```
pip install hmmlearn
```

```
Collecting hmmlearn
  Downloading hmmlearn-0.3.3-cp312-cp312-manylinux_2_17_x86_64.manylinux2014_x86_64.whl.metadata (3.0 kB)
Requirement already satisfied: numpy>=1.10 in /usr/local/lib/python3.12/dist-packages (from hmmlearn) (2.0.2)
Requirement already satisfied: scikit-learn!=0.22.0,>=0.16 in /usr/local/lib/python3.12/dist-packages (from hmmlearn) (1.6.1)
Requirement already satisfied: scipy>=0.19 in /usr/local/lib/python3.12/dist-packages (from hmmlearn) (1.16.3)
Requirement already satisfied: joblib>=1.2.0 in /usr/local/lib/python3.12/dist-packages (from scikit-learn!=0.22.0,>=0.16->hmmle
Requirement already satisfied: threadpoolctl>=3.1.0 in /usr/local/lib/python3.12/dist-packages (from scikit-learn!=0.22.0,>=0.16
Downloading hmmlearn-0.3.3-cp312-cp312-manylinux_2_17_x86_64.manylinux2014_x86_64.whl (165 kB)
   ━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━━ 166.0/166.0 kB 2.8 MB/s eta 0:00:00
Installing collected packages: hmmlearn
Successfully installed hmmlearn-0.3.3
```

```
import yfinance as yf
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from hmmlearn.hmm import GaussianHMM
```

```
sns.set_style("whitegrid")
plt.rcParams["figure.figsize"] = (12, 6)
```

```
# Downloading price data

tickers = ["TLT", "GLD", "SPY", "^VIX"]

close_prices = pd.DataFrame()

for ticker in tickers:
        print(f"Downloading {ticker}...")
        try:

            temp_data = yf.download(ticker, start="2005-01-01", progress=False, auto_adjust=True)
            close_prices[ticker] = temp_data['Close']
            print(f"  Success: {len(temp_data)} records")
        except Exception as e:
            print(f"  Error downloading {ticker}: {e}")

print(f"\nFinal price data shape: {close_prices.shape}")
print("First few rows:")
display(close_prices.head())

# Checking for missing values
print("\nMissing values:")
print(close_prices.isnull().sum())
```

```
Downloading TLT...
  Success: 5247 records
Downloading GLD...
  Success: 5247 records
Downloading SPY...
  Success: 5247 records
Downloading ^VIX...
  Success: 5247 records

Final price data shape: (5247, 4)
First few rows:
```

Date	TLT	GLD	SPY	^VIX
2005-01-03	45.395805	43.020000	81.847137	14.08
2005-01-04	44.920090	42.740002	80.847023	13.98
2005-01-05	45.160519	42.669998	80.289078	14.09
2005-01-06	45.191189	42.150002	80.697304	13.58
2005-01-07	45.293510	41.840000	80.581635	13.49

```
Missing values:
TLT     0
GLD     0
SPY     0
^VIX    0
dtype: int64
```

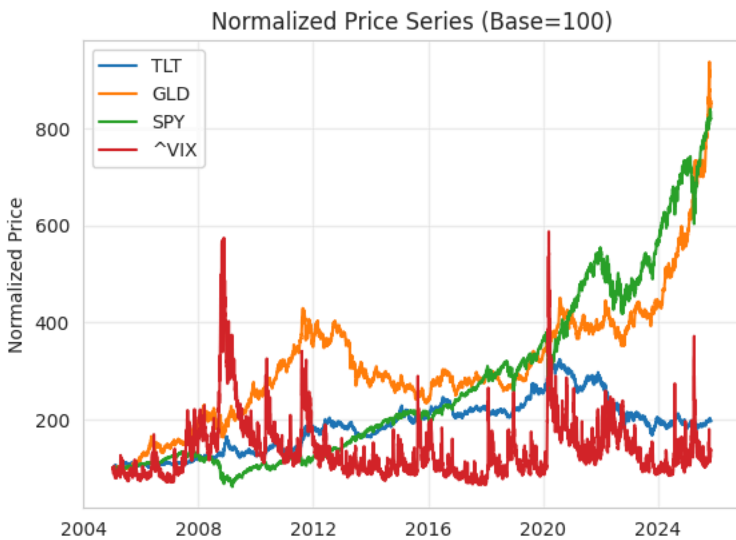
```
# Computing daily log-returns for ETFs and daily change for VIX

returns = pd.DataFrame()

# Calculating log returns for ETFs
returns["TLT"] = np.log(close_prices["TLT"] / close_prices["TLT"].shift(1))
returns["GLD"] = np.log(close_prices["GLD"] / close_prices["GLD"].shift(1))
returns["SPY"] = np.log(close_prices["SPY"] / close_prices["SPY"].shift(1))


returns["dVIX"] = close_prices["^VIX"].diff()

# missing values dropped
returns = returns.dropna()

print(f"\nReturns data shape: {returns.shape}")
print("First few rows of returns:")
display(returns.head())
```

```
Returns data shape: (5246, 4)
First few rows of returns:
```

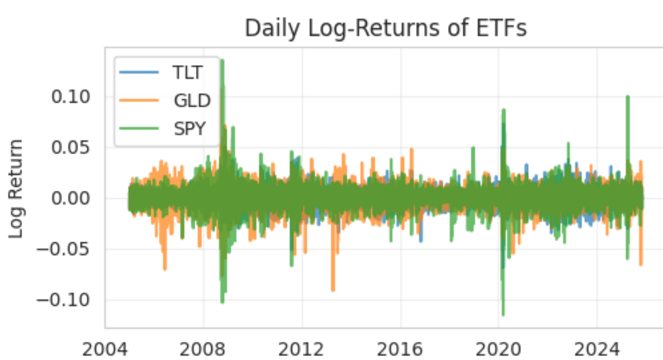
Date	TLT	GLD	SPY	dVIX
2005-01-04	-0.010535	-0.006530	-0.012295	-0.100000
2005-01-05	0.005338	-0.001639	-0.006925	0.110001
2005-01-06	0.000679	-0.012261	0.005072	-0.510000
2005-01-07	0.002262	-0.007382	-0.001434	-0.090000
2005-01-10	0.001580	0.002626	0.004717	-0.260000

```
# Plot of Normalized Price series (normalized to 100)
normalized_prices = (close_prices / close_prices.iloc[0]) * 100

plt.figure(figsize=(14, 10))

plt.subplot(2, 2, 1)
for column in normalized_prices.columns:
    plt.plot(normalized_prices.index, normalized_prices[column], label=column, linewidth=1.5)
plt.title('Normalized Price Series (Base=100)')
plt.ylabel('Normalized Price')
plt.legend()
```

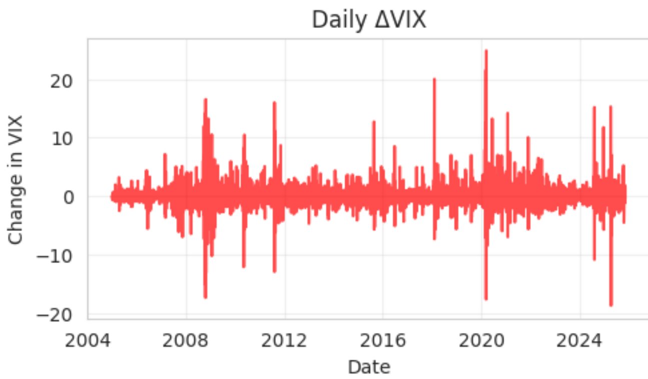
```
plt.grid(True, alpha=0.3)
```



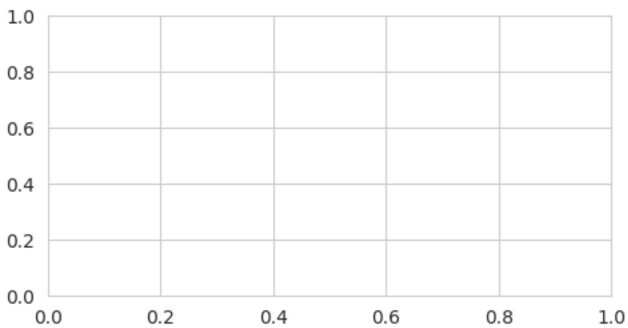

```
# Plot 2: ETF returns
plt.subplot(2, 2, 2)
plt.plot(returns.index, returns["TLT"], label="TLT", alpha=0.7)
plt.plot(returns.index, returns["GLD"], label="GLD", alpha=0.7)
plt.plot(returns.index, returns["SPY"], label="SPY", alpha=0.7)
plt.title("Daily Log-Returns of ETFs")
plt.ylabel("Log Return")
plt.legend()
plt.grid(True, alpha=0.3)
```

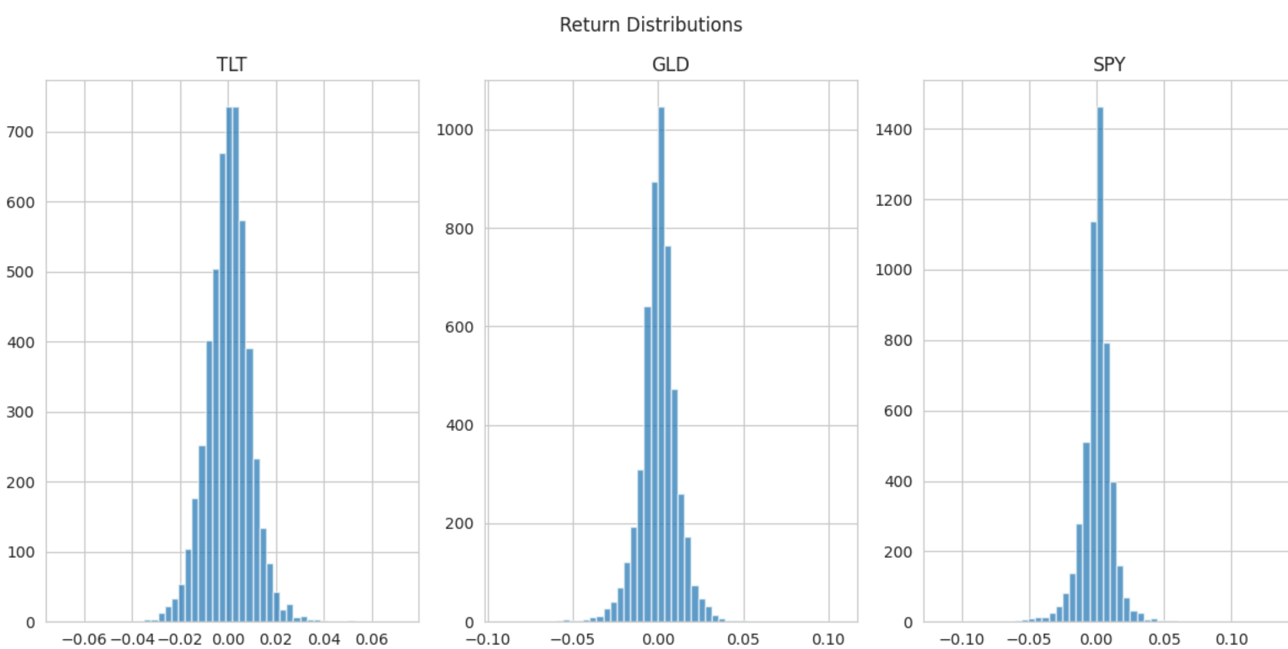



```
# Plot of ΔVIX
plt.subplot(2, 2, 3)
plt.plot(returns.index, returns["dVIX"], color='red', alpha=0.7)
plt.title("Daily ΔVIX")
plt.xlabel("Date")
plt.ylabel("Change in VIX")
plt.grid(True, alpha=0.3)
```



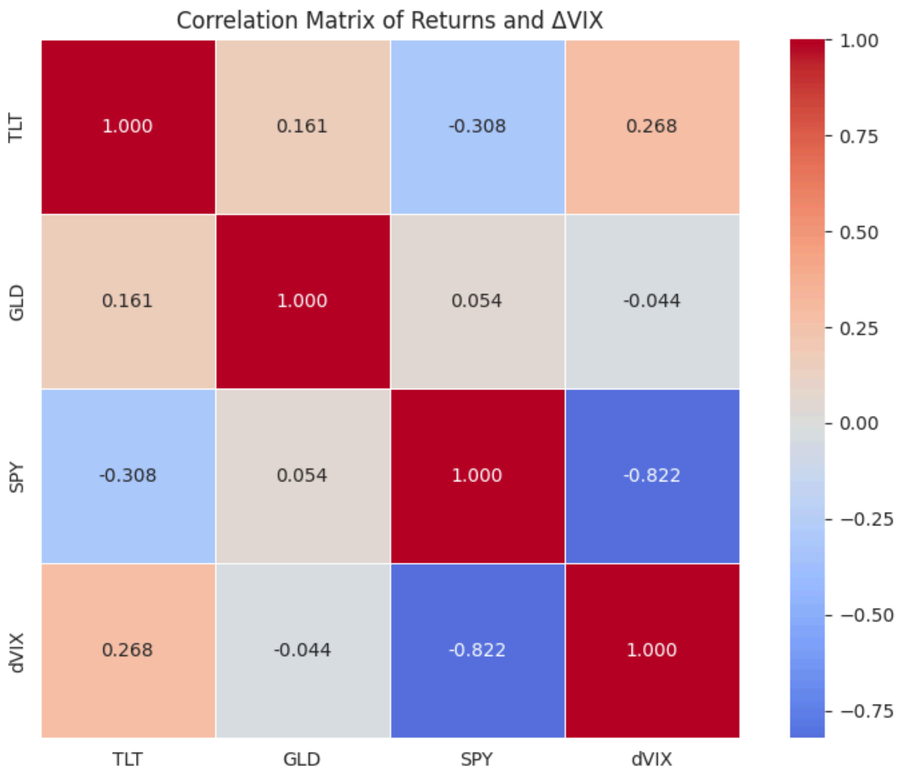

```
# Plot of Return distributions
plt.subplot(2, 2, 4)
returns[["TLT", "GLD", "SPY"]].hist(bins=50, alpha=0.7, layout=(1, 3))
plt.suptitle("Return Distributions")
plt.tight_layout()

plt.show()
```






```
#  Correlation analysis

correlation_matrix = returns.corr()

plt.figure(figsize=(8, 6))
sns.heatmap(correlation_matrix, annot=True, cmap='coolwarm', center=0,
            square=True, linewidths=0.5, fmt='.3f')
plt.title("Correlation Matrix of Returns and ΔVIX")
```

```
plt.tight_layout()
plt.show()

print("Correlation Matrix:")
print(correlation_matrix)
```



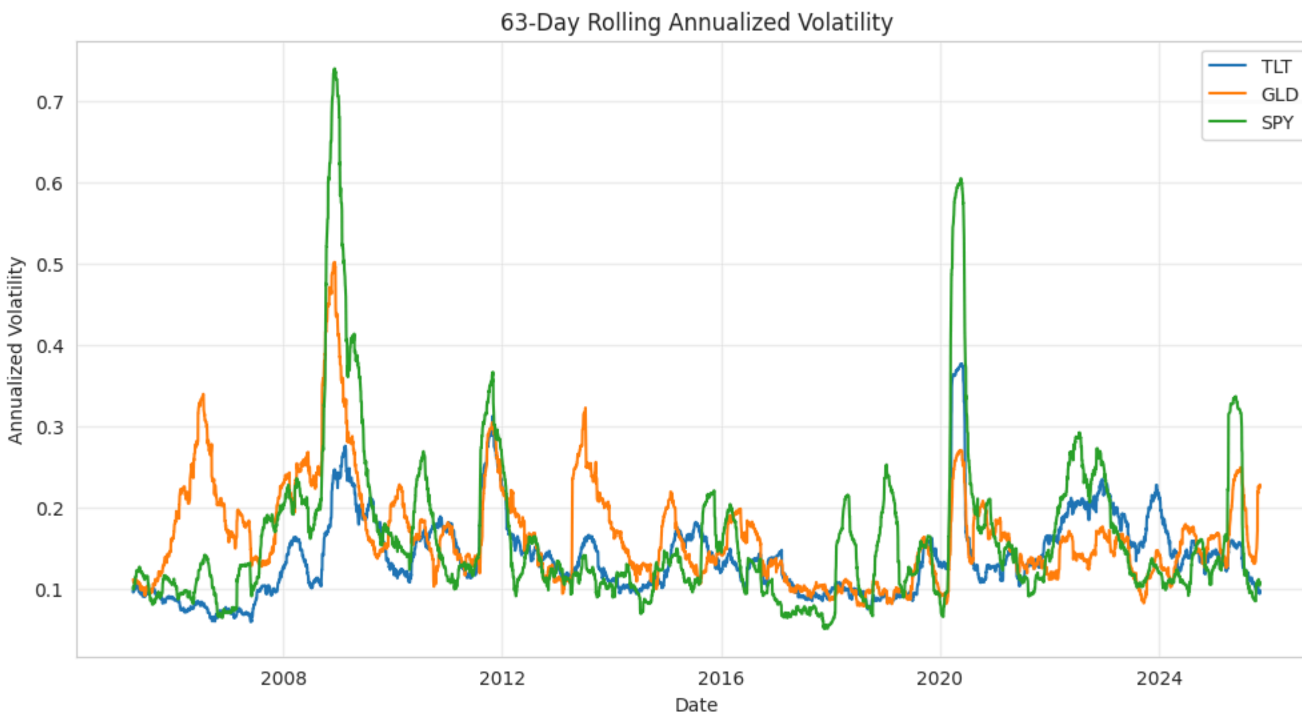

```
Correlation Matrix:
           TLT       GLD       SPY      dVIX
TLT   1.000000  0.160833 -0.308254  0.267848
GLD   0.160833  1.000000  0.054301 -0.044080
SPY  -0.308254  0.054301  1.000000 -0.821685
dVIX  0.267848 -0.044080 -0.821685  1.000000
```

```
#  Statistical analysis

print("\nDetailed Statistics:")
stats_summary = pd.DataFrame({
    'Mean_Daily_Return': returns.mean(),
    'Std_Daily_Return': returns.std(),
    'Annualized_Volatility': returns[['TLT', 'GLD', 'SPY']].std() * np.sqrt(252),
    'Annualized_Sharpe': returns[['TLT', 'GLD', 'SPY']].mean() / returns[['TLT', 'GLD', 'SPY']].std() * np.sqrt(252),
    'Min_Daily_Return': returns.min(),
    'Max_Daily_Return': returns.max(),
    'Skewness': returns.skew(),
    'Kurtosis': returns.kurtosis()
})

print(stats_summary.round(4))
```

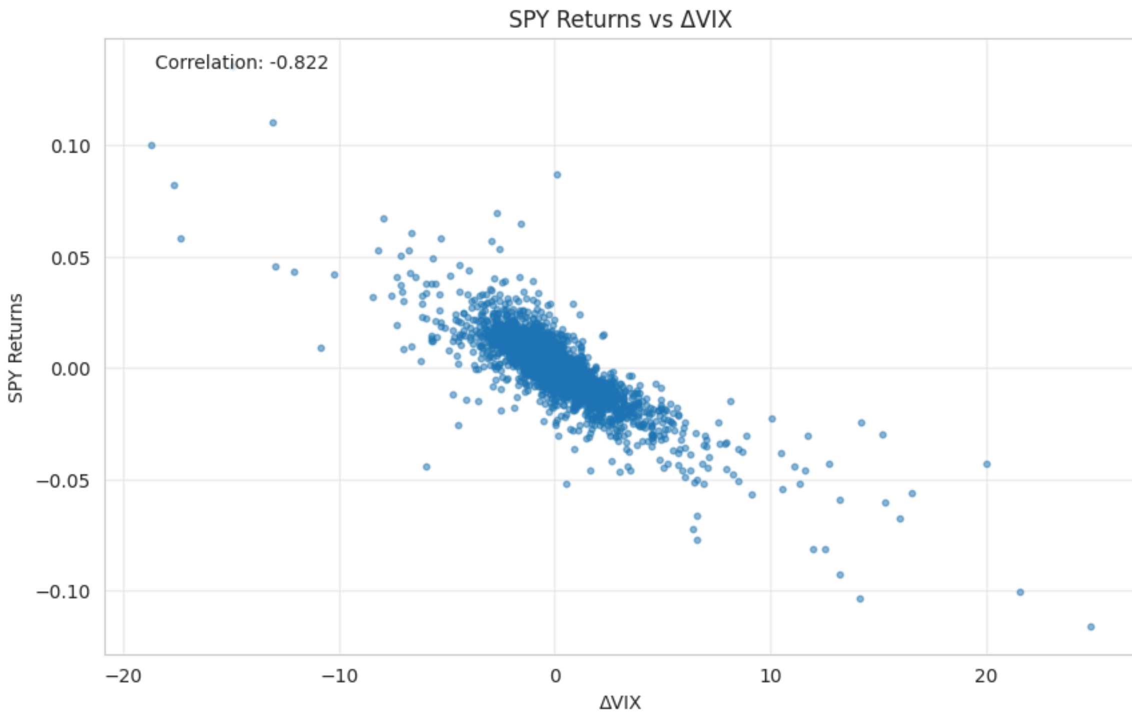
```
Detailed Statistics:
      Mean_Daily_Return  Std_Daily_Return  Annualized_Volatility  \
GLD              0.0004            0.0111                 0.1768
SPY              0.0004            0.0120                 0.1911
TLT              0.0001            0.0093                 0.1471
dVIX             0.0010            1.9172                    NaN

      Annualized_Sharpe  Min_Daily_Return  Max_Daily_Return  Skewness  \
GLD              0.5835           -0.0919            0.1070   -0.3283
SPY              0.5289           -0.1159            0.1356   -0.3044
TLT              0.2219           -0.0690            0.0725    0.0038
dVIX                NaN          -18.7100           24.8600    1.4457

      Kurtosis
GLD     5.8974
SPY    14.6799
```

```
TLT      3.3894
dVIX    25.9493
```

```
#  Rolling volatility analysis

rolling_volatility = returns[['TLT', 'GLD', 'SPY']].rolling(window=63).std() * np.sqrt(252)

plt.figure(figsize=(12, 6))
plt.plot(rolling_volatility.index, rolling_volatility)
plt.title("63-Day Rolling Annualized Volatility")
plt.xlabel("Date")
plt.ylabel("Annualized Volatility")
plt.legend(['TLT', 'GLD', 'SPY'])
plt.grid(True, alpha=0.3)
plt.show()
```




```
# VIX vs SPY returns scatter plot
plt.figure(figsize=(10, 6))
plt.scatter(returns["dVIX"], returns["SPY"], alpha=0.5, s=10)
plt.xlabel("ΔVIX")
plt.ylabel("SPY Returns")
plt.title("SPY Returns vs ΔVIX")
plt.grid(True, alpha=0.3)

corr = returns["dVIX"].corr(returns["SPY"])
plt.text(0.05, 0.95, f'Correlation: {corr:.3f}', transform=plt.gca().transAxes,
         bbox=dict(boxstyle="round,pad=0.3", facecolor="white", alpha=0.8))

plt.show()
```



Step 2

```
# STEP 2: HIDDEN MARKOV MODEL FOR ΔVIX


dVIX = returns["dVIX"].dropna().values.reshape(-1,1)


#  Fitting 2-state HMM

model_2 = GaussianHMM(n_components=2, covariance_type='full', n_iter=200)
model_2.fit(dVIX)

states_2 = model_2.predict(dVIX)

print("\n 2-State HMM Parameters ")
print("Means:", model_2.means_.flatten())
print("Variances:", [np.diag(cov) for cov in model_2.covars_])
print("Transition Matrix:\n", model_2.transmat_)
```

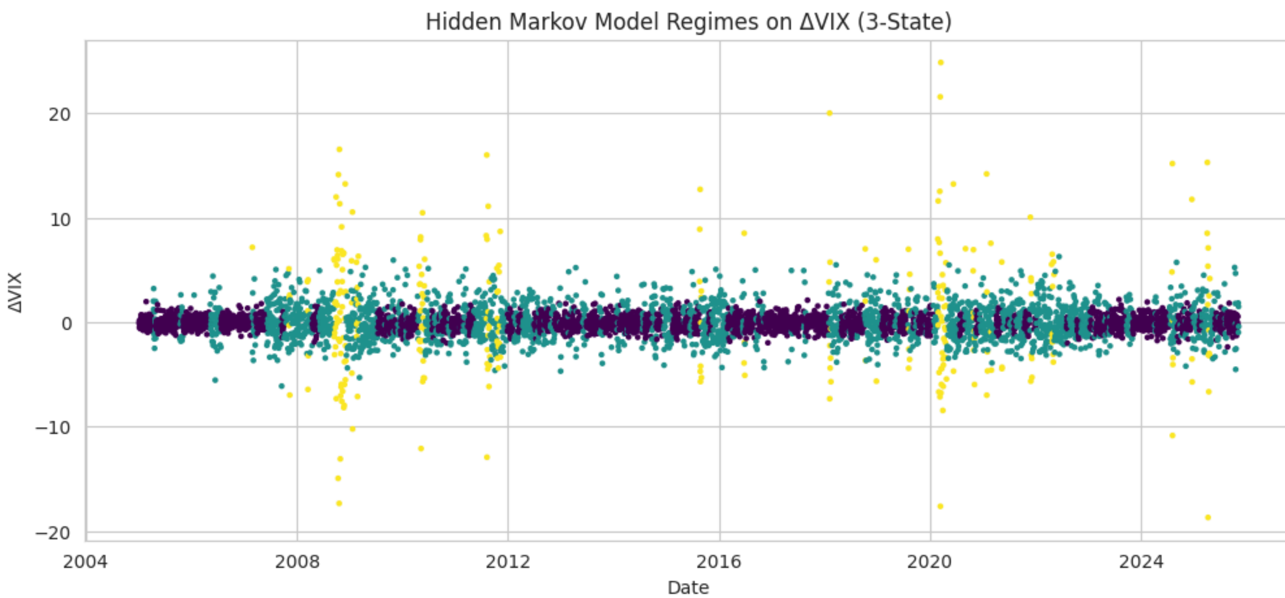
```
--- 2-State HMM Parameters ---
Means: [ 0.22528967 -0.07448549]
Variances: [array([12.52761492]), array([0.67524709])]
Transition Matrix:
 [[0.88205505 0.11794495]
 [0.03968083 0.96031917]]
```

```
#  Fit 3-state HMM

model_3 = GaussianHMM(n_components=3, covariance_type='full', n_iter=200)
model_3.fit(dVIX)

states_3 = model_3.predict(dVIX)

print("\n--- 3-State HMM Parameters ---")
print("Means:", model_3.means_.flatten())
print("Variances:", [np.diag(cov) for cov in model_3.covars_])
print("Transition Matrix:\n", model_3.transmat_)
```

```
--- 3-State HMM Parameters ---
Means: [-0.0620278  -0.01347739  0.62548442]
```

```
Variances: [array([0.37757506]), array([2.98231782]), array([35.50867948])]
Transition Matrix:
 [[9.38348545e-01 6.10991618e-02 5.52293693e-04]
 [7.34957017e-02 9.09075640e-01 1.74286579e-02]
 [1.39521041e-14 1.25728700e-01 8.74271300e-01]]
```

```
# Plotting 3-state HMM states on ΔVIX

plt.figure(figsize=(12,5))
plt.scatter(returns.index, returns["dVIX"], c=states_3, cmap="viridis", s=5)
plt.title("Hidden Markov Model Regimes on ΔVIX (3-State)")
plt.xlabel("Date")
plt.ylabel("ΔVIX")
plt.show()
```



Step 3

```
# Log-likelihoods
ll_2 = model_2.score(dVIX)
ll_3 = model_3.score(dVIX)

# Parameter count for Gaussian HMM:
def hmm_params(n_states):
    k = n_states

    return k + k + k*(k-1) + (k-1)

k2 = hmm_params(2)
k3 = hmm_params(3)

# AIC and BIC
AIC_2 = 2*k2 - 2*ll_2
AIC_3 = 2*k3 - 2*ll_3

BIC_2 = k2*np.log(len(dVIX)) - 2*ll_2
BIC_3 = k3*np.log(len(dVIX)) - 2*ll_3

comparison = pd.DataFrame({
    "Model":["2-State HMM","3-State HMM"],
    "Log-Likelihood":[ll_2, ll_3],
    "AIC":[AIC_2, AIC_3],
    "BIC":[BIC_2, BIC_3]
})

print("Model Comparison:")
display(comparison)
```

Model Comparison:

	Model	Log-Likelihood	AIC	BIC
0	2-State HMM	-8945.333565	17904.667129	17950.623677
1	3-State HMM	-8607.098368	17242.196736	17334.109832

Extracting state from model

```
states = states_3              # predicted states from Step 2B
returns["state"] = states      # attach states to return dataframe
```

Computing average ETF returns in each regime:

```
group_stats = returns.groupby("state")[["TLT","GLD","SPY"]].agg(["mean","std"])
print("State-Conditional ETF Return Stats:")
display(group_stats)
```

State-Conditional ETF Return Stats:

	TLT		GLD		SPY	
	mean	std	mean	std	mean	std
state						
0	-0.000137	0.007503	0.000450	0.009365	0.001318	0.005556
1	0.000250	0.009660	0.000351	0.011484	-0.000035	0.012160
2	0.001673	0.017325	0.000476	0.020023	-0.004749	0.033809

Bar Chart: Mean Return per State

```
mean_returns = returns.groupby("state")[["TLT","GLD","SPY"]].mean()

mean_returns.plot(kind="bar", figsize=(10,5))
plt.title("State-Conditional Mean Returns")
plt.ylabel("Average Daily Return")
plt.xlabel("HMM State")
plt.legend(["TLT","GLD","SPY"])
plt.show()
```

STEP 4: DESIGN ROTATION STRATEGY, SIGNALS, & STRATEGY RETURNS

```
if "state" not in returns.columns:
    raise ValueError("returns DataFrame must have 'state' column from Step 2/3")

state_mean = returns.groupby("state")[["TLT","GLD","SPY"]].mean()
state_std  = returns.groupby("state")[["TLT","GLD","SPY"]].std()

print("State-conditional mean returns (daily):")
display(state_mean)


etf_list = ["TLT","GLD","SPY"]

def top_n_weights(state_row, policy="100/0"):
    """
    state_row: pandas Series of mean returns for ETFs in the state (index: ETF names)
    policy: "100/0" or "60/40"
    returns: dict mapping ETF -> weight (floats summing to 1)
    """
    sorted_etfs = state_row.sort_values(ascending=False)
    top1 = sorted_etfs.index[0]
    top2 = sorted_etfs.index[1]
    if policy == "100/0":
        weights = {e: 1.0 if e == top1 else 0.0 for e in etf_list}
    elif policy == "60/40":
        weights = {e: 0.0 for e in etf_list}
        weights[top1] = 0.6
        weights[top2] = 0.4
    else:
        raise ValueError("Unknown policy. Use '100/0' or '60/40'.")
    return weights

# Creating a mapping table: state for both policies
alloc_table = []
for s in state_mean.index:
    row = {"state": int(s)}
    mr = state_mean.loc[s]
    w100 = top_n_weights(mr, policy="100/0")
    w6040 = top_n_weights(mr, policy="60/40")

    for etf in etf_list:
        row[f"{etf}_100p"] = w100[etf]
        row[f"{etf}_60_40p"] = w6040[etf]
    alloc_table.append(row)

alloc_df = pd.DataFrame(alloc_table).set_index("state").sort_index()
print("\nState → Allocation mapping (Policy A = 100/0, Policy B = 60/40):")
display(alloc_df)

# Creating daily allocation signals using 1-day execution lag
weights_100 = pd.DataFrame(index=returns.index, columns=etf_list, dtype=float)
weights_6040 = pd.DataFrame(index=returns.index, columns=etf_list, dtype=float)


for t in range(len(returns)):
    if t == 0:

        weights_100.iloc[t] = [0.0, 0.0, 0.0]
        weights_6040.iloc[t] = [0.0, 0.0, 0.0]
    else:
        prev_state = returns["state"].iloc[t-1]
        # retrieving mapping
        w100 = alloc_df.loc[prev_state, [f"{etf}_100p" for etf in etf_list]].values
        w100 = dict(zip(etf_list, w100))
        w6040 = alloc_df.loc[prev_state, [f"{etf}_60_40p" for etf in etf_list]].values
        w6040 = dict(zip(etf_list, w6040))
        weights_100.iloc[t] = [w100[e] for e in etf_list]
        weights_6040.iloc[t] = [w6040[e] for e in etf_list]

# Attaching weights into returns for inspection
weights_100.index = returns.index
weights_6040.index = returns.index
```

```
print("\nSample allocations (first 8 rows) — Policy A (100/0):")
display(weights_100.head(8))

print("\nSample allocations (first 8 rows) — Policy B (60/40):")
display(weights_6040.head(8))


etf_returns = returns[etf_list].copy()


strat_ret_100 = (weights_100.fillna(0).values * etf_returns.values).sum(axis=1)
strat_ret_6040 = (weights_6040.fillna(0).values * etf_returns.values).sum(axis=1)


returns["strat_100p"] = strat_ret_100
returns["strat_60_40p"] = strat_ret_6040

print("\nSample strategy returns (first 8 rows):")
display(returns[["TLT","GLD","SPY","state","strat_100p","strat_60_40p"]].head(8))



cum = (1 + returns[["TLT","GLD","SPY","strat_100p","strat_60_40p"]]).cumprod() - 1
plt.figure(figsize=(12,6))
plt.plot(cum.index, cum["strat_100p"], label="Strategy 100% top ETF")
plt.plot(cum.index, cum["strat_60_40p"], label="Strategy 60/40 top-two")
plt.plot(cum.index, cum["SPY"], label="Buy-and-hold SPY")
plt.plot(cum.index, cum["TLT"], label="Buy-and-hold TLT")
plt.title("Cumulative Returns (visual check) — Strategy vs ETFs")
plt.xlabel("Date")
plt.ylabel("Cumulative Return")
plt.legend()
plt.show()
```

```
State-conditional mean returns (daily):
```

state	TLT	GLD	SPY
0	-0.000137	0.000450	0.001318
1	0.000250	0.000351	-0.000035
2	0.001673	0.000476	-0.004749

```
State → Allocation mapping (Policy A = 100/0, Policy B = 60/40):
```

state	TLT_100p	TLT_60_40p	GLD_100p	GLD_60_40p	SPY_100p	SPY_60_40p
0	0.0	0.0	0.0	0.4	1.0	0.6
1	0.0	0.4	1.0	0.6	0.0	0.0
2	1.0	0.6	0.0	0.4	0.0	0.0

```
Sample allocations (first 8 rows) — Policy A (100/0):
```

Date	TLT	GLD	SPY
2005-01-04	0.0	0.0	0.0
2005-01-05	0.0	0.0	1.0
2005-01-06	0.0	0.0	1.0
2005-01-07	0.0	0.0	1.0
2005-01-10	0.0	0.0	1.0
2005-01-11	0.0	0.0	1.0
2005-01-12	0.0	0.0	1.0
2005-01-13	0.0	0.0	1.0

```
Sample allocations (first 8 rows) — Policy B (60/40):
```

Date	TLT	GLD	SPY
2005-01-04	0.0	0.0	0.0
2005-01-05	0.0	0.4	0.6
2005-01-06	0.0	0.4	0.6
2005-01-07	0.0	0.4	0.6
2005-01-10	0.0	0.4	0.6
2005-01-11	0.0	0.4	0.6
2005-01-13	0.0	0.4	0.6

```
Start coding or generate with AI.
```

Step 5

```
Sample strategy returns (first 8 rows):
```

TLT	GLD	SPY	state	strat_100p	strat_60_40p

```
# Helper functions

def annualized_return(r):
    """Annualized mean using 252 trading days"""
    return (1 + r).prod()**(252/len(r)) - 1

def annualized_vol(r):
    """Annualized volatility"""
    return r.std() * np.sqrt(252)

def sharpe_ratio(r, rf=0):
    """Assume risk-free = 0 unless provided"""
    ann_ret = annualized_return(r)
    ann_vol = annualized_vol(r)
    return ann_ret / ann_vol if ann_vol > 0 else np.nan
```

```
def max_drawdown(cum_curve):
    """cum_curve = cumulative growth, (1+r).cumprod()"""
    peak = np.maximum.accumulate(cum_curve)
    dd = (cum_curve - peak) / peak
    return dd.min()
```

```
#  5.1 Computing Benchmark Strategies


bh_spy = returns["SPY"]

weights_eq = pd.DataFrame(index=returns.index, columns=["TLT","GLD","SPY"])
current_weights = np.array([1/3,1/3,1/3])

for i, d in enumerate(returns.index):

    if i == 0 or d.month != returns.index[i-1].month:
        current_weights = np.array([1/3,1/3,1/3])
    weights_eq.iloc[i] = current_weights

# daily equal-weight returns
eq_ret = (weights_eq.values * returns[["TLT","GLD","SPY"]].values).sum(axis=1)
returns["eq_weight"] = eq_ret
```

```
# 5.2 Compute cumulative returns

strats = {
    "Buy&Hold SPY" : bh_spy,
    "Equal-Weight" : returns["eq_weight"],
    "Strategy 100%" : returns["strat_100p"],
    "Strategy 60/40" : returns["strat_60_40p"],
}
```