Design Theory for Relational Databases

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Introduction

- There are always many different schemas for a given set of data.
- E.g., you could combine or divide tables.
- How do you pick a schema?
  Which is better?
  What does “better” mean?
- Fortunately, there are some principles to guide us.

Database Design Theory

- It allows us to improve a schema systematically.
- General idea:
  - Express constraints on the relationships between attributes
  - Use these to decompose the relations
- Ultimately, get a schema that is in a "normal form" that guarantees good properties.
- "Normal" in the sense of conforming to a standard.
- The process of converting a schema to a normal form is called normalization.

Part I:
Functional Dependency Theory

A poorly designed table

part	manufacturer	manAddress	seller	sellerAddress	price
1983	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	5.59
8624	Lee Valley	102 Vaughn	ABC	1229 Bloor W	23.99
9141	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	12.50
1983	Hammers 'R Us	99 Pinecrest	Walmart	5289 St Clair W	4.99

Perhaps:

- Every part has 1 manufacturer
- Every manufacturer has 1 address
- Every seller has 1 address

If so, instances will have redundant data.

Principle: Avoid redundancy

- Redundant data can lead to anomalies.

part	manufacturer	manAddress	seller	sellerAddress	price
1983	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	5.59
8624	Lee Valley	102 Vaughn	ABC	1229 Bloor W	23.99
9141	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	12.50
1983	Hammers 'R Us	99 Pinecrest	Walmart	5289 St Clair W	4.99

- Update anomaly: if Hammers `R Us moves and we update only one tuple, the data is inconsistent!

part	manufacturer	manAddress	seller	sellerAddress	price
1983	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	5.59
8624	Lee Valley	102 Vaughn	ABC	1229 Bloor W	23.99
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- Deletion anomaly: if ABC stops selling part 8624 and Lee Valley makes only that one part, we lose track of its address!

Definition of FD

- Suppose R is a relation, and X and Y are subsets of the attributes of R.
- $X \rightarrow Y$ asserts that:
    - If two tuples agree on all the attributes in set X, they must also agree on all the attributes in set Y.
- We say that "$X \rightarrow Y$ holds in R", or "X functionally determines Y."
- An FD constrains what can go in a relation.

More formally…

$A \to B$ means:

$\forall$tuples t_1, t_2,

$$(t_1[A] = t_2[A]) \Rightarrow (t_1[B] = t_2[B])$$

Or equivalently:

$\neg\exists$tuples t_1, t_2 such that

$$(t_1[A] = t_2[A]) \wedge (t_1[B] \neq t_2[B])$$

Generalization to multiple attributes

$A_1 A_2 \ldots A_m \rightarrow B_1 B_2 \ldots B_n$ means:

$\forall$tuples t_1, t_2,

$$(t_1[A_1] = t_2[A_1] \wedge \ldots \wedge t_1[A_m] = t_2[A_m]) \Rightarrow$$
$$(t_1[B_1] = t_2[B_1] \wedge \ldots \wedge t_1[B_n] = t_2[B_n])$$

Or equivalently:

$\neg\exists$tuples t_1, t_2 such that

$$(t_1[A_1] = t_2[A_1] \wedge \ldots \wedge t_1[A_m] = t_2[A_m]) \wedge$$
$$\neg (t_1[B_1] = t_2[B_1] \wedge \ldots \wedge t_1[B_n] = t_2[B_n])$$

FDs in our parts table

part	manufacturer	manAddress	seller	sellerAddress	price
1983	Hammers 'R Us	99 Pinecrest	ABC	1229 Bloor W	5.59

etc.

We can now express relationships as FDs

- Every part has 1 manufacturer becomes
  part → manufacturer
- Every manufacture has 1 address becomes
  manufacturer → manAddress
- Every seller has 1 address becomes
  seller → sellerAddress

Why “functional dependency”?

- $X \rightarrow Y$ is a “dependency” because

  the value of Y depends on the value of X.

- It is “functional” because

  there is a mathematical function that takes a value for X and gives a *unique* value for Y. (It’s not a typical function; just a lookup.)

Equivalent sets of FDs

- When we write a set of FDs, we mean that all of them hold.
- We can very often rewrite sets of FDs in equivalent ways.
- When we say S_1 is equivalent to S_2 we mean that:

  S_1 holds in a relation iff S_2 does.

Splitting rules for FDs

- Can we split the RHS of an FD and get multiple, equivalent FDs?

- Can we split the LHS of an FD and get multiple, equivalent FDs?

Coincidence or FD?

- An FD is an assertion about *every* instance of the relation.
- You can't know it holds just by looking at one instance.
- You must use knowledge of the domain to determine whether an FD holds.

FDs are closely related to keys

- Suppose K is a set of attributes for relation R.
- Our old definition of superkey:

  a set of attributes for which no two rows can have the same values.

- A claim about FDs:

  K is a superkey for *R* iff

  K functionally determines all of *R*.

FDs are a generalization of keys

Every attribute

- Superkey:
  $X \rightarrow \boxed{R}$
- Functional dependency:
  $X \rightarrow \boxed{Y}$

Not necessarily every attribute

- An FD can be more subtle.

Inferring FDs

- Given a set of FDs, we can often infer further FDs.
- This will be handy when we apply FDs to the problem of database design.
- Big task: given a set of FDs,

  infer *every* other FD that must also hold.
- Simpler task: given a set of FDs,

  check whether *a given* FD must also hold.

Examples

- If $A \rightarrow B$ and $B \rightarrow C$ hold,
  must $A \rightarrow C$ hold?
- If $A \rightarrow H$, $C \rightarrow F$, and $FG \rightarrow AD$ hold,
  must $FA \rightarrow D$ hold?
  must $CG \rightarrow FH$ hold?
- If $H \rightarrow GD$, $HD \rightarrow CE$, and $BD \rightarrow A$ hold,
  must $EH \rightarrow C$ hold?
- Aside: we are not generating new FDs, but testing a specific possible one.

Method 1: Prove an FD follows using first principles

- You can prove it by referring back to
  - The FDs that you know hold, and
  - The definition of functional dependency.

Method 2: Prove an FD follows using the Closure Test

- Assume you know the values of the LHS attributes, and figure out everything else that is determined.
- If it includes the RHS attributes, then you know that LHS $\rightarrow$ RHS
- This is called the closure test.

Y is a set of attributes, S is a set of FDs.

Return the closure of Y under S.

Attribute_closure(Y, S):

Initialize Y^+ to Y

Repeat until no more changes occur:

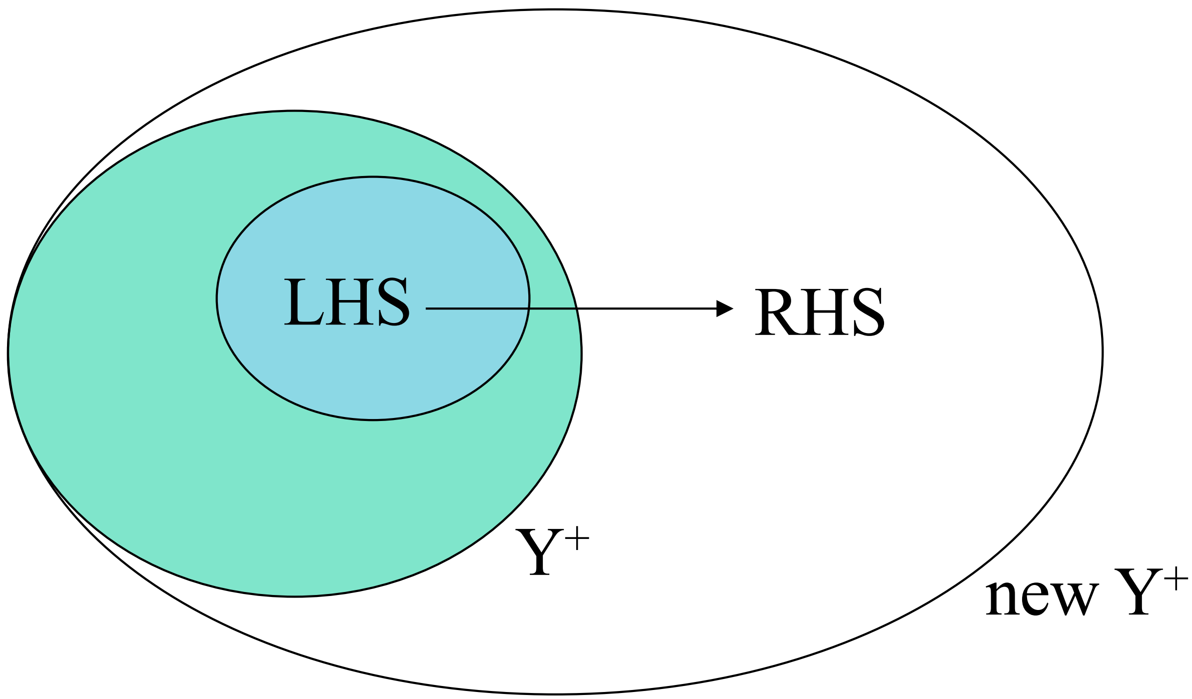
If there is an FD LHS $\rightarrow$ RHS in S

such that LHS is in Y^+:

Add RHS to Y^+

Return Y^+

Visualizing attribute closure



If LHS is in Y^+ and LHS $\rightarrow$ RHS holds, we can add RHS to Y^+

S is a set of FDs; LHS $\rightarrow$ RHS is a single FD.

Return true iff LHS $\rightarrow$ RHS follows from S.

FD_follows(S, LHS $\rightarrow$ RHS):

  Y^+ = Attribute_closure(LHS, S)

  return (RHS is in Y^+)

Projecting FDs

- Later, we will learn how to normalize a schema by decomposing relations.
  - This is the whole point of this theory.
- We will need to know what FDs hold in the new, smaller, relations.
  - We must project our FDs onto the attributes of our new relations.

S is a set of FDs; L is a set of attributes.

Return the projection of S onto L:

all FDs that follow from S and involve only attributes from L.

Project(S, L):

Initialize T to {}.

For each subset X of L:

Compute X^+ *Close X and see what we get.*

For every attribute A in X^+:

If A is in L: *$X \rightarrow A$ is only relevant if A is in L (we know X is).*

add $X \rightarrow A$ to T.

Return T.

A few speed-ups

- No need to add $X \rightarrow A$ if A is in X itself. It's a trivial FD.
- These subsets of X won't yield anything, so no need to compute their closures:
  - the empty set
  - the set of all attributes
- Neither are big savings, but …

A big speed-up

- If we find that X^+ = all attributes, we can ignore any superset of X.

    It can only give use "weaker" FDs (i.e., FDs with more on the LHS).

- This is a big time saver!

Projection is expensive

- Even with these speed-ups, projection is still expensive.
- Suppose R_1 has n attributes.
  How many subsets of R_1 are there?

Minimal Basis

- We saw earlier that we can very often rewrite sets of FDs in equivalent ways.
- Example: $S_1 = \{A \to BC\}$ is equivalent to $S_2 = \{A \to B, A \to C\}$.
- Given a set of FDs S, we may want to find a minimal basis: A set of FDs that is equivalent, but has
  - no redundant FDs, and
  - no FDs with unnecessary attributes on the LHS.

S is a set of FDs. Return a minimal basis for S.

Minimal_basis(S):

1. Split the RHS of each FD
2. For each FD $X \rightarrow Y$ where $|X| \geq 2$:

   If you can remove an attribute from X and get an FD that follows from S:

   Do so! (It's a stronger FD.)
3. For each FD *f*:

   If S – {*f*} implies *f*:

   Remove *f* from S.

Some comments on minimal basis

- Often there are multiple possible results.

  Depends on the order in which you consider the possible simplifications.

- After you identify a redundant FD, you must not use it when computing subsequent closures.

… and perhaps less intuitive

- When you are computing closures to decide whether the LHS of an FD

  $$X \rightarrow Y$$

  can be simplified, continue to use that FD.

- You must do steps (2) and (3) in that order.
  - Otherwise, must repeat until no changes occur.