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P.1 CTFT:

$$x(t)=\int_{-\infty}^{+\infty}\int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt\,e^{+j2\pi ft}df$$

$$\begin{cases} X(f)=\int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}dt \\ x(t)=\int_{-\infty}^{+\infty} X(f)e^{+j2\pi ft}df \end{cases}$$

DTFT:

$$x[n]=\int_{-\frac{1}{2}}^{+\frac{1}{2}}\sum_{n=-\infty}^{+\infty} x[n]e^{-j2\pi fn}\,e^{+j2\pi fn}df$$

$$\begin{cases} \tilde{X}(f)=\sum_{n=-\infty}^{+\infty} x[n]e^{-j2\pi fn} \\ x[n]=\int_{-\frac{1}{2}}^{+\frac{1}{2}} \tilde{X}(f)e^{+j2\pi fn}df \end{cases}$$

P.2 (1) $X_\Delta(f)=X(f)*f_s\sum_{k=-\infty}^{+\infty}\delta(f-kf_s)$

pf.
$$X_\Delta(f)=\int_{-\infty}^{+\infty} x_\Delta(t)e^{-j2\pi ft}dt$$
$$=\int_{-\infty}^{+\infty} x(t)\times\sum_{k=-\infty}^{+\infty}\delta(t-nT_s)e^{-j2\pi ft}dt$$
$$=\int_{-\infty}^{+\infty} x(t)f_s\sum_{k=-\infty}^{+\infty}e^{\frac{1}{T_s}\cdot j2k\pi t}e^{-j2\pi ft}dt$$
$$=f_s\sum_{k=-\infty}^{+\infty}\int_{-\infty}^{+\infty} x(t)e^{-j2\pi(f-kf_s)t}dt$$
$$=f_s\sum_{k=-\infty}^{+\infty}X(f-kf_s)$$
$$=f_s\big(X(f)+X(f-f_s)+X(f-2f_s)+\cdots$$
$$+X(f+f_s)+X(f+2f_s)+\cdots\big)$$
$$=f_s\big(X(f)*\delta(t)+X(f)*\delta(f-f_s)+X(f)*\delta(f-2f_s)$$
$$+\cdots+X_1(f)*\delta(f+f_s)+X_2(f)*\delta(f+2f_s)+\cdots\big)$$
$$=f_s\sum_{k=-\infty}^{+\infty}X(f)*\delta(f-kf_s)$$

$$= X(f) * f_s \sum_{k=-\infty}^{+\infty} \delta(f - kf_s) \#$$

2. Claim: $X_\Delta(f) = \tilde{X}(\hat{f} = \frac{f}{f_s})$

proof of claim: since $x_\Delta(t) = x(t) \times \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$

$$X_\Delta(f) = \int_{-\infty}^{+\infty} x_\Delta(t) e^{-j2\pi ft} dt$$

$$= \int_{-\infty}^{+\infty} x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \times e^{-j2\pi ft} dt$$

$$= \sum_{n=-\infty}^{+\infty} x(nT_s) e^{-j2\pi \frac{f}{f_s} n} \int_{-\infty}^{+\infty} \delta(t - nT_s) dt$$

$$= \sum_{n=-\infty}^{+\infty} x(nT_s) e^{-j2\pi \frac{f}{f_0} n}$$

also, $\tilde{X}_1(\hat{f}) = \sum_{n=-\infty}^{+\infty} x[n] \cdot e^{-j2\pi \hat{f} n}$

We conclude that $X_\Delta(f) = \tilde{X}_1(\hat{f} = \frac{f}{f_s})$

Q.E.D.

Problem 3.

1. (1) $X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt$

$$= \int_{-\infty}^{+\infty} e^{-at} \cos 2\pi f_0 t \, u(t) e^{-j2\pi ft} dt$$

$$= \int_0^{\infty} e^{-at} \cos 2\pi f_0 t \, e^{-j2\pi ft} dt$$

$$= \frac{1}{2} \int_0^{\infty} e^{-(a + j2\pi(f - f_0)t)} + e^{-(a + j2\pi(f + f_0))t} dt$$

$$= \frac{1}{2}\left(-\frac{1}{a+j2\pi(f-f_0)}e^{-(a+j2\pi(f-f_0))t} + \frac{1}{-(a-j2\pi(f+f_0))}e^{-(a+j2\pi(f+f_0))t}\right)\Bigg|_0^{+\infty}$$

$$= \frac{1}{2}\left(\frac{1}{a+j2\pi(f-f_0)}+\frac{1}{a+j2\pi(f+f_0)}\right)$$

$$= \frac{1}{2}\cdot\frac{2a+j2\pi(f-f_0)+j2\pi(f+f_0)}{(a+j2\pi(f-f_0))(a+j2\pi(f+f_0))}$$

$$= \frac{a+j2\pi f}{a^2+ja2\pi(f+f_0)+ja2\pi(f-f_0)+(j2\pi)^2(f^2-f_0^2)}$$

$$= \frac{a+j2\pi f}{(a+j2\pi f)^2+(2\pi f_0)^2}$$

$\#$

(2) $$X_\Delta(f) = X(f) * \sum_{n=-\infty}^{+\infty}\delta(f-nf_s)$$

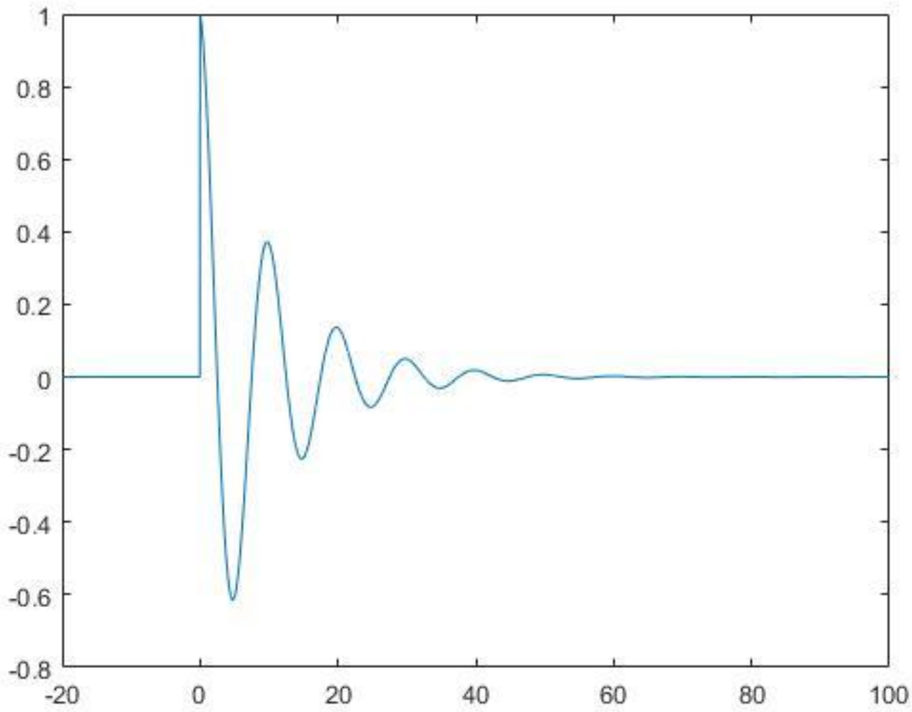
$$= f_s\sum_{m=-\infty}^{+\infty}X(f-mf_s)\ ,\ \because f_s = 1$$

$$= \sum_{m=-\infty}^{+\infty}X(f-m)$$

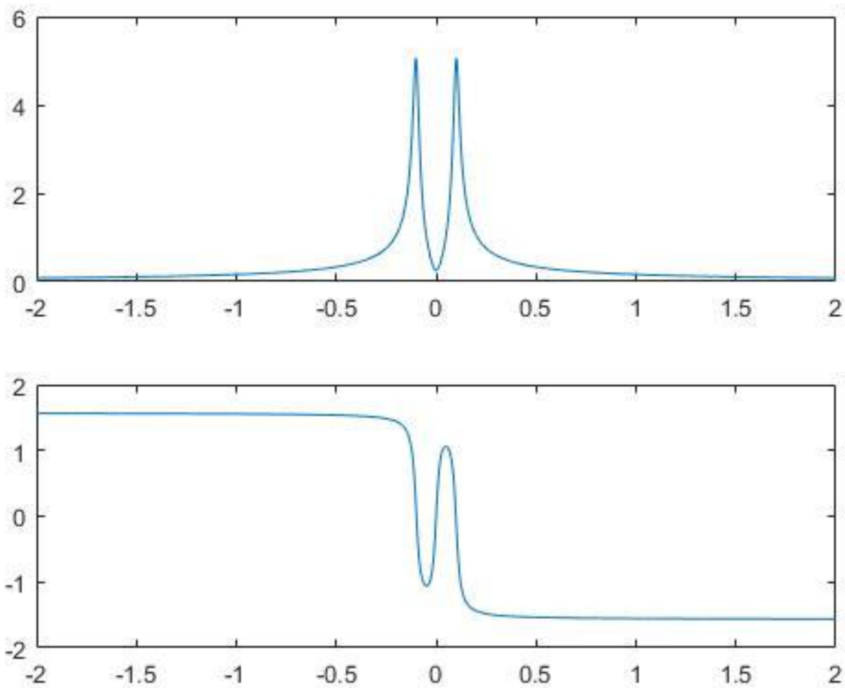
(4) 在時域上來看，振盪週期變小，訊号像被压縮了；在頻域上來看，figure 4頻譜平移了0.5單位變成figure 10，而figure 2半的頻譜被拉伸成8.

(由上而下依序為 figure1~12)

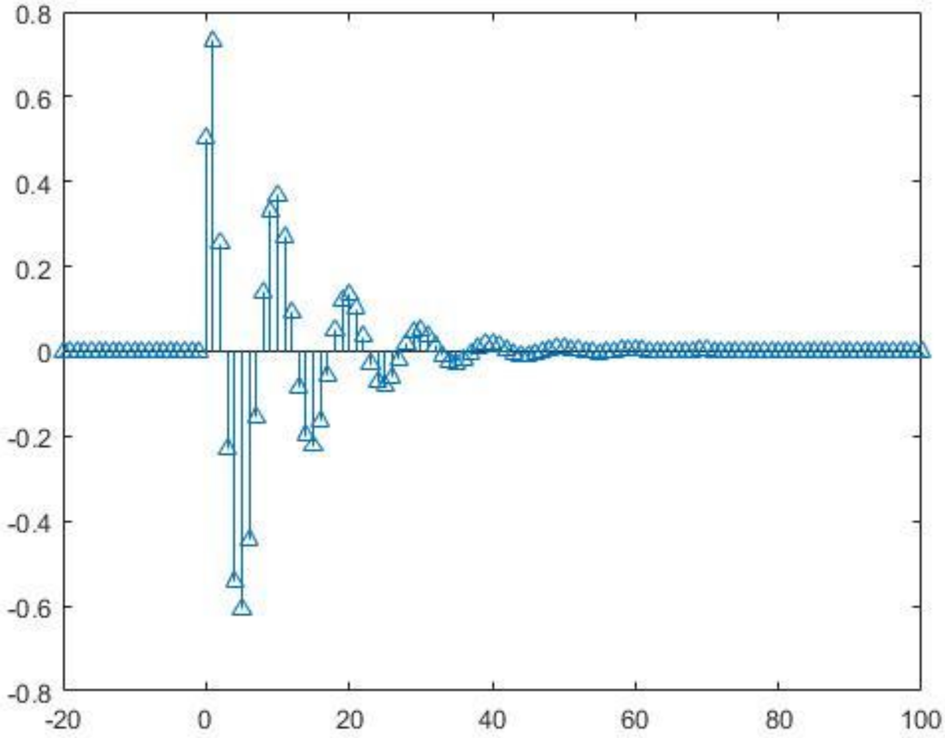
2. a=0.1 f_0=0.1



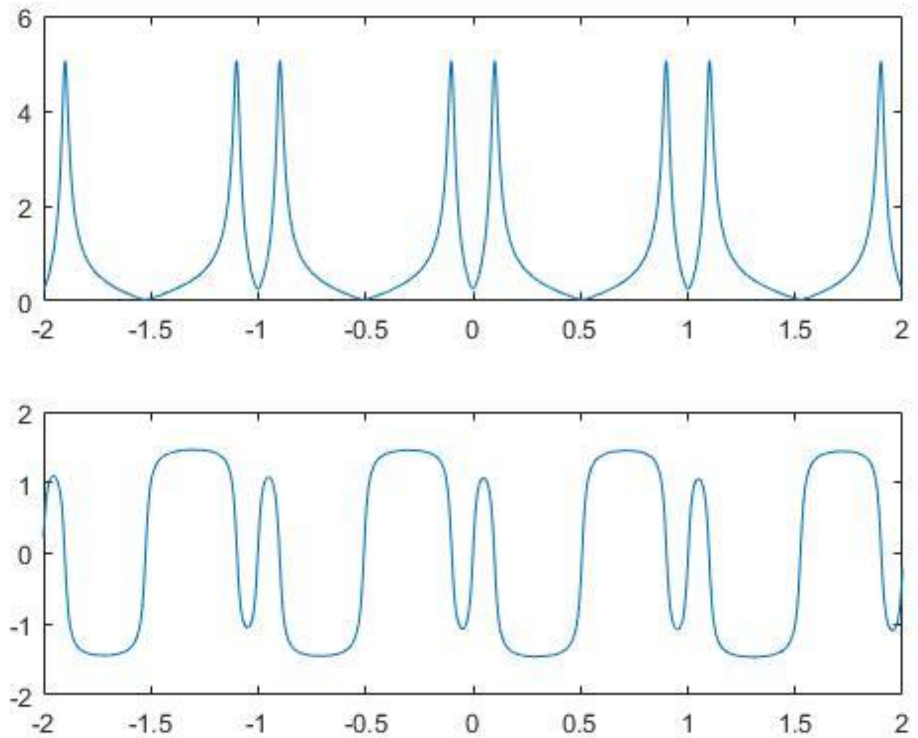
▲x(t)



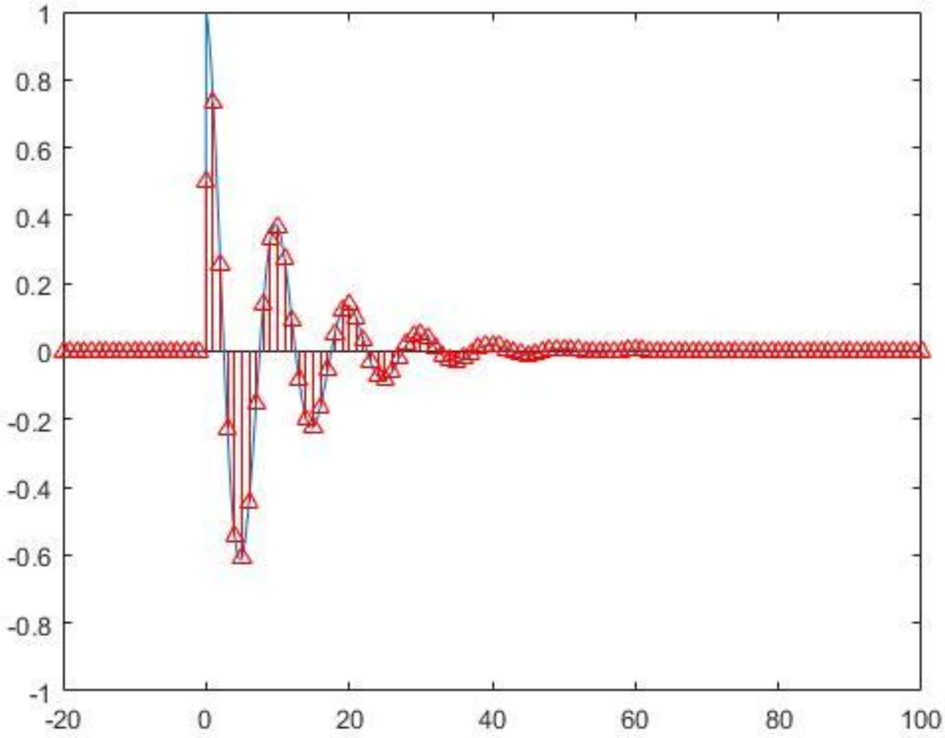
▲X(f)



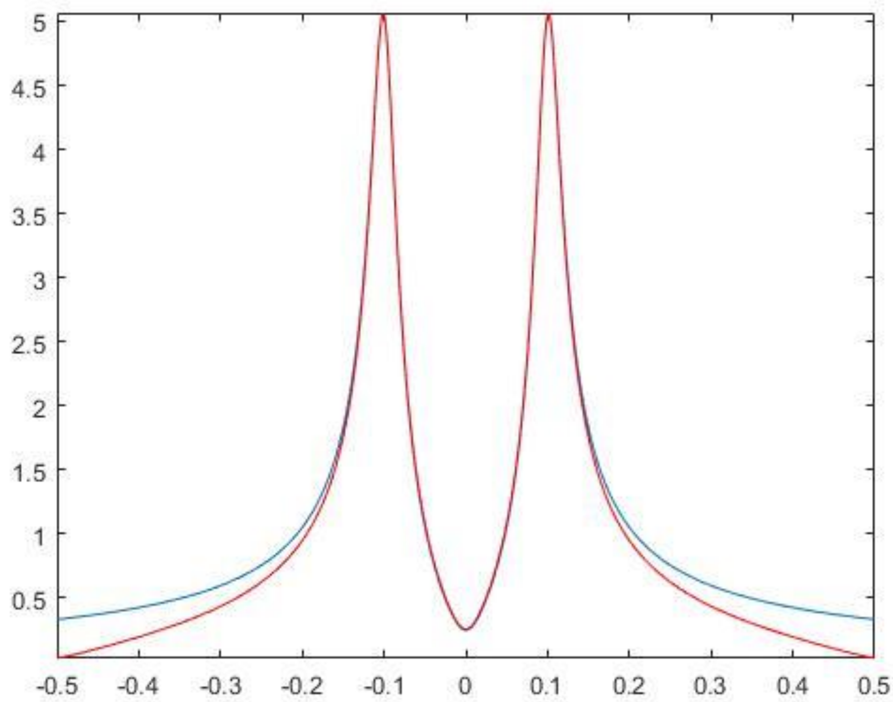
▲$x_\Delta(t)$



▲$X_\Delta(f)$

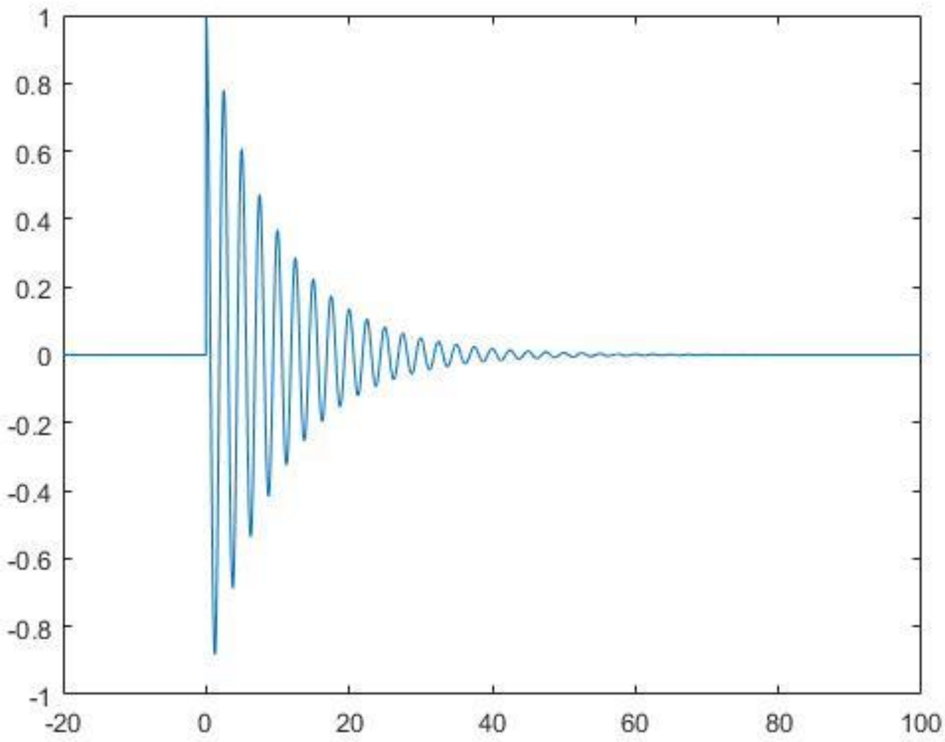


▲Compare of $\mathrm{x(t)}$ and $x_{\Delta}(t)$

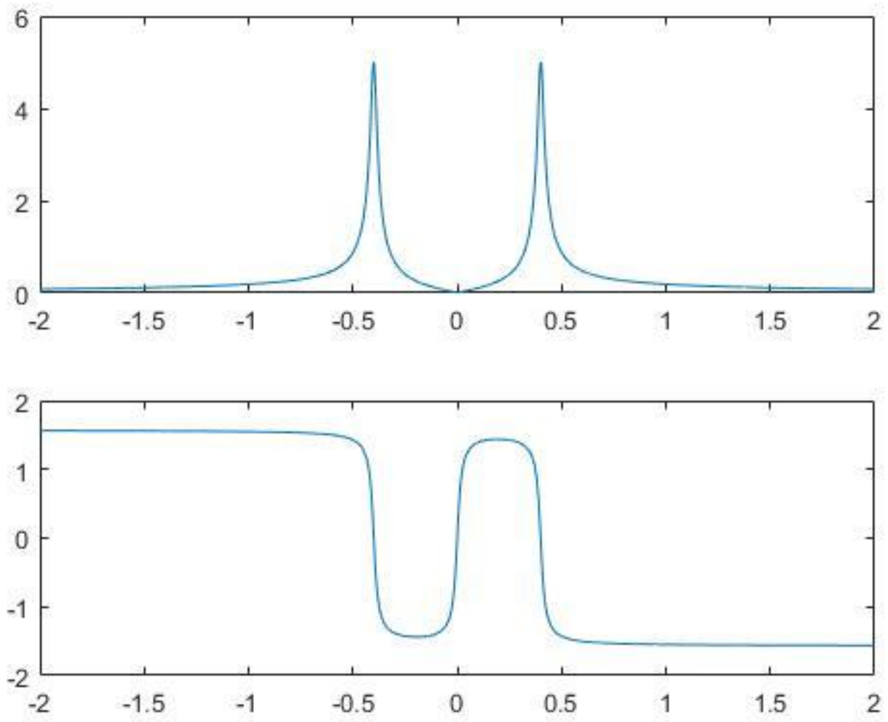


▲Compare of $\mathrm{X(f)}$ and $X_{\Delta}(f)$

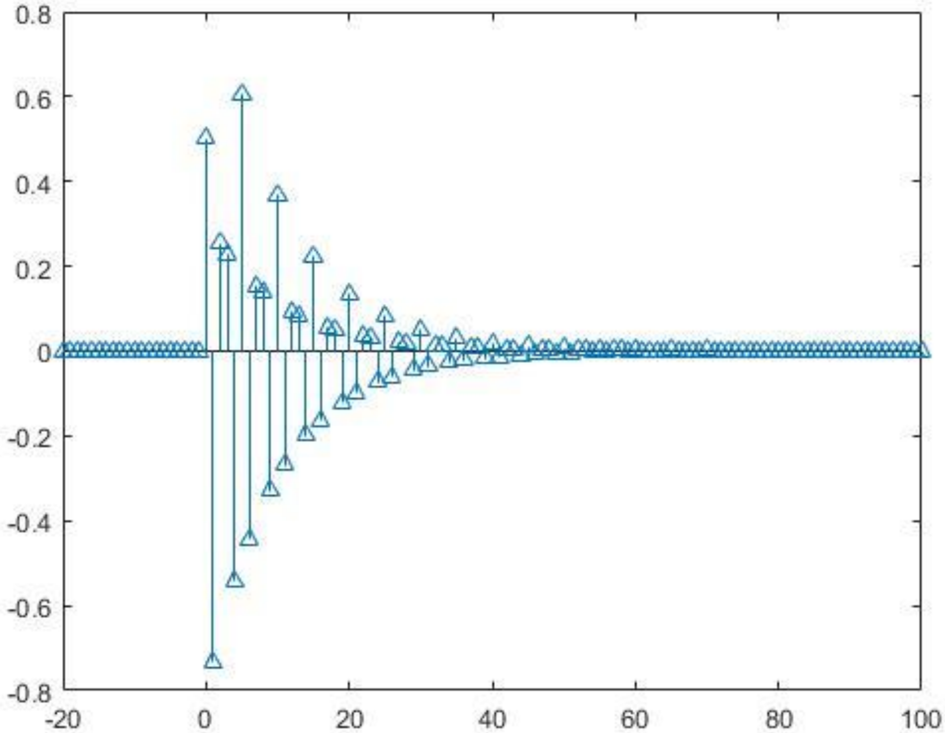
3. $a=0.1$ $f_0=0.4$



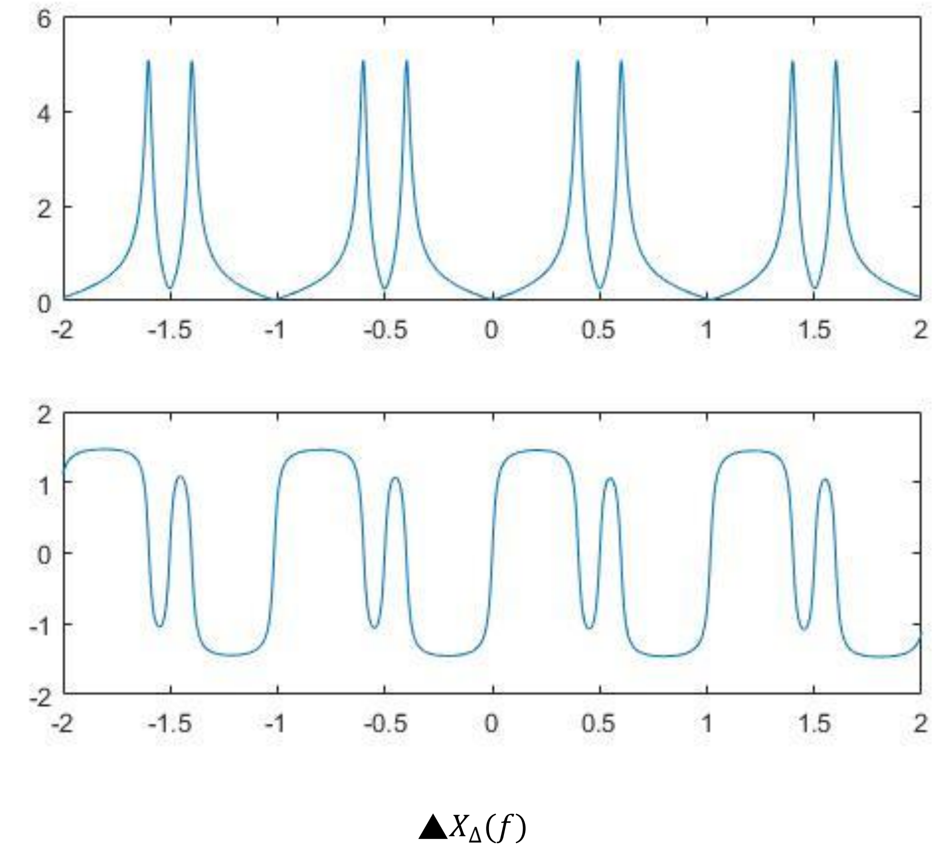
▲$x(t)$



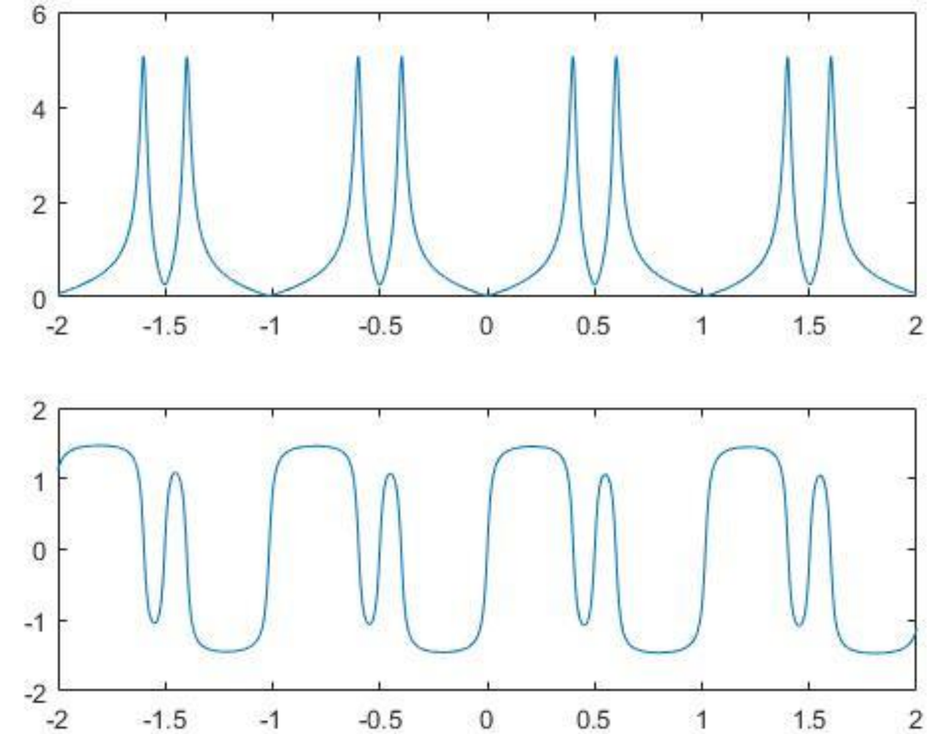
▲$X(f)$



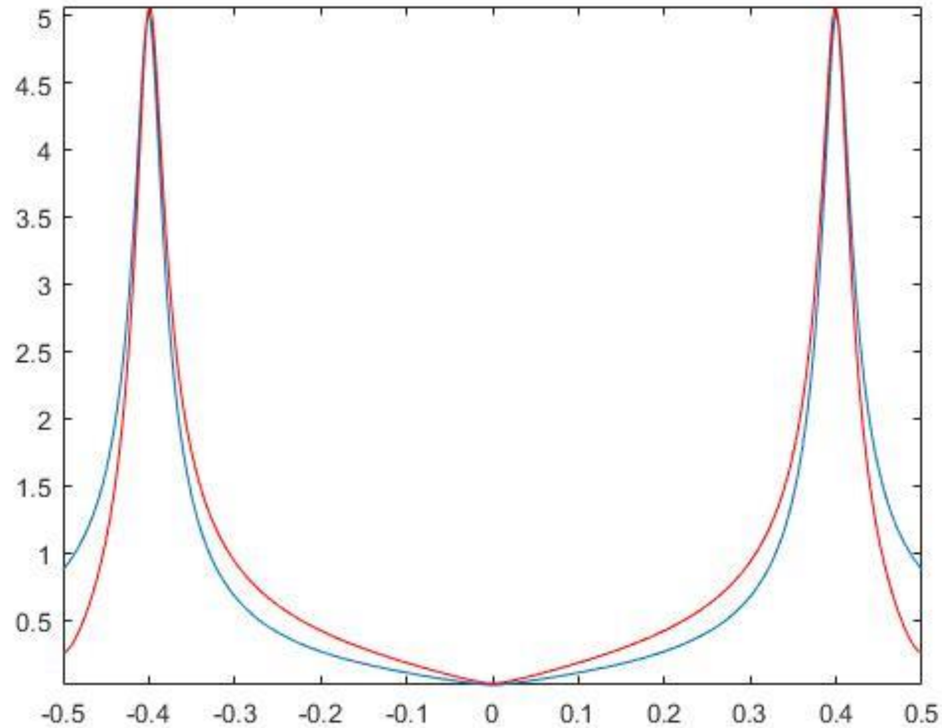
▲$x_{\Delta}(t)$



▲$X_{\Delta}(f)$



▲Compare of $\mathrm{x(t)}$ and $x_{\Delta}(t)$



▲Compare of $\mathrm{X(f)}$ and $X_{\Delta}(f)$