

SOEN 321

Exercise 1 (With Solution)

Prob. 1 Consider the affine cipher in which the ciphertext c is given by $C = aP + b \pmod{26}$. The cryptanalyst observed the following plaintext/ciphertext pairs (p,c) : (1,10) and (2,17).

1. Recover the key (a,b) used in the encryption system above.
2. What is the ciphertext corresponding to the plaintext $p=3$?

Answer: $a=7, b=3, \text{ ciphertext}=24$

Prob. 2 Consider the Hill cipher in which the ciphertext is related to the plaintext using the form

$$\begin{pmatrix} c_1 & c_2 \end{pmatrix} = \begin{pmatrix} p_1 & p_2 \end{pmatrix} \begin{pmatrix} k_1 & k_2 \\ k_3 & k_4 \end{pmatrix} \pmod{26}$$

The cryptanalyst observed the following plaintext/ciphertext pairs $(p_1 \ p_2)/(c_1 \ c_2)$: (1 2)/(16 23) and (3 3)/(1 16).

Determine the key corresponding to this system.

$$\text{Key} = \begin{bmatrix} 2 & 5 \\ 7 & 9 \end{bmatrix}$$

Prob. 3

a) Evaluate the following:

$$\gcd(621, 345)$$

Ans. 69

$$\gcd(11316, 1221)$$

Ans. 3

$$23^{-1} \pmod{67}$$

Ans. 35

$$32^{-1} \pmod{167}$$

Ans 47

$$\gcd(16, 56)$$

$$\gcd(161, 535)$$

$$161^{-1} \pmod{536}$$

$$16^{-1} \pmod{533}$$

Prob. 4

Find x that simultaneously satisfy the following congruent equations

a)

$$x \equiv 3 \pmod{7}$$

$$x \equiv 5 \pmod{11}$$

$$x \equiv 9 \pmod{13}$$

Ans. $x=269$

b)

$$x \equiv 2 \pmod{7}$$

$$x \equiv 3 \pmod{11}$$

Ans. $x=58$

Prob. 5

Consider an RSA system with $p=7$, $q=11$ and $e=13$. Find the plaintext corresponding to $c=17$.

Ans. $d=37$ and $m=52$

Prob. 6

Consider an RSA system in which the attacker knows that n_1 and n_2 has the form $n_1=pq_1=16637$ and $n_2=pq_2=17399$. Show how the attacker can break this system.

Ans. Eve evaluates $p=\gcd(n_1, n_2)$