App Logic

June 3, 2025

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1 Historical Backtest Engine: Mathematical Framework & Methodology

1.1 Core Philosophy & Approach

The Historical Backtest Engine operates on the principle of **realistic simulation** - every calculation mirrors real-world margin trading mechanics with institutional-grade precision. Unlike simplified backtests that ignore borrowing costs and margin requirements, this engine models the complete lifecycle of a leveraged position with daily granularity.

1.2 Data Architecture & Flow

1.2.1 Primary Data Sources

The engine uses your `ETFs and Fed Funds Data.xlsx` as the authoritative source, containing: - **Daily ETF Prices**: SPY and VTI closing prices - **Dividend Payments**: Quarterly dividend distributions
- **Fed Funds Rates**: Historical Federal Reserve rates for realistic interest calculations

1.2.2 Data Processing Logic

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Data Flow: Excel → Pandas DataFrame → Date-Indexed Time Series → Daily Simulation Loop
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The system converts all data to datetime-indexed pandas DataFrames, enabling efficient vectorized operations and precise date-based filtering for any user-specified backtest period.

1.3 Margin Account Mechanics

1.3.1 Account Type Specifications

Reg-T Account (Regulation T): - **Maximum Leverage**: 2:1 (50% initial margin requirement)
- **Maintenance Margin**: 25% of position value - **Interest Rate**: Fed Funds Rate + 1.5% spread

Portfolio Margin Account: - **Maximum Leverage**: 7:1 (dynamic initial margin requirement)
- **Maintenance Margin**: 15% of position value
- **Interest Rate**: Fed Funds Rate + 2.0% spread (higher due to increased risk)

1.3.2 Initial Position Setup

When a user specifies an investment amount and leverage, the system calculates:

$$\text{Cash Investment} = \frac{\text{Total Position Size}}{\text{Leverage Ratio}}$$

$$\text{Margin Loan} = \text{Total Position Size} - \text{Cash Investment}$$

$$\text{Initial Shares} = \frac{\text{Total Position Size}}{\text{Initial ETF Price}}$$

Example: \$10M position with 4x leverage - Cash Investment = \$10M ÷ 4 = \$2.5M - Margin Loan = \$10M - \$2.5M = \$7.5M
- Shares (if SPY = \$400) = \$10M ÷ \$400 = 25,000 shares

1.4 Interest Rate Calculations

1.4.1 Dynamic Interest Rate Model

The system uses **real historical Fed Funds rates** rather than static assumptions:

$$\text{Daily Margin Rate} = \frac{\text{Fed Funds Rate} + \text{Account Spread}}{365}$$

$$\text{Daily Interest Cost} = \text{Outstanding Margin Loan} \times \text{Daily Margin Rate}$$

1.4.2 Compounding Effect

Interest compounds daily, meaning the margin loan grows continuously:

$$\text{New Margin Loan} = \text{Previous Margin Loan} + \text{Daily Interest Cost}$$

This creates a realistic cost structure where longer holding periods and higher leverage dramatically impact returns through cumulative interest expenses.

1.5 Dividend Reinvestment Logic

1.5.1 Automatic Reinvestment Process

When dividends are paid (typically quarterly), the system:

1. **Calculates Dividend Income**:

$$\text{Dividend Received} = \text{Current Shares} \times \text{Dividend Per Share}$$

2. **Reinvests Immediately**:

$$\text{Additional Shares} = \frac{\text{Dividend Received}}{\text{Current ETF Price}}$$

3. **Updates Share Count**:

$$\text{New Total Shares} = \text{Previous Shares} + \text{Additional Shares}$$

This compounding effect means the position grows over time, increasing both potential gains and margin requirements.

1.6 Daily Portfolio Valuation

1.6.1 Core Portfolio Metrics

Each trading day, the system recalculates:

$$\text{Portfolio Value} = \text{Current Shares} \times \text{Current ETF Price}$$

$$\text{Equity} = \text{Portfolio Value} - \text{Outstanding Margin Loan}$$

$$\text{Leverage Ratio} = \frac{\text{Portfolio Value}}{\text{Equity}}$$

1.6.2 Maintenance Margin Requirements

$$\text{Required Maintenance Margin} = \text{Portfolio Value} \times \text{Maintenance \%}$$

Where Maintenance % = 25% for Reg-T, 15% for Portfolio Margin

1.7 Margin Call Detection

1.7.1 Margin Call Trigger Condition

A margin call occurs when:

$$\text{Current Equity} < \text{Required Maintenance Margin}$$

1.7.2 Margin Call Price Calculation

The system calculates the ETF price at which a margin call would trigger:

$$\text{Margin Call Price} = \frac{\text{Outstanding Margin Loan}}{\text{Current Shares} \times (1 - \text{Maintenance \%})}$$

This forward-looking calculation helps assess risk before it materializes.

1.8 Performance Metrics Calculations

1.8.1 Total Return

$$\text{Total Return \%} = \frac{\text{Final Equity} - \text{Initial Cash Investment}}{\text{Initial Cash Investment}} \times 100$$

1.8.2 Compound Annual Growth Rate (CAGR)

$$\text{CAGR \%} = \left(\frac{\text{Final Equity}}{\text{Initial Cash Investment}}\right)^{\frac{1}{\text{Years}}} - 1 \times 100$$

Where Years = Trading Days ÷ 252 (approximate trading days per year)

1.8.3 Maximum Drawdown

$$\text{Drawdown}_t = \frac{\text{Equity}_t - \text{Peak Equity}_t}{\text{Peak Equity}_t} \times 100$$

$$\text{Maximum Drawdown} = \min(\text{All Drawdowns})$$

Peak Equity is the rolling maximum equity value up to time t.

1.8.4 Annual Volatility

$$\text{Daily Returns}_t = \frac{\text{Equity}_t - \text{Equity}_{t-1}}{\text{Equity}_{t-1}}$$

$$\text{Annual Volatility} = \text{std(Daily Returns)} \times \sqrt{252} \times 100$$

1.8.5 Sharpe Ratio

$$\text{Sharpe Ratio} = \frac{\text{CAGR}}{\text{Annual Volatility}}$$

(Assuming zero risk-free rate for simplicity)

1.9 Daily Simulation Loop Logic

1.9.1 Sequential Processing

For each trading day, the engine executes this sequence:

1. **Load Market Data**: Get ETF price, dividend payment, Fed Funds rate
2. **Calculate Interest**: Apply daily interest to outstanding margin loan
3. **Process Dividends**: If dividend paid, reinvest automatically
4. **Update Portfolio**: Recalculate portfolio value and equity
5. **Check Margin**: Determine if margin call triggered
6. **Store Results**: Save all metrics for analysis and visualization

1.9.2 State Persistence

Critical state variables carry forward between days: - **Outstanding Margin Loan** (growing with interest) - **Total Shares Owned** (increasing with dividend reinvestment) - **Cumulative Interest Paid - Margin Call History**

1.10 Risk Assessment Framework

1.10.1 Multi-Dimensional Risk Analysis

The engine evaluates risk across several dimensions:

1. **Leverage Risk**: Higher leverage amplifies both gains and losses
2. **Interest Rate Risk**: Rising rates increase borrowing costs
3. **Market Risk**: ETF price volatility affects margin requirements
4. **Liquidity Risk**: Margin calls may force disadvantageous timing

1.10.2 Risk Indicators

Price Drop Tolerance:

$$\text{Max Price Drop \%} = \left(1 - \frac{\text{Margin Call Price}}{\text{Current Price}}\right) \times 100$$

Interest Coverage Ratio:

$$\text{Coverage Ratio} = \frac{\text{Annual Dividend Income}}{\text{Annual Interest Cost}}$$

1.11 Scenario Analysis Capabilities

1.11.1 Historical Stress Testing

The engine can test strategies through actual market crises: - **2008 Financial Crisis**: Extreme volatility and drawdowns - **2020 COVID Crash**: Rapid market decline and recovery
- **2022 Rate Hikes**: Rising interest rate environment

1.11.2 Leverage Impact Analysis

By comparing identical strategies with different leverage ratios, users can see the non-linear relationship between leverage and risk:

$$\text{Risk Multiplier} \neq \text{Leverage Multiplier}$$

Higher leverage creates disproportionate risk due to: - Compounding interest costs - Increased margin call probability - Reduced flexibility during market stress

1.12 Accuracy & Validation

1.12.1 Realistic Assumptions

The engine incorporates realistic constraints: - **No Slippage**: Assumes perfect execution (slight optimistic bias) - **Infinite Liquidity**: No market impact from position size - **Immediate Reinvestment**: Dividends reinvested at closing price - **Daily Interest**: Interest calculated and compounded daily

1.12.2 Data Integrity

All calculations use **actual historical data**: - Real ETF prices from 1993/2001 onwards - Actual dividend payment dates and amounts - Historical Federal Reserve interest rates - No synthetic or modeled data

1.13 Visualization Mathematics

1.13.1 Multi-Panel Chart Structure

Panel 1 - Portfolio Evolution: - Portfolio Value vs. Equity over time - Margin call events marked as red X symbols

Panel 2 - Drawdown Analysis: - Running drawdown percentage from peak equity - Filled area showing underwater periods

Panel 3 - Interest Rate Environment: - Fed Funds rate vs. actual margin rate paid - Visual correlation between rate changes and performance

1.13.2 Interactive Analytics

Each chart point contains metadata: - Exact date and values - Margin loan outstanding - Shares owned - Interest rate environment

1.14 Strategic Insights Generated

1.14.1 Key Learnings

The backtesting reveals critical insights:

1. **Interest Rate Sensitivity**: How rate changes affect leveraged returns
2. **Dividend Impact**: Quantifies the compounding benefit of reinvestment

3. **Crisis Performance**: How strategies perform during market stress
4. **Optimal Leverage**: Risk-adjusted leverage ratios for different market conditions
5. **Timing Effects**: How entry timing affects long-term outcomes

1.14.2 Decision Framework

Users can evaluate: - **Risk Tolerance**: Maximum acceptable drawdown levels - **Return Expectations**: Realistic performance targets - **Time Horizon**: How holding period affects outcomes - **Market Conditions**: When leverage enhances vs. hurts performance

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1.15 The Mathematical Beauty

This engine transforms raw market data into actionable intelligence through the elegant interplay of:

- **Compound Growth**: Both beneficial (dividends) and detrimental (interest)
- **Risk-Return Dynamics**: Non-linear relationships revealed through simulation
- **Market Reality**: Real costs, real constraints, real outcomes

- **Temporal Precision**: Daily granularity capturing market microstructure

The result: A mathematically rigorous, empirically grounded tool that demystifies leveraged investing through the lens of historical reality.

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