

Problem Set 10, May 14, 2022

(Duality)

Prove the following property from the lecture slides:

If f is closed and convex, then for any \mathbf{x}, \mathbf{y} ,

$$\begin{aligned}\mathbf{y} \in \partial f(\mathbf{x}) &\Leftrightarrow \mathbf{x} \in \partial f^*(\mathbf{y}) \\ &\Leftrightarrow f(\mathbf{x}) + f^*(\mathbf{y}) = \mathbf{x}^\top \mathbf{y}\end{aligned}$$