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Seminar II

The SSH Model

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Abstract

We introduce the SSH model, one of the simplest examples of a topological insulator, in order to demonstrate a number of properties, characteristic for such materials, such as the bulk-boundary correspondence and the existence of topological phases. We further generalise these properties to other models with chiral symmetry.

Contents

1 Introduction **1**

2 The SSH model **1**
2.1 The bulk physics 2
2.2 Edge states 4

3 Generalisation **6**
3.1 Chiral symmetry 6
3.1.1 Consequences 6
3.1.2 Chiral symmetry in the SSH model 7
3.2 Topological invariance of edge states 8

4 Conclusion **9**

5 References **10**

1 Introduction

It has been known for a long time that most physical systems in solid state physics can be classified into (semi)conductors and insulators, depending on the existence and value of the system's energy gap. Much more recently, a further classification of insulators has been introduced with the discovery of topological insulators that possess different insulating phases, where the order parameter is an integer topological invariant. Due to its discreteness, we cannot easily pass from one phase to another without doing something abrupt, such as closing the energy gap. In non-trivial topological phases, robust edge states with energy in the bulk energy gap appear. In the first chapter the Su-Schrieffer-Heeger (SSH) model is presented. It is a simple model, where many of the typical properties of the topological insulators arise. In the following chapter we discuss the bulk and boundary physics, respectively, and their connection with topology. Finally, in the third chapter, the results obtained from the SSH model are generalised to a wider range of topological insulators, obeying the chiral symmetry, which is also introduced. We conclude the seminar with the fifth chapter, summarizing the main ideas.

2 The SSH model

The Su-Schrieffer-Heeger (SSH) model describes a 1-dimensional lattice (a chain), where the unit cells are composed of two sites, denoted by the letters A and B. In the model we consider an electron hopping between different sites. As seen in Fig. 1, the hopping between the sites inside the same unit cell is mediated by the hopping parameter v, while the hopping between the neighboring sites from different unit cells is mediated by (in general different) parameter w.

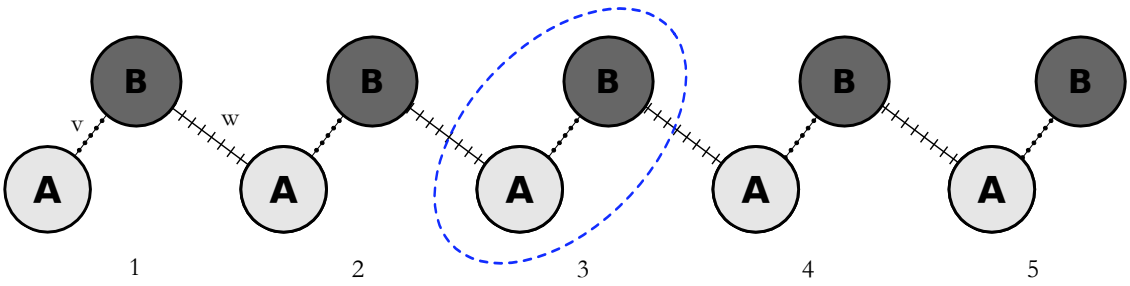

Figure 1: An atom chain corresponding to the SSH model. The light and dark grey circles correspond to type A and type B sites, respectively, with the unit cell encircled by a dotted line - the chain consists of five unit cells. The hopping within the unit cell is governed by the parameter v and is denoted by a line with small circle markers, while the hopping between the unit cells (parameter w) is denoted by a line with tick markers.

Formulating the above description results in the following Hamiltonian for the electron [1]:

$$\hat{H} = v \sum_{m=1}^{N} \left(|m, B\rangle\langle m, A| + \text{h. c.} \right) + w \sum_{m=1}^{N-1} \left(|m+1, A\rangle\langle m, B| + \text{h. c.} \right), \tag{1}$$

where $|m, \alpha\rangle = |m\rangle \otimes |\alpha\rangle$, with $m = 1, .., N$ and $\alpha \in \{A, B\}$ represents the electron state, localized on the sublattice α in the m-th unit cell. The total number of unit cells is N. Analogously we also split our Hamiltonian into the part acting on an external degree of freedom $|m\rangle$ and the part acting on an internal degree of freedom $|\alpha\rangle$:

$$\hat{H} = v \sum_{m=1}^{N} |m\rangle\langle m| \otimes \hat{\sigma}_x + w \sum_{m=1}^{N-1} \left(|m+1\rangle\langle m| \otimes \frac{\hat{\sigma}_x + i\hat{\sigma}_y}{2} + \text{h. c.} \right). \tag{2}$$

2.1 The bulk physics

Let us first disregard the boundary of the chain and focus on the bulk. Bulk is the long middle part of the chain and accounts for most of the physics in the thermodynamic limit $N \to \infty$. By standard arguments [2], bulk physics should not depend on what is happening at the boundary, so for its treatment we choose the simplest boundary conditions to work with - the periodic boundary conditions. The Hamiltonian describing the bulk physics $\hat{H}_{\text{bulk}}$ is therefore obtained by augmenting the previous Hamiltonian with the addition of a $w\ |N, B\rangle\langle 1, A|$ term (and its hermitian conjugate), that describes the interaction between the first and last unit cell of the original chain, transforming it into a periodic chain. We are looking for $2N$ (N unit cells with 2 sites each) eigenstates $|\Psi_n(k)\rangle$ with energies $E_n(k)$ of the bulk Hamiltonian:

$$\hat{H}_{\text{bulk}}|\Psi_n(k)\rangle = E_n(k)|\Psi_n(k)\rangle. \tag{3}$$

In the above equation we have already written the eigenenergies and eigenstates as a function of k, knowing that, due to periodicity of the chain, the bulk Hamiltonian is translation invariant, which allows us to invoke Bloch's theorem [2] to write out the eigenstates in the following form:

$$|\Psi_n(k)\rangle = |k\rangle \otimes |u_n(k)\rangle. \tag{4}$$

Above, we have introduced the Bloch function $|k\rangle$:

$$|k\rangle = \frac{1}{\sqrt{N}} \sum_{m=1}^{N} e^{imk}|m\rangle, \tag{5}$$

with k taking N different values in the first Brillouin zone. We now define the bulk momentum space Hamiltonian $\hat{H}(k) = \langle k|\hat{H}_{\text{bulk}}|k\rangle$, which acts on the internal degrees of freedom:

$$\hat{H}(k)|u_n(k)\rangle = E_n(k)|u_n(k)\rangle. \tag{6}$$

By expanding the internal part of the wavefunction in the sublattice basis $|u_n(k)\rangle = a_n(k)|A\rangle + b_n(k)|B\rangle$, $\hat{H}(k)$ becomes a two by two matrix and can therefore be written as a linear combination of the Pauli matrices and the identity:

$$H(k) = d_0(k)\mathbb{I} + d_x(k)\sigma_x + d_y(k)\sigma_y + d_z(k)\sigma_z. \tag{7}$$

It is through the vector $\vec{d}(k) = (d_x, d_y, d_z)(k)$ that the topological properties of the model arise [3]. It turns out that for the SSH model, $d_z(k)$ is equal to zero. As we increase k from 0 to 2π, the vector $\vec{d}(k)$ therefore traces a curve in the two dimensional (d_x, d_y) plane. Due to the periodicity of the Brillouin zone that curve is necessarily a loop, which we can topologically classify by an integer parameter known as the winding number around the origin ν [4]. As will become clearer on the examples, the winding number tells us how many times an oriented loop "winds" around the origin. Concretely, by the explicit construction of the bulk momentum space Hamiltonian we can extract the vector:

$$\vec{d}(k) = (v + w\cos k, w\sin k, 0). \tag{8}$$

The loops corresponding to some different choices of hopping parameters v, w can be seen in the second row of Fig. 2. For $w < v$ we have $\nu = 0$, while $w > v$ gives $\nu = 1$. The case of $w = v$ is special, because in this case the loop $\vec{d}(k)$ passes through the origin and the winding number around the origin is undefined. In the first row the dispersion relation for each case is plotted. It is obtained by diagonalizing the bulk momentum space Hamiltonian, which gives:

$$E(k) = \pm\sqrt{v^2 + w^2 + 2wv\cos k} \tag{9}$$

We can see, that for the cases $v \neq w$, a finite band gap between conducting and valence bands is present, which by definition corresponds to the insulator. At a special point $v = w$, where the winding number is undefined, the gap closes and the system acts as a conductor, allowing excitations from the valence to the conducting band at zero energy cost. A continuous transition between the two insulating states with different winding numbers is therefore only possible by passing through a conducting phase, a feature of the model which we will return to in a more general sense later on.

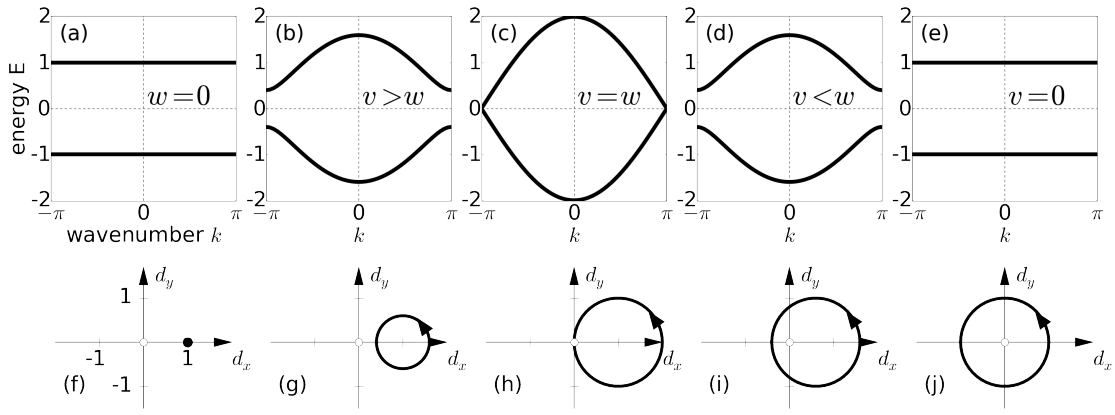

Figure 2: In the first row, the dispersion relation for various combinations of the hopping parameters v and w is displayed. On the second row the loops $\vec{d}(k)$ in (d_x, d_y) plane are drawn. The first two columns correspond to the case $\nu = 0$, the last two to the case $\nu = 1$, while in the middle column, the loop passes through the origin and ν is undefined.

On the above examples it is possible to easily deduce the winding number of a given loop by its intuitive definition of the amount of times it winds around the origin, but in general, the shapes of the loops can get complicated and it is helpful to have a systematic method [3] for the determination of ν from the plotted loop, which we will now describe. The following steps should be read alongside Fig. 3:

- Start by drawing $\vec{d}(k)$ for $k \in [0, 2\pi)$. Since $\vec{d}(k)$ is an oriented loop, we can define the left and the right side of the curve. We paint the left side one colour (here we choose the blue colour) and the other side another (our choice is red).
- Take an arbitrary curve $\mathcal{L}$ going from the origin to infinity. The simplest choice is a straight line.
- Start at the origin and trace the curve $\mathcal{L}$. Take note of the intersections of the two curves $\mathcal{L}$ and $\vec{d}(k)$.
- As you trace along $\mathcal{L}$, give an intersection a value of $+1$ if $\vec{d}(k)$ meets it from blue side and -1 otherwise.
- The winding number is then the sum of all the intersection values.

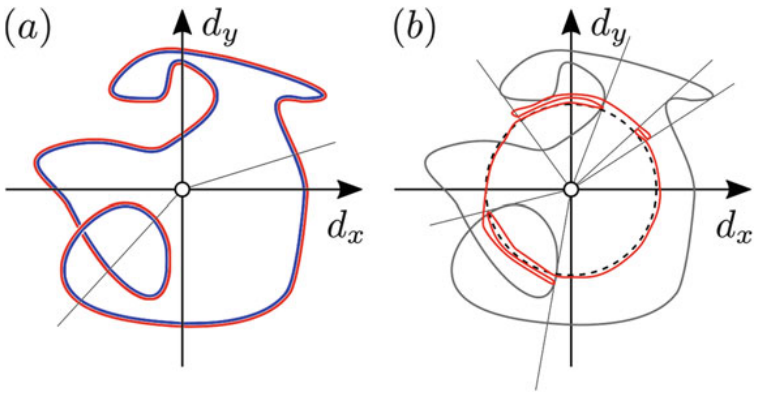

Figure 3: Determination of ν: On the left side it is assumed the loop is oriented counter-clockwise and its sides have been coloured accordingly. Two choices of straight lines $\mathcal{L}$ are also drawn and it is easy to see, that performing the above procedure gives the same result tracing either of them - the winding number is one. On the right we have projected our trajectory along the loop onto the unit circle, making the geometrical intuition of winding number as the amount of times a loop winds around the origin evident. Figure from: [3]

The winding number is a topological invariant, meaning that it does not change by continuous deformations of the curve $\vec{d}(k)$ avoiding the origin. From the above construction this can be easily seen by the following argument: If we continuously deform $\vec{d}(k)$ the following scenarios can happen: the intersections with $\mathcal{L}$ can just change their locations, which does not change the winding number, since the winding number only depends on the number of intersections, not on their locations. Additionally, new intersections can appear or the existing intersections can disappear, but that does not change the winding number either, since they can only appear/disappear pairwise with each of the two pairs meeting the curve $\mathcal{L}$ from a different colored side.

2.2 Edge states

Having explored the bulk physics, we now turn our attention to the edge states.

Let us begin with a simple example. For the moment, we assume a finite chain again and look at two limiting cases - the so called fully dimerized limits in which one of the hopping parameters is equal to zero and the chain falls apart into disconnected dimers as seen in Fig. 4.

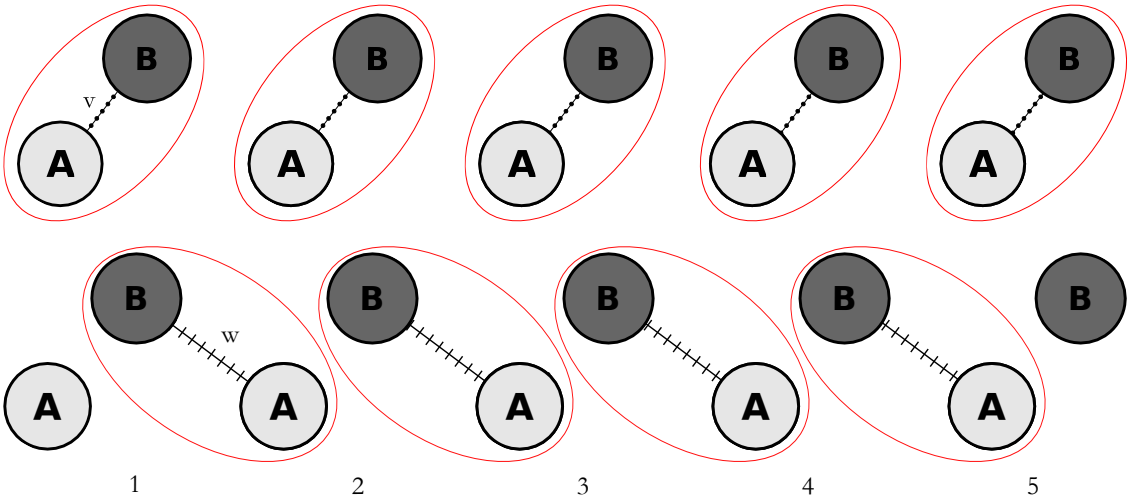

Figure 4: Two different fully dimerized limits of the SSH chain. The above picture shows the $w = 0$ case and the below one shows the $v = 0$ case. The resulting dimers are circled by a red line. In the $w = 0$ case every site is part of some dimer, while on the lower picture we can clearly see two sites that are not part of any dimers and correspond to the edge states.

In the first case we have $w = 0, v > 0$, which corresponds to the trivial phase ($\nu = 0$). The $2N$ eigenstates and eigenenergies are:

$$H(|m, A\rangle \pm |m, B\rangle) = \pm v(|m, A\rangle \pm |m, B\rangle). \tag{10}$$

The electron states are confined to the dimers, which coincide with the unit cell. In the second case $v = 0, w > 0$, which corresponds to the topological phase ($\nu = 1$) we have

$$H(|m, B\rangle \pm |m + 1, A\rangle) = \pm w(|m, B\rangle \pm |m + 1, A\rangle). \tag{11}$$

Again we are confined to the dimers, which now no longer correspond to the unit cells, but each contain two sites from neighboring lattice points. In the two cases the energy is equal to $\pm v$ or $\pm w$ and the energy bands are flat, which means that the group velocity of the electrons is zero [2], which makes sense, since they are confined to the individual dimers.

For the trivial phase, the above formula gives all the $2N$ states. For the topological phase however, the above formula only gives $2(N - 1)$ states. As we can see from Fig. 4, the remaining two states are edge states, which have the energy equal to zero:

$$H|1, A\rangle = H|N, B\rangle = 0 \tag{12}$$

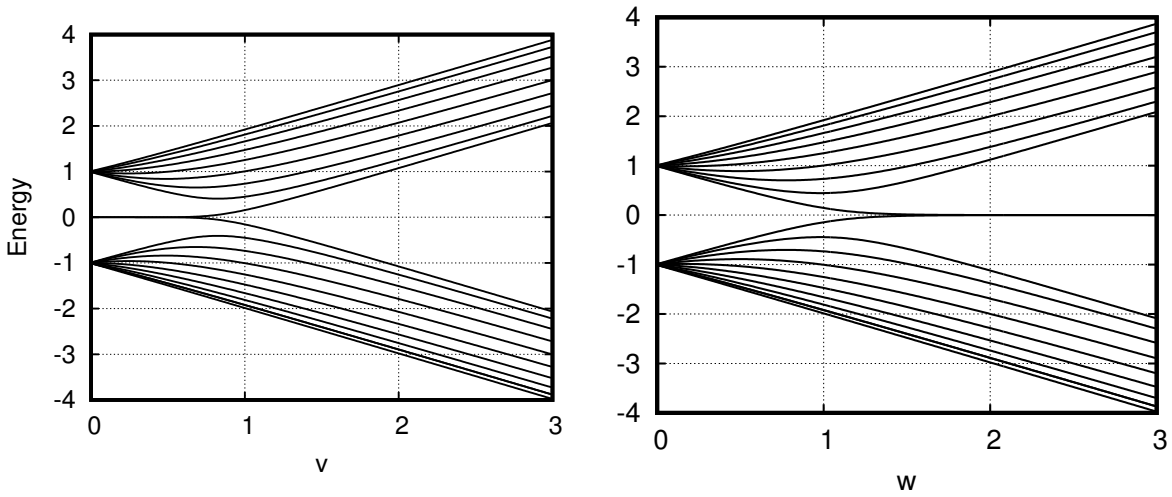

Figure 5: The calculated eigenenergies of the SSH model as the hopping parameters are changed. On the left we keep $w = 1$ fixed and change v, while on the right $v = 1$ is fixed and w is changed. In both cases we notice the edge states in the bulk gap appearing in the topological phase. Figure from: [5]

In the general case, we define the edge states as the states that are localized. This definition is sensible, since the states corresponding to the bulk - the eigenstates of the bulk hamiltonian - are all delocalized. Additionally we may recognise a state as an edge state if its energy is in the gap of the bulk hamiltonian dispersion. Next, let us look at what happens to those states as we move from the fully dimerized limit. Fig. 5 shows the energy levels of the system as we change one of the hopping parameters. Notice that we have edge states - states with energy in the bulk gap - for as long as we are in a topological phase. This is a simple example of the bulk-boundary correspondence [6] that is one of the characteristic properties of the topological insulators. In this case, it tells us that the topological invariant - the winding number ν of the bulk - is connected to the number of the edge states. $\nu = 0$ corresponds to no edge states, while $\nu = 1$ corresponds to two edge states - one on each side of the chain. We will talk about the bulk-boundary correspondence in a more general sense in the next section.

In Fig. 6 some of the eigenstates of the finite chain have been plotted. We see the edge states really are exponentially localized and, interestingly, only have non zero values on one of the sublattices A or B on each side, while the delocalized bulk state has equal support on both sublattices through the chain. This is also a more general feature that we will further explore in the next section.

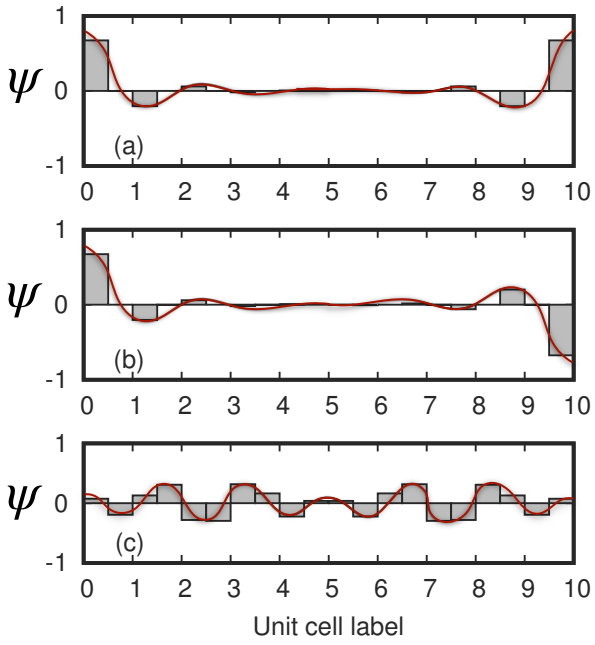

Figure 6: The above two plots are of an eigenstate with zero energy. Notice it is exponentially localized on the edges and only has support on one of the two sublattices, unlike the finite energy state on the third plot, which has equal support on both sublattices and is delocalized. Figure from: [5]

3 Generalisation

3.1 Chiral symmetry

Now we would like to generalise the properties we have mentioned so far to a wider class of physical systems. In quantum mechanics, we are often dealing with cases where our Hamiltonian posseses certain symmetries. We say that a unitary operator $\hat{U}$ represents a symmetry of the Hamiltonian $\hat{H}$ if $\hat{U}$ commutes with $\hat{H}$ [7] or equivalently if:

$$\hat{U}\hat{H}\hat{U}^\dagger = \hat{H}. \tag{13}$$

The property we will be concerned with is a so called chiral symmetry, which is defined in a slightly different way. An operator $\hat{\Gamma}$ represents a chiral symmetry of the Hamiltonian $\hat{H}$ if the following holds [3]:

- $\hat{\Gamma}$ is both unitary and hermitian and
$$\hat{\Gamma}\hat{H}\hat{\Gamma} = -\hat{H} \tag{14}$$
- $\hat{\Gamma}$ is a local operator. Just as in SSH case, we assume our system consists of unit cells and matrix elements of $\hat{\Gamma}$ between states, corresponding to different cells should vanish $\langle m, \alpha|\hat{\Gamma}|m', \alpha'\rangle = 0, m \neq m'$. This condition will always be satisfied if we can write $\hat{\Gamma}$ as a direct sum of unitary operators $\hat{\gamma}$ that act on internal degrees of freedom - each unit cell.
$$\hat{\Gamma} = \bigoplus_{m=1}^{N} \hat{\gamma} \tag{15}$$
- The final requirement is that the chiral symmetry has to be robust. Our Hamiltonian can depend on several parameters, which we will pack into a single vector ξ. Robustness then means that
$$\forall \xi: \quad \hat{\Gamma}\hat{H}(\xi)\hat{\Gamma} = -\hat{H}(\xi), \tag{16}$$
so the chiral symmetry condition must hold for each choice of these parameters, with the chiral symmetry operator $\hat{\Gamma}$ itself being independent of them. For the case of the SSH model, ξ would consist of all the $2N$ hopping parameters v_m and w_m, where $m = 1, 2, \ldots, N$. In this seminar we have always put $v_m = v, w_m = w$, but in order to discuss the chiral symmetry, we need to allow them to vary in order to verify that the above relation still holds and the robustness property is satisfied.

Having defined the chiral symmetry, let us look at its consequences.

3.1.1 Consequences

First let us look at the chiral symmetry from a different point of view, which will have a better physical interpretation. We can define the operators

$$\hat{P}_A = \frac{1}{2}(\hat{\mathbb{I}} + \hat{\Gamma}) \quad \hat{P}_B = \frac{1}{2}(\hat{\mathbb{I}} - \hat{\Gamma}). \tag{17}$$

Such operators obey $\hat{P}_A + \hat{P}_B = \hat{\mathbb{I}},\ \hat{P}_A\hat{P}_B = 0,\ \hat{P}^2_{A,B} = \hat{P}_{A,B}$. The operators $\hat{P}_{A,B}$ are therefore orthogonal projectors. We can use them to rewrite $\hat{\Gamma}\hat{H}\hat{\Gamma} = -\hat{H}$ as

$$\hat{P}_A\hat{H}\hat{P}_A = \hat{P}_B\hat{H}\hat{P}_B = 0 \tag{18}$$

or

$$\hat{H} = \left(\hat{P}_A + \hat{P}_B\right)\hat{H}\left(\hat{P}_A + \hat{P}_B\right) = \hat{P}_A\hat{H}\hat{P}_B + \hat{P}_B\hat{H}\hat{P}_A, \tag{19}$$

which means the chiral symmetry can equivalently be viewed as the Hamiltonian only allowing transitions from a certain subspace (A/B) to another (B/A), never back to itself, while the two subspaces A and B together make up the whole lattice.

Another consequence, that we have already seen appear in the SSH model, is that the spectrum of the Hamiltonian is symmetric: if we have a state with energy E, we also have a state with energy $-E$. This follows from:

$$\hat{H}|\Psi\rangle = E|\Psi\rangle \Rightarrow \hat{H}\hat{\Gamma}|\Psi\rangle = -\hat{\Gamma}\hat{H}|\Psi\rangle = -\hat{\Gamma}E|\Psi\rangle = -E\hat{\Gamma}|\Psi\rangle, \tag{20}$$

where we have simply used the properties of the operator $\hat{\Gamma}$.

In the case that $E \neq 0$, the above calculation implies that if we have an eigenstate $|\Psi\rangle$, with energy E, the state $\hat{\Gamma}|\Psi\rangle$ is also an eigenstate of the Hamiltonian with the energy $-E$. Furthermore the two states are orthogonal, since they are two eigenstates with different energies. So the following also holds:

$$0 = \langle\Psi|\hat{\Gamma}\Psi\rangle = \langle\Psi|\hat{P}_A|\Psi\rangle - \langle\Psi|\hat{P}_B|\Psi\rangle, \tag{21}$$

which implies that the states with $E \neq 0$, have the same support on both the subspaces A and B.

Let's look at the states with $E = 0$. Those states can be chosen to have support on only one of the two subspaces, since

$$\hat{H}|\Psi\rangle = 0 \Rightarrow \hat{H}\hat{P}_{A,B}|\Psi\rangle = \hat{H}(|\Psi\rangle \pm \hat{\Gamma}|\Psi\rangle) = 0. \tag{22}$$

3.1.2 Chiral symmetry in the SSH model

Having derived a few properties of the systems with chiral symmetry, let us make some connections to the SSH model again. It can be easily checked that the operator $\hat{\gamma} = \hat{\sigma}_z$ represents the chiral symmetry of the SSH Hamiltonian. The operator simply acts as the identity on all the states on sublattice A, while multiplying the states on sublattice B by -1. The projectors $\hat{P}_{A,B}$ turn out to be:

$$\hat{P}_A = \sum_{m=1}^{N} |m, A\rangle\langle m, A| \quad \hat{P}_B = \sum_{m=1}^{N} |m, B\rangle\langle m, B|. \tag{23}$$

The subspaces A and B corresponds to the two sublattices in the SSH model A and B. A consequence of the chiral symmetry stating that the Hamiltonian does not induce transitions from one sublattice to itself is clear from the structure of the SSH Hamiltonian itself, since it only contains terms of the form $|m, B\rangle\langle m', A|$. This continues to hold if the hopping parameters depend on position $v = v(m)$, which means the operator $\hat{\Gamma}$ has the required property of robustness. Let us now mention the connection between the chiral symmetry of the SSH Hamiltonian and the winding number ν we have introduced earlier. The defining relation for the chiral symmetry reads:

$$\hat{\sigma}_z\hat{H}(k)\hat{\sigma}_z = -\hat{H}(k). \tag{24}$$

By again writing the bulk momentum space Hamiltonian as a linear combination of Pauli matrices, we can see that the above requires:

$$d_z(k) = 0, \tag{25}$$

meaning that the fact that our system posseses a chiral symmetry with a symmetry operator $\hat{\sigma}_z$ directly implies that $\vec{d}(k)$ describes a loop in a two dimensional (d_x, d_y) plane. A fundamental result from homotopy theory [4] is that the space $\mathbb{R}^n$ with $n = 2$ has loops with non trivial winding numbers around the origin, while in the case $n > 2$ all loops have a trivial winding number. The chiral symmetry here was therefore necessary for us to even be able to have different topological phases.

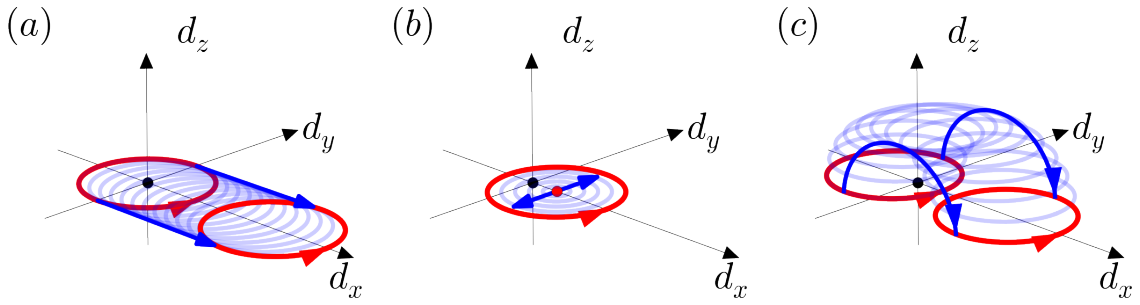

Figure 7: The three different ways of changing the winding number in the SSH model. The first two examples close the bulk energy gap by either moving or stretching the loop, while the third one breaks chiral symmetry. Figure from: [3]

As seen in Fig. 7, In the SSH model, the winding number can only be changed by either pulling the loop through the origin (and closing the bulk energy gap in the process) or moving the loop through the third dimension d_z (and breaking the chiral symmetry).

We will now generalise those notions and define the adiabatic deformation of insulating Hamiltonians. We say that an insulating Hamiltonian is adiabatically deformed if [3]:

- Its parameters are changed continuously.
- The relevant symmetries of the system are maintained.
- The bulk gap around $E = 0$ remains open.

We say that two Hamiltonians are adiabatically equivalent (or adiabatically connected) if there exists an adiabatic deformation between them. We call the integer number characterizing the adiabatically equivalent Hamiltonians a topological invariant. In the SSH model this is simply the winding number ν, and the adiabatically equivalent Hamiltonians are those with the same ν. The relevant symmetry in this case is the chiral symmetry. As seen before, ν can take on one of the two values, which means we have only two types of Hamiltonians up to adiabatic equivalence. The corresponding phase diagram can be seen in Fig. 8.

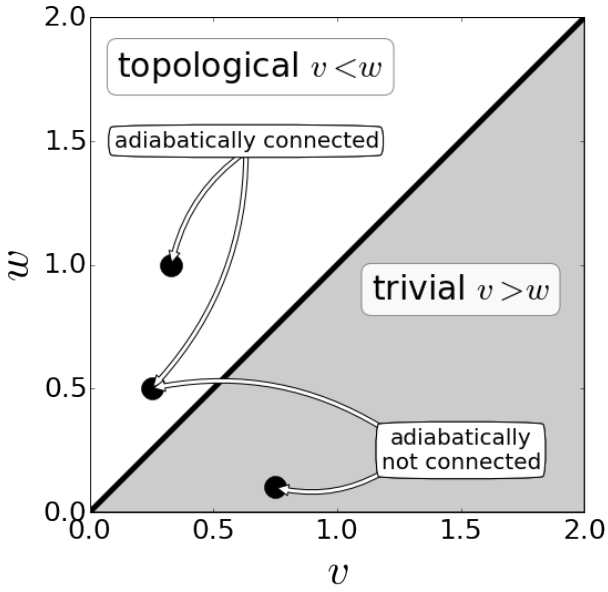

Figure 8: The phase diagram associated with SSH model, where each phase is characterized by a topological invariant. Figure from: [3]

3.2 Topological invariance of edge states

Previously, we have seen that in the fully dimerized limit, the SSH Hamiltonian has one edge state on each edge of the chain in the topological phase and zero in the trivial phase. The number of edge states on each edge in the SSH model is therefore also a topological invariant. We will now see that this remains the case in a general one dimensional chain with chiral symmetry.

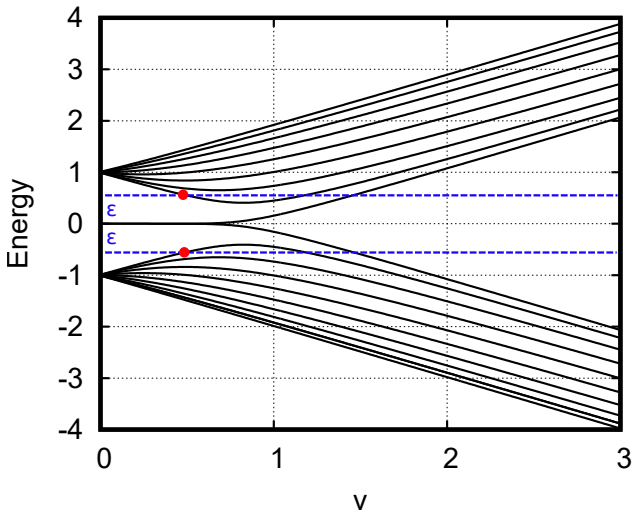

Figure 9: An example of a choice of the energy window used in the proof of the topological invariance of the edge states. The conclusions hold as long as the chosen window lies completely within the energy gap. On the above example that stops being the case at the points marked with the red dots, where the energy window first touches the bulk energy bands. Adapted from: [5]

Consider the eigenstates with support on the left edge of the chain, such that their energies E obey $-\varepsilon < E < \varepsilon$, where $\varepsilon > 0$ is small enough, so that all those energies E are within the bulk energy gap. As stated before, states with energies within the bulk energy gap are edge states, since being in the gap, they do not appear in the bulk physics. In the energy window $(-\varepsilon, \varepsilon)$ we can have zero energy states, which can, due to the chiral symmetry, be restriced to one of the sublattices A or B. Let us denote the number of states on the sublattice A by N_A and on the sublattice B by N_B. We will now show that the quantity $N_A - N_B$ remins unchanged as we adiabatically deform the Hamiltonian, if during the deformation its bulk energy gap remains larger than 2ε. The deformation can bring nonzero energy states to zero energy ones. Since the system has chiral symetry, non zero energy states come in pairs $|\Psi\rangle$, $\Gamma|\Psi\rangle$, each state having the same absolute value of energy with a different sign. So if one non-zero energy state becomes a zero energy state, the other must as well. As a consequence of the chiral symmetry the two new zero energy states can be chosen to take the form $|\Psi\rangle \pm \Gamma|\Psi\rangle$, where each of the two states is restricted to live on one sublattice each. The number $N_A - N_B$ is therefore unchanged, since both the numbers N_A and N_B increase by one. The opposite transformation, where a zero energy state becomes a non zero energy state, as the Hamiltonian is adiabatically deformed, works the same way (albeit in opposite direction). Consequently, $N_A - N_B$, the net number of edge states on sublattice A at the left edge, is a topological invariant.

We have shown that, in the addition to the order parameter (such as the winding number), characterizing the different topological phases, the number $N_A - N_B$ is also a topological invariant. In the case of the SSH model, we have seen that those two numbers are the same, $N_A - N_B = \nu$. This remains the case in modified versions of the SSH model, such as models with longer range interactions [6], and is known as the bulk-boundary correspondence. The bulk-boundary correspondence makes an appearance in other topological insulator systems as well in different forms, but the main idea is always the connection between the topological invariants of the bulk and the number of edge states [8].

4 Conclusion

We have given a brief overview of the topological insulator physics, demonstrating some main ideas on the SSH model. We have seen that the bulk Hamiltonian has a structure that can be connected to the topology of loops in a plane, characterized by a topological invariant - the winding number. Shifting our attention to the edge of the insulator, we have seen that the number of edge states on one side coincides with the winding number - an example of the bulk-boundary correspondence, a recurring feature of topological insulators. The notion of the chiral symmetry of the Hamiltonian has then been defined and we showed how many

properties, that we have seen in the SSH model arise from it, generalising the results to a broader range of systems.

The subject of topological insulators is much vaster than what was covered in this seminar and still a very active area of research. Many different materials have been experimentally observed to possess the properties of topological insulators [8] with different defining symmetries (such as, time reversal symmetry) and topological invariants (such as, the Chern number). Higher dimensional topological insulators' edge states can conduct current with little resistance, even in the presence of impurities [8], which could have major applications in technology and engineering.

5 References

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