CS111 ASSIGNMENT 4

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Problem 1: Give an asymptotic estimate, using the Θ-notation, of the number of letters printed by the algorithms given below. Give a complete justification for your answer, by providing an appropriate recurrence equation and its solution.

(a) **algorithm** PrintAs(n)
```
if n ≤ 1 then
    print("A")
else
    for j ← 1 to n^3
        do print("A")
    for i ← 1 to 5 do
        PrintAs(⌊n/2⌋)
```

(b) **algorithm** PrintBs(n)
```
if n ≥ 4 then
    for j ← 1 to n^2
        do print("B")
    for i ← 1 to 6 do
        PrintBs(⌊n/4⌋)
    for i ← 1 to 10 do
        PrintBs(⌈n/4⌉)
```

(c) **algorithm** PrintCs(n)
```
if n ≤ 2 then
    print("C")
else
    for j ← 1 to n
        do print("C")
    PrintCs(⌊n/3⌋)
    PrintCs(⌊n/3⌋)
    PrintCs(⌊n/3⌋)
    PrintCs(⌊n/3⌋)
```

(d) **algorithm** PrintDs(n)
```
if n ≥ 5 then
    print("D")
    print("D")
  if (x ≡ 0 (mod 2)) then
      PrintDs(⌊n/5⌋)
      PrintDs(⌈n/5⌉)
      x ← x + 3
    else
      PrintDs(⌈n/5⌉)
      PrintDs(⌊n/5⌋)
      x ← 5x + 3
```

In part (d), variable x is a global variable initialized to 1.

Solution 1:
For all of the equations, we will be utilizing the master theorem $T(n) = a * T(\frac{n}{b}) + c * n^d$ which, will help us solve for all of the algorithms. With the master theorem, we will also be checking the cases for each of the problems.

Let $T(n)$ be a function defined on the non-negative integers. The function $T(n)$ satisfies the recurrence relation:

$$T(n) = a * T(\frac{n}{b}) + c * n^d$$

where:

- Let $a \geq 1$, $b > 1$, $c > 0$, and $d \geq 0$
- Suppose that T(n) satisfies the recurrence

Then, $T(n)$ has the following asymptotic bounds:

1. $f(n) = \Theta(n^d)$ if $a < b^d$
2. $f(n) = \Theta(n^d \log n)$ if $a = b^d$

3. $f(n) = \Theta(n^{\log_b a)}$ if $a > b^d$

a)
Recurrence: $A(n) = 5A(\frac{n}{2}) + n^3$
Apply Master Theorem with: $a = 5, b = 2, c = 1, d = 3$
Plug in the numbers: $5 < 2^3 = 5 < 9$
Case: $a < b^d$
Solution: $\Theta(n^3)$

b)
Recurrence: $B(n) = 16B(\frac{n}{4}) + n^2$
Apply Master Theorem with: $a = 16, b = 4, c = 1, d = 2$
Plug in the numbers: $16 = 4^2 = 16 = 16$
Case: $a = b^d$
Solution: $\Theta(n^2 \log n)$

c)
Recurrence: $C(n) = 4C(\frac{n}{3}) + n$
Apply Master Theorem with: $a = 4, b = 3, c = 1, d = 0$
Plug in the numbers: $4 > 3^0 = 4 > 1$
Case: $a > b^d$
Solution: $\Theta(n^{\log_3 4})$

d)
This one is unique, so how we solve this is with $n \geq 5$ it either goes in the mod or the else case, but because the operations are practically the same (printDs n/5 twice), the recurrence equations are practically the same.
Recurrence: $D(n) = 2D(\frac{n}{5}) + 2$
Apply Master Theorem with: $a = 2, b = 5, c = 2, d = 0$
Plug in the numbers: $2 > 5^0 = 2 > 1$
Case: $a > b^d$
Solution: $\Theta(n^{\log_5 2})$

Problem 2: We have three sets A, B, C with the following properties:

(a) $|B| = 2|A|$, $|C| = 3|A|$,

(b) $|A \cap B| = 18$, $|A \cap C| = 20$, $|B \cap C| = 24$,

(c) $|A \cap B \cap C| = 11$,

(d) $|A \cup B \cup C| = 129$.

Use the inclusion-exclusion principle to determine the number of elements in A. Show your work.

Solution 2:
Alright so in the equation we know the following:

(a) $|B| = 2|A|$, $|C| = 3|A|$,

(b) $|A \cap B| = 18$, $|A \cap C| = 20$, $|B \cap C| = 24$,

(c) $|A \cap B \cap C| = 11$,

(d) $|A \cup B \cup C| = 129$.

Knowing this we need to find A. Now we can set up the inclusion-exclusion principle.

$|A \cup B \cup C| = |A| + |B| + |C| - (|A \cap B| + |A \cap C| + |B \cap C|) + |A \cap B \cap C|$
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
Substitute all of the numbers in:

$129 = |A| + 2|A| + 3|A| - 18 - 20 - 24 + 11$
$129 = 6|A| - 62 + 11$
$129 = 6|A| - 51$
$129 + 51 = 6|A|6|A| = 180$
$|A| = \frac{180}{6}$
$|A| = 30$

Problem 3: A company, Nice Inc., will award 45 fellowships to high-achieving UCR students from four different majors: computer science, biology, political science and history. They decided to give fellowship awards to at least 8 students majoring in computer science and at most 8 biology majors. The number of political science and history majors should be between 5 and 12 students each. How many possible lists of awardees are there? You need to give a complete derivation for the final answer, using the method developed in class. (Brute force listing of all lists will not be accepted.)

Solution 3:
For this problem, we need to list the possible awardees. Let's set the inequalities for this equation.

$$\begin{cases} w + x + y + z = 15 \\ CS : 8 \leq w \Rightarrow 0 \leq w' \leq 27 \\ BIO : 0 \leq x \leq 8 \Rightarrow 0 \leq x \leq 8 \\ History : 5 \leq y \leq 12 \Rightarrow 0 \leq y' \leq 7 \\ Political\ Science : 5 \leq y \leq 12 \Rightarrow 0 \leq z' \leq 7 \end{cases}$$

$S_{total} = \binom{27+4-1}{4-1} = \binom{30}{3} = \frac{30 \times 29 \times 28}{6} = 4060$ (w/o Constraints)
$m = 27$ (There are 27 possible students in total)
$k = 4$ (It is 4 Partitions since there are 4 majors.)

$S(w' \geq 28 \wedge x \geq 9, y' \geq 8, z' \geq 8)$
$= S(w' \geq 28) + S(x \geq 9) + S(y' \geq 8) + S(z' \geq 8) - S(w' \geq 28 \wedge x \geq 9) - S(w' \geq 28 \wedge y' \geq 8) - S(w' \geq 28 \wedge z' \geq 8) - S(x \geq 9 \wedge y' \geq 8) - S(x \geq 9 \wedge z' \geq 8) - S(y' \geq 8 \wedge z' \geq 8) + S(w' \geq 28 \wedge x \geq 9 \wedge y' \geq 8) + S(w' \geq 28 \wedge x \geq 9 \wedge z' \geq 8) + S(w' \geq 28 \wedge y \geq 8 \wedge z' \geq 8) + (x \geq 9 \wedge y' \geq 8 \wedge z' \geq 8) - S(w' \geq 28 \wedge x \geq 9 \wedge y' \geq 8 \wedge z' \geq 8)$

$= \binom{27-28+3}{3} + \binom{27-9+3}{3} + 2\binom{27-8+3}{3} - 2\binom{13}{3} - \binom{14}{3} + \binom{5}{3}$
$= \binom{2}{3} + \binom{21}{3} + 2\binom{32}{3} - 2\binom{13}{3} - \binom{14}{3} + \binom{5}{3}$
$= 1330 + 2(1540) - 2(286) - 364 + 10$
$= 3483$
$4060 - 3484 = 576$

Academic integrity declaration. The homework papers must include at the end an academic integrity declaration. This should be a short paragraph where you briefly explain *in your own words* (1) whether you did the homework individually or in collaboration with a partner student (if so, provide the name), and (2) whether you used any external help or resources.

We did this homework as partners. We got help from Ezekiel and Alice Thai on our homework and we have gotten help from the lecture notes and the discussion notes also on how to set up the equations. We also used YouTube in order to get a more in-depth look into the topic. We have also gotten a bit of help from Professor Strzheletska for clarification on some parts.

Submission. To submit the homework, you need to upload the pdf file to Gradescope. If you submit with a partner, you need to put two names on the assignment and submit it as a group assignment. Remember that only LaTeX papers are accepted.