Ordinary differential equation

Differential equⁿ: An equation involving one or more differentials or differential coefficients is called differential equation.

$$\frac{dy}{dx}$$ $x \rightarrow$ independent variable

$y =$ dependent variable

2 types —

Ordinary differential equⁿ: An equation involving one independent variable, one dependent variable and derivative of dependent variable with respect to a ~~sigle~~ single independent variable is called an ordinary differential equation.

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + by = 0$$

Partial differential equation: An equation involving one dependent variable, two or more independent variables and partial differential coefficient of dependent variable with respect to more than one independent variable is called partial differential equation.

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3z$$

Chap-1

Ordinary differential equation

Differential equ^n: An equation involving one or more differentials or differential coefficients is called differential equation.

$$\frac{dy}{dx}$$ $x \to$ independent variable

$y =$ dependent variable

2 types —

Ordinary differential equ^n: An equation involving one independent variable, one dependent variable and derivative of dependent variable with respect to a ~~sigle~~ single independent variable is called an ordinary differential equation.

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + by = 0$$

Partial differential equation: An equation involving one dependent variable, two or more independent variables and partial differential coefficient of dependent variable with respect to more than one independent variable is called partial differential equation.

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 3z$$

20-21

Order of diff. equⁿ: The order of the highest order differential coefficient involved in a differential equⁿ is called the order of the differential equⁿ.

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0 \rightarrow \text{order} = 2$$

20-21

Degree of differential: The degree or power of the higest order derivative of a differential equⁿ is called the degree of a diff. equⁿ.

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0 \rightarrow \text{degree} = 1$$

Ex:

(i) $\frac{dy}{dx} = e^x + x + 1$; order = 1, degree = 1

(ii) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$; order = 2, degree = 1

(iii) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 5 = 0$; order = 2, degree = 1

Solution of a Differential Equation:

A solution of a differential equation is a function that satisfies the equation when substituted into it. Solution can be general or particular.

1) General solution: It forms of functions with arbitary constant.

11) Particular solution: can be obtained by assign values to the constant in general equation.

Example:

$$\frac{dy}{dx} = 2x$$

or, $\int dy = 2\int x\,dx$ | c = arbitary constant

$\therefore y = x^2 + c$

Here, General solution : $y = x^2 + c$

particular solution : $y = x^2 + k_1$, $y = x^2 + k_2$

k = numeric values

20-21

* Some applications of diff. equ^n.

=>

Physics: Used to design model motion, heat, and waves application.

Economics: Applied in modeling financial markets, interest rates and economic growth.

Engineering: Used in control systems, design electrical circuits.

Chemistry: Helps to describe the reaction rates.

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\frac{x}{a} + c$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right| + c$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left|x+\sqrt{a^2+x^2}\right| + c$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right| + c$$

$$\int \sqrt{a^2+x^2}\,dx = \frac{x\sqrt{a^2+x^2}}{2} + \frac{a^2}{2} \ln\left|x+\sqrt{a^2+x^2}\right| + c$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a} + c$$

$$\int \sqrt{x^2-a^2}\,dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2} \ln\left|x+\sqrt{x^2-a^2}\right| + c$$

Chap-2

Equation of First order and First Degree

* Equation reducible to the form in which variables are ~~separated~~. separable.

田 $\frac{dy}{dx} = f(ax+by+c)$

Let $ax+by+c = v$

or, $a + b\frac{dy}{dx} = \frac{dv}{dx}$

or, $\frac{dy}{dx} = \frac{1}{b}\left(\frac{dv}{dx} - a\right)$

Here,

$$\frac{dy}{dx} = f(ax+by+c)$$

$\therefore \frac{1}{b}\left(\frac{dv}{dx} - a\right) = f(v)$

or, $\frac{dv}{dx} - a = b\,f(v)$

or, $\frac{dv}{dx} = a + bf(v)$

or, $\frac{dv}{a+bf(v)} = dx$

Integrating, $\int \frac{dv}{a+b\,f(v)} = \int dx$

20-21

$\frac{dy}{dx} = (4x+y+1)^2$

$\Rightarrow$ Let, $4x+y+1 = v$

or, $4 + \frac{dy}{dx} = \frac{dv}{dx}$

$\therefore \frac{dy}{dx} = \frac{dv}{dx} - 4$

Here,

$\frac{dy}{dx} = (4x+y+1)^2$

$\therefore \frac{dv}{dx} - 4 = v^2$

or, $\frac{dv}{dx} = v^2 + 4$

$\therefore \frac{dv}{v^2+4} = dx$

$$\int \frac{1}{x^2+a^2}\,dx$$

$$= \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

Integrating, $\int \frac{dv}{v^2+4} = \int dx$

or, $\frac{1}{2}\tan^{-1}\frac{v}{2} = x + C$

or, $\frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right) = x + C$

$\therefore$ ~~2tan x = 4x+y+1~~ $4x+y+1 = 2\tan 2(x+C)$ (Ans.)

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$\boxplus$ $(x-y)^2 \frac{dy}{dx} = a^2$ $\quad$ '20

$\Rightarrow$ Let,

$$x-y = v$$

$$\text{or, } 1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore \frac{dy}{dx} = 1 - \frac{dv}{dx}$$

Here,

$$(x-y)^2 \frac{dy}{dx} = a^2$$

$$\therefore v^2\left(1-\frac{dv}{dx}\right) = a^2$$

$$\text{or, } 1 - \frac{dv}{dx} = \frac{a^2}{v^2}$$

$$\text{or, } \frac{dv}{dx} = 1 - \frac{a^2}{v^2} = \frac{v^2-a^2}{v^2}$$

$$\text{or, } \frac{v^2}{v^2-a^2} dv = dx$$

$$\int \frac{a^2}{v^2-a^2} dv = a^2 \int \frac{1}{v^2-a^2}$$

$$= a^2 \cdot \frac{1}{2a} \ln \frac{v-a}{v+a} + c$$

$$= \frac{a}{2} \ln \frac{v-a}{v+a} + c$$

$$\int \frac{1}{x^2-a^2} dx$$

$$= \frac{1}{2a} \cdot \ln \left| \frac{x-a}{x+a} \right| + c$$

Integrating, $\int \frac{v^2 dv}{v^2-a^2} = \int dx$

$$\text{or, } \int \frac{v^2-a^2+a^2}{v^2-a^2} dv = x$$

$$\text{or, } \int dv + \int \frac{a^2}{v^2-a^2} dv = x$$

$$\text{or, } v + a^2 \cdot \frac{1}{2a} \ln \left| \frac{v-a}{v+a} \right| = x$$

$$\text{or, } x-y + \frac{a}{2} \ln \left| \frac{x-y-a}{x-y+a} \right| = x$$

$$\therefore y = \frac{a}{2} \ln \left| \frac{x-y-a}{x-y+a} \right|$$ Ans

— o —

20-21

$(x+y)^2 \frac{dy}{dx} = a^2$

=> Let $x+y = v$

or, $1 + \frac{dy}{dx} = \frac{dv}{dx}$

$\therefore \frac{dy}{dx} = \frac{dv}{dx} - 1$

Here

$(x+y)^2 \frac{dy}{dx} = a^2$

$\therefore v^2 \left(\frac{dv}{dx} - 1 \right) = a^2$

or, $\frac{dv}{dx} = 1 + \frac{a^2}{v^2}$

or, $\frac{dv}{dx} = \frac{v^2+a^2}{v^2}$

or, $\frac{v^2}{v^2+a^2} dv = dx$

Integrating, $\int \frac{v^2}{v^2+a^2} dv = \int dx$

or, $\int \frac{v^2+a^2-a^2}{v^2+a^2} dv = x$

or, $\int dv - \int \frac{a^2}{v^2+a^2}\,dv = x$

$\int \frac{1}{a^2+x^2}\,dx$

$= \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

or, $v - a^2\frac{1}{a}\tan^{-1}\frac{v}{a} = x + c$

or, $x+y - \frac{a^2}{a}\tan^{-1}\frac{x+y}{a} = x + c$

$\therefore\ y = a\tan^{-1}\left(\frac{x+y}{a}\right) + c$ (Ans:)

—o—

Homogenous differential equ^n

If the two functions $f_1(x,y)$ and $f_2(x,y)$ are homogenous functions of x and y of the same degree then the equation of the form $\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)}$ is called Homogenous Differential Equation.

$$\frac{dy}{dx} = \frac{f_1(x,y)}{f(x,y)}$$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{f_1(v)}{f_2(v)}$$

Ex-1) $(x^2+y^2)dx + 2xy\,dy = 0$

=> putting, $y = vx$

$$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now,

$$(x^2+y^2)\,dx = -2xy\,dy$$

or, $$\frac{dy}{dx} = \frac{x^2+y^2}{-2xy}$$

$$\text{or, } v + x\frac{dv}{dx} = -\frac{x^2 + v^2x^2}{2x \cdot vx}$$

$$\text{or, } x\frac{dv}{dx} = -v - \frac{1 + v^2}{2v}$$

$$\text{or, } x\frac{dv}{dx} = \boxed{\cancel{-2v^2 - 2v - 2}} \; \frac{-2v^2 - 1 - v^2}{2v}$$

$$\text{or, } x\frac{dv}{dx} = \frac{-3v^2 - 1}{2v}$$

$$\text{or, } \frac{1}{x}dx = \frac{2v}{-3v^2 - 1}dv$$

Integrating,

$$\int \frac{1}{x}dx = -\int \frac{2v}{3v^2 + 1}dv$$

Let $1 + 3v^2 = z$

$\therefore 6vdv = dz$

$2vdv = \frac{dz}{3}$

$$\text{or, } \log x = -\int \frac{dz/3}{z}$$

$$\boxed{\text{or, } \cancel{\log x = -\frac{1}{3}\log z}}$$

$$\text{or, } \log x + \frac{1}{3}\log z = \log c$$

$$\text{or, } \log x + \frac{1}{3}\log(1 + 3v^2) = \log c$$

$$\text{or, } \log x(1 + 3v^2)^{1/3} = \log c$$

$$\therefore x(1 + 3v^2)^{1/3} = c \quad \text{(Ans)}$$

Ex-4) $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{y^2}{xy - x^2}$

$y = vx$

$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

or, $v + x\frac{dv}{dx} = \frac{v^2x^2}{x.vx - x^2}$

or, $v + x\frac{dv}{dx} = \frac{v^2}{v-1}$

or, $\frac{1}{x}dx = \frac{v-1}{v}dv$

Integrating, $\int\frac{1}{x}dx - \int\frac{v-1}{v}dv = 0$

or, $\log x - \int dv + \int\frac{1}{v}dv = 0$

or, $\log x - v + \log v = \log c$

$\boxed{\text{or, } \cancel{\log x} - \frac{y}{x} + \cancel{\log\frac{y}{x}} = \cancel{\log c}}$

or, $\log vx - v = \log c$

$\boxed{v = \log e^v}$

or, $\log vx = \log ce^v$

or, $vx = ce^v$

or, $\frac{y}{x}.x = ce^{\frac{y}{x}}$

$\therefore y = ce^{y/x}$ (Ans:)

Equation reducible to Homogenous From

$$\frac{dy}{dx} = \frac{ax+by+c}{a'x+b'y+c'}$$

If $\frac{a}{a'} = \frac{b}{b'}$ Then, $x = X+h$, $y = Y+K$

$$\frac{dy}{dx} = \frac{dY}{dX}$$

Now we get,

$$\frac{dY}{dX} = \frac{aX+bY+(ah+bK+c)}{a'X+b'Y+(a'h+b'K+c')}$$

We choose h and K such that $ah+bK+c=0$

$a'h+b'K+c'=0$

$$\therefore \frac{dY}{dX} = \frac{aX+bY}{a'X+b'Y}$$ is homogenous

equ^n can be solved putting $Y = vX$

Ex-1) $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$

$\frac{1}{2} \neq \frac{2}{1}$

Here, $\frac{1}{2} \neq \frac{2}{1}$

put $x = X+h$, $y = Y+k$

$$\frac{dY}{dX} = \frac{X+2Y+(h+2k-3)}{2X+Y+(2h+k-3)}$$

We choose h and k such that,

$$h+2k-3=0$$
$$2h+k-3=0$$

$\therefore h=1, \quad k=1$

$$\therefore \frac{dY}{dX} = \frac{X+2Y}{2X+Y}$$

$Y = vX$

$\therefore \frac{dY}{dX} = v + X\frac{dv}{dX}$

$$\text{or, } v + X\frac{dv}{dX} = \frac{X+2vX}{2X+vX}$$

$$\text{or, } v + X\frac{dv}{dX} = \frac{1+2v}{2+v}$$

$$\text{or, } X\frac{dv}{dX} = \frac{1+2v-2v-v^2}{2+v}$$

$$= \frac{1-v^2}{2+v}$$

$$\text{or, } \frac{1}{X}dX - \frac{2+v}{1-v^2}dv = 0$$

Integrating, $\int \frac{1}{x} dx - \int \frac{2+v}{1-v^2} dv = 0$

$1-v^2 = z$

$-2v\,dv = dz$

or, $\log x - 2\int \frac{1}{1-v^2} dv - \int \frac{v}{1-v^2} dv = 0$

~~or, $\log x - 2\int \frac{1}{z} dz$~~

or, $\log x - 2 \cdot \frac{1}{2} \log\left(\frac{1+v}{1-v}\right) - \left(-\frac{1}{2}\right)\int \frac{dz}{z} \cancel{dz} = 0$

or, $\log x - \log\left(\frac{1+v}{1-v}\right) + \frac{1}{2} \log z = \log c$

or, $\log X = \log\frac{1+v}{1-v} - \frac{1}{2}\log(1-v^2) + \log c$

or, $X = c\cdot\frac{1+v}{1-v}\cdot\frac{1}{\sqrt{1-v^2}}$

or, $X = c\,\frac{\sqrt{1+v}}{(1-v)^{3/2}}$

or, $X^2(1-v)^3 = c^2(1+v)$

or, $X^2\left(1-\frac{Y}{X}\right)^3 = c^2\left(1+\frac{Y}{X}\right)$

or, $X^2\left(\frac{X-Y}{X}\right)^3 = c^2\cdot\frac{X+Y}{X}$

or, $(X-Y)^3 = c^2(X+Y)$

or, $(x-1-y+1)^3 = c^2(x+y-2)$

$\therefore (x-y)^3 = c^2(x+y-2)$ (Ans)

$x = X+1$

$y = Y+1$

Ex-2) $(3x-7y-3)\frac{dy}{dx} = 3y-7x+7$

$\frac{-3}{3} \neq \frac{-7}{-7}$

$$\frac{dy}{dx} = \frac{3y-7x+7}{3x-7y-3} = \frac{-7x+3y+7}{3x-7y-3}$$

Here, $\frac{-7}{3} \neq \frac{3}{-7}$

putting $x = X+h, \quad y = Y+k$

$$\therefore \frac{dY}{dX} = \frac{-7X+3Y+(-7h+3k+7)}{3X-7Y+(3h-7k-3)}$$

We choose ~~has~~ h and k such that

$-7h+3k+7=0$

$3h-7k-3=0$

$\therefore h=1, \quad k=0$

$$\therefore \frac{dY}{dX} = \frac{-7X+3Y}{3X-7Y}$$

let,

$Y = vX$

$\frac{dY}{dX} = v + X\frac{dv}{dX}$

or, $v + X\frac{dv}{dX} = \frac{-7X+3vX}{3X-7vX} = \frac{-7+3v}{3-7v}$

or, $X\frac{dv}{dX} = \frac{3v-7}{3-7v} - v$

$$= \frac{3v-7-3v+7v^2}{3-7v}$$

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$$= \frac{7v^2-7}{3-7v}$$

or, $\frac{x}{7}\frac{dv}{dx} = \frac{v^2-1}{3-7v}$

or, $\frac{7}{x}dx = \frac{3-7v}{v^2-1}dv$

or, $\frac{7}{x}dx = -\left(\frac{2}{v-1}+\frac{5}{v+1}\right)dv$

or, Integrating,

$$\int\frac{7}{x}dx = -\int\frac{2}{v-1}dv - \int\frac{5}{v+1}dv$$

or, $7\log x = -2\log(v-1) - 5\log(v+1) + \log c$

or, $\log x^7 + \log(v-1)^2 + \log(v+1)^5 = \log c$

or, $\log x^7\cdot(v-1)^2\cdot(v+1)^5 = \log c$

or, $x^7(v-1)^2(v+1)^5 = c$

or, $x^7\left(\frac{Y}{X}-1\right)^2\left(\frac{Y}{X}+1\right)^5 = c$

or, $(Y-X)^2(Y+1)^5 = c$

$x = X+1$

$y = Y+0$

or, $(y-1-x+1)^2 (y-1+x-1)^5 = c$

or, $(y-x)^2 (x+y-2)^5 = c$

equn solution imp* 20-21 19-20

Linear Differential equations

$$\frac{dy}{dx} + Py = Q$$

P, Q are the function of x or constant

Multiply the both sides by $e^{\int P dx}$

$$\therefore e^{\int Pdx}\frac{dy}{dx} + Py\,e^{\int Pdx} = Q e^{\int Pdx}$$

[Intigrating factor, $I.F = e^{\int Pdx}$]

or, $\frac{d}{dx}\left[e^{\int Pdx}\cdot y\right] = Q e^{\int Pdx}$

Integrating both side,

$$e^{\int Pdx}\cdot y = \int Q e^{\int Pdx}\,dx + C$$

$$\boxed{IF\cdot y = \int Q\; I.F\; dx + c}$$

Ex-1) $(1-x^2)\frac{dy}{dx} - xy = 1$

or, $\frac{dy}{dx} - \frac{xy}{(1-x^2)} = \frac{1}{1-x^2}$ [Linear form]

Here,

$P = -\frac{x}{1-x^2}$

$\therefore$ I.F. $= e^{\int -\frac{x}{1-x^2}dx} = e^{\frac{1}{2}\int -\frac{2x}{1-x^2}dx} = e^{\frac{1}{2}\log(1-x^2)} = \sqrt{1-x^2}$

Hence the solution,

$y \cdot \sqrt{1-x^2} = \int \frac{1}{1-x^2} \cdot \sqrt{1-x^2}\, dx + c$

$= \int \frac{1}{\sqrt{1-x^2}} dx + c$

$= \sin^{-1}x + c$

$\therefore y = \frac{\sin^{-1}x}{\sqrt{1-x^2}} + c$

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Ex-2(a) $x\frac{dy}{dx}+2y=x^2\log x$

or, $\frac{dy}{dx}+\overset{P}{\frac{2}{x}}y=\overset{Q}{x\log x}$

Here,

$P=\frac{2}{x}$

$\therefore IF = e^{\int\frac{2}{x}dx}$

$= e^{2\log x}$

$= x^2$

$$\int x\log x\,dx$$

$$= \log x\cdot(\int x\,dx)+\int\left(\int\log x\,dx\cdot\int x\,dx\right)dx$$

$$= \log x + \int\left(\frac{1}{x}\cdot\frac{x^2}{2}\right)dx$$

$$= \log x+\frac{1}{2}\int x\,dx$$

$$= \log x+\frac{x^2}{4}+c$$

Hence the solution,

$y\cdot x^2=\int x\log x\cdot x^2dx+c$

or, $x^2y=\int x^3\log x\,dx+c$

$=\log x\cdot\frac{x^4}{4}-\int\left(\frac{1}{x}\cdot\frac{x^4}{4}\right)dx+c$

$=\log x\cdot\frac{x^4}{4}-\frac{1}{4}\frac{x^4}{4}+c$

$=\frac{1}{4}x^4\log x-\frac{1}{16}x^4+c$

$\therefore y=\frac{1}{4}x^2\log x-\frac{1}{16}x^2+cx^{-2}$

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Ex-2(b) $x\frac{dy}{dx} + 2y = x^4$

or, $\frac{dy}{dx} + \frac{2}{x}y = x^3$

Here,

$P = \frac{2}{x}$

$\therefore I.F = e^{\int \frac{2}{x}dx}$

$= e^{\log x^2}$

$= x^2$

Hence the solution;

$y \cdot x^2 = \int x^3 \cdot x^2 dx + c$

or, $x^2 y = \frac{x^6}{6} + c$

$\therefore 6y = x^4 + cx^{-2}$

Bernoulii Equation

$$\frac{dy}{dx} + Py = Qy^n$$

where P and Q are functions of x or constants.

Diving both side by y^n,

$$y^{-n}\frac{dy}{dx} + Py^{1-n} = Q \quad \text{——} ①$$

Let, $y^{1-n} = v$

$$\text{or, } (1-n)\, y^{-n}\frac{dy}{dx} = \frac{dv}{dx}$$

Then ① becomes,

$$\frac{1}{1-n}\frac{dv}{dx} + Pv = Q$$

$$\text{or, } \frac{dv}{dx} + P(1-n)v = (1-n)Q$$

which is the linear equ^n of v and x.

$$\text{I.F.} = e^{\int P(1-n)\,dx}$$

$$\boxed{\text{IF}\cdot v = \int (1-n)Q \cdot \text{IF}\, dx + C}$$ → IF বসাবো এবং v এর মান replace করে দিবো।

Ex-1) $\frac{dy}{dx} = x^3y^3 - xy$

or, $\frac{dy}{dx} + xy = x^3y^3$

or, $y^{-3}\frac{dy}{dx} + xy^{-2} = x^3$ [Divide both side by y^3]

— ①

Let, $y^{-2} = v$

or, $-2y^{-3}\frac{dy}{dx} = \frac{dv}{dx}$

$\therefore$ $y^{-3}\frac{dy}{dx} = -\frac{1}{2}\frac{dv}{dx}$

from ① we get,

$$-\frac{1}{2}\frac{dv}{dx} + xv = x^3$$

or, $\frac{dv}{dx} - 2xv = -2x^3$

$\therefore$ I.F $= e^{\int -2x.dx}$

$= e^{-2\frac{x^2}{2}}$

$= e^{-x^2}$

Hence the solution,

$$e^{-x^2} \cdot v = \int -2x^3 \cdot e^{-x^2}\, dx + c$$

or, $$ve^{-x^2} = -2\int x^3 e^{-x^2}\, dx + c$$

$$= -2\left[\cancel{x^3} e^{-x^2} \cdot \frac{x^4}{4} - \int \left(e^{-x^2}(-2x) \cdot \frac{x^4}{4}\right) dx + c\right.$$

$$= -2\left[\frac{x^4}{4} \cdot e^{-x^2} + \_\_\_\right.$$

Do it yourself -

Chapter - 3

20-21 / 19-20

Equation of First order and First Degree

Equ^n of first degree -

$$Mdx + Ndy = 0$$

Working rule —

If $Mdx + Ndy = 0$ satisfy the condition

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

3 rules

Then it is exact.

$$\boxed{\int M dx + \int (N \text{ free from } x)\, dy = C}$$

Working rules

1) integrate M with regard to x regarding y as constant.

11) Find out those term in N which are free from x and integrate them with regard to y.

111) add the two expressions so obtained and equate the sum to an arbitory Constant.

Ex-1) $(y^4 + 4x^3y + 3x)\,dx + (x^4 + 4xy^3 + y + 1)\,dy = 0$

$\Rightarrow$ Let, $M = y^4 + 4x^3y + 3x$

$\boxed{\cancel{N = x^4 + 4xy^3 + y + 1}}$

$\therefore \frac{\partial M}{\partial y} = 4y^3 + 4x^3$

$N = x^4 + 4xy^3 + y + 1$

$\therefore \frac{\partial N}{\partial x} = 4x^3 + 4y^3$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Then equ^n is exact.

Now,

$\int (y^4 + 4x^3y + 3x)\,dx + \int (y+1)\,dy = c$

or, $y^4x + 4y \cdot \frac{x^4}{4} + 3\frac{x^2}{2} + \frac{y^2}{2} + y = c$

$\therefore xy^4 + x^4y + \frac{3}{2}x^2 + \frac{y^2}{2} + y = c$

Ex-2) $x(x^2+y^2-a^2)\,dx + y(x^2-y^2-b^2)\,dy = 0$

or, $(x^3+xy^2-a^2x)\,dx + (x^2y - y^3 - b^2y)\,dy = 0$

$\Rightarrow$ Let,

$M = x^3 + xy^2 - a^2x$

$\therefore \frac{\partial M}{dy} = 2xy$

$N = x^2y - y^3 - b^2y$

$\therefore \frac{\partial N}{\partial x} = 2xy$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Then equ^n is exact.

Now,

$$\int (x^3 + xy^2 - a^2x)\,dx + \int(-y^3 - b^2y)\,dy = c$$

or, $\frac{x^4}{4} + y^2\frac{x^2}{2} - a^2\frac{x^2}{2} - \frac{y^4}{4} - b^2\frac{y^2}{2} = c$

$\therefore x^4 + 2x^2y^2 - 2a^2x^2 - y^4 - 2b^2y^2 = C$

Ex-3) $(x^2 - 2xy + 3y^2)\,dx + (4y^3 + 6xy - x^2)\,dy = 0$

$\Rightarrow$

Let, $M = x^2 - 2xy + 3y^2$

$\therefore \frac{\partial M}{dy} = 6y - 2x$

$N = 4y^3 + 6xy - x^2$

$\therefore \frac{\partial N}{\partial x} = 6y - 2x$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Then equ^n is exact.

Now,

$$\int (x^2 - 2xy + 3y^2)\, dx + \int 4y^3\, dy = C$$

or, $\frac{x^3}{3} - 2y\frac{x^2}{2} + \cancel{y^3} + 3y^2x + 4 \cdot \frac{y^4}{4} = C$

$\therefore x^3 - 3x^2y + 3y^4 \cancel{= C} + 9y^2x = C$

Ex-4) $(x - 2e^y)\, dy + (y + x\sin x)\, dx = 0$

or, $(y + x\sin x)dx + (x - 2e^y)dy = 0$

$\Rightarrow$ Let $M = y + x\sin x$

$\therefore \frac{\partial M}{\partial y} = 1$

$N = x - 2e^y$

$\therefore \frac{\partial N}{\partial x} = 1$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ then equ^n is exact.

Now,

$$\int (y + x\sin x)dx + \int (-2e^y)\, dy = C$$

or, $xy + x \cdot (-\cos x) - \int (-\cos x)\, dx + (-2)e^y = C$

$\therefore\ xy + \sin x - x\cos x - 2e^{y} = c$

Ex-5)

$$x\,dx + y\,dy = \frac{a^2(x\,dy - y\,dx)}{x^2+y^2}$$

or, $$x\,dx + y\,dy = \frac{a^2 x}{x^2+y^2}\,dy - \frac{a^2 y}{x^2+y^2}\,dx$$

or, $$\left(x + \frac{a^2 y}{x^2+y^2}\right)dx + \left(y - \frac{a^2 x}{x^2+y^2}\right)dy = 0$$

Let,

$$M = x + \frac{a^2 y}{x^2+y^2}$$

$$\therefore \frac{\partial M}{\partial y} = \frac{(x^2+y^2)a^2 - a^2 y\cdot 2y}{(x^2+y^2)^2} = \frac{(x^2+y^2)a^2 - 2a^2y^2}{(x^2+y^2)^2}$$

$$= \frac{a^2(x^2-y^2)}{(x^2+y^2)^2}$$

$$N = y - \frac{a^2 x}{x^2+y^2}$$

$$\therefore \frac{\partial N}{\partial x} = -\frac{(x^2+y^2)a^2 - a^2x\cdot 2x}{(x^2+y^2)^2} = \frac{a^2(x^2-y^2)}{(x^2+y^2)^2}$$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Then equn is exact

$$\int\left(x + \frac{a^2 y}{x^2+y^2}\right)dx + \int y\,dy = c$$

or, $$\frac{x^2}{2} + a^2 y\,\frac{1}{y}\tan^{-1}\frac{x}{y} + \frac{y^2}{2} = c$$

$$\therefore x^2 + 2a^2 \tan^{-1}\frac{x}{y} + y^2 = c \quad \text{(Ans:)}$$

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Chap-3

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Ex-6) $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$

Let,

$$M = 1+e^{\frac{x}{y}}$$

$$\therefore \frac{\partial M}{\partial y} = e^{\frac{x}{y}} \cdot \left(\frac{y.0 - x}{y^2}\right) = e^{x/y} \cdot \left(-\frac{x}{y^2}\right)$$

$$N = e^{x/y}\left(1-\frac{x}{y}\right)$$

$$\therefore \frac{\partial N}{\partial x} = e^{x/y}\left(-\frac{x}{y^2}\right) + \left(1-\frac{x}{y}\right) e^{x/y}\left(-\frac{x}{y^2}\right)$$

$$= e^{x/y} \cdot \left(\frac{x}{y^2}\right) - \frac{y-x}{y} \cdot \frac{x}{y^2} \cdot e^{\frac{x}{y}}$$

$$\therefore \frac{\partial N}{\partial x} = e^{x/y} \cdot \left(-\frac{1}{y}\right) + \left(1-\frac{x}{y}\right) \cdot e^{x/y} \cdot \frac{1}{y}$$

$$= e^{x/y}\left\{\frac{1}{y} - \frac{x}{y^2} - \frac{1}{y}\right\}$$

$$= e^{x/y}\left(-\frac{x}{y^2}\right)$$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Then equ^n is exact.

Now, $\int (1+e^{\frac{x}{y}})dx + \int 0\,dy = c$

or, $x + \frac{e^{x/y}}{1/y} = c$

$\therefore x + ye^{x/y} = c$ (Ans.)

Ex-7) $[\cos x \tan y + \cos(x+y)]dx + [\sin x \sec^2 y + \cos(x+y)]dy = 0$

Here,

$M = \cos x \tan y + \cos(x+y)$

$\therefore \frac{\partial M}{\partial y} = \cos x . \sec^2 y + - \sin(x+y) . 1$

$= \cos x . \sec^2 y - \sin(x+y)$

$N = \sin x \sec^2 y + \cos(x+y)$

$\therefore \frac{\partial N}{\partial x} = \sec^2 y \cos x - \sin(x+y)$

since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Then equ^n is exact.

Now,

$$\int \{\cos x \tan y + \cos(x+y)\} dx + \int 0\, dy = c$$

or, $\int (\cos x \tan y)\, dx + \int \cos(x+y)\, dx = c$

or, $\tan y \sin x + \sin(x+y) = c$

$\therefore \sin x \tan y + \sin(x+y) = c$ (Ans)

Ex-8) $(\cos x \tan y - \sin x \sec y)dx + (\sin x \sec^2 y + \cos x \tan^2 y \operatorname{cosec} y)\, dy = 0$

Here,

$M = \cos x \tan y - \sin x \sec y$

$\therefore \dfrac{\partial M}{\partial y} = \cos x \sec^2 y - \sin x \sec y \tan y$

$N = \sin x \sec^2 y + \cos x \tan^2 y \operatorname{cosec} y$

$\therefore \dfrac{\partial N}{\partial x} = \cos x \sec^2 y - \sin x \tan^2 y \operatorname{cosec} y$

$= \cos x \sec^2 y - \sin x \sec y \tan y$

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since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Then the equn is exact.

Now,

$$\int (\cos x \tan y - \sin x \sec y)\,dx + \int 0\,dy = c$$

or, $\sin x . \tan y + \cos x \sec y = c$ (Ans:)

Ex:-9)

$$(\sin x \cos y + e^{2x})\,dx + (\cos x \sin y + \tan y)\,dy = 0$$

Here,

$M = \sin x \cos y + e^{2x}$

$\therefore \frac{\partial M}{\partial y} = - \sin x . \sin y$

$N = \cos x \sin y + \tan y$

$\therefore \frac{\partial N}{\partial x} = - \sin x \sin y$

since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Then the equn is exact.

Now,

$$\int (\sin x \cos y + e^{2x})\, dx + \int \tan y \, dy = c$$

or, $\cos y . (-\cos x) + \frac{1}{2} e^{2x} + \log(\sec y) = c$

$\therefore -\cos x \cos y + \frac{1}{2} e^{2x} + \log(\sec y) = c$

(Ans;)

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Integrating factor

If an equ^n becomes exact after it has multiplied by a function of x and y, then such a function is called an integrating factor.

Rules of finding an integrating factor:

rule-1: If $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$, a function of x, then $e^{\int f(x)\,dx}$ is an integrating factor.

rule-2: If $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = f(y)$, a function of y, then $e^{\int - f(y)}$ is an integrating factor.

Ex-1) $\cancel{(x+y^2)dy + (y-x^2)dx = 0}$

$$(x^2+y^2+x)\,dx + xy\,dy = 0$$

Let
$M = x^2+y^2+x$

$\therefore \frac{\partial M}{\partial y} = 2y$

$N = xy$

$\therefore \frac{\partial N}{\partial x} = y$

Since $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$; equ^n is not exact.

However,
$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2y-y}{xy} = \frac{1}{x}$$, a function of x alone.

Hence
$$IF = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

Multiplaying by IF,

$$x(x^2+y^2+x)\,dx + x^2y\,dy = 0$$

exact equ^n,

$$\int (x^3+xy^2+x^2)\,dx + \int 0\,dy = C$$

or, $\frac{x^4}{4} + y^2\frac{x^2}{2} + \frac{x^3}{3}$ $\cancel{+\frac{x^2y^2}{2} + c}$ $+ c_1 = C$

$\cancel{\therefore \frac{x^4}{4} + \frac{1}{2}x^2y^2 + \frac{1}{3}x^3 + c_1 = C \text{ (Ans)}}$

$$3x^4 + 6x^2y^2 + 4x^3 = C$$ (Ans)

Ex-2) $(x^2+y^2+1)\,dx - 2xy\,dy = 0$
$\quad M \qquad\qquad\qquad N$

Let $\cancel{x^2+y^2+1=0}$ $M = x^2+y^2+1$

$$\therefore \frac{\partial M}{\partial y} = 2y$$

$$N = -2xy$$

$$\therefore \frac{\partial N}{\partial x} = -2y$$

Since $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ Then equ^n is not exact.

However,

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2y+2y}{-2xy} = -\frac{2}{x}, \text{ a function of } x$$

$$\therefore I.F = e^{\int -\frac{2}{x}dx}$$

$$= e^{-2\log x}$$

$$= x^{-2}$$

Multiplying by IF,

$$x^{-2}(x^2+y^2+1)\,dx - 2x^{-1}y\,dy = 0$$

exact equ^n,

$$\cancel{\int (2+y^2x)dx - \int 0\,dy = c}$$

$$\int (1+x^{-2}y^2+x^{-2})\,dx - \int 0\,dy = c$$

or, $x + y^2\frac{x^{-1}}{-1} + \frac{x^{-1}}{-1} = c$

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or, $x - \frac{y^2}{x} - \frac{1}{x} = c$

or, $x^2 - y^2 - 1 = cx$

$\therefore x^2 + y^2 = cx + 1$ (Ans:)

Ex-3) $(x^2+y^2)dx - 2xy\,dy = 0$

Let
$M = x^2+y^2$

$\therefore \frac{\partial M}{\partial y} = 2y$

$N = -2xy$

$\therefore \frac{\partial N}{\partial x} = -2y$

Since $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ Then equn is not exact.

However, $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2y+2y}{-2xy} = -\frac{2}{x}$, a function of x.

$IF = e^{-\int \frac{2}{x} dx}$

$= e^{-2\log x}$

$= x^{-2}$

Multiplying by $\frac{1}{x^2}$

$$\frac{1}{x^2}(x^2+y^2)\,dx - 2.\frac{1}{x^2}y\,dy = 0$$

or, $$\left(1+\frac{y^2}{x^2}\right)dx - 2\frac{y}{x^2}dy = 0$$

Exact equation,

$$\int\left(1+\frac{y^2}{x^2}\right)dx - \int 0\,dy = c$$

or, $$x + y^2.\frac{x^{-1}}{-1} = c$$

or, $$x - \frac{y^2}{x} = c$$

$$\therefore x^2 - y^2 = cx \quad (\text{Ans:})$$

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Linear differential equation with coefficient

* $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$

The equation is

$$(D^3 + 6D^2 + 11D + 6)y = 0$$

Let $y = e^{mx}$ then $\frac{dy}{dx} = me^{mx}$, $\frac{d^2y}{dx^2} = m^2e^{mx}$, $\frac{d^3y}{dx^3} = m^3e^{mx}$

The equation becomes-

$$(m^3 + 6m^2 + 11m + 6)e^{mx} = 0$$

Auxiliary equation is -

$$m^3 + 6m^2 + 11m + 6 = 0$$

Sloving the equation -

$$m = -1, \quad m_2 = -2, \quad m_3 = -3$$

The complete solution is -

$$y = c_1e^{-x} + c_2e^{-2x} + c_3e^{-3x}$$

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Ex-1) $$\frac{d^3y}{dx^3} - 13\frac{dy}{dx} - 12y = 0$$

The equation is -

$$(D^3 - 13D - 12)y = 0$$

Let, $y = e^{mx}$ then $\frac{dy}{dx} = me^{mx}$, $\frac{d^3y}{dx^3} = m^3 e^{mx}$

The equation becomes -

$$(m^3 - 13m - 12)e^{mx} = 0$$

Auxilary equation is -

$$m^3 - 13m - 12 = 0$$

Solving the equation -

$$m = -1, \ m = -3, \ m = 4$$

The complete solution,

$$y = c_1 e^{-x} + c_2 e^{-3x} + c_3 e^{4x}$$

Ex-2) $$(D^3 + 6D^2 + 11D + 6)y = 0$$

Let $y = e^{mx}$, then $D^3y = m^3 e^{mx}$, $D^2y = m^2 e^{mx}$, $Dy = me^{mx}$

The equation becomes -

$$(m^3 + 6m^2 + 11m + 6)e^{mx} = 0$$

Auxiliary equation is —

$$m^3+6m^2+11m+6=0$$

Solving the equation —

$$m=-1,-2,-3$$

The complete solution is —

$$y=c_1e^{-x}+c_2e^{-2x}+c_3e^{-3x}$$

Ex-3) $\dfrac{d^4y}{dx^4}-\dfrac{d^3y}{dx^3}-9\dfrac{d^2y}{dx^2}-11\dfrac{dy}{dx}-4y=0$

The equation is -

$$(D^4-D^3-9D^2-11D-4)y=0$$

Let $y=e^{mx}$ then, $\dfrac{dy}{dx}=me^{mx}$, $\dfrac{d^2y}{dx^2}=m^2e^{mx}$, $\dfrac{d^3y}{dx^3}=m^3e^{mx}$,

$$\frac{d^4y}{dx^4}=m^4e^{mx}$$

The equation becomes —

$$(m^4-m^3-9m^2-11m-4)e^{mx}=0$$

Auxiliary equation is —

$$m^4-m^3-9m^2-11m-4=0 \Rightarrow (m+1)^3(m-4)$$

Solving the equation —

$$m=-1,-1,-1,4$$

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The complete solution is—

$$y = (c_1 + c_2 x + c_3 x^2)e^{-x} + c_4 e^{4x}$$

Ex-4) $(D^4 + 5D^2 + 6)y = 0$

Auxiliary equation is—

$$D^4 + 5D^2 + 6 = 0$$

or, $(D^2+3)(D^2+2) = 0$

or, $D = \pm\sqrt{3}i, \pm\sqrt{2}i$

Complete solution is—

$$y = c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x + c_3 \cos\sqrt{2}x + c_4 \sin\sqrt{2}x$$

—o—

Ex-5) $(D^4 - D^3 - D + 1)x = 0$

Auxiliary equation is—

$$D^4 - D^3 - D + 1 = 0$$

or, $D^3(D-1) - 1(D-1) = 0$

or, $(D-1)(D^3-1) = 0$

or, $(D-1)^2(D^2+D+1) = 0$

$\therefore D = 1, -1/2, 1, \pm\frac{\sqrt{3}}{2}i$

$\cancel{D^2 + D + 1 = 0}$

$\cancel{D^2 + 2\cdot\frac{1}{2}\cdot D + \left(\frac{1}{2}\right)^2 + 1 - \left(\frac{1}{2}\right)^2}$

$\cancel{\left(D + \frac{1}{2}\right)^2 - \frac{3}{2}}$

$\cancel{\Rightarrow \left(D + \frac{1}{2} + \sqrt{\frac{3}{4}}\right)\left(D + \frac{1}{2} - \sqrt{\frac{3}{4}}\right)}$

$\cancel{= \left(D + \frac{1+\sqrt{3}}{2}\right)\left(D + \frac{1-\sqrt{3}}{2}\right)}$

$\cancel{D = -1, 1, \frac{-1-\sqrt{3}}{2}, \frac{-1+\sqrt{3}}{2}}$

The solution is

$$y = (c_1 + c_2 x)e^{x} + e^{-x/2}\left[c_3 \cos\frac{\sqrt{3}}{2}x + c_4 \sin\frac{\sqrt{3}}{2}x\right]$$

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Introduction and Introductory Concepts

20-21
Statistics : Statistics is a branch of applied mathematics that involves the collection, description, analysis and inference of conclusions from quantitative data.

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* function of Statistics:

i) collection of data

ii) Organization of data

iii) Presentation of data

iv) Analysis of data

v) Interpretation and drawing inference.

Collect
↓
Organized
↓
Presentation
↓
Analysis
↓
Drawing inference

~~* Limitations of statistics~~

~~i) Does not focus on Indiv~~

* Characteristic Features of Statistics (PCI)

1) Statistics deals with aggregate of individuals rather than with individuals. Per capital income of a country is a statistical information because it is an information about the population.

II) Statistics deals with variation

III) Statistics deals with only numerically specified population.

iv) Statistical inferences are drawn with the probability of uncertainty.

v) The logic used in statistical inference is inductive.

* Uses of statistics:

1) Statistics in Agriculture ✓

II) ,, ,, Economics ✓

III) ,, ,, Planning ✓

IV) ,, ,, Biology ✓

V) ,, ,, Trade and commerce. ✓

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* Limitation of Statistics

i) Does not Focus on Individuals : satistics deals with data from groups and populations, not individuals.

ii) Results Are True on Average: Statistical results are generally accurate for a group but may not always hold true for every individual case.

iii) Based on sampling: statistics often relies on data collected through sample surveys.

Chap-2

Variable and frequency Distribution

Variable: Measurable characteristics of a population that may vary from element to element either in magnitude or in ~~quant~~ quality are called variables.

Two types — quantitative & qualitative.

i) **Quantitatives Variables:** Variables characteristics, whose values are expressed numerically, are known as quantitative variables. Ex: Height / weight, of students, weight of tomato, number of grapes per bunch etc.

ii) **Qualitative variable:** Some variables, which express the quality of population elements, cannot be numerical measured but can be classified or categorised, then are called qualitative variable. Ex: merit of students, type of farmers, type of fishes (sea fish, river fish) etc.

Frequency: Frequency refers to the number of times an event occurs within a given dataset or population. It's often used to analyzed distributions and patterns in data.

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Frequency Distribution: Arrangement of observational data according to frequencies of the observations is called frequency distribution.

Construction of Frequency Distribution: Imp point

Steps in constructing a frequency distribution below –

i) **Finding the range:** The difference between the higest value and the lowest value is called the range. It is denoted by R.

Range = Higest value – Lowest value.

ii) **Decision About the Number of Classes:** There is no hard and fast rule for choosing the number of classes. It generally expected to limit between 7 and 15. However M.A. Sturge's formula gives a guideline for desired number of classes.

$$K = 1 + 3.322 \log_{10} N$$

Here,
N = Total number of observations
K = desired number of classes

iii) <u>Choosing the class Interval:</u>

$$\text{Class Interval, } C = \frac{\text{Range}}{\text{No. of classes}}$$

iv) <u>Counting of Frequencies:</u> For convenience of counting the number of observations falling within each class tally marks are used.

<u>Ex: 2.2</u>

Higest value = 120

lowest value = 52

Range, R = 120 − 52 = ~~88~~ 68

Here,
Number of observations, N = 80

According to Sturge's formula -

$$K = 1 + 3.322 \log_{10} 80$$

$$= 7.322$$

The data set may be grouped in about 8 classes.

Now $C = \frac{R}{K}$

$= \frac{68}{8} = 8.5 \approx 9$

It will be convenient to take 10 as the class interval. The lowest observation is 52, it is convenient to start from 50.

The frequency distribution will be -

Class interval	Tally mark	Frequency	Cumulative frequency Ascending	Cumulative frequency Descending
50-60	𝍸	5	5	80
60-70	𝍸 IIII	9	14	75
70-80	𝍸 𝍸 III	13	27	66
80-90	𝍸 𝍸 𝍸 𝍸	20	47	47
90-100	𝍸 𝍸 𝍸 IIII	19	66	27
100-110	𝍸 IIII	9	75	14
110-above	𝍸	5	80	5

* Graphical Representation of frequency Distribution:

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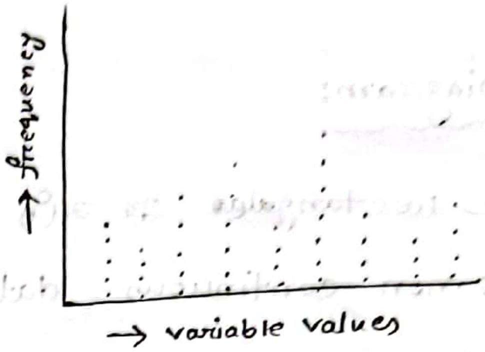
1) Dot Frequency Diagram

The X-axis is used for the variable values and the Y-axis is for the frequency.



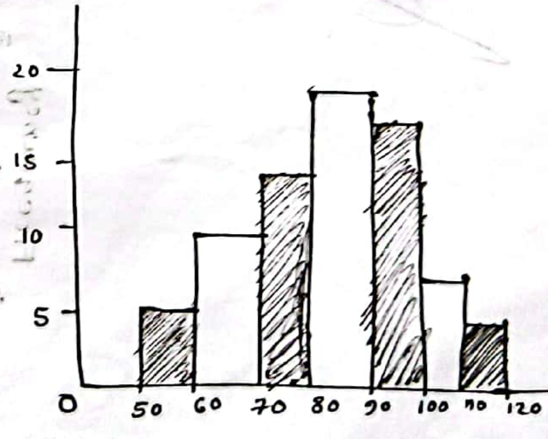
If we indicate the frequencies of each variable value by dots, the resulting diagram is known as dot frequency diagram.

11) Histogram

→ bar এর মতো কিন্তু কোন gap নাই-

→ continuous data এর জন্য ব্যবহার করা হয়। 5-10, 10-20 এমন class interval হবে।

For continuous variable, the rectangles such drawn are attached to adjacent rectangles at both-sides and the resulting graph is known as histogram.



∴ frequency density.

$$f_d = \frac{f}{c}$$

c = class interval

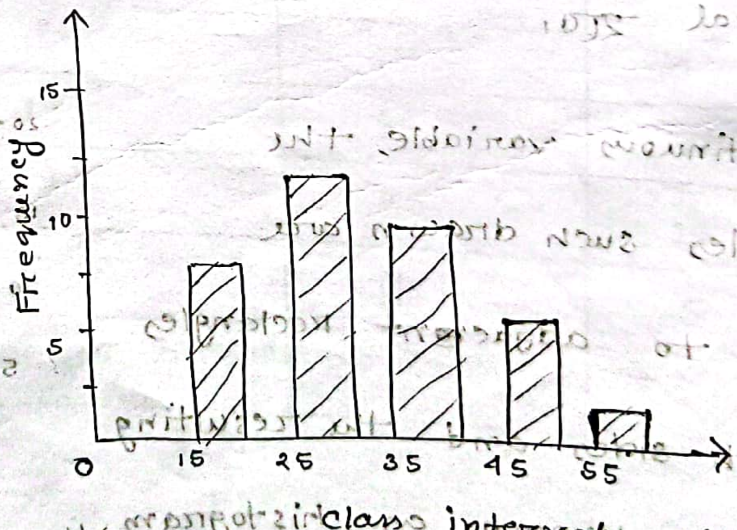
f = no. of families

Bar Diagram:

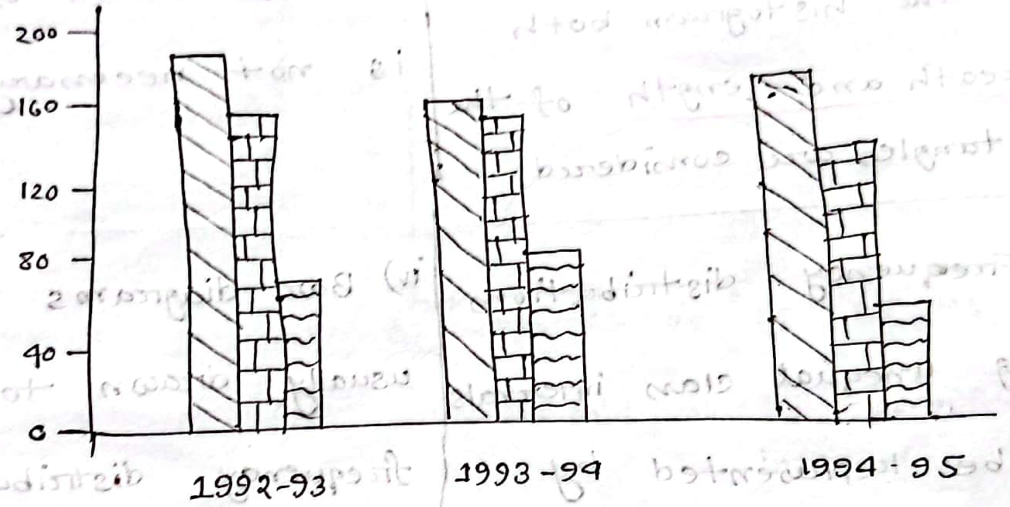
→ rectangular এর মধ্যে gap আছে

→ non continuous data এর ক্ষেত্রে ব্যবহার করা হয়। class interval হবে 5–10, 11–20, 21–30 এরকম

"For discrete variables a gap exists between the upper limit of a class and the lower limit of the following class and the adjacent rectangles are not attached to each other. The graph is known as bar diagram."



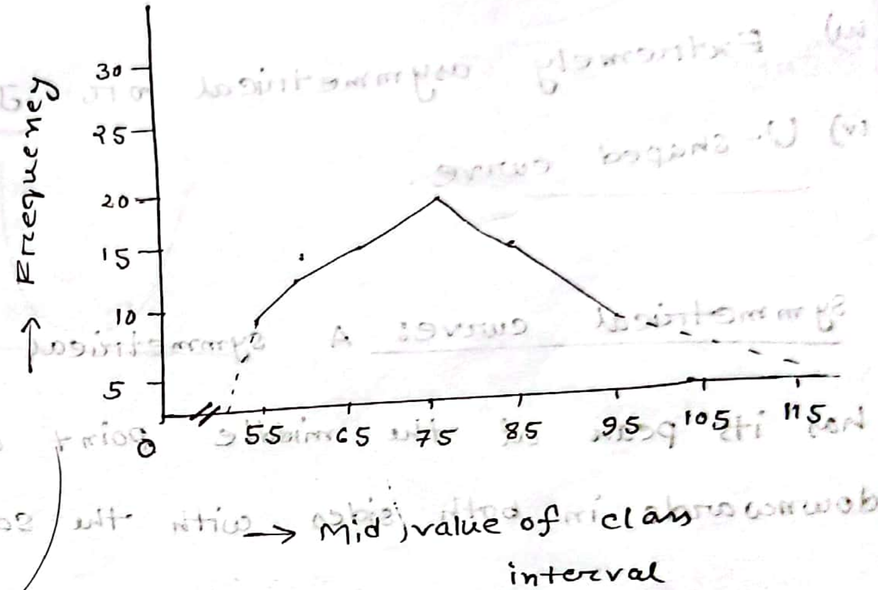
Multiple Bar Diagram: Data on several variables in respect of different places or timepoints may be represented by multiple bar diagrams. The simple bars for the variables corresponding to a place or time-point are constructed side by side (without gap).



Multiple bar diagram for pulses production.

Histogram	Bar diagram
i) Histograms are used to represent continuous frequency distributions	i) Bar diagrams are used to represent discrete frequency distributions.
ii) used to represent frequency distributions only	ii) —— frequency distribution, data on different place or time points
iii) In drawing the rectangles of the histogram both breath and length of the rectangles are considered	iii) consideration of the breath is not necessary.
iv) Frequency distributions having unequal class intervals may be represented by histograms: in such ~~case~~ case the breath of the rectangles are unequal	iv) Bar diagrams are not usually drawn to represent frequency distributions having unequal class intervals.

Frequency polygon: A frequency polygon is a straight line graph of class frequency plotted against class midpoint. It is almost identical to a histogram, which is used to compare sets of data or to display a comulative frequency distribution.



Frequency polygon can also be obtained by joining the mid points of the vertical lines of the rectangles of the histogram.

Frequency curve: In drawing frequency polygons, the consecutive points are connected by straight lines. If the points are connected by a free hand smooth curve, the resulting graph is known as frequency curve.

~~Frequency~~

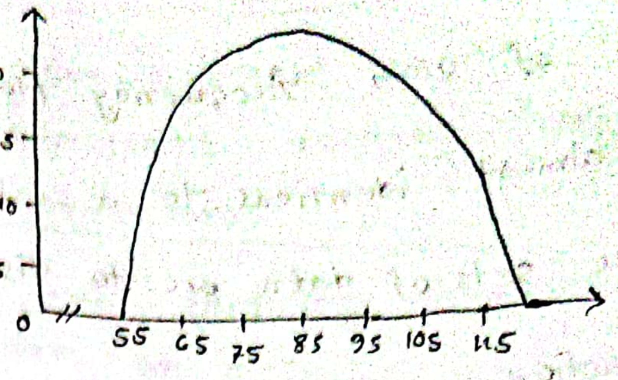
Forms of Frequency curves:

i) Symmetrical curve

ii) Moderately asymmetrical or skew curve

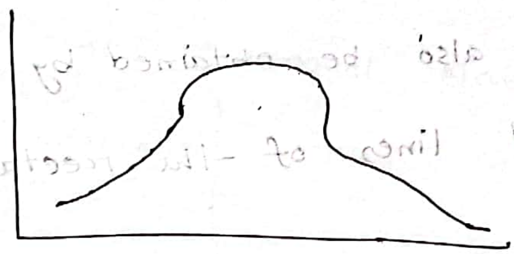
iii) Extremely asymmetrical or J-shaped curve

iv) U-shaped curve.

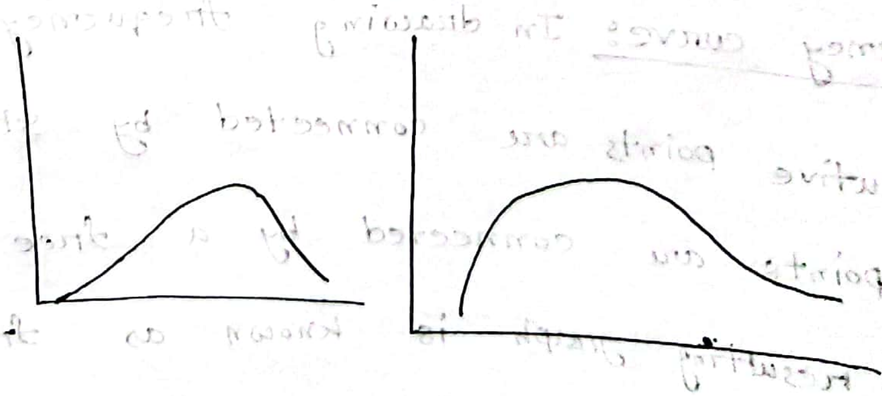


Frequency curve

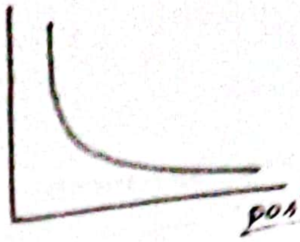
Symmetrical curve: A symmetrical curve is one which has its peak at the middle point and gradually moves downwards in both sides with the same rate of decrease.



Moderately Asymmetrical or Skew curve:



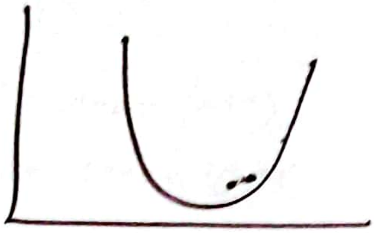
Extremely Asymmetrical or J-shaped Curve:



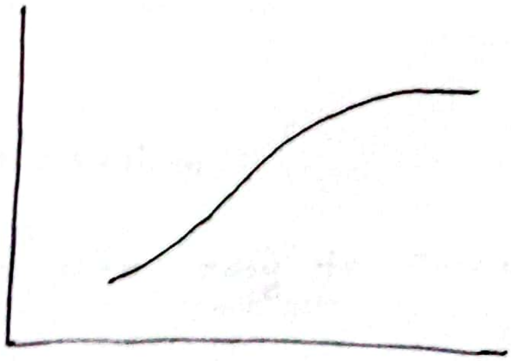
positive

negative

U-shaped curve



Cummulative Frequency curve or the ogive:



Chapter-3

Central Tendency and its Measures

Def: In statistics, a central tendency is a central or typical value for a probability distribution.

Different measures of central tendency:

1) Mean
   - a) Arithmetic Mean (AM)
   - b) Geometric Mean (GM)
   - c) Harmonic mean (HM)

2) Median and quantiles

3) Mode

Characteristics of CT

- It should be ~~be~~ rigidly defined
- ——— be based on all the observations.
- ——— be readily comprehensible and easy to calculate
- ——— suitable for further algebraic treatment.
- ——— least affected by sampling fluctuations.

Arithmetic Mean

Arithmetic mean of a set of observations is there sum divided by the number of observations.

Two types—

i) Simple Arithmetic:

The arithmetic mean,

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

In case of frequency distribution,

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \dots + f_nx_n}{f_1 + f_2 + \dots + f_n}$$

$$= \frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i}$$

Where

$$\sum_{i=1}^{n} f_i = N$$

$$= \frac{1}{N}\sum_{i=1}^{n} f_ix_i$$

ii) weighted Arithmetic Mean:

$$\bar{x} = \frac{x_1w_1 + x_2w_2 + \dots + x_nw_n}{w_1 + w_2 + \dots + w_n} = \frac{\sum_{i=1}^{n} w_ix_i}{W}$$

Where

$$W = \sum_{i=1}^{n} w_i$$

Advances

Disadvantages

Uses

Properties of Arithmetic Mean

~~Ex-3.1)~~

Ex-3.1) : Direct Method.

Daily wages T.k.	Number of workers f_i	Mid Value x_i	$f_i x_i$
50-55	5	52.5	262.5
55-60	10	57.5	575.0
60-65	25	62.5	1562.5
65-70	35	67.5	2362.5
70-75	15	72.5	1087.5
75-80	7	77.5	542.5
80-85	3	82.5	247.5
	100		6640.0

$$\therefore \bar{x} = \frac{1}{N}\sum f_i x_i = \frac{1}{100}(6640.0) = 66.40 \text{ T.k.}$$

$\therefore$ Average daily wage is Tk. 66.40 ।

Indirect Method:

Daily wages (Tk.)	Numbers of Workers (f_i)	Mid value (x_i)	New variable $u_i = \frac{x_i - 67.5}{5}$	f_iu_i
50 - 55	5	52.5	-3	-15
55 - 60	10	57.5	-2	-20
60 - 65	25	62.5	-1	-25
65 - 70	35	67.5	0	0
70 - 75	15	72.5	1	15
75 - 80	7	77.5	2	14
80 - 85	3	82.5	3	9
	100			-22

New variable, $\boxed{u_i = \frac{x_i - a}{h}}$

where,
$a = 67.5$
$h = 5$

Now, $\bar{u} = \frac{1}{N} \sum f_iu_i$

$= \frac{1}{100}(-22)$

$= -0.22$

$\therefore \bar{x} = a + h\bar{u} = 67.5 + 5(-0.22) = 66.40$

$\therefore$ Average daily wage is Tk. 66.40.

Geometric Mean (GM).

Geometric mean of a set of n non-zero positive observations is the nth root of their product.

The Geometric Mean,

$$GM = \text{Antilog}\left\{\frac{1}{n}\sum_{i=1}^{n}\log x_i\right\}$$

$$= \sqrt[N]{x_1^{f_1}\cdot x_2^{f_2}\cdots\cdots x_n^{f_n}}$$

In case of frequency distribution,

$$GM = \text{Antilog}\left[\frac{1}{N}\sum_{i=1}^{n} f_i \log x_i\right]$$

where, $N = \sum_{i=1}^{n} f_i$

Advantages

disadvantages

uses

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Ex-3.2) For computation of geometric mean, we construct the following table:

Rate of change of yield (%)	Frequency (f_i)	Mid value (x_i)	$\log x_i$	$f_i \log x_i$
0-5	1	2.5	0.39794	0.39794
5-10	2	7.5	0.87506	1.75012
10-15	4	12.5	1.09691	4.38764
15-20	2	17.5	1.24304	2.48608
20-25	1	22.5	1.35218	1.35218
	$\sum f_i = N = 10$			$\sum f_i \log x_i = 10.37396$

$$\therefore GM = \text{Antilog}\left\{\frac{1}{N}\sum f_i \log x_i\right\}$$

$$= \text{Antilog}\left\{\frac{1}{10}\left(10.37396\right)\right\}$$

$10^{\square}$

$$= 10.9 \text{ (Approx.)}$$

$\therefore$ The average rate of change of yield of the new variety of wheat is 10.9%.

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Harmonic Mean (HM)

Harmonic mean of a set of non-zero observations is the reciprocal of the arithmetic mean of the reciprocals of the given values.

The Harmonic Mean,

$$HM = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

In case of frequency distribution,

$$HM = \frac{N}{\sum_{i=1}^{n} \left(\frac{f_i}{x_i}\right)}$$

Advantages

disadvantages

uses

Ex: 3.3: We construct the following table:

Profit per share (Tk)	Frequency (f_i)	Mid value (x_i)	f_i/x_i
0-5	1	2.5	0.4000
5-10	2	7.5	0.2667
10-15	4	12.5	0.3200
15-20	2	17.5	0.1143
20-25	1	22.5	0.0444
Total	10		1.1454

$$\therefore HM = \frac{N}{\sum \frac{f_i}{x_i}} = \frac{10}{1.1454} = 8.73$$

The average profit per share is Tk. 8.73

— . —

Relationship among AM, GM and HM: 20-21

→ For two non-zero positive observations:

i) $\boxed{A \geq G \geq H}$

ii) $\boxed{AH = G^2}$

A = Arithmetic Mean

G = Geo ______

H = Har ______

Proof: Let the two non-zero positive observations be x_1 and x_2

By definition, $A = \dfrac{x_1+x_2}{2}$; $G = \sqrt{x_1 x_2} = (x_1 \cdot x_2)^{1/2}$

and $H = \dfrac{1}{\frac{1}{2}\left(\frac{1}{x_1}+\frac{1}{x_2}\right)} = \dfrac{1}{\frac{x_1+x_2}{2x_1x_2}} = \dfrac{2x_1x_2}{x_1+x_2}$

(i) Since any square quantity is always non-negative,

$$\therefore \left(\sqrt{x_1}-\sqrt{x_2}\right)^2 \geq 0$$

$$\text{or, } x_1+x_2-2\cdot\sqrt{x_1x_2} \geq 0$$

$$\text{or, } \frac{x_1+x_2}{2} \geq \sqrt{x_1x_2} \quad \text{--- (1)}$$

$$\therefore A \geq G \quad \text{--- (2)}$$

Again from equn (1) –

$$x_1+x_2 \geq 2\sqrt{x_1x_2}$$

$$\text{or, } 2\sqrt{x_1x_2} \leq x_1+x_2$$

$$\text{or, } \frac{2\sqrt{x_1x_2}}{x_1+x_2} \leq 1$$

Multiplying both sides by $\sqrt{x_1x_2}$ we get

$$\frac{2\sqrt{x_1x_2}\,\sqrt{x_1x_2}}{x_1+x_2} \leq \sqrt{x_1x_2}$$

$$\text{or, } \frac{2x_1x_2}{x_1+x_2} \leq \sqrt{x_1x_2}$$

$$\therefore H \leq G \quad \text{--- (3)}$$

From (2) and (3) $\Longrightarrow$ $A \geq G \geq H$ (proved)

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(ii) $A.H = \frac{x_1 + x_2}{2} \times \frac{2x_1 x_2}{x_1 + x_2} = x_1 x_2 = (\sqrt{x_1 x_2})^2$

$\therefore AH = G^2$ (proved)

Median (মধ্যমা)

The median of a distribution is the value of the variable which divides the distribution into two equal parts if arranged in order of magnitude.

Ex: 1 3 3 6 7 8 9 / 1 2 3 3 4 6 8 9

3.5

$$\therefore Me = L_m + \frac{\frac{N}{2} - F_m}{f_m} \times h$$

Where,

L_m = lower limit of the median class

N = total frequency

f_m = frequency of the median class

F_m = cumulative frequency of the pre-median class.

h = length of median class

Advantages

Disadvantages

uses

Quantiles

Quartiles also are some positional or location measures of the distribution. Quartiles are those values in a series, which divide the whole distribution into a number of equal parts when the series is arranged in order of magnitude of observations.

following quantiles ——

i) 3 Quartiles: Q_i $(i=1,2,3)$; divide the whole distribution into four equal parts.

$$Q_i = L_i + \frac{\frac{iN}{4} - F_i'}{f_i} \times h \quad ; \quad i = 1,2,3$$

ii) 9 Deciles: D_j $(j=1,2,\ldots 9)$; divide the whole distribution into 10 equal parts.

$$D_j = L_j + \frac{\frac{jN}{10} - F_j'}{f_j} \times h \quad ; \quad j = 1,2,\ldots,9$$

iii) 99 Percentiles: P_k $(k=1,2,\ldots,99)$; divide the whole distribution into 100 equal parts.

$$P_k = L_k + \frac{\frac{kN}{100} - F_k'}{f_k} \times h \quad ; \quad k = 1,2,\ldots,99$$

Here, i, j, k indicate the order of quantiles, deciles and percentiles respectively ; F_i, F_j and F_k are respectively the cumulative frequencies of class preceding the ith quantile, jth decile and kth percentile classes.

Graphical Location of Median and Quantiles

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Mode (M_0)

Mode of the distribution is that value of the variate for which the frequency is the maximum.

$$M_0 = L + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h$$

L = lower limit of modal class

f_0 = frequency of modal class

f_1 = frequency of pre-modal class

f_2 = frequency of post-modal class

Advantages

disadvantages

uses

Location of Mode

Comparison Among the Measures of Central Tendency

20.21 13.20

Ex - 3.4)

Class	Frequency	C.f
50-60	5	5
60-70	9	14
70-80	13	27
80-90	20	47
90-100	19	66
100-110	9	75
110 and over	5	80

$N = 80$

Here $N = 80$

Computation of Median:

$\frac{N}{2} = \frac{80}{2} = 40$th observation lies in the class (80-90)

$\therefore$ (80-90) is the median class

$$\therefore Me = L_m + \frac{\frac{N}{2} - F_m}{f_m} \times h$$

$$= 80 + \frac{40 - 27}{20} \times 10$$

$$= 86.5$$

$L_m = 80$
$N/2 = 40$
$F_m = 27$
$f_m = 20$
$h = 10$

Computation of Quartiles:

$\frac{N}{4} = \frac{80}{4} = 20^{th}$ observation lies in the class (70-80)

$\therefore$ (70-80) is the lower quartile (Q_1) class

$$\therefore Q_1 = L_1 + \frac{\frac{1N}{4} - F_1'}{f_1} \times h$$

$$= 70 + \frac{20-14}{13} \times 10$$

$$= 74.62 \text{ (app.)}$$

Again, $\frac{3N}{4} = \frac{3(80)}{4} = 60^{th}$ observation lies in the class (90-100)

$\therefore$ (90-100) is the upper quartile (Q_3) class.

$$Q_3 = L_3 + \frac{\frac{3N}{4} - F_3'}{f_3} \times h$$

$$= 90 + \frac{60-47}{19} \times 10$$

$$= 96.84$$

$L_3 = 90$

$\frac{3N}{10} = 60$

$F_3 = 47$

$f_3' = 19$

$h = 10$

Computation of Deciles:

$\frac{4N}{10} = \frac{4(80)}{4} = 32^{th}$ observation lies in (80-90)

$\therefore$ (80-90) is the 4th deciles (D_4) class

$$\therefore D_4 = L_4 + \frac{\frac{4N}{10} - F_4'}{f_4} \times h$$

$$= 80 + \frac{32-27}{20} \times 10$$

$$= 82.50$$

$L_4 = 80$

$\frac{4N}{10} = 32$

$F_4' = 27$

$f_4 = 20$

$h = 10$

Computation of percentiles:

$\frac{70N}{100} = \frac{70(80)}{100} = 56^{th}$ observation lies in the class (90-100)

$\therefore$ (90-100) is the 70th percentiles (P_{70}) class

$$\therefore P_{70} = L_{70} + \frac{\frac{70N}{100} - F'_{70}}{f_{70}} \times h$$

$$= 90 + \frac{56-47}{19} \times 10$$

$$= 94.74 \text{ (app.)}$$

$L_{70} = 90$

$\frac{70N}{100} = 56$

$F'_{70} = 47$

$f_{70} = 19$

$h = 10$

Computation of Mode:

Here (80-90) is the modal class because maximum frequency (20) lies in that class

$$\therefore M_o = L + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h$$

$$= 80 + \frac{20-13}{2 \times 20 - 13 - 19} \times 10$$

$$= 88.75$$

$L = 80$

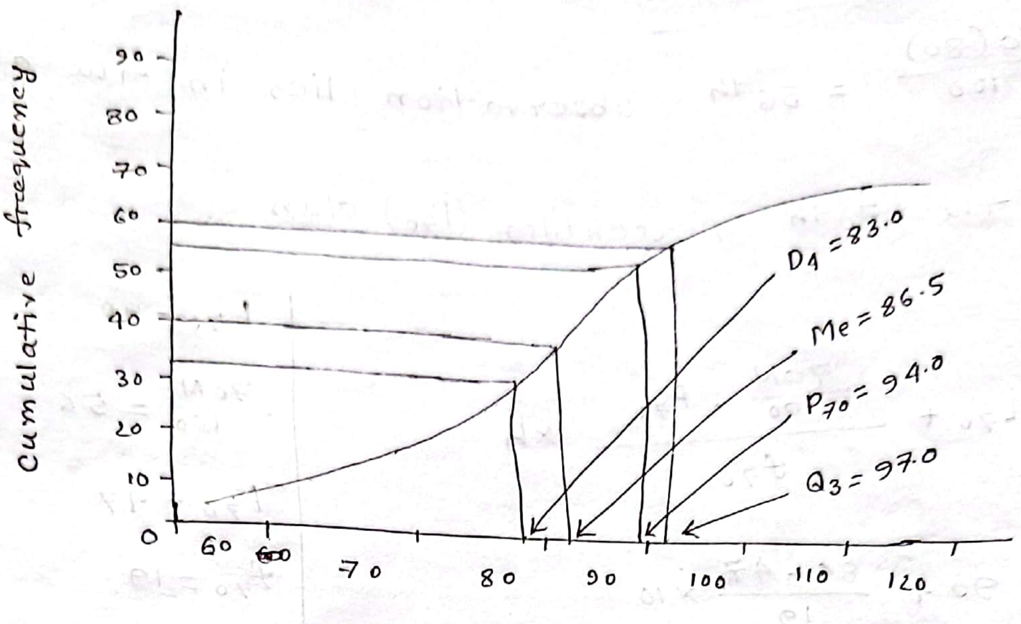
$f_0 = 20$

$f_1 = 13$

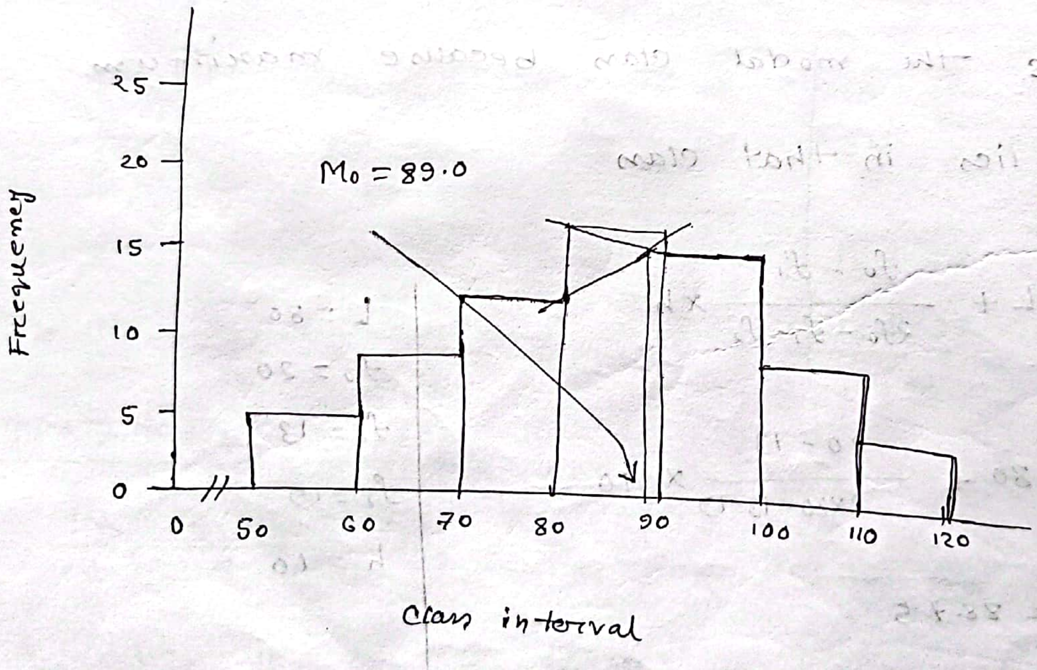
$f_2 = 19$

$h = 10$

i) Graphical location of quantiles from ogive:



ii) Location of mode from histogram.



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Chap - 4

Desperion, nature and shape of frequency Distribution

Measurement of Desperion

Absolute Measures একক আছে

- Range
- Quartile deviation
- Mean Deviation
- Standard Deviation

Relative Measures একক নেই

- Co-efficient of Range
- Co-efficient of Quartile Deviation
- Co-efficient of mean deviation
- Co-efficient of variation

Standard Deviation —

$$\sigma = \sqrt{\frac{1}{N}\left\{\sum f_i x_i^2 - \frac{(\sum f_i x_i)^2}{N}\right\}}$$

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}$$

20-21

Advantages of standard deviation

Central frequency is characteristic is same

→ rigidly defined

→ based upon all the observations

→ less affected by sampling fluctuation

→ suitable for further algebric treatments.

Disadvantages

→ not rigidly comprehensible

→ affected by the extreme values

→ cannot be computed in case of distributions having open ended class interval

20-21

20.2.1

Some properties of Standard Deviation:

i) Standard deviation is independent of change of origin but not of scale.

ii) Standard deviation is the least possible root mean square deviation.

iii) For two observations, Standard deviation is the half of the range.

Working formula of Standard Deviation

Here,

$$\sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$= \sum x_i^2 - 2\bar{x}\sum x_i + \sum \bar{x}^2$$

$$= \sum x_i^2 - 2\bar{x}\sum x_i + n\bar{x}^2$$

$$= \sum x_i^2 - 2\left(\frac{\sum x_i}{n}\right)\sum x_i + n\left(\frac{\sum x_i}{n}\right)^2$$

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$$= \sum x_i^2 - 2\frac{(\sum x_i)^2}{n} + \frac{(\sum x_i)^2}{n}$$

$$= \sum x_i^2 - \frac{\sum x_i^2}{n}$$

$$\therefore \sigma^2 = \frac{1}{n}\sum (x_i - \bar{x})^2$$

$$= \frac{1}{n}\left\{\sum x_i^2 - \frac{\sum x_i^2}{n}\right\}$$

$$= \frac{1}{n}\sum x_i^2 - \left(\frac{\sum x_i}{n}\right)^2$$

$$\therefore \sigma = \sqrt{\frac{1}{n}\sum x_i^2 - \left(\frac{\sum x_i}{n}\right)^2}$$

In case of grouped data

$$\sigma = \sqrt{\frac{1}{N}\sum f_i x_i^2 - \left(\frac{\sum f_i x_i}{N}\right)^2}$$ where, $N = \sum_{i=1}^{n} f_i$

$$= \sqrt{\frac{1}{N}\left\{\sum f_i x_i^2 - \frac{(\sum f_i x_i)^2}{N}\right\}}$$

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Ex - 4.2

Direct Method:

Class interval	frequency f_i	Mid value of class x_i	$f_i x_i$	$f_i x_i^2$
50-60	5	55	275	15125
60-70	9	65	585	38025
70-80	13	75	975	73125
80-90	20	85	1700	144500
90-100	19	95	1805	171475
100-110	9	105	945	99225
110-120	5	115	575	66125
Total	N = 80		6860	607600

Standard deviation,

$$\sigma = \sqrt{\frac{1}{N}\left\{\sum f_i x_i^2 - \frac{(\sum f_i x_i)^2}{N}\right\}}$$

$$= \sqrt{\frac{1}{80}\left\{607600 - \frac{(6860)^2}{80}\right\}}$$

$$= \sqrt{\frac{19355}{80}}$$

$$= 15.554$$

Indirect Method:

We change the origin to $x=85$ and scale by dividing by 10

Class interval	Mid value of class x_i	frequency f_i	$u_i = \frac{x_i - 85}{10}$	f_iu_i	$f_iu_i^2$
50-60	55	5	-3	-15	45
60-70	65	9	-2	-18	36
70-80	75	13	-1	-13	13
80-90	85	20	0	0	0
90-100	95	19	1	19	19
100-110	105	9	2	18	36
110-120	115	5	3	15	45
Total		N = 80		6	194

$$\sigma_u = \sqrt{\frac{1}{N}\left\{\sum f_iu_i^2 - \frac{(\sum f_iu_i)^2}{N}\right\}}$$

$$= \sqrt{\frac{1}{80}\left\{(194)^2 - \frac{6^2}{80}\right.}$$

$$= \sqrt{\frac{1}{80}(193.55)}$$

$$= 1.5554$$

$$\therefore \sigma_x = h\sigma_u = 10 \times 1.5554 = 15.554$$

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Co-efficient of Range

The percentage ratio of range and sum of maximum and minimum observation is known as co-efficient of Range.

$$CR = \frac{x_m - x_l}{x_m + x_l} \times 100\%$$

where,

x_m = the highest value of the data

x_l = " lowest " " "

Co-efficient of variation

Co-efficient of variation of a set of data is the ratio of the standard deviation to mean expressed as percentage -

$$C.V = \frac{\sigma x}{\bar{x}} \times 100\%$$

Moments: Moments are const. which are used to determine some characteristics of frequency distributions.

Two types $\begin{cases} \text{Central moment} \to \text{Moments about the mean} \\ \text{Raw moments} \to \text{Moments arbitrary value} \end{cases}$

If $x_1, x_2 \cdots x_n$ occur with frequencies $f_1, f_2, \ldots, f_n$ respectively the rth central moment,

$$\mu_r = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^r}{N}$$

Skewness – lack of symmetry

for symmetrical distribution, mean = mode = median

Karl Pearson's β and γ co-efficient

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \qquad \beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\gamma_1 = \pm\sqrt{\beta_1} \qquad \boxed{\gamma_2 = \beta_2 - 3}$$

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Measures of skewness

co-efficient of skewness,

$$S_k = \frac{(\beta_2+3)\sqrt{\beta_1}}{2(5\beta_2-6\beta_1-9)}$$

2021 (C.H.S)

Kurtosis

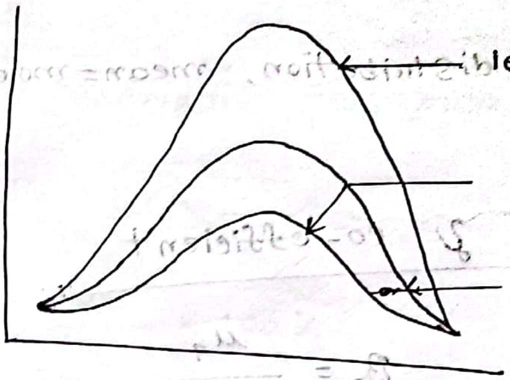
$$\gamma_2 = \beta_2 - 3$$

If a distribution has,

i) $\beta_2 > 3$ (lepto kurtic)

ii) $\beta_2 < 3$ (platy kurtic)

iii) $\beta_2 = 3$ (mesokurtic)



$$\bar{x} = \frac{1}{N}\sum_{i=1}^{n} f_i x_i$$

central moment, $\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - \bar{x})^r$

Karl pearson's β and γ co-efficient

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \qquad \beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\gamma_1 = \pm\sqrt{\beta_1} \qquad \gamma_2 = \beta_2 - 3$$

Co-efficient of skewness,

$$S_k = \frac{\sqrt{\beta_1}\,(\beta_2+3)}{2(5\beta_2 - 6\beta_1 - 9)}$$

Kurtosis, $\gamma_2 = \beta_2 - 3$

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— o —

20-21 5(c)

* Short note about moments, skewness and kurtosis —

Moments: Moments are constant which are used to determine ~~the~~ some characteristics of frequency distribution.

Two types —

1) Central moment: moments about the mean

$$(\mu)\quad \mu_r = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^r}{N}$$

ii) Raw moment: moments about arbitrary value

$$(\mu)\quad \mu'_r = \frac{\sum_{i=1}^{n} f_i (x_i - a)^r}{N}$$

Skewness: means lack of symmetry.

For symmetrical distribution,

Mean = Mode = Median

Two types —

i) Negative skewness: $\bar{x} < Me < Mo$

ii) positive skewness: $\bar{x} > Me > Mo$

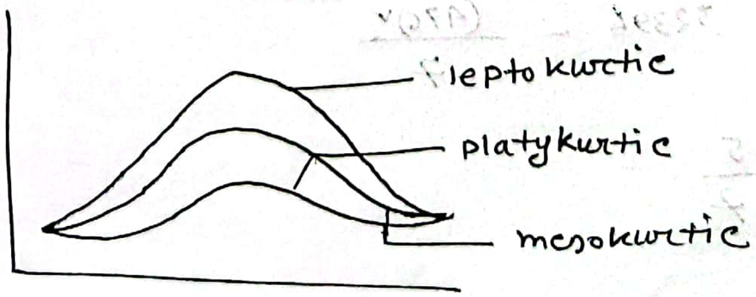
Kurtosis: A statistical measure to describe the distribution of ~~observation~~ observed data around mean.

Three types —

i) $\beta > 3$ (leptokurtic)

ii) $\beta_2 < 3$ (platykurtic)

iii) $\beta_2 = 3$ mesokurtic



46)

Amount of credit (lac Tk.)	No of branches f_i	Mid value x_i	$f_i x_i$	$x_i - \bar{x}$	$f_i(x_i-\bar{x})$	$f_i(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^3$	$f_i(x_i-\bar{x})^4$
0-5	1	2.5	2.5	-10	-10	100	-1000	10000
5-10	2	7.5	15.0	-5	-10	50	-250	1250
10-15	4	12.5	50.0	0	0	0	0	0
15-20	2	17.5	35.0	5	10	50	250	1250
20-25	1	22.5	22.5	10	10	100	1000	10000
Total	N=10	~~125.0~~	125.0	0	0	300	0	22500

$$\bar{x} = \frac{1}{N} \sum f_i x_i = \frac{1}{10} \times 125 = 12.5$$

$\mu_1 = \frac{1}{N} \sum f_i (x_i - \bar{x}) = \frac{1}{10} \times 0 = 0$ ($\mu_1 = 0$ always)

$\mu_2 = \frac{1}{N} \sum f_i (x_i - \bar{x})^2 = \frac{1}{10} \times (300) = 30.0$

$\mu_3 = \frac{1}{N} \sum f_i (x_i - \bar{x})^3 = \frac{1}{10} \times 0 = 0$

and $\mu_4 = \frac{1}{N} \sum f_i (x_i - \bar{x})^4 = \frac{1}{10} \times 22500 = 2250.0$

Now,

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{0}{(30)^3} = 0$$

∴ Coefficient of ~~skewness~~ skewness,

$$S_K = \frac{\sqrt{\beta_1}\,(\beta_2 + 3)}{2(5\beta_2 - 6\beta_1 - 9)} = 0$$

Hence the distribution is symmetrical.

Again $\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{2250}{(30)^2} = 2.5 < 3$

$\therefore \gamma_2 = \beta_2 - 3 = 2.5 - 3 = -0.5$

Since $\gamma_2 < 0$. The curve is platykurtic.

∴ The distribution is symmetrical and platykurtic.

19-20

* Importance of measure of dispersion —

1) M.O.D is needed to know representativeness of the observations of distribution.

11) Help to control the deviation of data

111) give the comparative picture of different distributions

1v) help to control the quality of industrial product.

v) important for time series data such as- rainfall, temperature.

Chap - 5

Probability and Probability theory

Experiment: An act that can be repeated under certain conditions is known as an experiment.

Sample Space: In a random experiment, the collection of all possible outcomes is called sample space and an element of the sample space is called sample points.

Event → 19-20

Conditional event: If the occurrence of an event depends on the occurrence or not occurrence of another event in the same sample space, it is called a conditional event.

Mutually Exclusive Events: If a coin is tossed the occurrence of Flower and Leaf are mutually exclusive because a single throw of a coin cannot result these two events simultaneously.

Independent Events: If two coins are tossed, flower or leaf in one coin and leaf or flower in the other coin are independent events.

Mathematical or Classical Definition of Probability:

If an experiment results in 'n' exhaustive, mutually exclusive and equally likely cases 'm' of which are favourable to a particular event A, the probability of the event A is defined as the ratio of the favourable cases to the total number of equally likely cases.

$$\text{Symbolically, } P(A) = \frac{m}{n}$$

Since the number of favourable cases can range from 0 to n, the probability of an event will range from 0 to 1, that is

$$\boxed{0 \leq P \leq 1}$$

If all the cases are favourable to the event A so that $P(A) = 1$; A is called a certain or sure event.

and if $P(A)=0$, A is called and impossible event.

Ex - 5.1) When thrown, a die may show any of the faces numbered 1, 2, 3, 4, 5 and 6.

$\therefore$ the equally likely cases, $n = 6$

Here, the even number faces 2, 4 and 6.

$\therefore$ the favourite number cases, $m = 3$

$\therefore$ Probability, $P = \frac{m}{n} = \frac{3}{6} = \frac{1}{2}$

Ex - 5.2)

If two sides of a coin are denoted by F (Flower) and L (Leaf), there will be $2^3 = 8$ outcomes.

FFF, FFL, FLF, FLL, LFF, LFL, LLF, LLL

Cases of at least two flowers

= cases of 2 flowers + cases of 3 flowers

$\therefore$ No. of favourite cases is $= 3 + 1 = 4$

$\therefore n = 8$

$\therefore P = \frac{m}{n} = \frac{4}{8} = \frac{1}{12}$

— o —

~~Law of probability~~

Ex- 5.3) In simultaneous throw of 2 dice, the sum of the faces are:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

The equally likely cases are $6^2 = 36$

The numbers divisible by 3 are 3, 6, 9 and 12

In the above table, 3 appears 2 times

6 appears 5 times

9 appears 4 times

12 appears 1 times

So the number of favourable cases are 2+5+4+1 = 12

Hence the required probability

$$P = \frac{12}{36} = \frac{1}{3}$$

Ex - 5.4)

In a pack there are 52 cards.

So the equally likely cases are 52.

i) There are 13 spades in a pack of cards. So the number of favourable cases is 13.

∴ The required probability of spade is,

$$P = \frac{13}{52} = \frac{1}{4}$$

ii) There are 4 queens. That is favourable number of cases is 4.

Hence the required probability is $P = \frac{4}{52} = \frac{1}{13}$

iii) There are 20 honours cards in a pack. So the number of favourable cases is 20.

Hence the required probability is $P = \frac{20}{52} = \frac{5}{13}$

Ex-5.5) Total number of balls are $4+6=10$

2 balls can be drawn from 10 balls in $\binom{10}{2} = 45$ ways

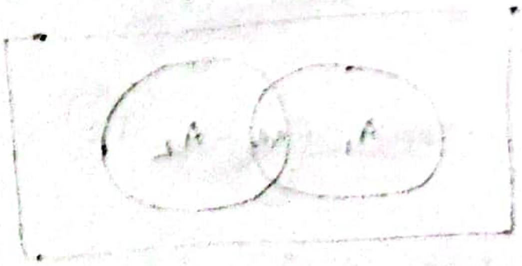
Hence the equally likely cases are 45.

Let A be the event that both the balls are red.

out of 4 red balls 2 balls can be drawn in $\binom{4}{2} = 6$ ways

Hence the number of favouriable cases are 6.

$\therefore$ The required probability is $P(A) = \frac{6}{45} = \frac{2}{15}$



Additive Law of probability of Two events

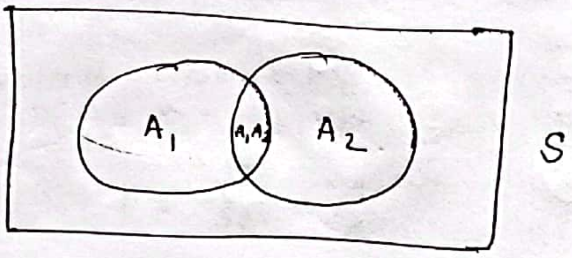
Probability of any one of the events A_1 and A_2 is the sum of the probabilities of A_1 and A_2 minus the probability of the compound event $(A_1 A_2)$.

Symbolically,

$$P(A_1 + A_2) = P(A_1) + P(A_2) - P(A_1 A_2)$$

Proof:

Let the happening of the two events A_1 and A_2 be shown in the following venn Diagram.



Suppose the number of equally likely cases in the sample space be n of which m_1 cases are favourable to the event A_1, m_2 cases are favourable to A_2 and m cases are favourable to both A_1 and A_2.

By definition, $P(A_1) = \frac{m_1}{n}$, $P(A_2) = \frac{m_2}{n}$ and

$$P(A_1 A_2) = \frac{m}{n}$$

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Now cases favourable to either A_1 or A_2 are

$m_1 + m_2 - m$

$\therefore$ probability of any one of A_1 and A_2 is

$$P(A_1 + A_2) = \frac{m_1 + m_2 - m}{n}$$

$$= \frac{m_1}{n} + \frac{m_2}{n} - \frac{m}{n}$$

$$\therefore P(A_1 + A_2) = P(A_1) + P(A_2) - P(A_1 A_2)$$

* If the events A_1 and A_2 are $\boxed{\text{independent}}$:

$$P(A_1 + A_2) = P(A_1) + P(A_2) - P(A_1)\,P(A_2)$$

Since $P(A_1 A_2) = P(A_1)\,P(A_2)$ if A_1 and A_2 are independent.

* If the events A_1 and A_2 are mutually $\boxed{\text{exclusive}}$:

The probability of any one of them is the sum of the probabilities of the individual events.

That is $P(A_1 + A_2) = P(A_1) + P(A_2)$

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* Extension of the Additive Law of Probabity:

$$P(A_1+A_2+A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1A_2) - P(A_1A_3) - P(A_2A_3) + P(A_1A_2A_3)$$

Ex - 5.6)

As given,

$$P(A) = \frac{75}{100} = \frac{3}{4}$$

and,

$$P(B) = \frac{50}{100} = \frac{1}{2}$$

According to the additive law of probability

$P(A+B) = P(A) + P(B) - P(A)P(B)$ [Since A and B are independent $P(AB) = P(A)P(B)$]

The required probability is

$$P = P(A+B) = \frac{3}{4} + \frac{1}{2} - \frac{3}{4} \times \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{2} - \frac{3}{8}$$

$$= \frac{7}{8}$$

Ex - 5.7)

Let A be the event that the card will be an Ace and B be the event that it will be the queen of diamond.

As there are a total of 52 cards in a pack, the number of equally likely cases is 52.

In a pack of cards, there are, in all, four Aces.

Hence, $P(A) = \frac{4}{52} = \frac{1}{13}$

There is only one queen of diamond.

Hence $P(B) = \frac{1}{52}$

Since there is no scope that a card can be at the same time an Ace and the Queen of diamond, events A and B are mutually exclusive; that is

$$P(AB) = 0$$

According to the additive law of probability,

$$P(A+B) = P(A) + P(B)$$

Hence the required probability is

$$P = P(A) + P(B) = \frac{1}{13} + \frac{1}{52} = \frac{5}{52}$$

Linear Regration: If the variable x and y is a bivariate distribution are related, we will find that the points in a scatter diagram will cluster around a curve called regression curve. If the curve is a straight line, it is called the line of regression and the regression is known as linear regression. In the case of of bivariate distribution, the co-efficient of regression of y on x is denoted by B_{yx} and that of x on y by B_{xy}.

* The regression co-efficient of y on x is b_{yx} and that of x on y is b_{xy}.

$$b_{yx} = \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sum(x_i-\bar{x})^2} = \frac{\sum x_iy_i - \frac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}$$

and

$$b_{xy} = \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sum(y_i-\bar{y})^2} = \frac{\sum x_iy_i - \frac{\sum x_i \sum y_i}{n}}{\sum y_i^2 - \frac{(\sum y_i)^2}{n}}$$

* Geometrical Interpretation of Regression y on x —

a = Intercept

$b = \tan\theta$ = Slop

which is the required co-efficient of regression of y on x.

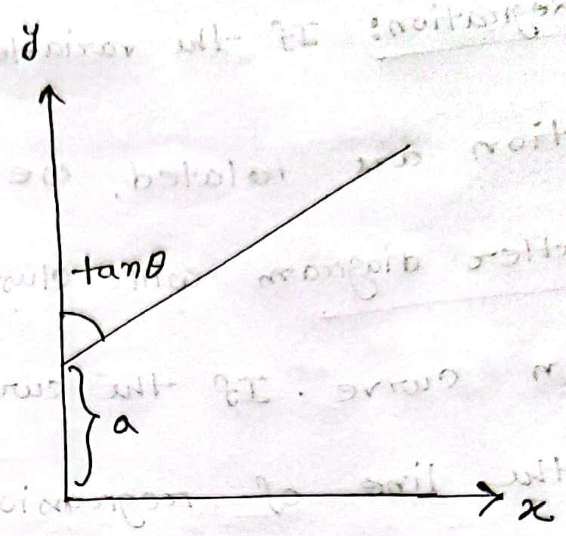

Fig: Regression line of y on x

For regression equation,

$$\boxed{y = a + bx}$$

when $x = 0$, $y = a$

$$\boxed{\text{ঢাল}, b = m = \frac{y_2 - y_1}{x_2 - x_1}}$$ বাক্য

- b_{yx} or b, the regression co-efficient of y on x indicates the change in the value of y (dependent variable) for unit change of x (independent variable)

[Similarly, b_{xy}, the regression co-efficient of x on y, indicates the change in the value of x corresponding to a unit change in the value of y.]

- b is positive when x and y change in the same direction and b is negative when x and y change in opposite direction.

- $b = 0$, y does not change for changing x.

- $b = 1$, one unit change of x results one unit change in the value of y.

- $b = 2$, 1 unit change of x results 2 unit change in the value of y.

∴ Co-effiient of regression indicates the change of dimension/degree and direction with respect to the change in the independent variable.

— o —

Ex- 1) 13-20

Estimation of Regression equation : Table for calculation

x	y	x^2	y^2	xy
1	52.5	1	2765.25	52.50
2	58.7	4	3445.69	117.40
3	65.0	9	4225.00	195.00
4	70.2	16	4928.04	280.80
5	75.4	25	5885.16	377.00
6	81.1	36	6577.21	486.60
7	87.2	49	7603.84	610.40
8	95.5	64	9120.25	764.00
9	102.2	81	10444.84	919.80
10	108.0	100	11664.00	1080.00
Total = 55	795.8	385	66450.28	4883.50

$$\bar{x} = \frac{\sum x_i}{n} = \frac{55}{10} = 5.5$$

and $$\bar{y} = \frac{\sum y}{n} = \frac{795.8}{10} = 79.58$$

Now

$$b = b_{yx} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}$$

$$= \frac{4883.5 - \frac{55 \times 795.8}{10}}{385 - \frac{(55)^2}{10}}$$

$$= 6.14$$

and $a = \bar{y} - b\bar{x} = 79.58 - (6.14)(5.5)$

$= 45.81$

∴ Regression equation of weight on age is

$y = a + bx = 45.81 + 6.14x$

Now estimated weight at the age of 6.5 week is

$y_{6.5} = 45.81 + 6.14 \times 6.5$

$= 85.72$ gm.

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Correlation

Correlation: The mutual linear relationship present between the variables of a bivariate population is termed as Correlation. ~~the variables are to have the same~~

~~Characteristics of the~~

A measure of intensity or degree of linear relationship between two variables is called coefficient of Corelation.

coefficient of correlation between two random variables x and y of a bivariate population is denoted by ρ_{xy} or ρ and that between the random variables x and y of a sample is denoted by r_{xy} or r.

Mathematically,

$$r \text{ or } r_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

$$= \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\left\{\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right\}\left\{\sum y_i^2 - \frac{(\sum y_i)^2}{n}\right\}}}$$

Properties of correlation coefficient:

- Correction coefficient is independent of change of origin and scale.
- The value of correlation coefficient lies between -1 and +1.

  That is, $\boxed{-1 \leq r \leq +1}$

- Correlation ~~of~~ coefficient is the geometric mean of two regression coefficients.
- Correlation coefficient is symmetric with respect to the dependence of the variables.
- The value of correlation coefficient is very much influenced by large items, if they are present in data.

* Correlation coefficient is the geometric mean of two regression coefficients.

Proof: Regression coefficient of y on x is

$$b_{yx} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$ and that of x on y is

$$b_{xy} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(y_i - \bar{y})^2}$$

Now,

$$r_{xy} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}}$$

$$= \sqrt{\frac{\{\sum(x_i - \bar{x}) \sum(y_i - \bar{y})\}^2}{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}}$$

$$= \sqrt{\frac{\{\sum(x_i - \bar{x})(y_i - \bar{y})\}}{\sum(x_i - \bar{x})^2}} \times \sqrt{\frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(y_i - \bar{y})^2}}$$

$$= \sqrt{b_{yx}\, b_{xy}}$$

= Geometric mean of b_{yx} and b_{xy}

Note 1: $r \leq 1$

$$\text{or, } \sqrt{b_{yx} b_{xy}} \leq 1$$

$$\text{or, } b_{yx} b_{xy} \leq 1$$

$$\text{or, } b_{yx} \leq \frac{1}{b_{xy}}$$

That is, if one of the regression coefficient is greater than 1, the other must be less than 1 and vice varsa because their product cannot be exceed 1.

Note 2: We know that geometric mean of two value cannot be exceed their arithmetic mean.

Hence,

$$r_{xy} = \sqrt{b_{yx} b_{xy}} \leq \frac{b_{yx} + b_{xy}}{2}$$

$$\text{or, } \frac{b_{yx} + b_{xy}}{2} \geq r_{xy}$$

That is, arithmetic mean of the regression coefficients is greater than the correlation coefficient.

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- **Geometrical Interpretation of Correlation coefficient:**

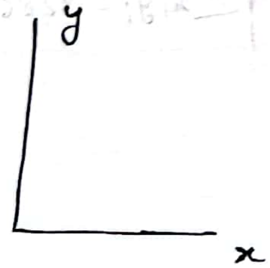
The value of correlation coefficient may be negative, zero and positive.

- **When correlation coefficient is negative ($r<0$):** In this case, both the variables change in the opposite direction i.e., ~~oth~~ with an increase in one variable, there is a decrease in the ~~ord~~ other variable or vice-versa.

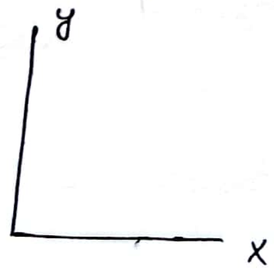
- **When correlation coefficient is positive ($r>0$):**

In this case, both the variables are changed in the same direction i.e, an increase or decrease in one variable results to an increase or a decrease in the other variable.

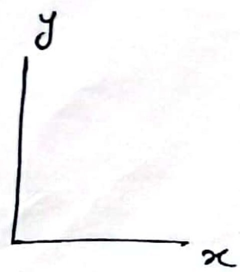
- The scatter diagrams in three various cases are :-



$r>0$



$r=0$



$r<0$

- When $r=0$, There is no relation between the two variables that is the variable are uncorrelated.

- When $r=\pm 1$, existing correlation is called perfect correlation.

— o —

Ex-8.2

Computation of correlation coefficient by Direct Method:

x_i	y_i	x_i^2	y_i^2	$x_i y_i$
70	72	4900	5184	5040
66	68	4356	4624	4488
68	69	4624	4761	4692
71	69	5041	4761	4899
69	72	4761	5184	4968
65	67	4225	4489	4355
67	66	4489	4356	4422
$\sum x_i = 476$	$\sum y_i = 483$	$\sum x_i^2 = 32396$	$\sum y_i^2 = 33359$	$\sum x_i y_i = 32864$

Now,

$$r_{xy} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\left\{\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right\}\left\{\sum y_i^2 - \frac{(\sum y)^2}{n}\right\}}}$$

$$= \frac{32864 - \frac{(476)(483)}{7}}{\sqrt{\left\{32396 - \frac{(476)^2}{7}\right\}\left\{33359 - \frac{(483)^2}{7}\right\}}}$$

$$= \frac{32864 - 32844}{\sqrt{(32396 - 32368)(33359 - 33327)}}$$

$$= \frac{20}{\sqrt{28 \times 32}}$$

$= 0.668$ (Approx.)

* Correlation Coefficient in Terms of Regression Coefficients:

Let use denote the correlation coefficient by r, coefficient of regression of y on x by b_{yx} and that of x on y by b_{xy}.

By definition,

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

$$= \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} \times \frac{\sqrt{\sum (x_i - \bar{x})^2}}{\sqrt{\sum (y_i - \bar{y})^2}}$$

$$= b_{yx} \frac{\sqrt{n}\, \sigma_x}{\sqrt{n}\, \sigma_y}$$ [σ_x and σ_y are the sd. of x and y respectively]

$$= b_{yx} \frac{\sigma_x}{\sigma_y}$$

$$\therefore b_{yx} = r \frac{\sigma_y}{\sigma_x}$$

Again,

$$r = \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum(x_i-\bar{x})^2 \sum(y_i-\bar{y})^2}}$$

$$= \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sum(y_i-\bar{y})^2} \times \frac{\sqrt{\sum(y_i-\bar{y})^2}}{\sum(x_i-\bar{x})^2}$$

$$= b_{xy} \frac{\sqrt{n}\,\sigma_y}{\sqrt{n}\,\sigma_x}$$

$$= b_{xy} \frac{\sigma_y}{\sigma_x}$$

$$b_{xy} = r \frac{\sigma_x}{\sigma_y}$$

— o —

* Regression vs Correlation

→ boi 214 Page

Regression	Correlation
1) Determine the estimate value of the dependent variable with respect to one or more independent variables.	1) means the linear relationship of two or more dependent variables with each other.
2) In regression, influenced variable is dependent and others are independent.	2) In correlation, the variables are dependent on each other. There is no concept of independent variables
3) Besides the linear relationship other relationship within the variables can be determined	3) only linear relationship can be determined.
4) only dependent variable needs to be random.	4) both the variables must be random
5) either $y = f(x)$ or $x = f(y)$ both may be true.	5) $y = f(x)$ or $x = f(y)$ are ~~not~~ to be true
6) $b_{yx} \neq b_{xy}$	6) $r_{xy} = r_{yx}$
7) No upper and lower limits.	7) $-1 \leq r \leq +1$

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Population: Statistical investigations usually aim at the assesment of the general magnitude and the study of variation with respect to certain characteristics of the individuals ~~belog~~ belonging to a group. Such a group of individuals under study is known as population.

Ex: All the farmers, students, domestic animals, birds, total forest area, total agricultural land etc. may constitute a population.

population may be finite or infinite.

Finite population: A population composed of a finite (Specific) number of elements is known as finite population.

Ex: Students of an institution, farmers in a country, number of livestocks etc.

Infinite population: A population composed of an infinite number of elements, which cannot be enumerated, is called infinite population.

Ex: Number of fishes in the river, number of stars in the

19-20

Sample: A sample is a small representative fraction of a population.

Ex: A small quantity of blood, not the whole, is collected for testing, the blood is sample where the total quantity of blood of a person is the population.

20-21, 19-20

Census: If data are collected on all the elements of a population, the process is known as census.

Ex: Detailed information on all the citizens of a country are collected usually in every ten years. This called population census.

Another example agricultural census.

20-21

Sample survey: Sample survey is a method by which detailed information on the population characteristics are collected on the basis of sample elements.

Ex: population parameters such as mean, standard deviation etc.

Pilot Survey: Small scale surveys are sometimes conducted in order to get quick primary information before census. Such a survey is known as pilot survey.

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* **Advantages of Sample Survey**: The main advantages of Sample survey ~~are over complete enumeration~~ are summerized below:

i) **Less time, Money and Labour**: In sample survey, a part of the population is considered. As a result, it can be obtained within a shot time, less cost and less labour.

ii) **Accuracy of Results**: For less involvement in sample survey, it is possible to employ more experienced and skilled manpower. As a result the information obtained from sample survey are more accurate and reliable.

iii) **Greater Scope**: Sometimes large number of skilled manpower and modern equipments required for census may not be ~~available~~ possible to make available. In such case sample survey is more suitable.

iv) Large population problem: Sometimes, the population may be very large (such as trees in the forest or fishes in the river). In such case sample survey is only ~~the~~ way because it is not possible to collect data on all ~~the~~ trees or fishes. ~~in the river~~.

20-21

5(b) uses of co-efficient of variation –

Co-efficient: refers to a set of data which is the ration of the standard deviation to mean expressed as percentage

$$C.V = \frac{\sigma x}{\bar{x}} \times 100\%$$

use

- Risk Management: In finance and economics, the C.V. is used to measure the risk per unit of return. A lower C.V. indicates less risk while higher C.V. indicates more risk.

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- Quality control: In manufacturing and engineering C.V. is used to determine quality and reliability of the product.

- Medical and Biological studies: It often apply in biological and medical research such as blood pressure, growth rates etc.

20-21

5)-c)

x	y	x^2	y^2	xy
70	72	4900	5184	5040
66	68	4356	4624	4488
68	69	4624	4761	4692
71	69	5041	4761	4899
69	72	4761	5184	4968
65	67	4225	4489	4355
67	66	4489	4356	4422
Total: 476	483	32396		32864

Now,

$$\bar{x} = \frac{\sum x_i}{n} = \frac{476}{7} = 68$$

$$\bar{y} = \frac{\sum y_i}{n} = \frac{483}{7} = 69$$

Now,

$$b_{yx} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}$$

$$= \frac{32864 - \frac{476 \times 483}{7}}{32396 - \frac{(476)^2}{7}}$$

$$= \frac{5}{7}$$

19-20

1(c) $\frac{dy}{dx} = (x+y)^2$

Let, $x+y = v$

or, $1 + \frac{dy}{dx} = \frac{dv}{dx}$

$\therefore \frac{dy}{dx} = \frac{dv}{dx} - 1$

Here,

$\frac{dv}{dx} - 1 = v^2$

or, $\frac{dv}{dx} = v^2 + 1$

or, $\frac{dv}{v^2+1} = dx$

Integrating,

$\int \frac{dv}{1+v^2} = \int dx$

or, $\tan^{-1} v = x + c$

or, $\tan^{-1}(x+y) = x + c$ (Ans.)

19-20

3(b)(i) $D^3 - 2D^2 - 4D + 8 = 0$

Let,

$y = e^{mx}$ then $\frac{dy}{dx} = me^{mx}$

$$\frac{d^2y}{dx^2} = m^2 e^{mx}$$

$$\frac{d^3y}{dx^3} = m^3 e^{mx}$$

The equ^n becomes —

$$(m^3 + 2m^2 - 4m + 8)e^{mx} = 0$$

Auxilary equation is –

$$m^3 - 2m^2 - 4m + 8 = 0$$

or, ~~$m^3 + 2m^2 - 4m -$~~

or, ~~$m^2(m+2) - 4(m+2)$~~

or, $m^3 + 2m^2 - 4m^2 - 8m + 8m$

or, $m^2(m+2) - 4m(m+2) + 8(m+2)$

12-20

4(b)

Class	Frequency	c.f
25-30	39	39
30-35	45	84
35-40	52	136
40-45	75	211
45-50	15	226
50-55	08	234
55-60	05	239

$N = 239$

Mode,

$$M_o = L + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h$$

$L = 40$

$f_0 = 75$

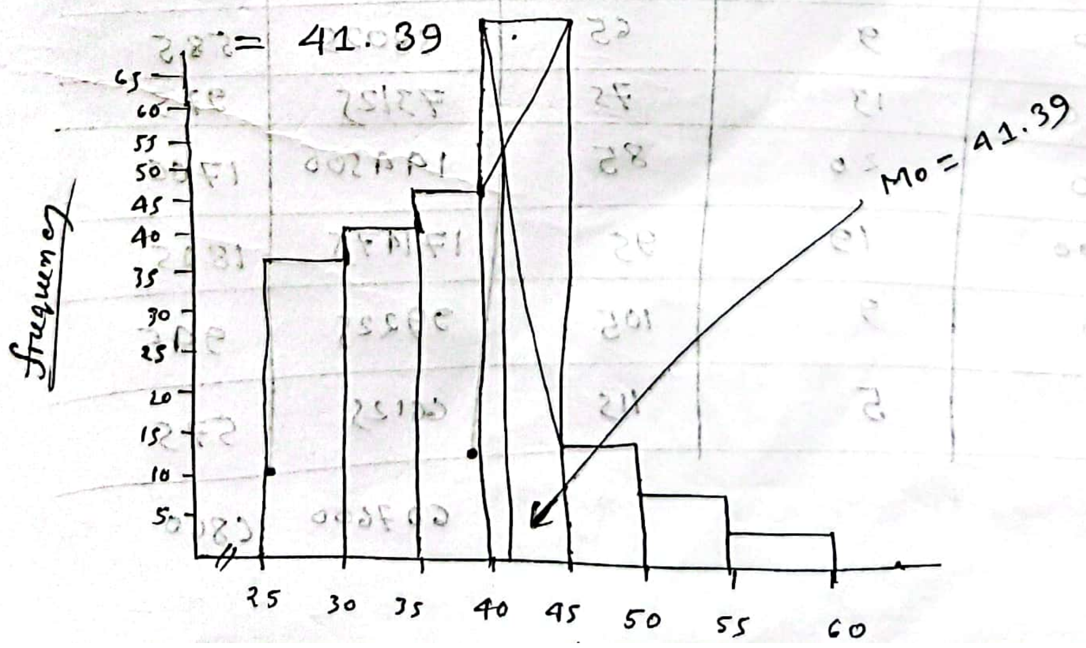
$f_1 = 52$

$f_2 = 15$

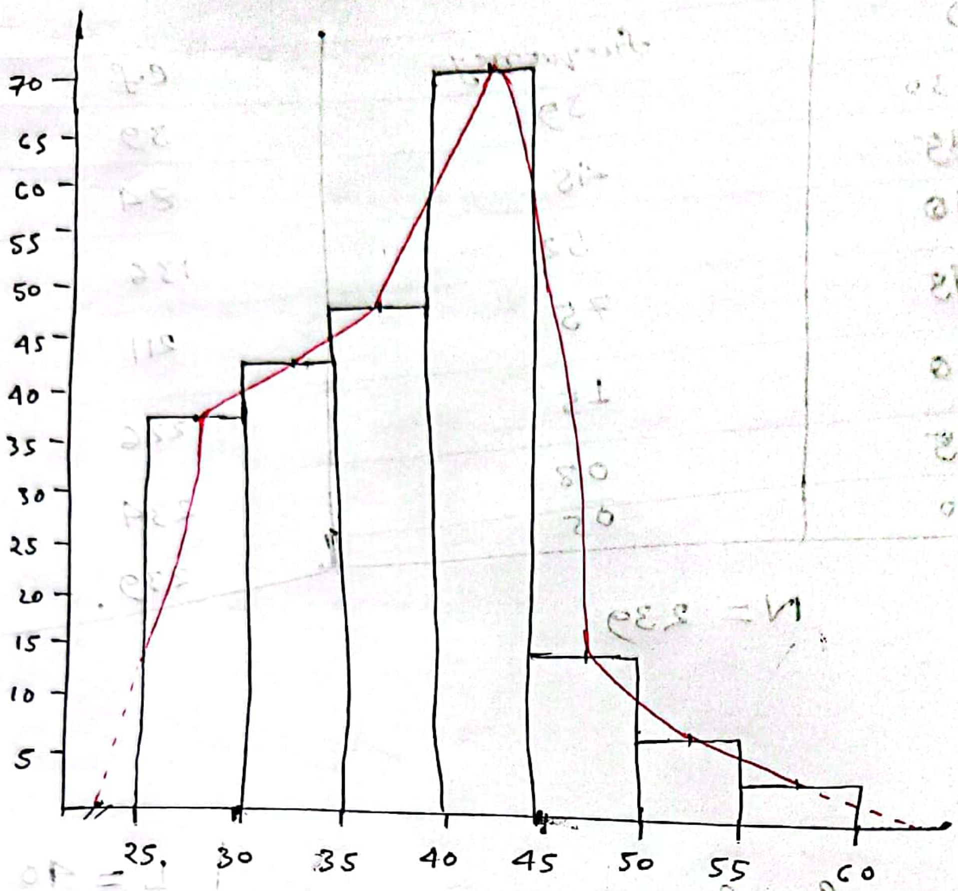
$h = 5$

$$= 40 + \frac{75 - 52}{(2 \times 75) - 52 - 15} \times 5$$

$= 41.39$



polygon



19-20

5(b)

Class	f_i	x_i	$f_i x_i^2$	$f_i x_i$
50-60	5	55	15125	275
60-70	9	65	38025	585
70-80	13	75	73125	975
80-90	20	85	144500	1700
90-100	19	95	171475	1805
100-110	9	105	99225	945
110-120	5	115	66125	575
			607600	6860

S.D, $\sigma = \sqrt{\frac{1}{N}\left\{\sum f_i x_i^2 - \frac{(\sum f_i x_i)^2}{N}\right\}}$

$= \sqrt{\frac{1}{80}\left\{60700 - \frac{(6860)^2}{80}\right\}}$

$= 19.24$

C.V, $= \frac{\sigma_x}{\bar{x}} \times 100\%$

$= \frac{19.24}{\frac{1}{80} \times 6860} \times 100$

$= 22.43\%$.

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Patuakhali Science and Technology University

Faculty of Computer Science and Engineering

3rd Semester (L-2, S-I) Final Examination of B.Sc. in Engg. (CSE), Jan-June-2023, Session: 2021-22

Course Code: MAT-211, Course Title: Mathematics-III

Marks-70, Time: 3 hours, Credit: 3.00

[Figure in the right margin indicates full marks. Split answering of any question is not recommended]

1-3+4-2

Answer any 5 of the following questions

1. a) Define Differential equation, order and degree of differential equation. 02

   b) Solve the differential equations 12

   (i) $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0$ (ii) $(x-y)^2\frac{dy}{dx} = a^2$ (iii) $y - x\frac{dy}{dx} = a(y^2 + \frac{dy}{dx})$

2. a) Define homogeneous differential equation. 02

   b) Solve the differential equations 08

   (i) $(x^2+y^2)dy = xydx$ (ii) $\frac{dy}{dx} = x^3y^3 - xy$

   c) Define Integrating factor. Solve the linear differential equation: $\frac{dy}{dx} + Py = Q$, where P and Q are the function of x or constant. 04

Pay the economics

3. a) Write some applications of differential equation. 02

   b) Solve the Exact differential equation: $(2y-x-1)dy + (2x-y+1)dx = 0$ 04

   Solve the following 08

   (i) $(D^3 - 3D^2 + 4D - 2)y = 0$ (ii) $(D^2 - 13D + 42)y = 0$

5.549

4. a) Define frequency distribution and write down the name of graphs that are used to represent the frequency distribution. 03

   b) The following frequency distribution shows the length of hilsa fish caught on a certain day at a certain point of the Padma: 05

   Class interval (Length in cm): 25-30, 30-35, 35-40, 40-45, 45-50, 50-55, 55-60

   No. of fishes caught: 39, 45, 52, 75, 15, 08, 05

   Draw (i) Histogram and locate the mode and (ii) Frequency polygon by the above distribution.

   c) The following frequency distribution below gives the cost of production of computers in different brands: 06

   Cost (Tk. in Lacs): 10-14, 14-18, 18-22, 22-26, 26-30, 30-34, 34-38, 38-42

   No. of Computers: 11, 27, 42, 45, 50, 55, 65, 70

   Compute quartiles Q_1, Deciles D_4 and Percentiles P_{80}

   23 27.49 37.82

5. a) Write short notes on Moments, Skewness and Kurtosis 03

   b) Calculate the standard deviation and co-efficient of variation from the following frequency distribution: 05

   Class Interval: 50-60, 60-70, 70-80, 80-90, 90-100, 100-110, 110-120

   Frequency: 05 09 13 20 19 09 05

   c) A distribution of short term computer credit disbursement from 10 branches of a bank is given below- 06

   2

   Amount of credit (Lac Tk.): 0-5, 5-10, 10-15, 15-20, 20-25

   No. of branches : 01 02 04 02 01

   Find the coefficients of skewness and kurtosis and thus comment on the shape and nature of the distribution.

6. a) Discuss about the terms: Event, Sample, Census and Pilot survey 04

$\sqrt{\frac{1}{N}\sum f_i x_i^2 - \frac{(\sum f_i x_i)^2}{n}}$

b) Establish the relation between correlation coefficient and regression coefficient 04

c) Per week weight (in pounds) of a calf from its birth is given bellow: 06

Age in week (x):	01	02	03	04	05	06	07	08	09	10
weight (g):	52.5	58.7	65.0	70.2	75.4	81.1	87.2	95.5	102.2	108.0

Estimate the least square regression of weight on age and also estimate the weight when the age in 8.5 weeks.

795.92

$$b_{xy} = \frac{\sum x_i y_i - \frac{(\sum x_i y_i)^2 \; \sum x_i \sum y_i}{n}}{\sum y_i^2 - \frac{(\sum y_i)^2}{n}}$$

0.1985

$y = a + bx$

100.232