DIF. GEOM. POPRAVKI

② b) $X(\alpha(Y)) = (\nabla^*_X \alpha)(Y) + \alpha(\nabla_X Y)$, $\nabla_{e_j} e_i = \sum_k A^k_{ij} e_k$

ČE VSTAVIMO $Y, X \in \{e_1, e_2\}$ $\alpha \in \{e^*_1, e^*_2\}$

KOT PREJ DOBIMO ENAČBE:

$$\nabla^*_{e_j} e^*_i = \sum_k B^k_{ij} e^*_k$$

$A^1_{11} = -B^1_{11}$ ①

$A^1_{21} = -B^2_{11}$ ②

$A^2_{12} = -B^1_{22}$ ③

$A^2_{22} = -B^2_{22}$ ④

$A^2_{11} = -B^1_{21}$ ⑤

$A^2_{21} = -B^2_{21}$ ⑥

$A^1_{12} = -B^1_{12}$ ⑦

$A^1_{22} = -B^2_{12}$ ⑧

∇ LEVI-CIVITA $\Rightarrow$ BREZ TORZIJE $\Rightarrow$ $\boxed{A^k_{ij} = A^k_{ji}}$

∇ LEVI CIVITA: $\mathcal{L}_Z g(X,Y) = g(\nabla_Z X, Y) + g(X, \nabla_Z Y)$

$X = e_i,\ Y = e_j,\ Z = e_k$

$$\underbrace{\mathcal{L}_{e_k}(\underbrace{g(e_i, e_j)}_{1 \text{ ALI } 0 \text{ (ONB)}})}_{\text{SMERNI ODVOD KONSTANTNE FUNKCIJE} \to 0} = g(\nabla_{e_k} e_i, e_j) + g(e_i, \nabla_{e_k} e_j)$$

$$\Rightarrow g\Big(\sum_l A^l_{ik} e_l, e_j\Big) = -g\Big(e_i, \sum_l A^l_{jk} e_l\Big)$$

$$\boxed{A^j_{ik} = -A^i_{jk}} \quad \forall i,j,k$$

$A^1_{11} = -A^1_{11} \Rightarrow \boxed{A^1_{11} = 0}$, $\boxed{A^2_{22} = 0}$

$\Rightarrow \boxed{A^i_{jk} = 0, \text{ ČE } i = j\ \ \cancel{\text{[illegible]}}\ \ j,k}$

$$\Rightarrow g\left(\sum_l A^l_{ik} e_l, e_j\right) = -g\left(e_i, \sum_l A^l_{jk} e_l\right)$$

$$\boxed{A^j_{ik} = -A^i_{jk}} \quad \forall\, i,j,k$$

$$A^1_{11} = -A^1_{11} \Rightarrow \boxed{A^1_{11}=0}, \boxed{A^2_{22}=0}$$

$$\Rightarrow \boxed{A^i_{jk} = 0, \text{ όΕ } i=j \quad i,k}$$

1. $B^1_{11} = A^1_{11} = 0$
2. $B^2_{11} = -A^1_{21} = A^2_{11}$
3. $B^1_{22} = -A^2_{12} = A^1_{22}$
4. $B^2_{22} = -A^2_{22} = 0$
5. $B^1_{21} = -A^2_{11} = A^1_{21}$
6. $B^2_{21} = -A^2_{21} = 0$
7. $B^1_{12} = -A^1_{12} = 0$
8. $B^2_{12} = -A^1_{22} = A^2_{12}$

$$\Rightarrow \boxed{A^k_{ij} = B^k_{ij}} \quad \forall\, i,j,k$$

③ $End(TS^2)$ VEKT. SV. ENDOMORFIZMOV TS^2

$\{e_1, e_2\}$ ONB T_pS^2 $\quad \mathcal{A}, \mathcal{B} \in End(T_pS^2)$ $\quad \langle \mathcal{A}, \mathcal{B} \rangle = Tr(AB^T)$ $\quad$ A, B STA $\mathcal{A}, \mathcal{B}$ V BAZI $\{e_1, e_2\}$

$E \subset End(T_pS^2)$, $E_p = \{ \mathcal{A} \in End(TS^2) \mid \mathcal{A}^* = \mathcal{A}, Tr(\mathcal{A}) = 0 \}$

KONSTRUIRAJ IZOMORFIZEM $\psi : S^2 \times \mathbb{R} \to E^{\perp}$

$Tr(\mathcal{A}) = 0 \Leftrightarrow Tr(A) = 0$

E_p OPIŠEMO Z $\{ A \in Mat(2 \times 2, \mathbb{R}) \mid A^T = A, Tr(A) = 0 \}$ $\quad$ $A^* = A \Leftrightarrow A^T = A$

ONB

~~TAKOJ~~

$\Rightarrow E_p = \left\{ \begin{bmatrix} a & b \\ b & -a \end{bmatrix} \,\middle|\, a, b \in \mathbb{R} \right\}$

$B \in End(T_pS^2)$, $B = \begin{bmatrix} c & d \\ e & f \end{bmatrix}$ POGOJ ZA $B \in E^{\perp}$: $\underset{\forall \mathcal{A} \in E}{0 =} \langle \mathcal{A}, B \rangle = Tr\left(\begin{bmatrix} a & b \\ b & -a \end{bmatrix} \begin{bmatrix} c & e \\ d & f \end{bmatrix} \right) =$

$= ac + bd + be - af = 0 = a(c-f) + b(d+e) = 0 \quad \forall a, b \in \mathbb{R}$

$\Rightarrow c = f, d = -e \Rightarrow B = \begin{bmatrix} c & d \\ -d & c \end{bmatrix} c, d \in \mathbb{R}$

$\Rightarrow E_p^{\perp} = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \,\middle|\, a, b \in \mathbb{R} \right\}$

$\psi : S^2 \times \mathbb{R}^2 \to E^{\perp}$ $\quad \psi(p, (x, y)) \to (p, \varphi_p(x, y))$ $\quad \varphi_p : \mathbb{R}^2 \to \{ \mathcal{A} \in End(T_pS^2)$, KATERE

$B = \begin{bmatrix} \cdot & \cdot \\ -d & c \end{bmatrix}, c, d \in \mathbb{R}$

$$\Rightarrow E_p^{\perp} = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \,\middle|\, a, b \in \mathbb{R} \right\}$$

$\Psi : S^2 \times \mathbb{R}^2 \to E^{\perp}$ $\quad \Psi(p, (x,y)) \to (p, \varphi_p(x,y))$ $\quad \varphi_p : \mathbb{R}^2 \to \{ \mathcal{A} \in \mathrm{End}(T_pS^2)$, KATERE MATRIKE SO $A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}, a, b \in \mathbb{R}$ V ON BAZI $\{e_1, e_2\}_p \}$

$\cancel{\varphi_p(x,y)}$

$$\varphi_p((x,y)) = \begin{array}{c c} & \begin{array}{cc} e_{1p} & e_{2p} \end{array} \\ & \begin{bmatrix} x & y \\ -y & x \end{bmatrix} \end{array} \begin{array}{c} e_{1p} \\ e_{2p} \end{array}$$

$$\Psi(p, (x,y)) = \left(p, \begin{bmatrix} x & y \\ -y & x \end{bmatrix}_p \right) \leftarrow$$

ZVEZNA, KER JE VSAKA KOMPONENTA ZVEZNA ($id : S^2 \to S^2$ JE ZVEZNA $\varphi_p : \mathbb{R}^2 \to \mathrm{End}(T_pS^2)$ ZVEZNA)

OZNAKA, DA JE MATRIKA V ON BAZI T_pS^2

INVERZ $\Psi^{-1}\left(p, \begin{bmatrix} x & y \\ -y & x \end{bmatrix}_p \right) = (p, (x,y)) \leftarrow$ SPET ZVEZNA

MATRIKA, KI PRIPADA NEKEMU ENDOMORFIZMU IZ $E^{\perp}$

$\Rightarrow \Psi$ IZOMORFIZEM