$ext{CS } 422/622 ext{ Project 4} \\ ext{Due } ext{11/29 } ext{11:59pm} ext{}$

Logistics: Logistics: You must implement everything stated in this project description that is marked with an implement tag. You may only use numpy and matplotlib pyplot. No late work will be accepted. I advise that you submit to Webcampus regularly and well before the deadline.

Deliverables: Each student should submit a single ZIP file, containing your project code (*.py files) and your README. Your zip file should be named lastname_firstname_project4.zip. For example, a zip file for Sara Smith might look like smith_sara_project4.zip. Your code should run without errors on the ECC linux machines. If your code does not run for a particular problem, you will lose 50% on that problem. If you submit code that plots/prints things, you will be docked points.

PCA (50 points)

File name: pca.py

Implement: You will implement four functions listed here and detailed below.

```
def compute_Z(X, centering=True, scaling=False)
def compute_covariance_matrix(Z)
def find_pcs(COV)
def project_data(Z, PCS, L, k, var)
```

def compute_Z(X, centering=True, scaling=False)

The above function will take the data matrix X, and boolean variables centering and scaling. X has one sample per row. Remember there are no labels in PCA. If centering is True, you will subtract the mean from each feature. If scaling is True, you will divide each feature by its standard deviation. This function returns the Z matrix (numpy array), which is the same size as X.

```
def compute_covariance_matrix(Z)
```

The above function will take the standardized data matrix Z and return the covariance matrix $Z^TZ=COV$ (a numpy array).

```
def find_pcs(COV)
```

The above function will take the covariance matrix COV and return the ordered (largest to smallest) principal components PCS (a numpy array where each column is an eigenvector) and corresponding eigenvalues L (a numpy array). You will want to use np.linalg.eig for this.

```
def project_data(Z, PCS, L, k, var)
```

The above function will take the standardized data matrix Z, the principal components PCS, and corresponding eigenvalues L, as well as a k integer value and a var floating point value. k is the number of principal components you wish to maintain when projecting the data into the new space. $0 \le k \le D$. If k=0, then we use the cumulative variance to determine the projection dimension. var is the desired cumulative variance explained by the projection. $0 \le v \le 1$. If v = 0, then k is used instead. Assume they are never both 0 or both > 0. This function will return Z_{star} , the projected data.

2 Application (50 points)

File name: compress.py

You will use grayscale face images for this application. They are located in the DATA/TRAIN and DATA/TEST directories provided with the project description. You may test on both of them.

Implement: You will implement two functions listed here and detailed below.

```
def compress_images(DATA,k)
def load_data(input_dir)
```

You will write an image compression function detailed below.

def compress_images(DATA,k)

This function will take the flattened image data (DATA) as input, along with k, the number of principal components to use. This function will use PCA to find the principal components of the face images. It will return the compressed data. $X_{compressed} = Z^*U^T$. Where Z^* is the projected data and U^T is the transpose of the principal components. This function will then output the compressed images into a directory named Output. You should use the os package to make sure that the directory is present and to create it if it does not exist. NOTE: Images have values from 0 to 255, so you will want to rescale them before saving them. You will also want to use the cmap='gray' option in pyplot.imsave to save them as grayscale images.

You will write the following helper functions:

def load_data(input_dir)

The above function takes the input directory as input, and outputs the DATA matrix. DATA will have one flattened image per column, so each column represents an image and one row represents the pixel values for every image at a particular location. This function will use pyplot.imread to load the images. Before you return DATA you will want to convert it to floating point.