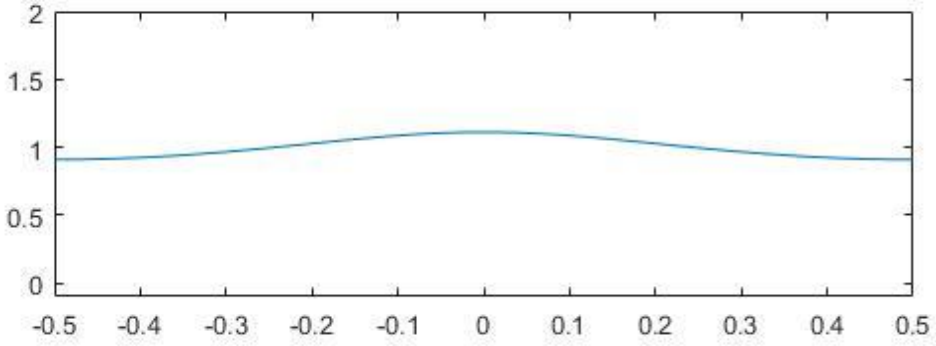
$$+ \frac{1}{4}\left|H(\tfrac{3}{8})\right|\cos\left(\frac{3\pi n}{4} + \angle H(\tfrac{3}{8})\right) \quad \#$$

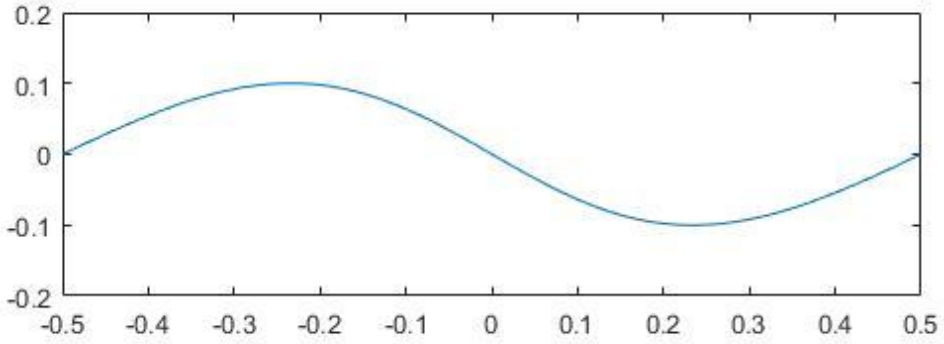
5. a 較大時，振動劇烈，振幅亦較大

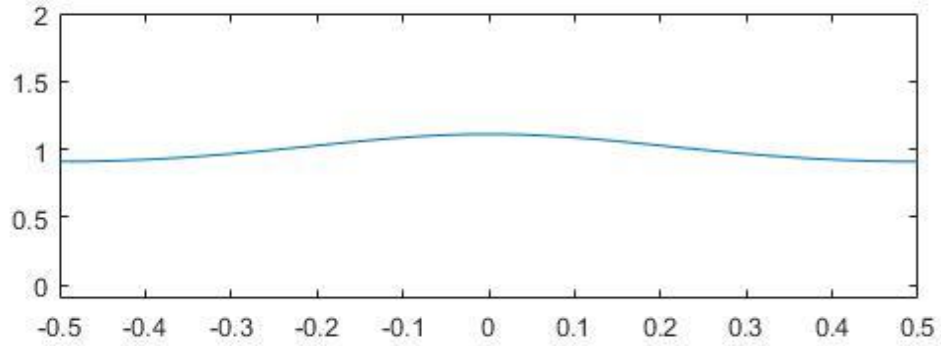
以下各圖片之由上而下之順序皆按照題目所問之先後的順序







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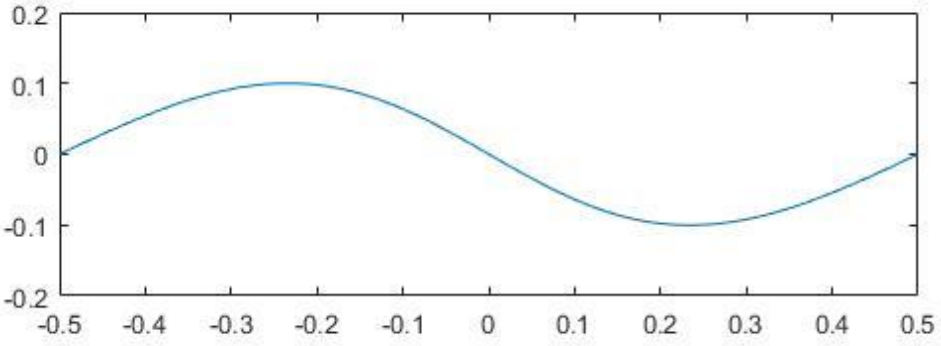
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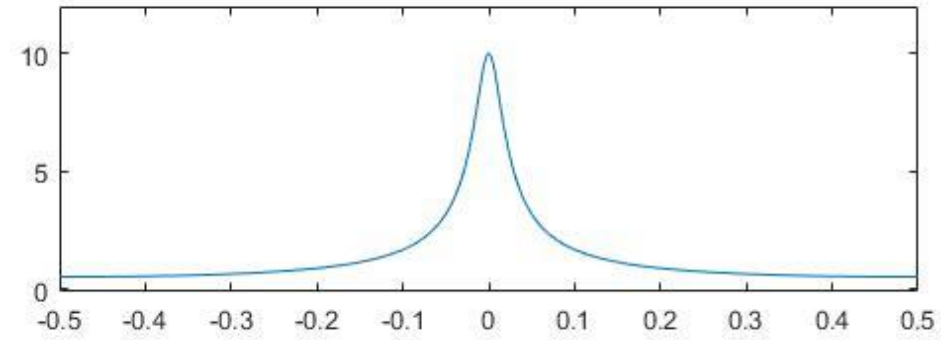
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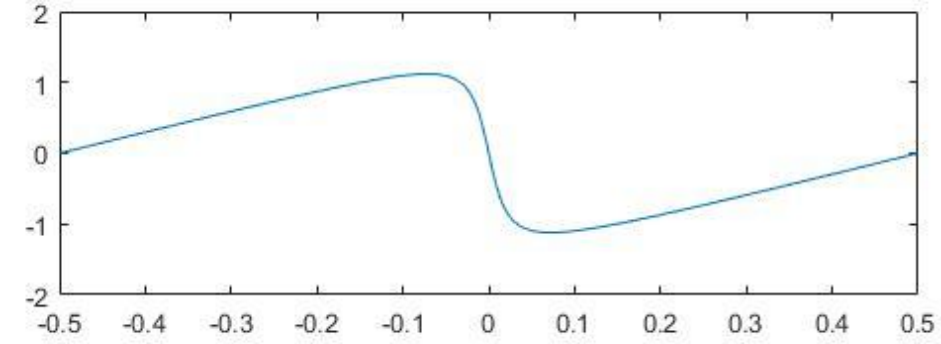
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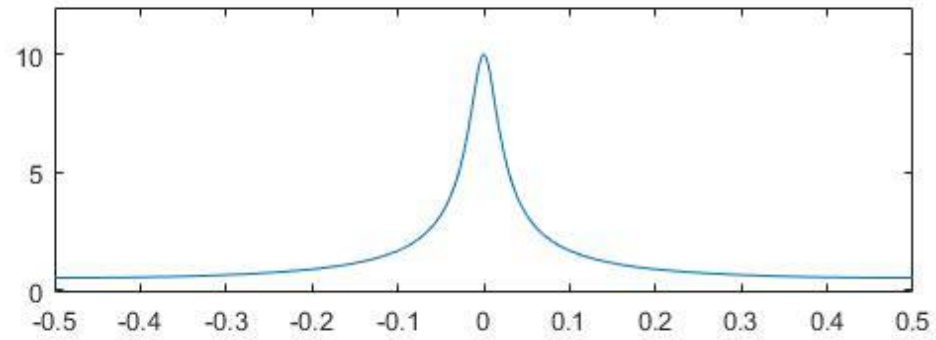
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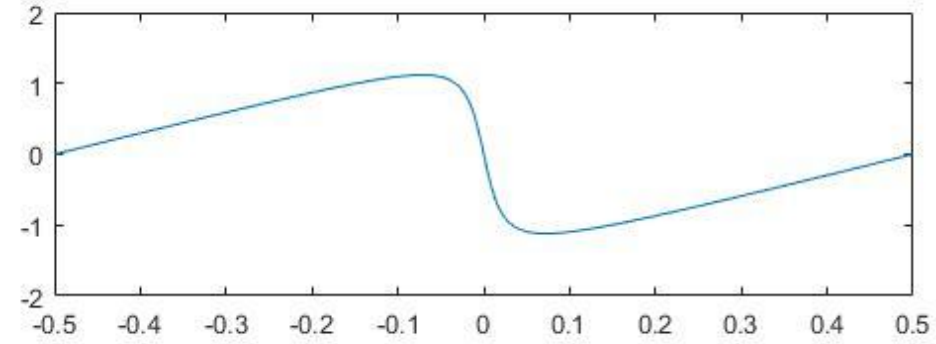
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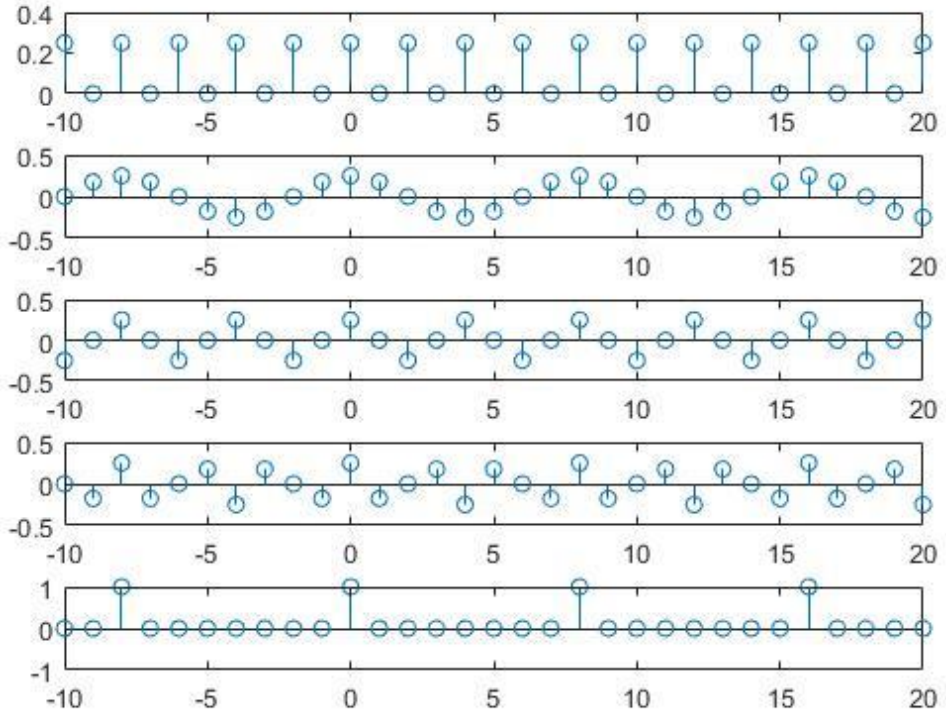
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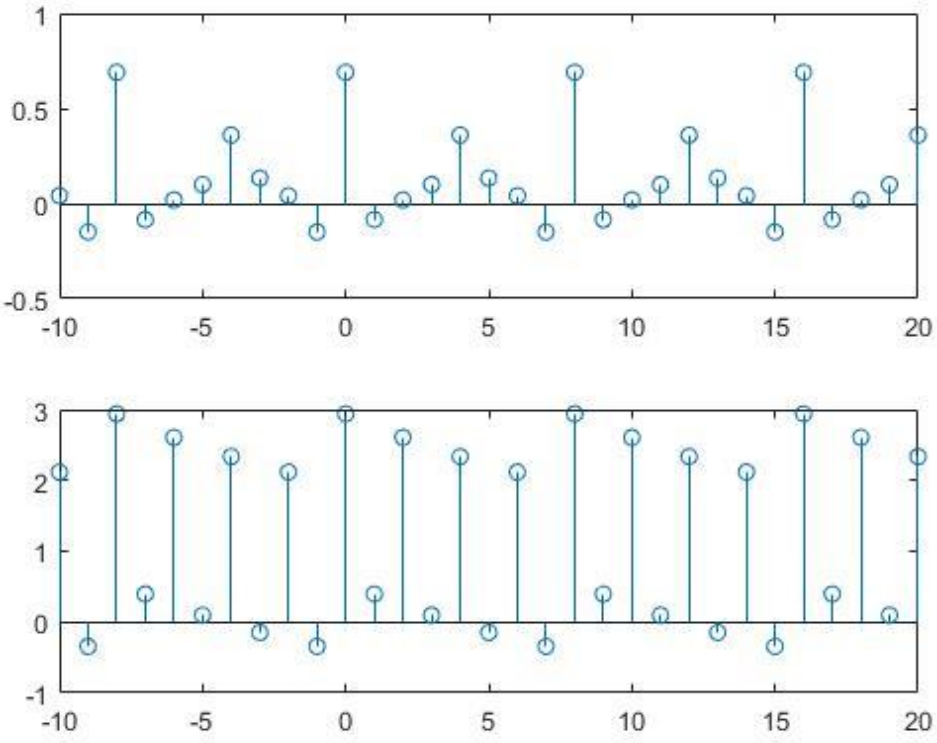
20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