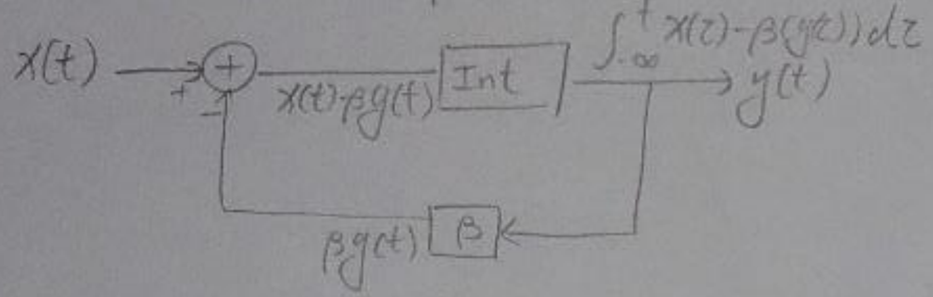
HW6 106061218 李[illegible]恩



1. Since $y(t) = \int_{-\infty}^{t} x(\tau)-\beta y(\tau)d\tau$, we conclude that $y'(t) = x(t)-\beta y(t)$ or $y'(t)+\beta y(t) = x(t)$ by Fundmental Thm of Calculus

2. To find $h(t)$, substitute $y(t)=h(t)$ and $x(t)=\delta(t)$, we have $h'(t)+\beta h(t) = \delta(t)$
To solve this equation, let $h(t)=w(t)u(t)$ plug into the ODE, where $u(t)$ represent unit step function. Hence
$w'u + wu' + \beta wu = \delta(t)$ —(*)
Recall that : $u'(t) = \delta(t)$, hence
$w(t)u'(t) = w(t)\delta(t) = w(0)\delta(t)$ (*) becomes
$(w'+\beta w)u + w(0)\delta(t) = \delta(t)$ Compare the terms of both side, one derives

$$\begin{cases} w'(t)+\beta w(t) = 0 \\ w(0) = 1 \end{cases}$$

Solving this initial value problem,
$w(t) = Ce^{-\beta t}$ and since $w(0)=1$, $C=1$

Hence $h(t) = e^{-\beta t} u(t)$

3. By properties of Fourier Transform,

$$\mathcal{F}\{h(t)\} = H(f) = e^{-\beta t} u(t)$$

and

$$\begin{cases} |H(f)| = \dfrac{1}{\sqrt{\beta^2 + 4\pi^2 f^2}} \\ \angle H(f) = -\tan^{-1} \dfrac{2\pi f}{\beta} \end{cases}$$

4. Expend $x(t)$ in Fourier series,

$$x(t) = \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{1}{2k-1} \sin(2k-1)t$$

$$= \frac{4}{\pi} \sin\left(2\pi \cdot \frac{1}{2\pi} t\right) + \frac{4}{\pi} \cdot \frac{1}{3} \sin\left(2\pi \cdot \frac{3}{2\pi} t\right)$$

$$+ \frac{4}{\pi} \cdot \frac{1}{5} \sin\left(2\pi \cdot \frac{1}{2\pi} \cdot 5t\right) + \cdots$$

Hence $y(t) = \frac{4}{\pi} \left| H\left(\frac{1}{2\pi}\right) \right| \sin\left(2\pi \cdot \frac{1}{2\pi} + \angle H\left(\frac{1}{2\pi}\right)\right)$

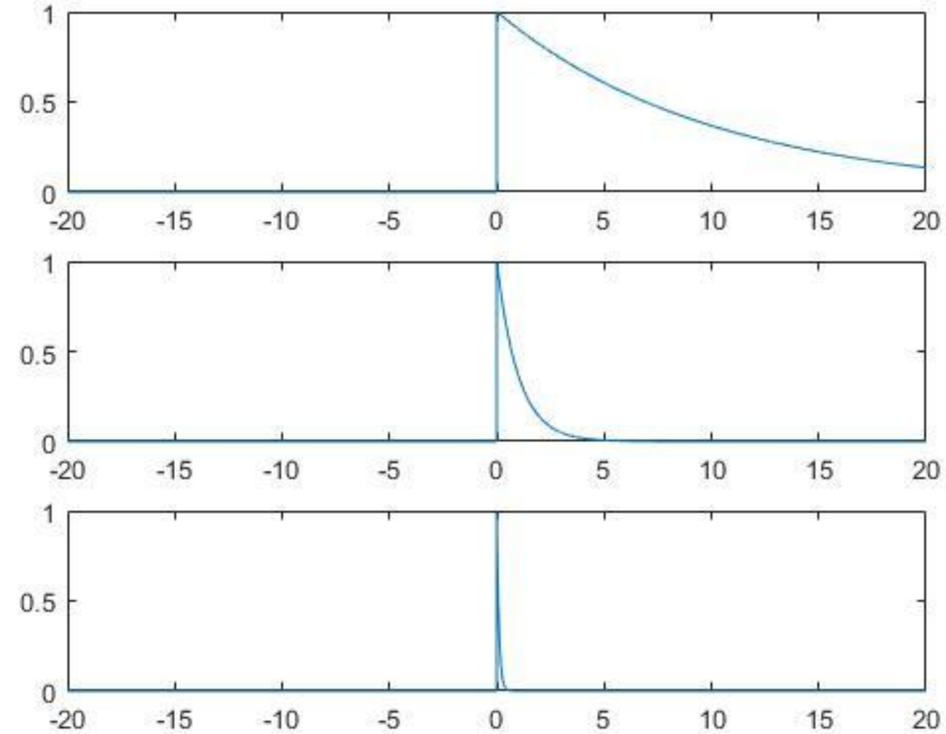
$$+ \frac{4}{\pi} \left| H\left(\frac{3}{2\pi}\right) \right| \sin\left(2\pi \cdot \frac{3}{2\pi} t + \angle H\left(\frac{3}{2\pi}\right)\right)$$

$$+ \cdots$$

We left the plot in MATLAB as follows.

5. In general, the larger the value of β is, the slower the curves declined

(這次嘗試用了数字系的风格写作業 XD)



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0
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1
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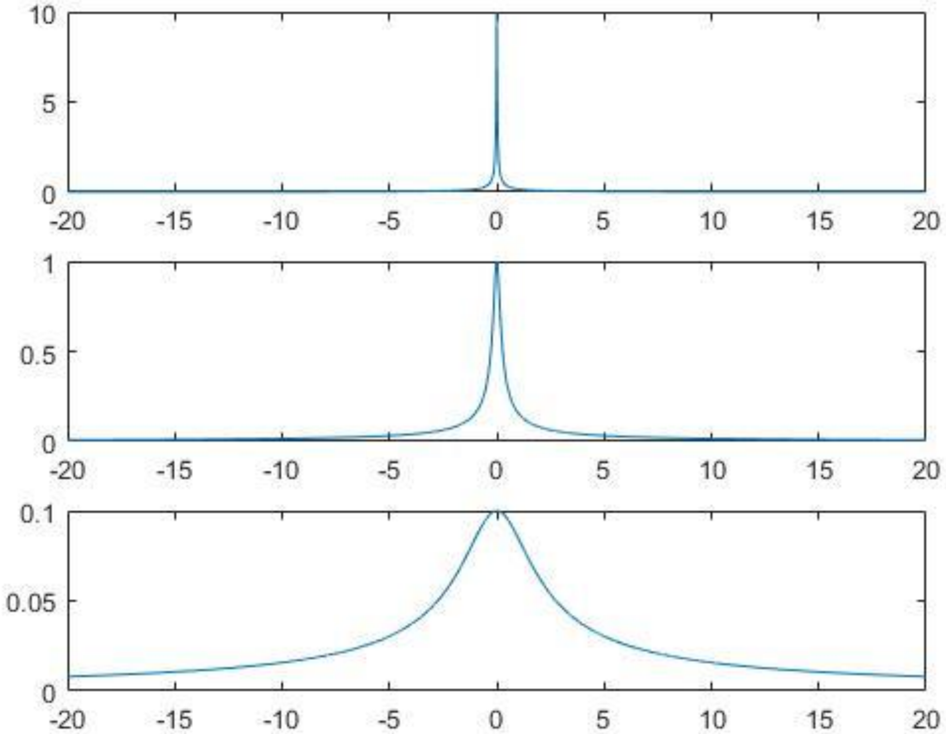
-20 -15 -10 -5 0 5 10 15 20



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0.05
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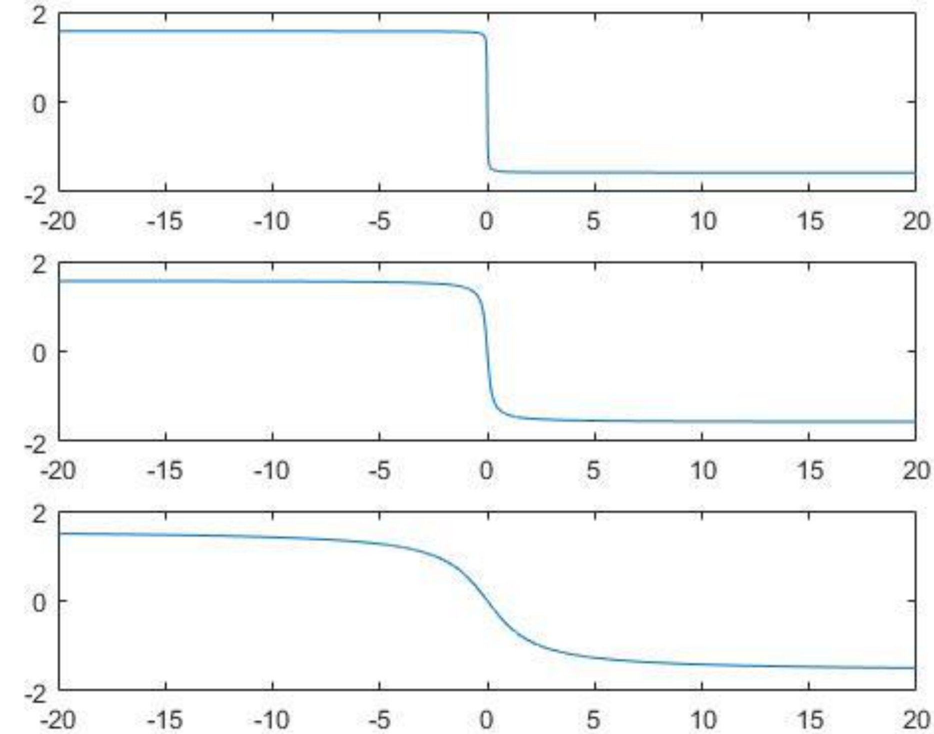
-20 -15 -10 -5 0 5 10 15 20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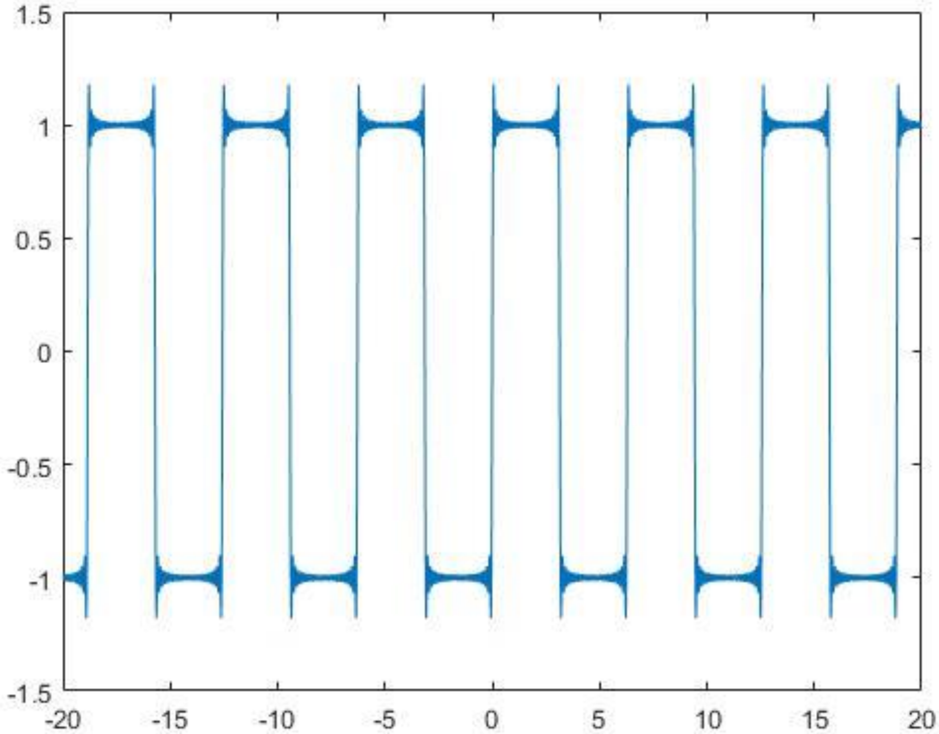
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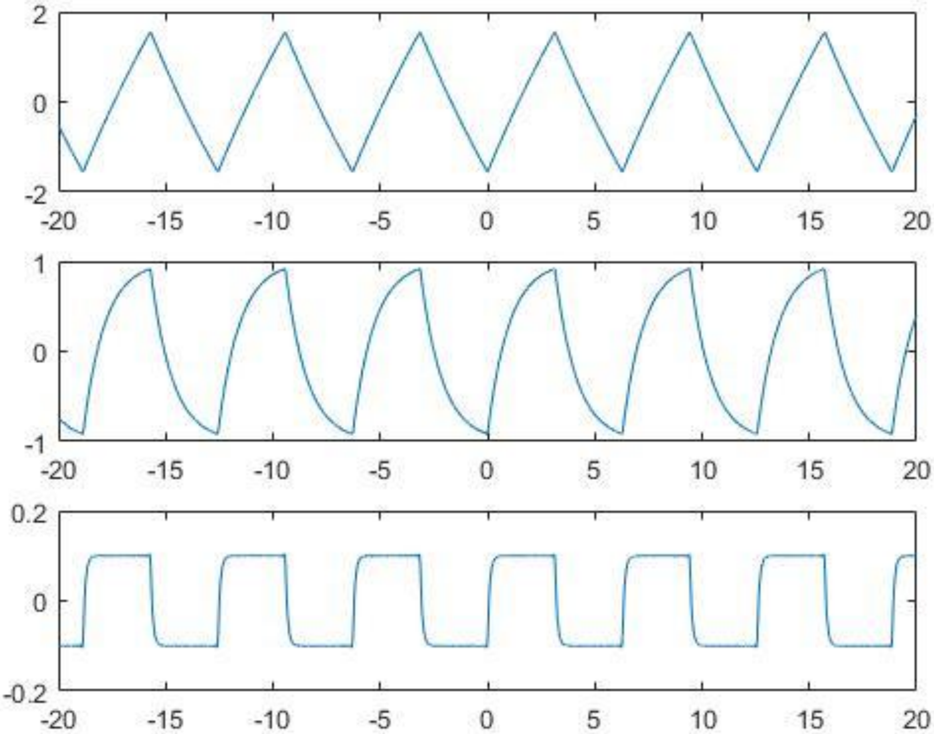
-20 -15 -10 -5 0 5 10 15 20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-1
-20 -15 -10 -5 0 5 10 15 20
0.2
0
-0.2
-20 -15 -10 -5 0 5 10 15 20