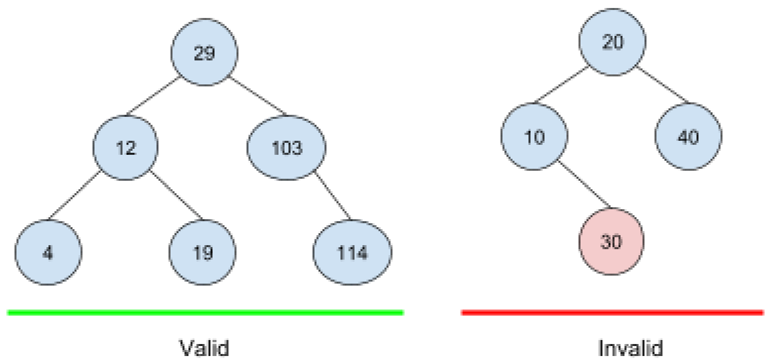
CS1332 Fall 2018 - Week 03

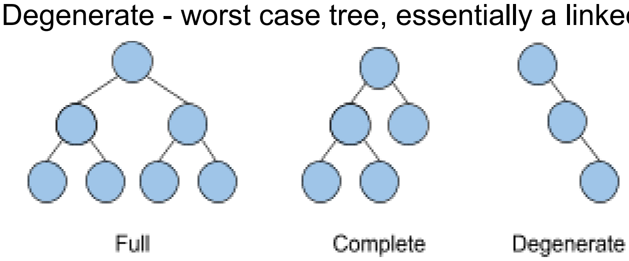
9/10/18

<u>Topics</u>

I. Binary Search Trees
   A. Binary tree review
      1. Binary trees nodes can have 0, 1 or 2 children
      2. Children are in positions defined as left and right children
      3. The root is the entry point for the tree (like the head of a linked list)
   B. Binary search trees are binary trees subject to an *order property*:
      1. All data in a node's left subtree are less than the data in the node and all data in a node's right subtree are greater than the data in the node
      2. data in left subtree < current node < data in right subtree

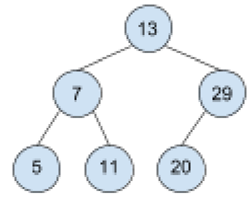


      3. Because of the order property, BSTs can only hold comparable data
         a) The data type must implement the Comparable interface with requires the implementation of the .compareTo() method
         b) If a.compareTo(b) < 0 → a < b
         c) If a.compareTo(b) > 0 → a > b
      4. Advantages of a BST:
         a) Searching is optimized: traverse fewer nodes to find data, every time you do a comparison, you can eliminate about half the data that you don't need to look at (like the binary search algorithm)
   C. Various shape properties of binary trees:
      1. Full - all nodes, except for the leaves, have 2 children, all searches will be guaranteed to be O(logn)
      2. Complete - leaves are filled level by level, left to right with no gaps
      3. Degenerate - worst case tree, essentially a linked list, O(n) operations



D. BST Traversals

1. Depth - go down the rabbit hole, follow one path as far as you can go

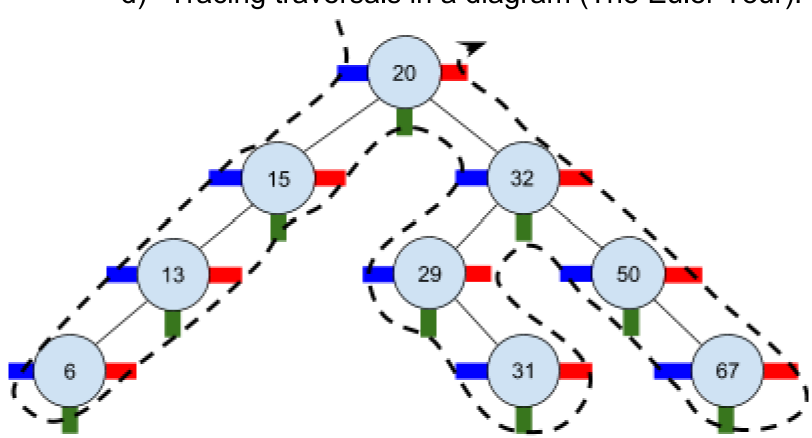


a) Pre-order
   (1) If current node is null → return
   (2) Else →
      (a) look at data (record it, print it, etc.)
      (b) recurse left
      (c) recurse right
   (3) Pre-order: 13, 7, 5, 11, 29, 20

b) Post-order
   (1) If current node is null → return
   (2) Else →
      (a) recurse left
      (b) recurse right
      (c) look at data
   (3) Post-order: 5, 11, 7, 20, 29, 13

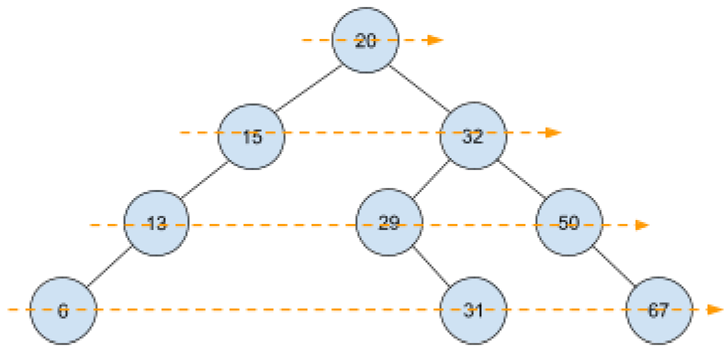
c) In-order
   (1) If current node is null → return
   (2) Else →
      (a) recurse left
      (b) look at data
      (c) recurse right
   (3) In-order: 5, 7, 11, 13, 20, 29

d) Tracing traversals in a diagram (The Euler Tour):



Pre-order: 20, 15, 13, 6, 32, 29, 31, 50, 67
Post-order: 6, 13, 15, 31, 29, 67, 50, 32, 20
In-order: 6, 13, 15, 20, 29, 31, 32, 50, 67

2. Breadth - goes one level at a time
    a) Level-order - not recursive, uses a queue and a while-loop
        (1) Add the root to the queue
        (2) While the queue is not empty →
            (a) Remove one node from the queue
            (b) Enqueue its left and right children (in that order)



9/12/18

<u>Topics</u>

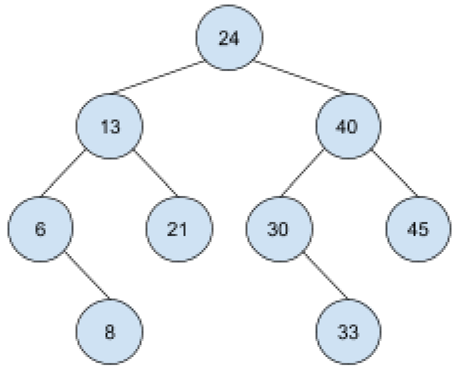
I. BST Operations
    A. Adding
        1. Start at the root → act like you are "searching" for the data
            a) If you reach a null node → add the data here
            b) If you reach a match → do nothing (we don't want duplicates)
        2. Example pseudocode:

Look-Ahead Technique (don't do this...)	Pointer Reinforcement Technique (do this!)

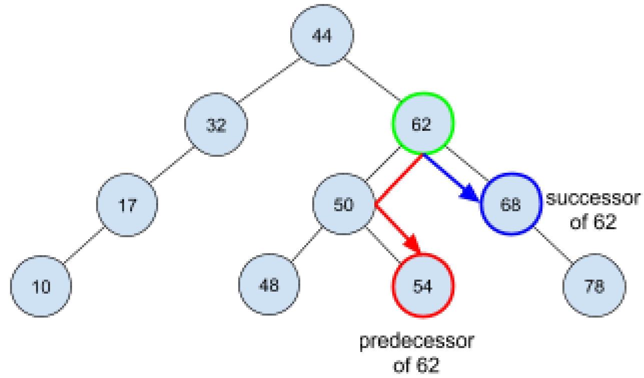
`void add(data) // public method`
`  if root == null`
`    root = new Node(data)`
`  else add(data, root)`
 	
`void add(data, node) // private`
`  if (data < node's data)`
`    if node.left == null`
`      node.left = new Node(data)`
`    else add(data, node.left)`
`  else if (data > node's data)`
`    if node.right == null`
`      node.right = new`
`Node(data)`
`    else add(data, node.right)`
`  else // data == node's data`
	`void add(data) // public method`
	`  root = add(data, node)`
	
	`Node add(data, node) // private`
	`  if (node == null)`
	`    return new Node(data)`
	`  else if (data < node's data)`
	`    node.left =`
	`add(data,node.left)`
	`  else if (data > node's data)`
	`    node.right = add(data, node.r)`
	`  return node`

`return`	

3. Tracing an example: add 24, 13, 6, 21, 40, 30, 8, 45, 33



B. Removing
   1. Cases, removing a node with:
      a) no children - easiest
      b) one child - easy
      c) two children - hardest
   2. No child remove case (eg. remove 45 from the tree above)
      a) Set the parent's pointer to the node to null
   3. One child remove case (eg. remove 6 from the tree above)
      a) Set the parent's pointer to the node to the node's child
   4. Two children remove case (eg. remove 24 from the tree above)
      a) We can't easily move pointers to it
      b) Instead of removing the node itself, we'll replace the node's data with the predecessor or successor and remove THAT node:
         (1) Predecessor - largest value still smaller than the current value; go one to the left, then as far right as possible
         (2) Successor - smallest value still larger that the current value; go one to the right, then as far left as possible
      c) We'll delegate the removal of an actual node to the node containing the predecessor or successor since these will always be 0 or 1 child remove cases
         (1) To remove 62, we'd replace its data with the pred. or succ. and delete the node which contained the pred. or succ.



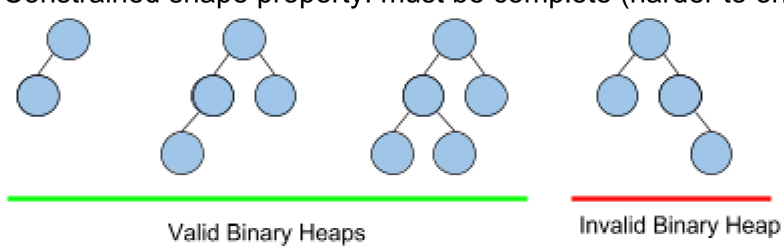
Activities

I. BST traversals reality check
II. Drawing a BST activity

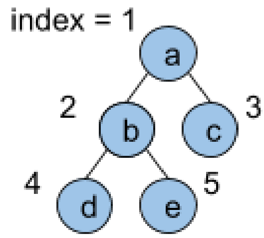
9/14/18

Topics

I. Exam Expectations
   A. Practice exams are online with “hints” on how to start each question
   B. Exams are thoroughly tested by TAs to ensure fairness and timing
   C. Exam 1 - historically the easiest
   D. Exam 2 - historically the hardest
   E. Exam 3 - difficulty between 1 and 2
   F. Piazza will be shut down the day of the exam (do not panic)
   G. We will only offer ONE makeup date (see syllabus for more details)
II. Binary Heaps
   A. A heap is a binary tree but NOT a binary search tree
      1. General shape property: nodes can have up to two children
      2. Constrained shape property: must be complete (harder to enforce)

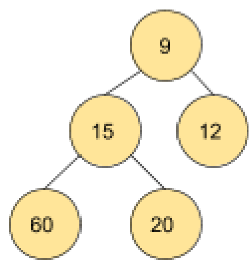


   B. Implementing a binary heap
      1. Since they are complete trees, they can be implemented with Arrays:
         a) For a node at index n:
            (1) its left child is at index n * 2
            (2) its right child is at index (n * 2) + 1
            (3) its parent is at index n / 2 (Java truncates decimals)



0	1	2	3	4	5
	a	b	c	d	e

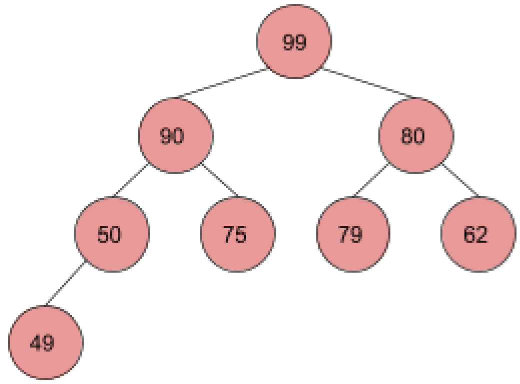
C. Min-Heap order property:
   1. Minimum heaps (min-heaps) keep the smallest data in the set at the root
   2. Children are always greater than the parent (there is not relationship between the siblings, i.e. the larger of the two doesn't need to be anywhere special)



0	1	2	3	4	5
	9	15	12	60	20

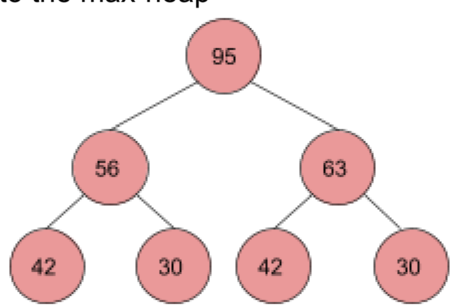
   3. The last element is at index size

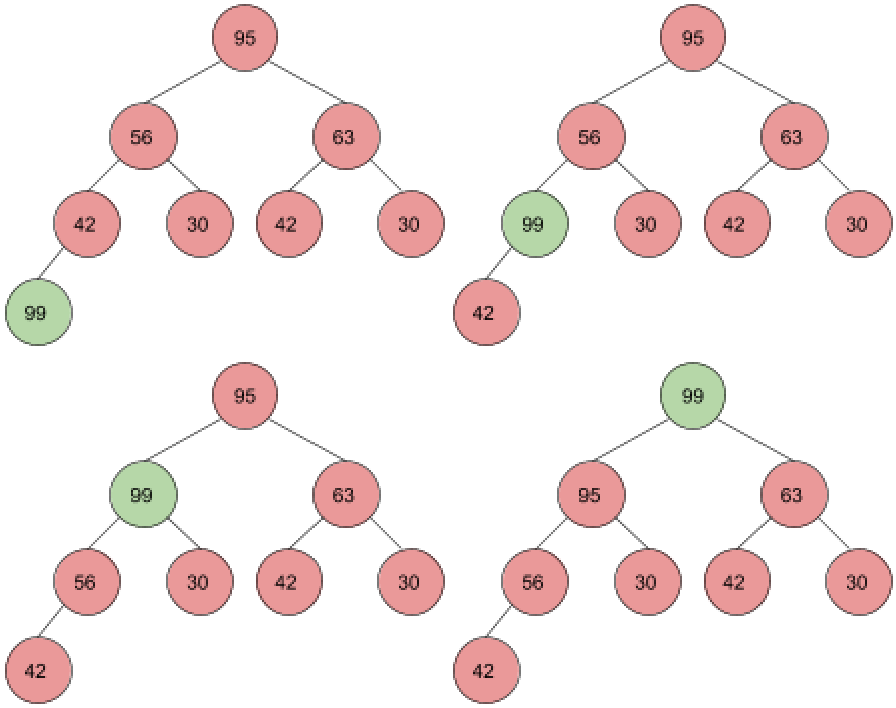
D. Max-Heap order property:
   1. Maximum heaps (max-heaps) keep the largest data in the set at the root
   2. Children are always less than the parent (also no relationship between siblings)



0	1	2	3	4	5	6	7	8
	99	90	80	50	75	79	62	49

E. Pros:
   1. Used to back Priority Queues → return the most urgent/important data
      a) Eg. emergency room waiting room, those with more urgent emergencies are pushed to the front of the line and tended to first
F. Cons:
   1. If you want to search for an element, you'd have to search through the entire backing structure (heaps are not like binary search trees)
G. Adding to a heap:
   1. Add to the end of the array (this keeps the shape in place)
   2. Heapify-up/up-heap (this restores the order)
      a) Compare the new data with its parent
      b) If the new data does not have the correct relationship with its parent, swap them
      c) Repeat this process until the new data is at the root or the new data has the correct relationship with its parent.
   3. Eg. Add 99 to the max-heap




95
56
63
42
30
42
30
99
95
56
63
99
30
42
30
42
95
99
63
56
30
42
30
42
99
95
63
56
30
42
30
42