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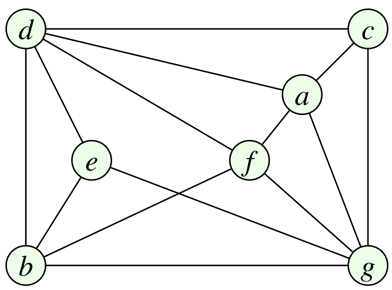
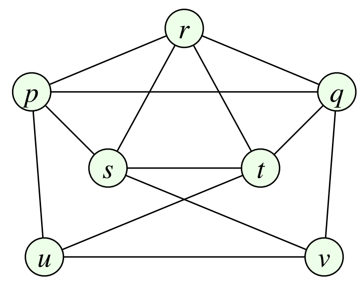
CS 111 ASSIGNMENT 5
due March 8, 2024

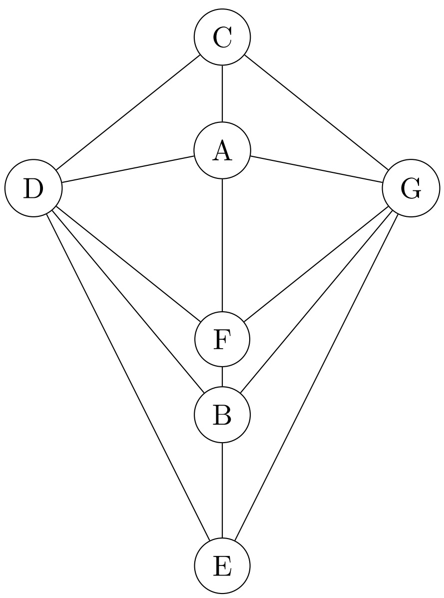
Problem 1: Determine whether the two graphs below are planar or not. To show planarity, give a planar embedding. To show that a graph is not planar, use Kuratowski's theorem.

Graph H:

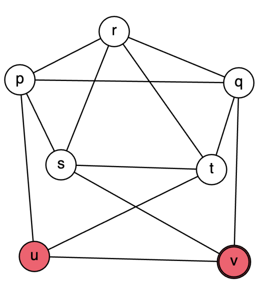


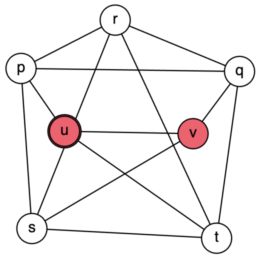
Graph G:

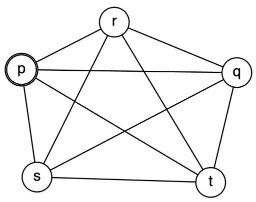




Solution 1:
Graph H is a Planar, all we have to do is move E to the outside and rearrange a bit.







Graph G is not planar because it contains a subgraph homeomorphic to K_5. If we move around the nodes U and V we can see that it is a subgraph homeomorphic to K_5.

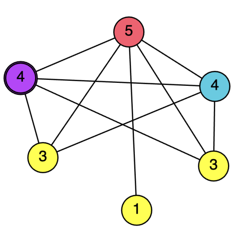
Problem 2: (a) For each degree sequence below, determine whether there is a graph with 6 vertices where vertices have these degrees. If a graph exists, (i) draw it, (ii) find the chromatic number and justify. If it doesn't, justify that it doesn't exist.
Note. To give a justification for the chromatic number, you need to give a coloring and explain why it's not possible to use fewer colors.

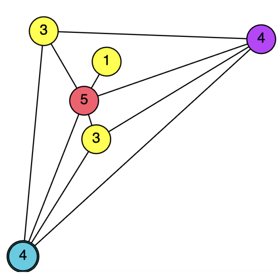
(a1) $5, 4, 4, 3, 3, 1$.
(a2) $5, 4, 3, 2, 2, 1$.
(a3) $4, 4, 4, 3, 3, 2$.

(b) For each degree sequence below, determine whether there is a planar graph with 6 vertices where vertices have these degrees. If a graph exists, (i) draw it, (ii) find the chromatic number and justify. If it doesn't, justify that it doesn't exist.

(b1) $5, 5, 4, 4, 4, 2$.
(b2) $3, 3, 3, 3, 3, 3$.

Solution 2:
$A_1 = 5 + 4 + 4 + 3 + 3 + 1 = 20$
Then we use the handshaking lemma to divide 20 by 2 to get our edges in this case we have 10 edges.

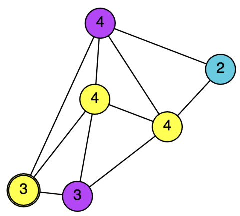




The chromatic number is 4 therefore it can be colored with 4 colors. It can't be colored with 4 colors since it contains the clique size of 4: 5(red), 4(purple), 4(blue) and the 3's and 1 (yellow)

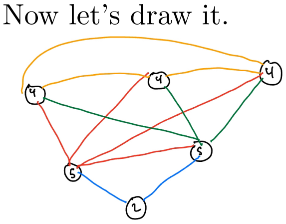
$A_2 = 5 + 4 + 3 + 2 + 2 + 1 = 17$
Then using the handshaking lemma 17 is divided by 2 which gives us 8.5, since it is odd, we prove that it can not exist.

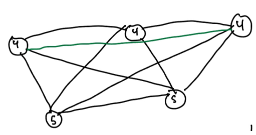
$A_3 = 4 + 4 + 4 + 3 + 3 + 2 = 20$
Then using the handshaking lemma again the edges would be 10.



The chromatic color is 3 therefore the graph can be colored with 3 colors. It can't be colored with 2 colors since it contains the clique size of 3: 4 and 3 (purple), 4,4, and 3 (yellow), 2 (blue).
$B_1 = 5 + 5 + 4 + 4 + 4 + 2 = 24$
Then using the handshake lemma, you get 12. After you would then use Euler Formula where $m \leq 3n – 6$ using this formula we have $m = 12$ and $n = 6$ so plugging everything in we get.

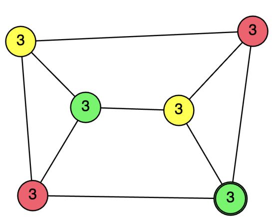
$12 \leq 3 \times 6 - 6$
$12 \leq 12$ Which shows that it can be a planar.
Now let's draw it.





The graph is not a planar, containing a subgraph homeomorphic to K_5. Even though we used the Euler inequality. This is basically a check to see if it is possible to see if it's a planar or not.

$B_2 = 3 + 3 + 3 + 3 + 3 + 3 = 18$
Then using handshaking lemma, we get 9 edges. Then $m = 9$ and $n = 6$. Then we test with Euler inequality.
$9 \leq 3 \times 6 - 6$
$9 \leq 12$ Which shows that it can be a planar.

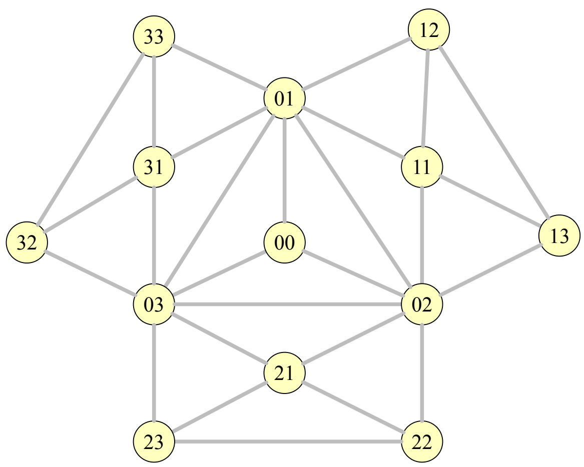


The chromatic number is 3, therefore the graph can be colored with 2 colors. It can't be colored with 3 colors since it contains the clique size of 3: two with 3(red), 2 with 3(yellow), and two with 3(green).

Problem 3:

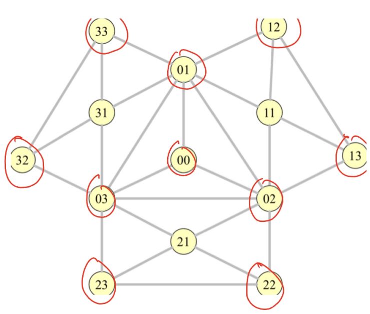
a) Does the graph shown below have an Euler tour? Give a complete justification for your answer.

b) Does the graph shown below have a Hamiltonian cycle? Give a complete justification for your answer.



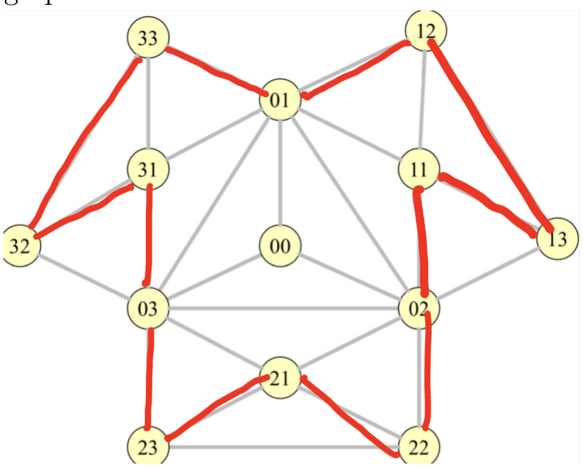
Solution 3:

a) This graph does not include an Euler tour. This is because the following vertices have an odd degree: 33,12,01,00,03,32,23,22,03, and 13. We know that if a graph has at least one odd degree, it cannot support an Euler tour. By definition, a graph G has an Euler tour if and only if all of its vertices are of even degree. This is derived from the handshake lemma, which states that if an edge enters a vertex, it must depart with another edge. Finally, this graph does not include an Euler Tour.



b) This can't be a Hamiltonian cycle, we can prove this by contradiction. Assume that the graph can be denoted by C. To visit the vertex 01, edges (01, 33), (01,12) have to be in C. Then, to visit vertices 32 and 13 of degree 3, edges (13,12) and (32,31) has to be in C, we have to ignore 01 going straight to 03 to 02 to include 31 and 13, so we will include 2 of the 4 edges. Then we have to visit 03 and 02, where (31,03) and (11,02) has to be in C, we can't go straight back to 01 with 31 and 11 as in a Hamiltonian cycle we need to include all of the vertices, so we need to include 2 of the 4 edges. Then from vertices 03 and 02 we need to visit vertices 23 and 22, edges (03,23) and (02,22) has to be in C, we can't go into 00 or else we won't include the rest of the vertices and we can't connect 03 and 02 to each other or it'll end the cycle and we can't go straight to 01 or it'll be repeating, this means that 2 of the 7 edges will be included. Then from vertices 23 and 22 we go to 21 to complete most of the cycle, edges (23,21),(22,23) has to be in C. But now we see we cannot interact or connect with the vertex 00 at all without repeating from where vertices 03 or 02 is, therefore proving that it is a contradiction and proving that it is not a Hamiltonian cycle.

In summary, it isn't a Hamiltonian cycle because it can't go to 00 without repeating itself on my graph.



Academic integrity declaration. The homework papers must include at the end an academic integrity declaration. This should be a short paragraph where you briefly explain *in your own words* (1) whether you did the homework individually or in collaboration with a partner student (if so, provide the name), and (2) whether you used any external help or resources.

We did this homework as partners. We got help from Ezekiel and Alice Thai on our homework and we have gotten help from the lecture notes,the discussion notes, and the solution notes for the finalon how to set up the answer to the equations on the graphs. Then for the graphs, some were drawn by Ben, and the others were made through Menthy Wu Graph Editor, without her graph editor we couldn't have finished it on time, not sponsoring but here is the link to the repository https://github.com/menthy-wu/graphEditor. Lastly, I wanna thank the person who is grading Ben's essay on Hamiltonian cycle.

Submission. To submit the homework, you need to upload the pdf file to Gradescope. If you submit with a partner, you need to put two names on the assignment and submit it as a group assignment. Remember that only LaTeX papers are accepted.